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TECHNICAL ARTICLE—PEER-REVIEWED

Experimental Investigation on Crack Localization in Steel and Composite Structures by Intersection of First Three Normalized Mode Shape Curves

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Abstract The aim of crack detection and localization is to minimize sudden failures of rotating machines and static structures while in operation. For this purpose, an impact hammer test is performed in cracked and no crack beams made of steel and composite material to obtain the change in natural frequencies and mode shapes. The impact hammer test is performed with the help of four-channel vibration analyzer, impact hammer and uniaxial accelerometer. A change in natural frequencies is used to detect the crack present in the steel and composite beams. Test data of first three natural frequencies obtained from an impact hammer test on different cracked steel and composite beams are used to train the normalized mode shapes algorithm and plot the mode shapes of first three natural frequencies. The intersection of first three normalized mode shapes is used to estimate the crack location with very high accuracy. From the experimental results, it is confirmed that the change in natural frequencies and intersection of first three normalized mode shapes are used for detection and localization of crack present in the steel and composite beams, respectively. When compared to other methods in the literature for detecting crack location based on mode shape curvature, the methodology used in this paper is more accurate.

Keywords Crack detection · Localization · Steel beams · Composite beams · Impact hammer test · Natural frequencies · Normalized mode shapes

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Introduction

Structural health monitoring is an effective and modern tool used to prevent premature failure of the mechanical structures. Several methods are used for structural health monitoring, with vibration-based condition monitoring being an effective tool for preventing mechanical structures from failing prematurely. Ram Prasad and Varsha Singh [1] predicted the looseness fault in driving end and non-driving end windings of the induction machine by experimental modal analysis. The experimental modal analysis is performed on the driving end and non-driving end windings of the induction machine to identify the looseness of the seventy-two stator slot structure. Aryayi et al. [2] studied change of the natural frequency of clamped-free shafts by both experimental and numerical methods and found that natural frequencies are decreasing with the increase of corrosion pits on the shaft. Pitting corrosion is a typical damage on structures, which causes stiffness reduction and mass loss, and these changes affect the vibrational behavior of corroded structures. Josue et al. [3] performed an experimental demonstration of the feasibility of the detection, localization, and length measurement of crack-type damage features on the curved surface of a thin-wall beam structure made from composite material using modal analysis. Short crack-type damage features, both longitudinal and transverse, were detected reliably, and the true length of the crack can be estimated from the damage signal. Elshamy et al. [4] carried out natural frequency tests on different cantilever beam specimen configurations by changing crack location and depth and found that crack position and depth are the most important parameters in crack investigation. Prashant et al. [5] performed an experimental modal analysis on a rectangular cantilever beam to obtain natural frequencies, mode shapes and modal damping. Initial excitation is given by an impact hammer connected to a vibration analyzer with NI Lab VIEW software, which was utilized to extract modal data. Good conformity was observed between theoretical and experimental results.

Nahvi and Jabbari [6] established an experimental approach for crack detection in cantilever beams by linear fracture mechanics theory and found that natural frequencies decrease when crack location moves toward the fixed end of the beam. The finite element model of the cracked beam is constructed and used to determine its natural frequencies and mode shapes. Agarwalla and Parhi [7] observed that crack hinders the performance of a machine. The dynamic behavior of a whole structure is affected due to the presence of a crack as the stiffness of that structural element is altered. They detected the open crack in cantilever beam by both experimental and numerical methods and concluded that natural frequencies and mode shapes are influenced by crack depth and crack location. Owolabi et al. [8] investigated the effects of cracks on the first three modes of vibrating beams and detected the presence of crack in beams through changes in the natural frequencies and amplitudes of the frequency response function. Maksimoc et al. [9] developed an effective and reliable computational technique for residual life estimation of cracked aircraft structural components using fracture mechanics and fatigue analysis. They have defined computational procedures for residual life estimation of aircraft structural components such as wing skin and attachment lugs under cyclic loads of constant amplitude and load spectrum. Mishra and Sahu [10] investigated the free vibration of industry-driven woven fiber glass/epoxy composite plates with different boundary conditions and concluded that the natural frequency was much lower for cantilever than simply supported and fully clamped boundary conditions. The modern modal testing and subsequent analysis with powerful computer and digital analysis system is an important tool for prediction of behavior of structures.

Pushparaj et al. [11] investigated the effect of changes in the matrix material, hybridization and laminate stacking sequence of composite beams on the natural frequencies and mode shapes. It is found that hybridization and the orientation of the outermost layer have a more significant effect on the natural frequencies and mode shapes. Lei et al. [12] presented the vibration characteristics of woven fiber composite beams with different thickness and concluded that the woven structures have a strong effect on the fiber volume fraction and the warp architectures of the composites. Dynamic mechanical analyzer and vibration test technique were used to reveal the dynamical behaviors of specimens in different frequencies of vibration. Wei and Yao [13] simulated the fatigue damage evolution in composite laminates and predicted fatigue life of the laminates with different layup sequences based on the fatigue characteristics of longitudinal, transverse and in-plane shear directions by finite element analysis method. The simulation results, including the fatigue life predicted and the residual stiffness, were coincident with the experimental results. Vaishali and Gaurang [14] used a modal curvature technique and a modal flexibility technique for detecting damage in reinforced concrete beams. A discontinuing element model method is used for simulating crack damage. It was found that these methods could find the location of the crack damage. It is observed that these methods effectively detected the existence of damage and can locate the position of damage for single and multiple damage scenarios for beams. Salawu [15] reviewed the various techniques for structural damage assessment through changes in modal frequency. The approach is since natural frequencies are sensitive indicators of structural integrity. Thus, an analysis of periodical frequency measurements can be used to monitor structural condition. This paper also explained the various methods for locating the geometric location of the damage using modal frequencies.

