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REVIEW ARTICLE

Evolutionary multi-objective optimization: some current research trends and topics that remain to be explored

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Abstract This paper provides a short review of some of the main topics in which the current research in evolutionary multi-objective optimization is being focused. The topics discussed include new algorithms, efficiency, relaxed forms of dominance, scalability, and alternative metaheuristics. This discussion motivates some further topics which, from the author's perspective, constitute good potential areas for future research, namely, constraint-handling techniques, incorporation of user's preferences and parameter control. This information is expected to be useful for those interested in pursuing research in this area.

Keywords evolutionary multi-objective optimization, evolutionary algorithms, multi-objective optimization, metaheuristics

1 Introduction

Evolutionary algorithms (EAs) are a population-based metaheuristic inspired on the "survival of the fittest" principle, whose use has become increasingly popular over the last three decades, mainly for optimization and classification tasks [1,2]. This popularity has given rise to a series of subdisciplines within the so-called evolutionary computation area. One of the subdisciplines that has experienced one of the fastest growth is evolutionary multi-objective optimization (EMO), which refers to the use of EAs for solving multi-objective problems (MOPs). A MOP has two or more (usually conflicting) objective functions that we wish to optimize simultaneously. Because of their nature, MOPs normally have several solutions rather than a single one[1](like in global optimization). Thus, the use of the population to conduct the search presents the advantage of allowing us to generate several solutions after a single run. Additionally, because of their heuristic nature, EAs are less susceptible to the specific features of a MOP (e.g., continuity) than mathematical programming techniques, and therefore their increasing popularity within different domains, mainly during the last 15 years [3−6].

The first implementation of a multi-objective evolutionary algorithm (MOEA) dates back to the mid-1980s [7,8]. Since then, many other MOEAs have been proposed, and an important number of publications have been released.[2] Readers interested in the historical development of this field, should refer to Ref. [9].

After 23 years of existence, EMO is now experiencing growing pains. With no doubt, this is a very popular discipline, but at the same time, it seems less friendly to newcomers. Producing original contributions has apparently become harder (e.g., at the level of a PhD thesis), and a lot of "work by analogy" is now commonly seen in a number of publications. This has led to some EMO researchers to raise an important question: **will we continue to do research in EMO**

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[1]A MOP will have a single solution only if the objectives have no conflict among them, in which case there is no need to use any sort of special approach, since the sequential optimization of each of the objectives, considered separately, will lead us to this single solution.

[2]The author maintains the EMO repository, which currently contains over 3400 bibliographical references, plus public-domain software, and a small database of EMO researchers. The EMO repository is located at: http://delta.cs.cinvestav.mx/˜ccoello/EMOO

during the next few years? This is precisely the focus of this paper, in which we will briefly discuss some of the topics that are currently the main focus of research in EMO and that, from the author's perspective, represent promising research venues for the years to come. Thus, the main hypothesis of this paper is that there still exist enough research topics for both novice and advanced researchers, if one looks carefully within the (now overwhelming) EMO literature. The main goal of this paper is precisely to provide some hints to get relatively quickly to these promising research topics.

The remainder of this paper is organized as follows. Section 2 presents some basic concepts on multi-objective optimization, which are provided in order to make this paper self-contained. The topics that, from the author's perspective, are more representative of the current research trends in the area are discussed in Section 3. Section 4 presents some additional topics that we believe are worth exploring in the future. Finally, Section 5 presents our conclusions.

2 Basic concepts

We are interested in solving problems of the type[3]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leqslant 0, \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0, \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, ..., m$, $j = 1, 2, ..., p$ are the constraint functions of the problem.

Now, we will provide some definitions that are required in order to make this paper more understandable.

Definition 2.1 Given two vectors $\vec{u}, \vec{v} \in \mathbb{R}^k$, we say that $\vec{u} \leqslant \vec{v}$ if $u_i \leqslant v_i$ for $i = 1, 2, ..., k$, and that $\vec{u} < \vec{v}$ if $\vec{u} \leqslant \vec{v}$ and $\vec{u} \neq \vec{v}$.

