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TECHNICAL ARTICLE—PEER-REVIEWED

Reliability Analysis for Load-Sharing Parallel Systems with No Failure of Components

Xinshui Yu · Tianxiang Yu · Kunling Song · Bifeng Song

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Abstract Conventional reliability analysis of load-sharing parallel systems is mainly based on the assumption that there is failure of components. When a component fails, the load redistributes among the remaining surviving components. But sometimes there is no failure of components during the usage of a load-sharing parallel system. When there is no failure of components, working load is dispatched among components, which means the load on each component is almost the same during the whole working process. The degradation process of each component is also almost the same. The reliability analysis of a load-sharing parallel system with no component fails can be simplified to the reliability analysis of a component. In this paper, three cases are studied. First, based on stress-strength interference model, estimation of component reliability with load application times is generated, which considered the nonlinear fatigue damage model. Then, Poisson process is used to describe reliability changed with time. Third, the reliability is studied when the working load is not constant but changed with time or load application times. Some examples are used to illustrate the application of these models, and the Monte Carlo simulation method is used as standard to verify the proposed models.

Keywords Load-sharing parallel system · Nonlinear fatigue damage model · Poisson process · Changed working load · No failure of components

X. Yu · T. Yu (✉) · K. Song · B. Song
School of Aeronautics, Northwestern Polytechnical University, Xi'an, China
e-mail: Tianxiangyu@nwpu.edu.cn

Introduction

For a mechanical system, the redundancy technique is a commonly used application which can improve the reliability of system, so the *k*/*n* redundancy systems are widely applied in many fields. Load-sharing parallel system is the simplest form of *k*/*n* redundancy system. Airplane's multi-engine system, bridge's wire cable, flow transmission system, task processing system and the body's kidney are all load-sharing parallel systems [1, 2]. So the evaluation of load-sharing parallel systems' reliability plays an important role in the normal operating process of mechanical systems. A significant body of research has accumulated in the reliability study of load-sharing systems. Almost all the studies assume there is component failure. In a load-sharing system, if a component fails, the total workload will be shared by the remaining components, resulting in an increased load shared by each surviving component [3, 4]. Lots of researchers use failure rate to analyze the reliability of load-sharing systems [5–10]. Amari et al. [8] provided a closed-form analytical solution for the reliability of tampered failure rate load-sharing systems with identical components where all components share the load equally. Tang et al. [11] described the increase in the same component's failure rate under different loads on the basis of capacity flow model and studied the reliability of the load-sharing system with different components. The methods mentioned above assumed the failure time distribution, for the components are exponential distribution. In order to overcome this limitation, Liu [12] developed a method to get the reliability of a load-sharing k-out-of-n: G system, whose components are nonidentical and the failure time distribution of each component can be the same or different. Zhang et al. [15] used the first-order method, which linearized the component limit-state function at the most probable point, to calculate the reliability of a load-sharing k-out-of-n: G system. Xie et al. [16] presented an approach, which is based on system-level load-strength interference analysis and a concept of 'conditional failure probability of component,' to calculate the system's reliability.

Regardless the lifetime's distribution, the reliability analysis methods of load-sharing systems mentioned above are almost based on the calculation of mechanical components' failure rate. However, in order to get the failure rates of mechanical components, large amount samples of load and strength degradation should be needed, which is hard to get. Because of the vibration, complex working environment and so on, the fatigue, corrosion and wear frequently appear in the working process of mechanical system. All of these reasons can degrade the component's strength, which will result in the variation of the reliability of components with load application times or time. So it is important to consider the strength degradation process in the reliability analysis of mechanical components [13]. Lots of researchers studied the reliability of load-sharing systems with degrading components. Yang et al. [17] combined the tampered failure rate model with a performance degradation model to analyze the reliability of load-sharing k-out-of-n system with degrading components. Gao et al. [13] calculated the system's reliability in terms of stress and strength parameters, which consider the degradation mechanism of mechanical components. Zhao et al. [14] analyzed the reliability of the load-sharing systems with identical components subject to continuous degradation.