Behera et al. [16] designed and analyzed a cantilever beam with a slanted open edge crack using experimental and FEM software. The impact of crack location and crack depth on modal attributes of a cantilever beam was also examined. It has been found that the natural frequencies decline as crack depth increases and natural frequency increases as the crack location distance increases from the fixed end. Shabani and Yusuf [17] studied the free vibration analysis of multilayered symmetric sandwich Timoshenko beams, made of functionally graded materials with two edges cracked. They calculated the natural frequency values of a cantilever beam using MATLAB code. From the results, it was concluded that the first four natural frequencies are dependent on the crack location and crack depth. Zizheng et al. [18] simulated propagation of crack growth in a plate with a hole using cracking elements method and found that this method can be used for the propagation of existing cracks as well as initiation of new crack. Chaphalkar et al. [19] studied the transverse vibrations of a fixed-free beam using experimental and numerical analysis. ANSYS and experimental modal analysis utilized to determine modal frequencies. It was observed that there is good conformity between results.

Hajializadeh et al. [20] performed a virtual health monitoring and residual life estimation of steel bridges. In his work, a structural health monitoring is integrated with bridge weigh in-motion estimations of the actual traffic stacking on a bridge to perform fatigue damage computations. The structure health monitoring system uses the ‘virtual monitoring’ concept, where all parts of the bridge that are not monitored directly using sensors are 'virtually' monitored using the load information and a calibrated finite element model of the bridge. Jian Kong [21] calculated the residual fatigue life of crane girder accurately with the help of artificial network model. It is based on two main theories, which are Miner fatigue cumulative damage theory and the theory of linear elastic fracture mechanics. The method proposed to estimate the residual life of crane in the present work is different from the traditional case, further effective estimation of residual life of cranes fatigue, and it is also the reasonable innovation on the basis of traditional test. Leonard et al. [22] proposed a methodology based on spectrograms of the free-decay responses for crack detection in the cracked cantilever beam. They showed a time drift of the frequency and damping. Signal processing such as the worm transform and phase spectrogram methods have been developed with enough accuracy to display the behavior of an uncracked beams. Sadettin [23] studied free and forced vibration analysis of single- and two-edged cracked beams and suggested that free vibration analysis provides suitable information for the detection of cracks, whereas forced vibration can detect only the single crack condition. However, dynamic response of the forced vibration better describes changes in crack depth and location than the free vibration. Hu and Madenci [24] predicted the fatigue life and residual strength of composite laminates and concluded that peridynamic predictions agree with the reduction in stiffness and strength as a function of the number of load cycles.

The review of the literature revealed that little work was found in locating cracks in steel and composite beams in literature. The present paper has developed a new and innovative technique to locate the crack in the steel and composite beams based on intersection of first three modes shapes obtained from an impact hammer test.

Fig. 1 Image of steel beam

Materials and Methods

Materials

An IS2062 mild steel beam with dimensions of 500 mm × 50 mm × 6 mm is considered in the present

Fig. 2 Image of the E-Glass composite beam

Table 2 Mechanical properties and chemical composition of composite beam

Material properties	Value	Material properties	Value
Young's modulus (E_x), MPa	35,000	Poisson's ratio (ν_{xy})	0.28
Young's modulus (E_Y), MPa	9000	Poisson's ratio (ν_{xz})	0.28
Young's modulus (E_Z), MPa	9000	Poisson's ratio (ν_{yz})	0.40
Shear modulus (G_{xy}), MPa	4700	SiO_2%	54
Shear modulus (G_{yz}), MPa	3500	Al_2O_3%	15
Shear modulus (G_{zx}), MPa	4700	CaO%	12

Fig. 3 Image of an impact hammer test on no cracked steel beam

Table 1 Mechanical properties and chemical composition of the steel beam

Material	Carbon%	Manganese%	Sulfur and phosphorous%	Tensile strength MPa	Yield strength, MPa	Density Kg/m^3	Young's modulus GPa
IS 2062 mild steel	0.23	1.50	0.05	460	240	7850	200