Definition 2.2 Given two vectors $\vec{u}, \vec{v} \in \mathbb{R}^k$, we say that $\vec{u}$ *dominates* $\vec{v}$ (denoted by $\vec{u} \prec \vec{v}$) iff $\vec{u} < \vec{v}$.

Definition 2.3 We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ ($\mathcal{F}$ is the feasible region) is *Pareto optimum* if there does not exist another $\vec{x} \in \mathcal{F}$ such that $\vec{f}(\vec{x}) \prec \vec{f}(\vec{x^*})$.

Definition 2.4 The *Pareto optimal Set* $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimum}\}.$$

The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called *nondominated*.

Definition 2.5 The *Pareto front* $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^n | \vec{x} \in \mathcal{P}^*\}.$$

We thus wish to determine the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy Eqs. (2) and (3).

3 Some of the current research trends

Based on an analysis of a sample of the specialized literature, we have selected the following list of topics, which seem to be representative of the main current research trends in EMO:

1) New algorithms
2) Efficiency
3) Relaxed forms of dominance
4) Scalability
5) Alternative metaheuristics

Each of these topics will be briefly discussed next.

3.1 New algorithms

In the early days of EMO, the design of new algorithms was a hot topic. However, from the many MOEAs that have been proposed in the specialized literature since Schaffer's Vector Evaluated Genetic Algorithm (VEGA) [8] (published in 1985), few have become widely used in the EMO community. The most popular nonelitist[4] MOEAs were: Multi-Objective Genetic Algorithm (MOGA) [10], Niched-Pareto Genetic Algorithm (NPGA) [11], and Nondominated Sorting Genetic Algorithm (NSGA) [12].

Although some notions of elitism had already been contemplated by some EMO researchers since the mid-1990s (see for example [13,14]), it was until the publication of the Strength Pareto Evolutionary Algorithm (SPEA) [15] in the late 1990s, that elitist MOEAs became common. Although several elitist MOEAs exist, few have become widely used (see for example [16,17]), and from them, one has become extremely popular: the Nondominated Sorting Genetic Algorith-II (NSGA-II) [18]. In fact, the popularity of this algorithm has created a new trend within EMO to propose mechanisms that improve (e.g., for a certain class of prob-

[3] Without loss of generality, we will assume only minimization problems.

[4] Elitism is an operator that retains the best solution from the population of an EA and passes it intact to the next generation. In EMO, elitism, however, involves ALL the nondominated solutions from the population, and is normally implemented using an external archive that filters solutions, such that only solutions that are nondominated with respect to all the previously evaluated populations are retained.

lems) its performance (see for example [19−22]).

It is important to note that MOEAs normally modify EAs in two ways: (1) they incorporate a selection mechanism based on Pareto optimality, and (2) they adopt a diversity preservation mechanism that avoids that the entire population converges to a single solution (as would normally occur because of the stochastic nature of EAs). Diversity preservation mechanisms have also evolved over the years, from naive fitness sharing schemes in which an individual is penalized for sharing the same "niche" with other individuals from the population (a niche is defined either in decision or in objective function space by adopting a certain niche radius from each individual, whose value is normally defined by the user) [23,24]. Over the years, other (more elaborate) schemes have been proposed: clustering [16,25], the adaptive grid [18], the crowded-comparison operator [15], and entropy [26−28], among others.

In spite of the previously indicated trends within this area, the design of algorithms is still an active area of research, although it is now much less popular than before. One of the current trends within this area is to adopt a selection mechanism based on some performance measure. For example, the Indicator-Based Evolutionary Algorithm (IBEA) [29] is intended to be adapted to the user's preferences by formalizing such preferences in terms of continuous generalizations of the dominance relation. This is a nice idea, since it avoids the need to provide an explicit diversity preservation mechanism. In order to achieve this, the optimization goal of IBEA is defined in terms of a binary performance measure (e.g., the additive ϵ-indicator [30]). Recently, the same authors introduced the Set Preference Algorithm for Multiobjective Optimization (SPAM) [31], which consists of a hillclimber based on the same idea of IBEA, but which turns out to be more general, since it is not restricted to a single binary performance measure (several of such performance measures can be used in sequence, and any type of set preference relation is acceptable). Within a similar line of thought, but without explicitly considering the incorporation of user's preferences, the S Metric Selection Evolutionary Multiobjective Optimization Algorithm (SMS-EMOA) [32,33] adopts a selection operator based on the hypervolume measure (also known as S metric [34,35]). There have also been multi-objective extensions of successful single-objective evolutionary optimizer, such as CMA-ES [36,37], which is invariant to rotation in its two versions (single- and multi-objective).