All of the studies are based on the assumption that the component of load-sharing parallel system has only two states: functioning or failed. But sometimes, the components will degrade instead of fail. In this paper, we will study the reliability analysis of load-sharing parallel system with no component fails and the system fails to meet a required performance threshold. This paper is organized as follows. In part 2, we discuss the load-sharing parallel systems with no failure of components when the systems cannot meet the requirement. We divide this situation into three cases: the relationship between reliability and load application times, the relationship between reliability and time and reliability analysis with increment process of working load. In part 3, some numerical examples are used to illustrate the application of the models under the three cases. Concluding remarks are discussed in part 4.

Load-Sharing Parallel Systems with No Failure of Components

For a mechanical load-sharing parallel system, components are usually chosen with identical property parameters. Traditional load-sharing parallel system models assume that all components share the working load with some load-sharing rules. When one of the components fails, the load will be automatically redistributed to the remaining components. The failure of each component is always sudden, which leads the whole system to stop working immediately. However, there are some situations that the components have no failure, but with the increased working time and working load application times, the strength of each component will degrade. Then, the whole performance of the system will degrade. In order to work normally, each system has a threshold. When the total degradation of the system reaches the threshold, the system will fail [2]. As in Fig. 1, four plates combined to form a torsional spring system, which is widely used in armored vehicles. These plates are identically distributed, which means these four plates equally share the total load. In the working process, the torsional spring system needs to be torqued time after time. After torqued some angle, the torsional spring can afford some certain torque, and the torque always needs to fit some requirements. In this system, these four plates will degrade gradually with the work load application times. The system will fail when the generated torque cannot meet the requirement. During the whole working process, there is no sudden failure of components.

For a load-sharing parallel system without failure of components, the system working load is allocated to the components by some sharing rules (e.g., equal load sharing, tapered load sharing, local load sharing, etc.) [24]. In this paper, we use the equal load sharing principle. In the process of working, the strength of components degrades along with application times of loads. Along with the strength degradation of components, the work load will be changed to balance the degradation. If one component degrades fast, the working load it allocated will be small, so the degradation of it will be slow. The whole working load allocated by other components will be large; then, the degradation of other components will be fast. Because of this, we can assume that the degradation level and working load shared by each component are almost the same during the working time of a system. When the strength is smaller than the desired stress of a component, which means the component will fail, the whole load-sharing parallel system will fail.

Because of the reasons mentioned above, the reliability analysis of a load-sharing parallel system with no failure of

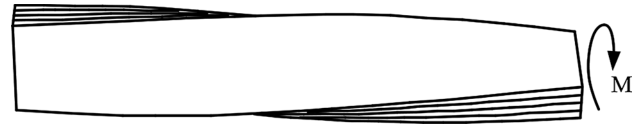

Fig. 1 Load-sharing parallel system of torsional spring

components can be simplified into the reliability analysis of each component of this system. Based on the actual working condition, three cases will be discussed in the following parts.

Dynamic Reliability Models with Nonlinear Strength Degradation

If the fatigue is considered to be the only one failure mode, then the process of load application will be discrete. And the load process can be characterized by two factors, which are application times and magnitude. When only the failure mode of fatigue is considered, the reliability analysis of mechanical components at a given time instant will be insignificance. The study of reliability with load application times will be more meaningful [18]. As we all know, when there is no appearance of load application, there will be no degradation of strength, which means that the strength degradation process is not continuous.

After n times of load application, the remaining strength of the mechanical components can be expressed as [19]:

$$r(n) = r_0[1 - D(n)]^a \quad \text{(Eq 1)}$$

where r_0, n and a are the initial strength, load application times and material parameter, respectively. $D(n)$ is the cumulative damage caused by load, the magnitude of it is determined by load application times n and the magnitude of load.

The damage caused by a load with a magnitude of s_i once can be got by the Miner rule, which can be expressed as:

$$D_i(1) = \frac{1}{N_i} \quad \text{(Eq 2)}$$

where N_i is the lifetime under the load s_i.