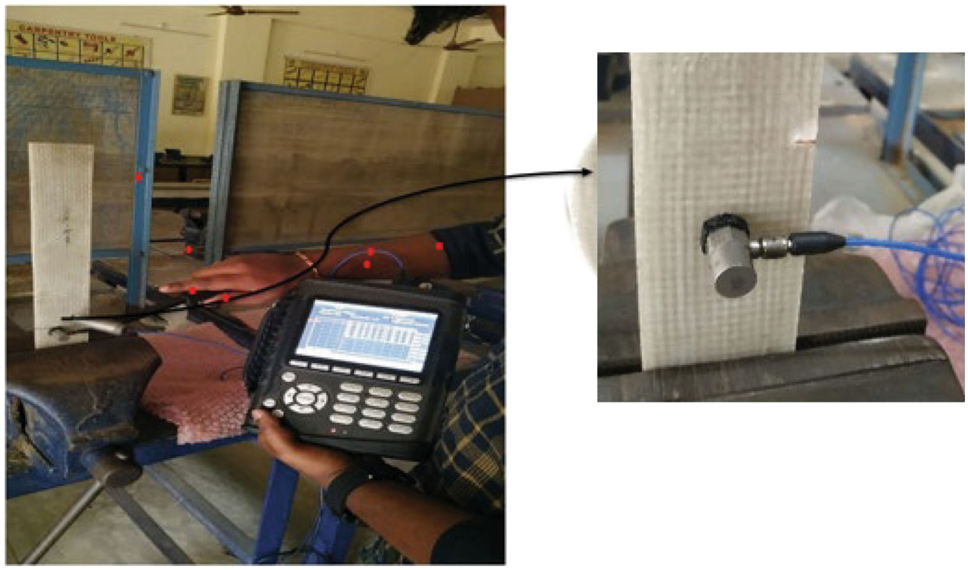

Fig. 4 Image of an impact hammer test on cracked composite beam

work. Figure 1 shows the image of steel beam, and Table 1 displays the mechanical properties and chemical composition of the steel beam. An E-Glass composite beam with dimensions of 300 mm × 50 mm × 3 mm is also considered in the present paper. Epoxy LY556 (Araldite) and Hardener HY951 (Aradur) in the ratio of 10:1 are used for fabrication of E-Glass composite beams by the hand layup method. Figure 2 shows the image of the composite beam, and Table 2 displays the mechanical properties and chemical composition of composite beam.

Impact Hammer Test on Steel and Composite Beams

The impact hammer test is conducted on a cracked and no crack beams made of steel and composite material with the help of fast Fourier transform (FFT) analyzer and an impact hammer with a sensitivity 10 mV/lbf. Accelerometers of sensitivity 107 mV/g with magnetic mounts are used for steel beams, and lightweight accelerometers of sensitivity 9.68 mV/g with adhesive mounts are used for composite beams. The lightweight accelerometer is selected for composite beam to avoid the effect of the mass of the accelerometer on the estimation of natural frequencies. The impact hammer test is carried out on clamped-free steel and composite beams. Different iterative impact hammer tests are carried out to obtain the best location of the accelerometer on the test beams for accurate and consistent results. From these iterations, it is found that the best location of accelerator is toward the end for no crack beams but near the crack for cracked beams. So, all impact hammer tests on both cracked and no crack beams made of steel and composite material are carried out in the present investigation by attaching the accelerometer accordingly.

Impact hammer and accelerometer are connected to chanel-1 and channel-2 of FFT analyzer. The data acquisition for all impact hammer test on both cracked and no crack beams made of steel and composite material is carried out in the frequency range of 0–500 Hz with a frequency resolution of 0.5 Hz. The input is given by an impact hammer at selected locations on the beam with a specified force to excite all frequencies in the range of 0–500 Hz. A piezoelectric acceleration sensor attached at selected locations on the beam is used to collect the response of the beam in the range of 0–500 Hz. The time domain signals obtained from the sensor and impact hammer are saved in the FFT analyzer, and then, these time domain signals are converted into frequency domain signals with the help of FFT algorithm. Post-analysis software called engineering data management software is used to plot the frequency response graphs and to estimate the natural frequencies. The time domain data collected from FFT analyzer are given as input to the engineering data management software. The Hanning windowing function is used to convert the time domain plot to a frequency response plot. The Hanning windowing function converts a time domain signal to a frequency response signal using the fast Fourier transform. Figure 3 shows the image of an impact hammer test on no cracked steel beam, and Fig. 4

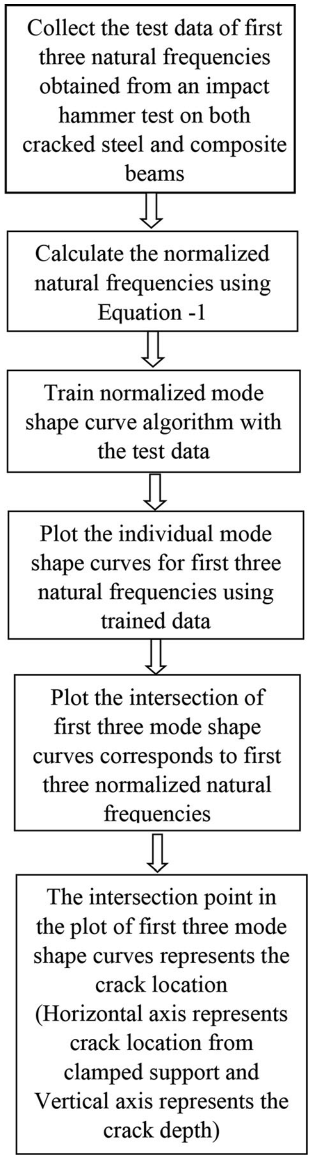

Fig. 5 Flowchart of normalized mode shape curve algorithm

displays the image of an impact hammer test on cracked composite beam. Beams are kept in a vertical position in the bench vice, as shown in Figs. 3 and 4 to avoid the effect of the mass of the accelerometer on estimation of natural frequencies.