Obviously, other types of MOEAs may also be developed inspired, for example, by concepts from mathematical programming (see for example the Nash Genetic Algorithm [38] and the ϵ-constraint Cultural Differential Evolution [39]), or by existing (single-objective) EAs (see for example the Multiobjective Cellular Genetic Algorithm [40,41] and the micro Genetic Algorithm for Multiobjective Optimization [42,43]). Clearly, much remains to be done regarding algorithm design, and a new generation of MOEAs is expected to arise in the future.

3.2 Efficiency

Several EMO researchers have addressed efficiency issues[5] (see for example [11,16,18,44]). If focused on algorithm design, one gets the impression that little can be done to improve efficiency, since the computational efficiency bounds of nondominance checking have been known for over thirty years [45]. Nevertheless, this is normally assumed by researchers, but few detailed studies of MOEA's algorithmic complexity and of the algorithms used to extract nondominated solutions from a set are currently available in the specialized literature (see for example [46,47]).

Interestingly, most EMO researchers have focused on an apparently easier way of increasing efficiency: the reduction of the number of individuals that are used for determining nondominance. Perhaps the first attempt to reduce the number of individuals involved in the Pareto ranking process of a MOEA is the selection mechanism of the Niched-Pareto Genetic Algorithm (NPGA) [11]. The NPGA uses binary tournament selection. However, instead of comparing fitness directly between two individuals (randomly chosen from the population), in this case a small sample of the population is randomly chosen (e.g., 10% of the total population size). Then, each of the two individuals participating in the tournament are compared with respect to the sample. If one of them turns out to be nondominated (with respect to the sample) and the other is dominated, then the nondominated individual wins the tournament and is selected as a parent. In any other case (i.e., both individuals are nondominated or both are dominated), the individual with less neighbors in its niche wins. Since the sample randomly chosen is smaller than the total population size, the NPGA never ranks an individual with respect to the entire population. This results in a faster algorithm. Another remarkable work in the same direction of the NPGA is the improved ranking procedure proposed by Jensen [44], which significantly reduces the computational complexity of the NSGA-II [18]. However, this approach is based on an algorithm that, as indicated before, is sensitive to the number of objectives [44]. There have also been proposals in which a very small population size

[5]By *efficiency*, we refer here to any sort of process that reduces the number of instructions performed in an algorithm (a MOEA in our case), without modifying the outcome produced by such algorithm.

is adopted, based on the concept of the micro-genetic algorithm [48], in which no more than five individuals are used in the population [42]. This sort of MOEA requires, however, of clever reinitialization schemes in order to avoid getting stuck during the search.

Nowadays, a more common research trend has been to focus on the design of MOEAs that reduce the number of objective function evaluations performed, under the assumption that such evaluations may be very expensive in some real-world applications (clearly, much more expensive than a Pareto ranking scheme). For that sake, EMO researchers have been adopting techniques such as surrogate models, which have long been used in engineering (see for example [49−53]). The main idea of surrogate models is to build an approximate model of the problem, which is cheap (computationally speaking) to evaluate. Their main problem is that these models evidently have errors with respect to the original function to be optimized and, sometimes, such an error may be very significant. Also, some of the current MOEAs that adopt this sort of scheme can only be applied to problems of low dimensionality (e.g., parEGO [52]). Another possible approach is to use previously gathered knowledge (e.g., based on previous evaluations of the fitness function), in order to adapt the recombination and mutation operators so that we can sample offspring in promising areas of the search space (this is the idea of cultural algorithms [54], which have been scarcely considered for multi-objective optimization [55]). Knowledge of past evaluations can also be used to build an empirical model that approximates the fitness function to optimize. This approximation can then be used to predict promising new solutions at a smaller evaluation cost than that of the original problem (see for example [52,56]). It is also possible to use fitness inheritance in order to reduce the number of evaluations of the objective functions. Fitness inheritance [57] works as follows: when assigning fitness to an individual, sometimes the objective function is evaluated as usual, but the rest of the time, the fitness of an individual is assigned as the average of the fitnesses of its parents, thus avoiding a fitness function evaluation based on the assumption of similarity of the individual to its parents. Fitness inheritance has been extended for multi-objective optimization by a few researchers (see for example [58,59]). For a more thorough discussion on the different knowledge incorporation schemes that have been adopted in MOEAs, the interested reader is referred to [60].