When the initial strength is determined, the remaining strength based on the Miner rule after n times of load application can be expressed as [18]:

$$\begin{aligned} r(n) &= r_0[1 - D(n)]^a = r_0\left(1 - \frac{n_0 n}{N_0}\right)^a \\ &= r_0\left(1 - \frac{n\int_{-\infty}^{\infty} s^m f_s(s)\mathrm{d}s}{C}\right)^a \end{aligned} \quad \text{(Eq 3)}$$

where m and C are material parameters.

And the reliability can be expressed as:

$$R(n) = \prod_{i=0}^{n-1}\left[\int_{-\infty}^{r_0\left(1-\frac{i\int_{-\infty}^{\infty} s^m f_s(s)\mathrm{d}s}{C}\right)^a} f_s(s)\mathrm{d}s\right] \quad \text{(Eq 4)}$$

In the model mentioned above, the cumulative damage caused by load is established by the Miner linear damage accumulation rule. But the Miner rule has some obvious drawbacks: the effects of loading sequence, load interaction and the damage contribution caused by those stresses below the fatigue limit are ignored. Because of these reasons, the results between the predicted and experimental value got by Miner rule always have difference. So, in this paper, we choose the nonlinear fatigue damage model proposed by Manson and Halford [20] to instead the Miner rule to overcome these drawbacks. In this nonlinear fatigue damage model, the damage caused by load s_i once can be expressed as:

$$D(1) = \left(\frac{1}{N_i}\right)^{BN_i^\beta} \quad \text{(Eq 5)}$$

where B and β are material parameters.

The relationship between the load magnitude s_i and the corresponding lifetime N_i, which can be got by the S–N curve theory of components, can be expressed as:

$$\sigma_i^m N_i = C, \quad N_i = \frac{C}{\sigma_i^m}, \quad \text{(Eq 6)}$$

where m and C are material parameters.

Then, Eq 5 can be expressed as:

$$D(1) = \left(\frac{\sigma_i^m}{C}\right)^{B\left(\frac{C}{\sigma_i^m}\right)^\beta} \quad \text{(Eq 7)}$$

When we know the probability density function (pdf) $f_s(s)$ of the random load, the damage caused by stress s_i once can be expressed as:

$$D(1) = \int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^\beta} f_s(s)\mathrm{d}s \quad \text{(Eq 8)}$$

The remaining strength considered nonlinear strength degradation with n load application times can be expressed as:

$$r(n) = r_0\left[1 - n\int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^\beta} f_s(s)\mathrm{d}s\right]^a \quad \text{(Eq 9)}$$

Then, after n times of load application, the reliability of the mechanical component can be derived as:

$$R(n) = \prod_{i=0}^{n-1}\left\{\int_{-\infty}^{r_0\left[1-i\int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^\beta} f_s(s)\mathrm{d}s\right]^a} f_s(s)\mathrm{d}s\right\} \quad \text{(Eq 10)}$$

Monte Carlo Simulation Method

In order to validate the proposed model, the Monte Carlo simulation method is carried out. The main steps of the Monte Carlo simulation are listed as follows:

Step 1: The load application times n and the total times of the Monte Carlo simulation trials k will be fixed firstly, and let $i = 0$, $m = 1$ and $j = 1$.
Step 2: Get the value of initial strength of the component r_0. Let $r_j = r_0$.
Step 3: Get the value of random load s_j.
Step 4: If $r_j > s_j$, let $i = i + 1$ and go to step 5; otherwise go to step 6.
Step 5: If $m = k$, go to step 7; otherwise let $m = m + 1$ and $j = 1$.
Step 6: If $j = n$, go to step 5; otherwise calculate remaining strength r, let $r_j = r$ and $j = j+1$, then go to step 3.
Step 7: Use the formula $R = 1 - i/k$ to get the reliability of the system.
Step 8: Stop the simulation.

Reliability Analysis over Time

In the working process of mechanical equipment or system, the working loads are always related with time. The working process of loads can be described by random process [21, 22].