Estimation of Crack Location

The normalized first three natural frequencies are calculated by Eq. 1. The intersection of first three normalized natural frequencies is obtained with the normalized mode

Table 3 Natural frequencies of no crack and cracked steel beam

	Natural frequency in Hz	
Mode	No crack beam	Cracked beam (Hz)
1	20.25	19.53
2	127.50	120.31
3	340.00	336.03

Table 4 Natural frequencies of no crack and cracked composite beam

	Natural frequency in Hz	
Mode	No crack beam (Hz)	Cracked beam (Hz)
1	25.00	20.31
2	153.13	140.63
3	418.12	405.86

shape curve algorithm with the set of data obtained from an impact hammer tests on cracked steel and composite beams. Normalized mode shape curve algorithm is trained with the modal data obtained by an impact hammer test on cracked steel and composite beams for crack localization. A normalized mode shape curve is a graph between crack depth and crack location for a particular modal frequency. Crack location is taken on the horizontal axis and crack depth on the vertical axis. Figure 5 shows the flowchart of normalized mode shape curve algorithm. Crack localization is estimated with the intersection of mode shapes for first three normalized natural frequencies of cracked steel and composite beams as per the flowchart explained in Fig. 5.

$$\text{Normalized modal frequency}(f_N) = \frac{\text{Natural frequency of cracked beam}}{\text{Natural frequency of no crack beam}}. \quad \text{(Eq 1)}$$

Results and Discussion

Crack Detection in Steel and Composite Beams

The impact hammer test is performed on both cracked and no crack beams made of steel and composite material to detect the presence of the crack. A transverse crack is made at 100 mm from the clamped end of the steel beam, and the