It is worth noting, however, that other approaches are possible, by using hybrid schemes. For example, in Ref. [61], a MOEA is used to produce a rough approximation of the Pareto front, and then a local search scheme based on rough sets theory is adopted to rebuild the missing portions of the Pareto front. In Ref. [62], a similar scheme is proposed, but using scatter search as the local search engine, instead. Clearly, the use of powerful local search schemes hybridized with MOEAs that can produce rough approximations of the Pareto front with a reduced number of evaluations [63], or with MOEAs that use special operators to accelerate convergence [64,65], is a very promising research topic.

3.3 Relaxed forms of dominance

In recent years, some researchers have proposed the use of relaxed forms of Pareto dominance as a way of regulating convergence of a MOEA [66]. From these proposals, the most popular is the so-called ϵ-dominance, which was introduced in Ref. [67]. This mechanism acts as an archiving strategy to ensure both properties of convergence towards the Pareto optimal set and properties of diversity among the solutions found. The idea is to use a set of boxes to cover the Pareto front, where the size of such boxes is defined by a user-defined parameter (called ϵ). Within each box, it is only allowed a single nondominated solution to be retained (e.g., the one closest to the lower lefthand corner, if both objectives are being minimized). Thus, by using a large value of ϵ, the user can speed up convergence, but at the sake of sacrificing the quality of the Pareto front approximation obtained. Conversely, if a high-quality approximation of the front is required, then a small value of ϵ must be adopted instead. The definition of ϵ, is then, quite important. Unfortunately, it is not straightforward to find the most appropriate value of ϵ that produces a certain (required) number of nondominated solution within an archive, when nothing is known in advance about the shape of the Pareto front. Also, to correlate the number of nondominated solutions desired with the value of ϵ chosen is not easy, and normally some preliminary runs are required in order to estimate the appropriate value. This makes it difficult to compare approaches that adopt ϵ with respect to MOEAs that do not use this concept. Additionally, because of its nature, this mechanism eliminates certain portions of the Pareto front (e.g., almost straight segments and the extremes of the Pareto front), which may be undesirable in some cases [68]. This, however, can be (at least partially) compensated by using geometrical assumptions about the possible shapes of the Pareto front, and adopting boxes of varying sizes (see for example [69]).

Several modern MOEAs have adopted the concept of ϵ-dominance (see for example [70−73]), and, mainly because of its nice mathematical properties, its use has become relatively popular in the last few years. However, much more work on this topic is expected to be developed in the years to come, both from a pragmatic and from a theoretical point of view.

3.4 Scalability

For several years, most EMO research focused on solving MOPs with only two or three objectives, and it was assumed that scaling such MOEAs to a larger number of objectives would be straightforward. However, several EMO researchers have found this assumption to be wrong [74–76]. One of the reasons for this is that the proportion of nondominated solutions in a population increases rapidly with the number of objectives. Indeed, in Ref. [77], it is shown that this number goes to infinity when the number of objectives approaches to infinity. This implies that in the presence of many objectives the selection of new solutions is carried out almost at random since a large number of the solutions are equally good in the Pareto sense [78]. This has made scalability an important research topic [79–82].

Currently, there are mainly two approaches to deal with problems involving many objectives: 1) to adopt relaxed forms of Pareto optimality by proposing an optimality relation that yields a solution ordering finer than that yielded by Pareto optimality (see for example [77,81,83,84]) and 2) to reduce the number of objectives of the original MOP, thus lowering the dimensionality to a reasonable value that can be handled by standard MOEAs [85,86]. Although the second of these types of approaches seems to be an attractive choice, the difficulties commonly associated with dimensionality reductions has made relaxed forms of Pareto optimality more popular in the literature [87]. Because of its relevance, an important increase of research in this area is expected to occur in the coming years.