Poisson Process

As an important process of counting, Poisson process can be used to describe the variation of the number of random loads over time. $N(t)$ is the total number of random loads during period (0, t), which fits the following conditions as [23]:

1. When $t = 0$, the number of random loads is zero, which means $N(0) = 0$;
2. In any period of time, the appearance of the load is independent of each other, which means $0 < t_1 < \cdots < t_n$, $N(t_1), N(t_2) - N(t_1), \ldots, N(t_n) - N(t_{n-1})$ are independent of each other.
3. The number of random loads has non-relationship with initial time, it only has relationship with the time period, which means $\forall s, t \geq 0, n \geq 0, P[N(s+t) - N(s) = n] = P[N(t) = n]$;
4. For any $t > 0$ and sufficiently small $\Delta t (\Delta t > 0)$, there is:

$$\begin{cases} P[N(t+\Delta t) - N(t) = 1] = \lambda(t)\Delta t + o(\Delta t) \\ P[N(t+\Delta t) - N(t) \geq 2] = o(\Delta t) \end{cases}$$

where $\lambda(t)$ is the intensity of the Poisson process. Then, the total number of random loads $N(t)$ can be seen as Poisson process with rate function $\lambda(t)$. And the probability of n times of loads at any time t can be expressed as:

$$P[N(t) - N(0) = n] = \frac{\left(\int_0^t \lambda(t)\mathrm{d}t\right)^n}{n!} \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) \tag{Eq 11}$$

Based on the Poisson process theory mentioned above and the total probability theorem, the reliability considered nonlinear strength degradation between the time *0* and t can be got by:

$$\begin{aligned} R(t) = {} & \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) + \sum_{n=1}^{\infty} \frac{\left(\int_0^t \lambda(t)\mathrm{d}t\right)^n}{n!} \\ & \times \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) \\ & \times \exp\left\{\prod_{i=1}^{n-1}\left[\int_{-\infty}^{r_0\left[1-i\int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^{\beta}} f_s(s)\mathrm{d}s\right]^a} f_s(s)\mathrm{d}s\right]\right\} \end{aligned} \tag{Eq 12}$$

Based on the Taylor expression of exponential function, which is:

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

Equation 12 can be simplified as:

$$\begin{aligned} R(t) = {} & \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) + \left[1 - \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right)\right] \\ & \times \exp\left\{\prod_{i=1}^{n-1}\left[\int_{-\infty}^{r_0\left[1-i\int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^{\beta}} f_s(s)\mathrm{d}s\right]^a} f_s(s)\mathrm{d}s\right]\right\} \end{aligned} \tag{Eq 13}$$

Due to manufacturing, materials and other factors, the initial strength has some dispersion. The pdf of r_0 $(f_{r_0}(r_0))$ can be used to denote the dispersion of initial strength. Then, the reliability can be expressed as:

$$\begin{aligned} R(t) = {} & \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) \int_{-\infty}^{\infty} f(r_0)\mathrm{d}r_0 \\ & + \sum_{n=1}^{\infty} \frac{\left(\int_0^t \lambda(t)\mathrm{d}t\right)^n}{n!} \times \exp\left(-\int_0^t \lambda(t)\mathrm{d}t\right) \\ & \times \exp\left\{\int_{-\infty}^{\infty} f_{r_0}(r_0)\left\{\prod_{i=0}^{n-1}\left[\int_{-\infty}^{r_0\left[1-i\int_{-\infty}^{+\infty}\left(\frac{s^m}{C}\right)^{B\left(\frac{C}{s^m}\right)^{\beta}} f_s(s)\mathrm{d}s\right]^a} f_s(s)\mathrm{d}s\right]\right\}\mathrm{d}r_0\right\} \end{aligned} \tag{Eq 14}$$

Reliability Analysis with Increment Process of Working Load

In the process of working, the working load may be not constant but changed with time or load application times. As in Fig. 2, three bars are fixed at one end and suffered a total load F at the other end. In the process of working, the

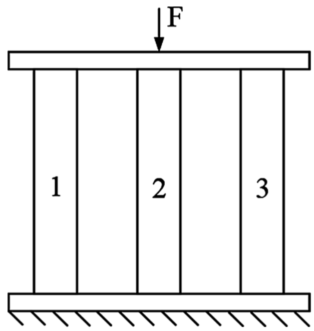

Fig. 2 Load-sharing parallel system of three bars

total working load F can be changed with time or work load application times.