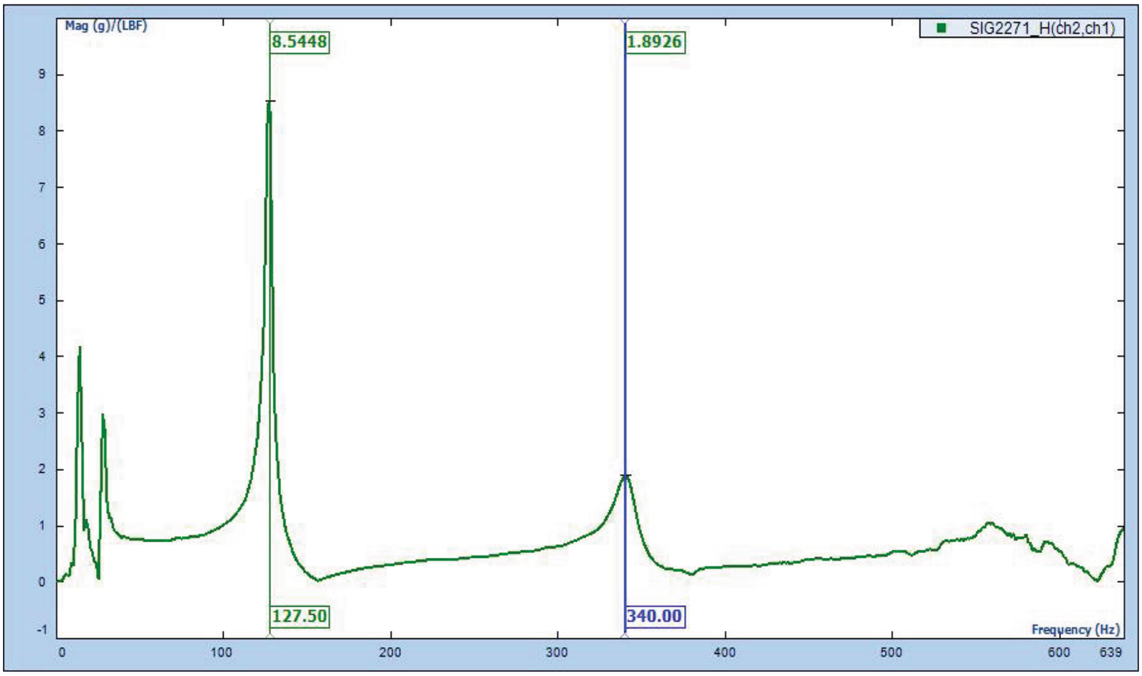

Fig. 6 Frequency response spectrum of no crack steel beam

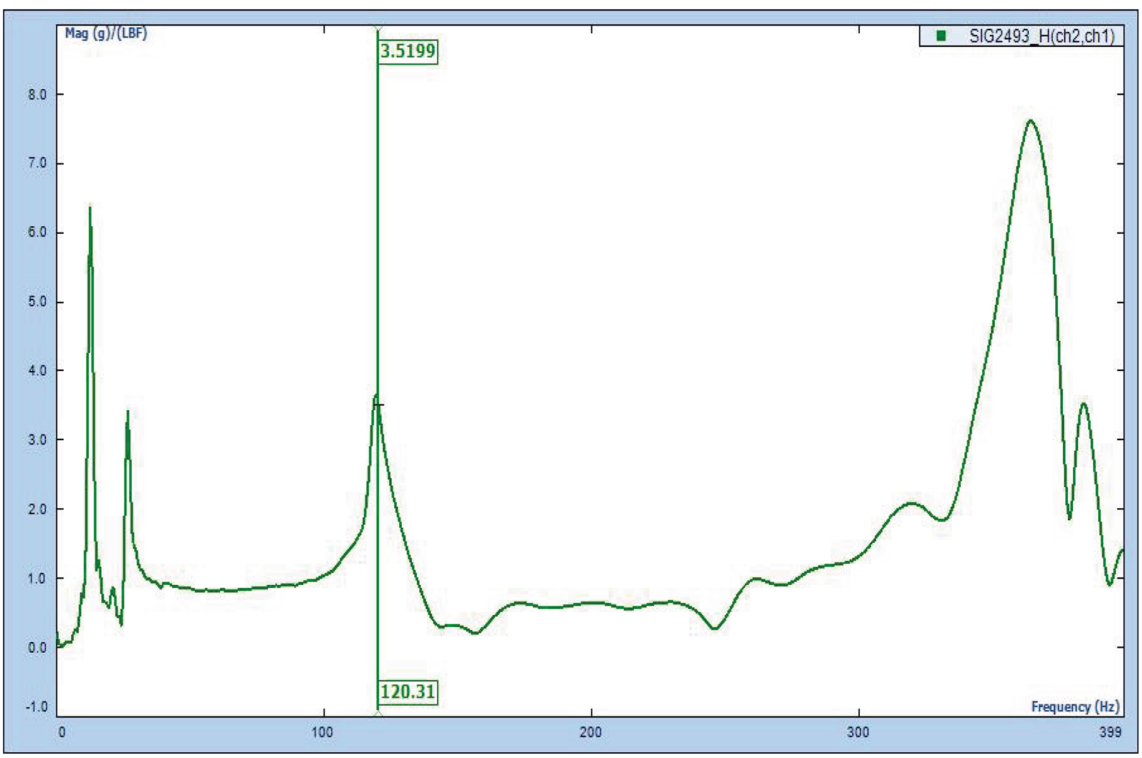

Fig. 7 Frequency response spectrum of cracked steel beam

depth of the crack is 2 mm. For a composite beam, a transverse crack is made at 50 mm from the clamped end of the composite beam and the depth of the crack is 5 mm. The natural frequencies of both cracked and no crack beams made of steel and composite material are compared to detect the presence of the crack. Table 3 shows the first