It is worth noting, however, that until recently, the focus of scalability studies has been high dimensionality in objective function space, but scalability in decision variable space is also worth studying [88,89].

3.5 Alternative metaheuristics

Relatively recently, several other biologically-inspired metaheuristics have been adapted to solve MOPs [3,90]:

- Artificial immune systems

From a computational perspective, our immune system can be seen as a distributed intelligent system, which is able to learn and retrieve knowledge previously acquired, in order to solve recognition and classification tasks [91]. Because of these features, researchers have developed computational models of our immune system and have used them for a variety of tasks, including classification, pattern recognition, and optimization [91–93]. Several multi-objective extensions of artificial immune systems have been proposed in the specialized literature (see for example [94–98]). Also, combinations of artificial immune systems and another type of approach have been proposed, aiming to solve specific types of MOPs (e.g., [99,100], in which the aim is to solve bi-objective flowshop scheduling problems). However, from the author's perspective, the potential of multi-objective artificial immune systems for solving classification and pattern recognition problems has not been fully exploited yet [101].

- Ant colony optimization

This is a metaheuristic inspired on the foraging behavior of real ants. It is a distributed, stochastic search procedure based on the indirect communication of a set (called "colony") of artificial ants, which mediate using artificial pheromone trails. These pheromone trails can be seen as distributed information which is used by the ants to construct their solutions to the problem at hand. Such pheromone trails are modified during the algorithm's execution, such that they reflect the search experience acquired by the ants. This metaheuristic is intended for solving difficult (both static and dynamic) combinatorial optimization problems, in which solutions can be generated through the use of a construction procedure [102–105]. There are several multi-objective extensions of ant colony optimization (ACO) algorithms (see for example [106–112]), but as multi-objective combinatorial optimization becomes more attractive for EMO researchers [113,114], it is expected that more multi-objective ACO approaches (and hybrids of ACO algorithms with MOEAs and other metaheuristics) are proposed in the near future.

- Particle swarm optimization

This metaheuristic is inspired on the choreography of a bird flock which aim to find food [115,116]. It can be seen as a distributed behavioral algorithm that performs (in its more general version) a multidimensional search. The implementation of the algorithm adopts a population of particles, whose behavior is affected by either the best local (i.e., within a certain neighborhood) or the best global individual. Particle swarm optimization (PSO) has been successfully used for both continuous nonlinear and discrete binary optimization [116–120]. An important number of multi-objective versions of PSO currently exist (see for example [121–127]). However, until relatively recently, most of the research had concentrated on producing new variations of existing algorithms, rather than on studying other (more interesting) topics, such as the role of the main components of PSO in multi-objective optimization. Some recent research in that direction has shown that certain components that had been traditionally disregarded (e.g., the leader selection mechanism and the parameters of the flight formula) play a key role in the performance of a multi-objective PSO algorithm [128,129]. This opens new paths for future research within this area.

- Scatter search

This approach was originally conceived as an extension of a heuristic called surrogate constraint relaxation, which was designed for solving integer programming problems [130]. The main idea of this approach is to adopt a series of different initializations to generate solutions. A reference set of solutions (the best found so far) is adopted, and then such solutions are "diversified" in order to generate new solutions within the neighborhood of the contents of the reference set. This sort of simple procedure is repeated until no further improvements to the contents of the reference set are detected. In the mid-1990s, some further mechanisms were added to the original scatter search algorithm, which allowed its extension to solve nonlinear, binary and permutation optimization problems [131]. These new applications triggered an important amount of research in the last few years [132,133]. Multi-objective extensions of scatter search are relatively recent, but have been steadily increasing [89,134−136]. Scatter search has a lot of potential for hybrid approaches, such as memetic MOEAs [137], since it can act as a powerful local search engine for tasks such as generating missing parts of a Pareto front [62]. Because of its flexibility and ease of use, scatter search is expected to become more commonly adopted in the near future, particularly when designing hybrid MOEAs that rely heavily on good local search engines.