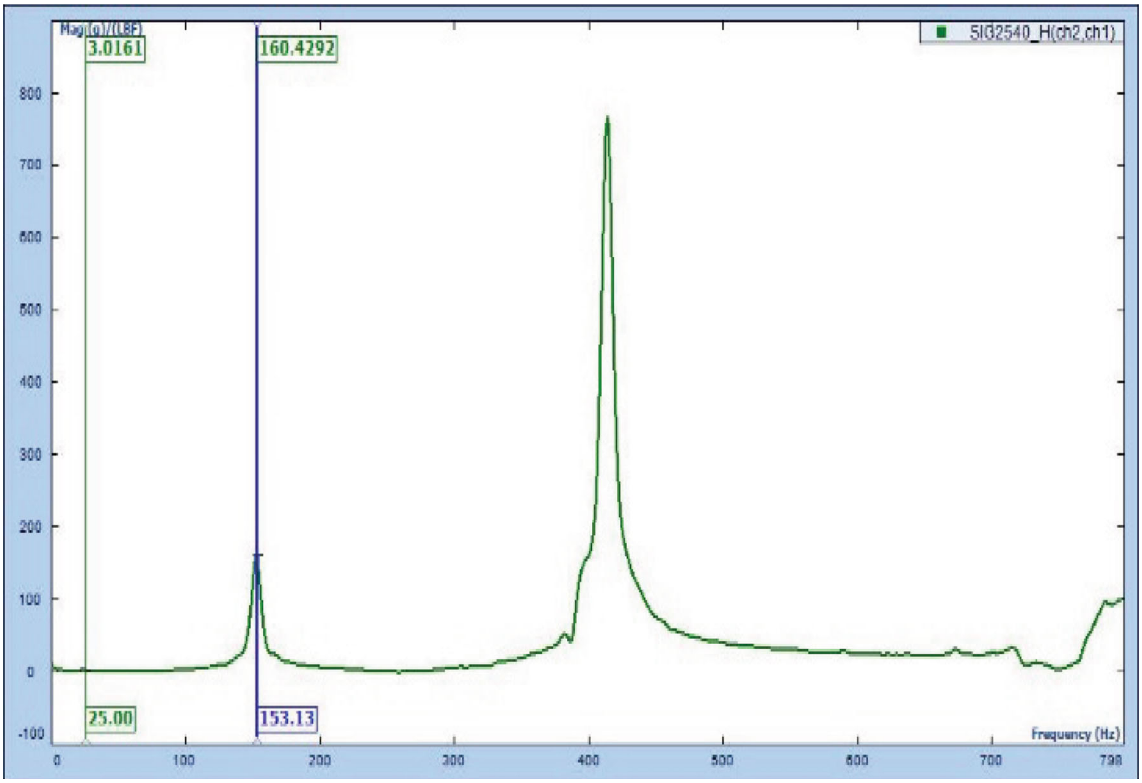

Fig. 8 Frequency response spectrum of no crack composite beam

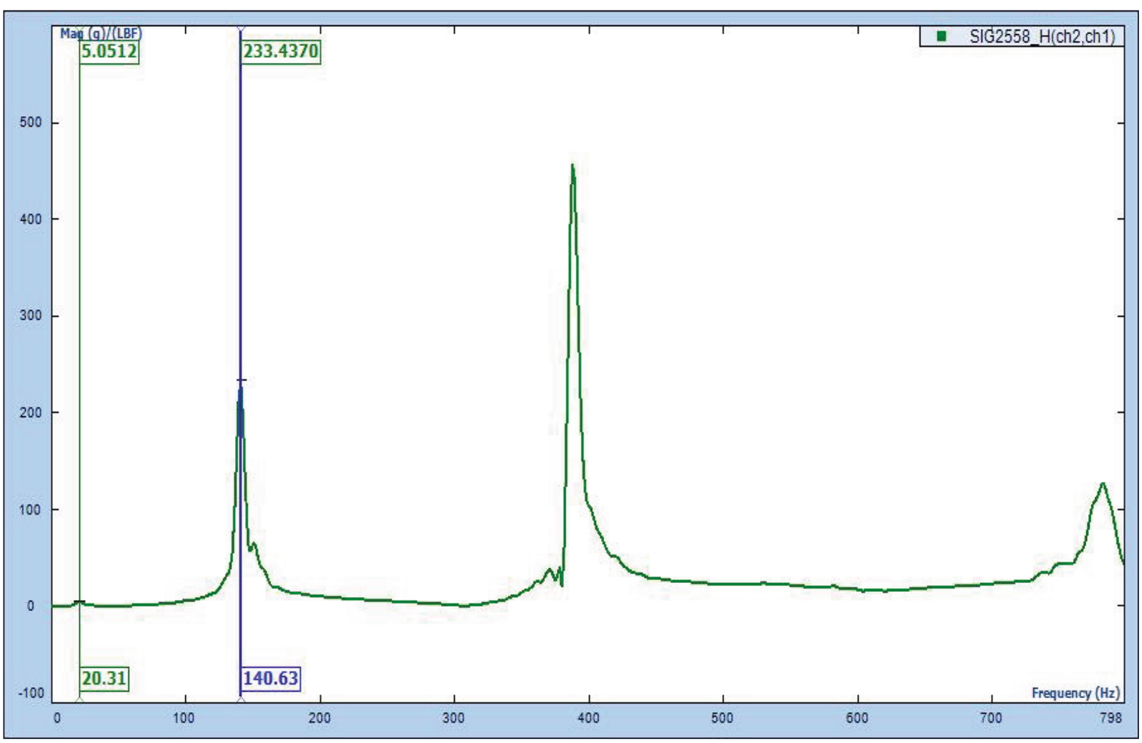

Fig. 9 Frequency response spectrum of cracked composite beam

Table 5 First three natural frequencies of cracked steel beams

Location of crack from fixed end in mm	Crack depth in mm	First natural frequency in Hz	Second natural frequency in Hz	Third natural frequency in Hz
No crack	No crack	20.25	127.50	340.00
100	2.00	19.53	120.31	336.03
100	2.50	18.92	120.29	334.81
100	3.00	18.63	120.28	333.45
150	2.00	19.32	120.03	333.55
150	2.50	19.15	119.79	330.60
150	3.00	18.95	119.46	326.87
200	2.00	19.39	119.22	335.64
200	2.50	19.30	118.35	334.10
200	3.00	19.18	117.24	332.18
250	2.00	19.43	118.40	337.54
250	2.50	19.39	117.29	337.54
250	3.00	19.31	115.25	337.54
300	2.00	19.48	118.69	334.98
300	2.50	19.46	117.61	333.27
300	3.00	19.43	115.70	330.39

Table 6 First three natural frequencies of cracked composite beams

Location of crack from fixed end in mm	Crack depth in mm	First natural frequency in Hz	Second natural frequency in Hz	Third natural frequency in Hz
No crack	No crack	25.00	153.13	418.12
50	5	20.31	140.63	405.86
50	10	20.06	140.47	404.48
50	15	19.66	140.15	402.14
100	5	20.36	140.52	405.94
100	10	20.03	139.25	402.74
100	15	20.26	136.83	403.91
150	5	20.39	140.20	406.10
150	10	20.34	138.91	405.31
150	15	20.26	136.83	403.91
200	5	20.40	139.41	405.90
200	10	20.38	137.88	405.16
200	15	20.43	140.21	406.17
250	5	20.42	140.63	406.42
250	10	20.42	140.48	406.36
250	15	20.43	140.21	406.17

three natural frequencies of both no crack and cracked steel beams. Table 4 displays the first three natural frequencies of both no crack and cracked composite beams. The crack present in the steel and composite beam affects the vibrational characteristics of the beam like modal frequencies, mode shapes and damping ratio. From Tables 3 and 4, it is evident that the natural frequencies of a cracked and no crack beams are different.

The first natural frequency of no crack steel beam is 20.25 Hz and for cracked steel beam is 19.53 Hz. The crack present in the steel beam reduces the beam's stiffness and causes variations in vibrational characteristics. These variations in frequency response function give useful data regarding a location and severity of damage. The first three natural frequencies of cracked steel and composite beams are lower than the respective no crack beams. The natural frequencies are dependent on the deflection, and the deflection depends on the stiffness. The natural frequencies are inversely proportional to deflection. The deflection of cracked beam is higher when compared with a respective no crack beam due to presence of crack. In other words, natural frequencies of cracked beams are lower when compared with respective no crack beams. Figures 6 and 7 show the frequency response spectrum of no crack and cracked steel beam, respectively. Figures 6 and 7 clearly indicate higher acceleration for unit force at the first three natural frequencies of cracked and no crack steel beam when compared to other frequencies in the spectrum. Figures 8 and 9 display the frequency response spectrum of a no crack and cracked composite beam, respectively. The first natural frequency of no crack composite beam is 25 Hz and for cracked composite beam is 20.31 Hz. The presence of crack in the composite beam reduces the beam's stiffness. So it is observed that the natural frequency of a cracked composite beam is much lower when compared with the no cracked composite beam.

Crack Localization in Steel and Composite Beams

The intersection of first three normalized natural frequencies is obtained using the trained normalized mode shape curve algorithm with the set of data obtained from an impact hammer tests on cracked steel and composite beams as explained in "Materials and Methods" section. Normalized mode shape curve algorithm is trained separately for steel and composite beams for crack localization. Table 5 shows the first three natural frequencies of cracked steel beams, and Table 6 shows the first three natural frequencies of cracked composite beams with various crack depths and locations. The values in Tables 5 and 6 are used to train the normalized mode shape curve algorithm for steel and composite beams separately. Normalized modal

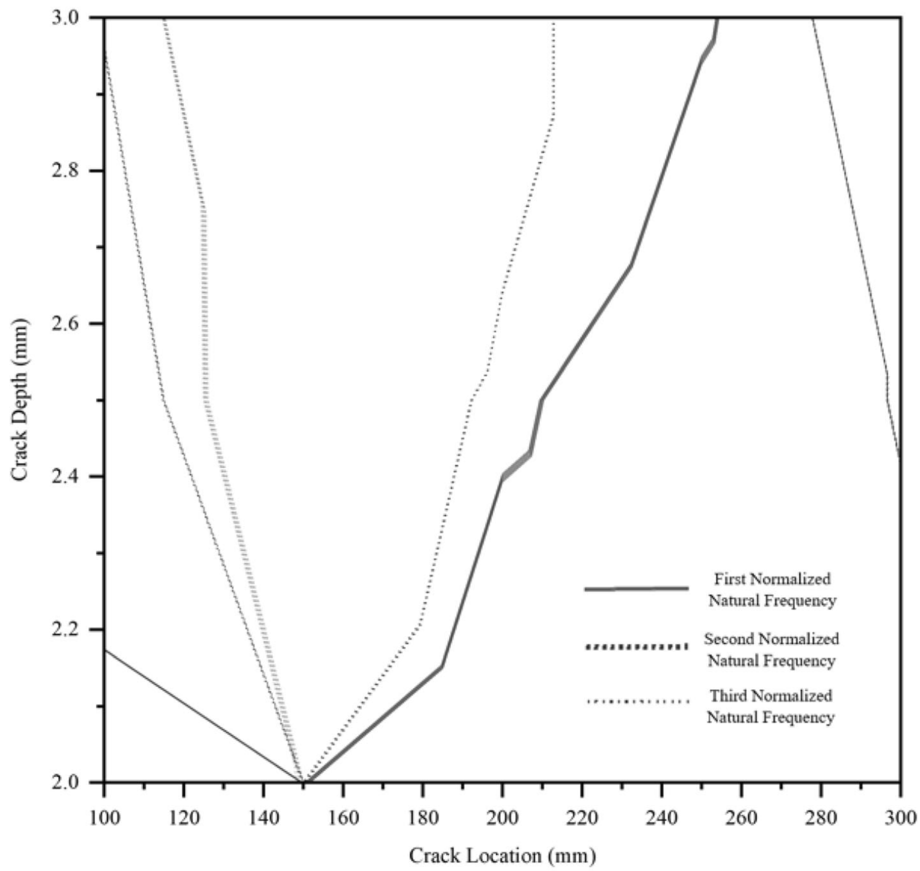

Fig. 10 Intersection point for first three normalized mode shape curves for cracked steel beam

Table 7 Comparison of actual and estimated crack location for steel beam

Description	Actual	Estimated	Error (%)
Crack location, mm	150	149.56	0.29
Crack depth, mm	2	2.01	0.50

frequencies are calculated for each crack depth and crack location for each mode using data from Tables 5 and 6.

A comparison is made between the actual crack location and predicted crack location for steel beam at 150 mm length and 2 mm depth. Figure 10 shows the intersection point for first three normalized mode shape curves for cracked steel beam at a crack location of 150 mm length and 2 mm depth. Figure 10 clearly shows the intersection point for first three mode shape curves of steel beam are at crack length 149.56 mm and depth 2.01 mm, which are very close to the actual location of the crack in steel beam. Table 7 shows the comparison of actual and estimated crack locations for steel beam. For composite beam, the crack localization is estimated with the first three normalized natural frequencies for a transverse cracked composite beam. A comparison is made between the actual crack location and predicted crack location for composite beam at 100 mm length and 10 mm depth. Figure 11 shows the intersection point for first three normalized mode shape curves for cracked composite beam for crack location at 100 mm length and 10 mm depth. In Fig. 11, the intersection point of first three mode shape curves of composite beam at length 100.21 mm and depth 25 mm was observed, which is very near to the actual location of the crack in composite beam. Table 8 shows the comparison of actual and estimated crack locations for composite beam. From the results, it is observed that crack location estimation results are close to the actual crack location for both steel and composite beams. Table 9 displays the comparison of the present work with recent work in the literature. From Table 9, it is confirmed that the present method predicts the crack location more accurately when compared with recent work in the literature.