4 What else remains to be done

Other topics that, from the author's perspective, are worth exploring within the next few years are the following:

- Constraint-handling

One of the research areas that has attracted a lot of interest in recent years has been the use of multi-objective optimization concepts to design constraint-handling mechanisms for (single-objective) EAs (see for example [138−142]). Interestingly, however, relatively few research has been done regarding the design of constraint-handling mechanisms for MOEAs (see for example [143−146]), in spite of the importance of constraints in real-world applications of MOEAs. Most of the current work has focused on extending the Pareto optimality relation in order to incorporate constraints (e.g., giving preference to feasibility over dominance, such that an infeasible solution is discarded even if it is nondominated). Also, the use of penalty functions that "punish" a solution for not being feasible are easy to incorporate into a MOEA [147]. However, topics such as the design of constraint-handling mechanisms for dealing with equality constraints,[6] the design of scalable test functions that incorporate constraints of different types (linear, nonlinear, equality, inequality), and the study of mechanisms that allow an efficient exploration of constrained search spaces in MOPs remain practically unexplored.

- Incorporation of user's preferences

In practical applications of MOEAs, users are normally not interested in a large number of nondominated solutions. Instead, they are usually only interested in a few types of tradeoffs among the objectives (e.g., perhaps only the solutions around the "knee" of the Pareto front are of interest to the user). Thus, if such user's preferences are incorporated into the selection mechanism of a MOEA, the search can be much more efficient (e.g., one can zoom in a certain region of the Pareto front and evolve the population only towards the area of interest) and the results more meaningful. Although some research has been done in this direction (see for example [148,149,141,150]), it is still relatively uncommon to report results of a MOEA that incorporates user's preferences. It is thus important that EMO researchers get closer to the extensive work done in Operations Research in this regard (see for example [151]).

- Parameter control

The design of mechanisms that allow an automated control of the parameters of a MOEA (by using, for example, online adaptation [152,153] or self-adaptation [154], so that the MOEA can adapt its parameters without any human intervention) has been scarcely explored by EMO researchers [43,155−160]. This is clearly a very challenging topic, due to the high nonlinear interaction among the parameters of an EA [161]. The goal of a parameterless MOEA is rarely discussed in the EMO literature [43], and alternative (perhaps more viable) schemes such as the use of internal restarts (in other words, the use of information from previous runs to improve performance of subsequent runs) is also scarcely addressed [161]. Additionally, studies that show the effect of the parameters of a MOEA in its performance are still lacking in the specialized literature (see for example [162]), and are a key aspect of algorithmic design.

Several other topics that are also very promising re-

[6] When dealing with equality constraints, the optimum lies on the boundary between the feasible and the infeasible regions. Therefore, the use of approaches that always favor feasible solutions over the infeasible ones are not effective in this case.

search paths will not be discussed due to obvious space limitations (for example, runtime analysis of MOEAs[7] [163,164], archiving techniques[8] [25,69,165,168] and convergence analysis[9] [169−171], just to name a few), but they serve as a good indicator of a healthy research field in which many things remain to be done.

5 Conclusions

This paper has attempted to provide a summary of the main topics in which EMO researchers are currently working, and which, from the author's perspective, provide several interesting challenges for the years to come. This aims to provide a quick reference for those interested in starting research in this field, so that they can get a very general picture of the current state of the area.

At the end of the paper, a few other topics are briefly discussed. Such topics also offer the potential to become very popular research areas within a few more years, and have remained relatively unexplored so far, thus offering important opportunities for newcomers. Hopefully, this general overview of the current and future status of the field will serve to maintain and increase the interest of researchers and practitiones in EMO, since such is the main goal of this paper.

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[7]Runtime analysis addresses the question of how long a certain algorithm takes to find the optimal solution for a specific problem or a class of problems.

[8]This refers to the use of special data structures for an efficient storage and retrieval of nondominated solutions (e.g., quadtrees [165,166], red-black trees [167], etc.).

[9]This refers to providing mathematical proofs of convergence of a MOEA under certain conditions.

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