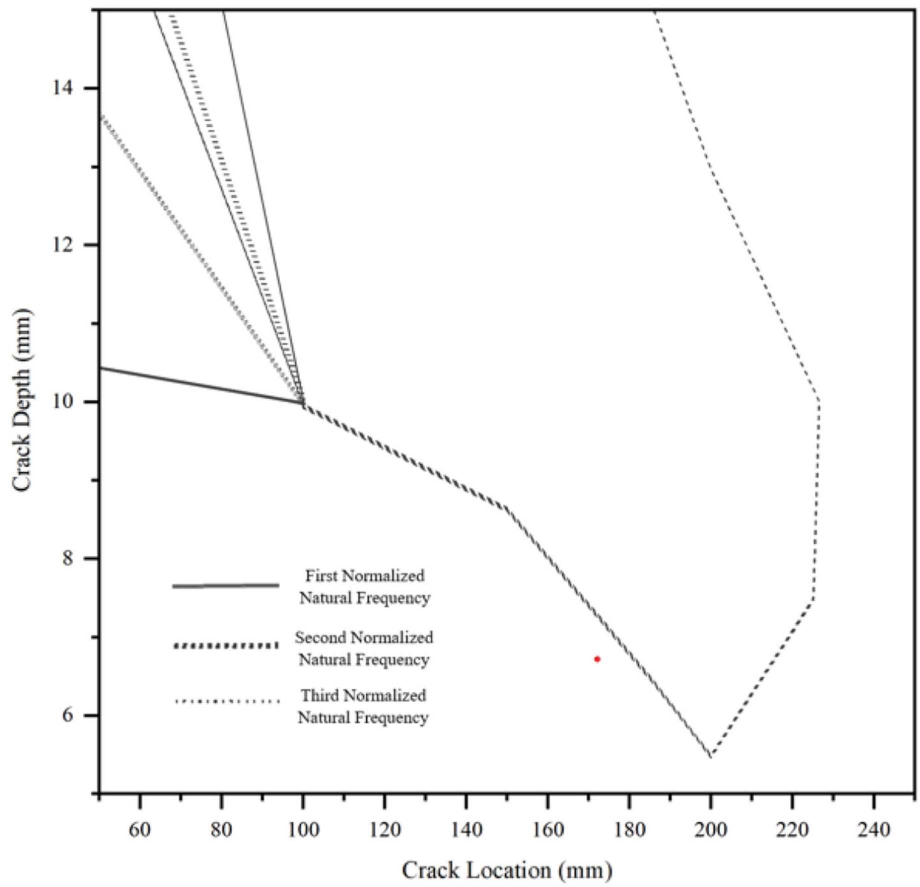

Fig. 11 Intersection point for first three normalized mode shape curves for cracked composite beam

Table 8 Comparison of actual and estimated crack location for composite beam

Description	Actual	Estimated	Error (%)
Crack location, mm	100	100.21	0.21
Crack depth, mm	10	9.98	0.20

Conclusions

Natural frequencies are estimated by an impact hammer test on both cracked and no crack beams made of steel and composite material. The first natural frequency of cracked and no crack steel beam is 19.53 Hz and 20.25 Hz, respectively, and for cracked and no crack composite beam is 20.31 Hz and 25.0 Hz, respectively. The presence of the crack in steel and composite beams is identified with the help of change in natural frequencies obtained from an impact hammer test on both cracked and no crack beams. The natural frequencies of steel and composite beams decrease with an increase in crack depth, and they increase with an increase in transverse crack location from the clamped end. The normalized mode shape curves are generated with the trained normalized mode shape curve algorithm with the set of data obtained by an impact hammer test on cracked steel and composite beams. For crack localization, the intersection point of the first three normalized natural frequencies of a cracked steel and composite beam is used. Crack location is estimated with reasonable accuracy using the intersection of first three normalized mode shape curves for steel and composite beams. It is found that there is a good agreement between the actual and estimated crack location as the percentage of error between them is less than 1%. The methodology presented in the current paper for estimation of crack location is more accurate when compared to various methods in recent literature based on mode shape curves.

Table 9 Comparison of the present work with recent work in the literature

Author	Crack detection and Crack localization method	Remarks
Cherrez et al. [3]	Mode shape difference	Predicted only crack detection, not found the crack location
Agarwalla and Parhi [7]	Modal Parameters	Predicted only crack detection, not found the crack location
Vaishali and Gaurang [14]	Modal curvature technique and modal flexibility technique of first three modes	Predicted only crack detection, not found the crack location
Behera et al. [16]	Synthesis of mode shapes	Predicted only crack detection, not found the crack location
Shkelzen and Yusuf [17]	Stiffness matrix for the crack	Predicted only effects of crack location and crack depth on first three modes, not found the crack location
Zizheng et al., [18]	Cracking elements method	Predicted only crack detection, not found the crack location
Leonard et al. [22]	Modal analysis algorithms	Predicted only crack detection with 6.2% error, not found the crack location
Sadettin [23]	Free and forced vibration analysis using ANSYS	Predicted only effects of crack location and crack depth on first three modes, not found the crack location
This work	Intersection of first three normalized mode shape curves	Predicted crack detection and crack location with less than 1% error

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