This means that the pdf of stress will be changed, which can be expressed as $f_s(s,t)$. Then, the reliability under the application of random load n times can be expressed as:

$$R(n) = \prod_{i=0}^{n-1} \left\{ \int_{-\infty}^{r_0} \left[1 - i \int_{-\infty}^{+\infty} \left(\frac{(s,t)^m}{C} \right)^B \left(\frac{C}{(s,t)^m} \right)^{\beta} f_s(s,t) \mathrm{d}s \right]^a f_s(s,t) \mathrm{d}s \right\} \quad \text{(Eq 15)}$$

And the reliability considered nonlinear strength degradation between the time interval (0, t) can be expressed as:

$$\begin{aligned} R(t) = {} & \exp\left(- \int_0^t \lambda(t) \mathrm{d}t \right) + \sum_{n=1}^{\infty} \frac{\left(\int_0^t \lambda(t) \mathrm{d}t \right)^n}{n!} \\ & \times \exp\left(- \int_0^t \lambda(t) \mathrm{d}t \right) \\ & \times \left\{ \prod_{i=1}^{n-1} \left[\int_{-\infty}^{r_0} \left[1 - i \int_{-\infty}^{+\infty} \left(\frac{(s,t)^m}{C} \right)^B \left(\frac{C}{(s,t)^m} \right)^{\beta} f_s(s,t) \mathrm{d}s \right]^a f_s(s,t) \mathrm{d}s \right] \right\} \end{aligned} \quad \text{(Eq 16)}$$

Numerical Examples

Case 1: Reliability under load application times with nonlinear strength degradation.

In this section, the material parameters are given by $m = 2$, $a = 1$ and $C = 10^8$ MPa^2. The constants B and β are 2/3 and 0.4 according to the research by Manson and Halford. The initial strength is normally distributed with a mean value $\mu(r_0)$ and a standard deviation of $\sigma(r_0)$. The stress at each load application follows the normal distribution with a mean value of $\mu(s)$ and a standard deviation of $\sigma(s)$. The value of the mean value and standard deviation of the initial strength and stress are listed in Table 1.

Table 1 Value of mean and standard deviation of stress and initial strength

$\mu(r_0)$ (MPa)	$\sigma(r_0)$ (MPa)	$\mu(s)$ (MPa)	$\sigma(s)$ (MPa)
600	10	500	20

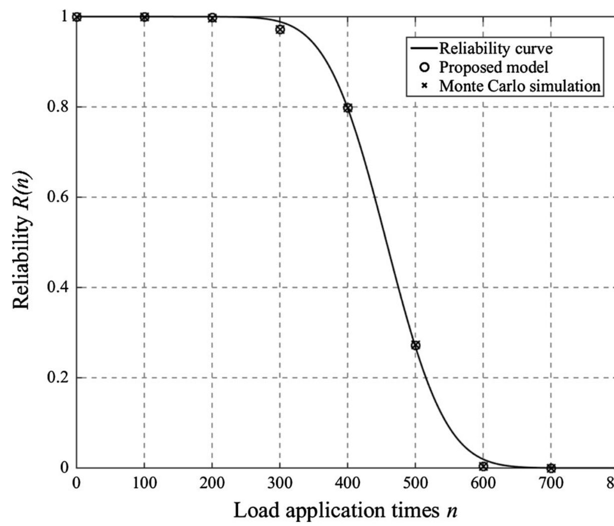

Fig. 3 Reliability with load application times from the proposed model and Monte Carlo simulation

The reliability with load application times got by Monte Carlo method and the proposed model (Eq 10) in this paper are listed in Fig. 3.

From this figure, we can see that the reliability, which considered the nonlinear strength degradation progress, decreases with load application times. The reliability calculated by the proposed method in this paper agrees well with the results calculated by the Monte Carlo simulation.

Case 2: Reliability over time.

Consider the random load follows the Poisson process with an intensity of 0.6 h^{-1}, the material parameters are the same as Case 1. The mean and standard deviation of stress and initial strength are also the same as Case 1. Then, the results of reliability over time calculated by the proposed model (Eq 12) in this paper are listed in Fig. 4.

From this figure, we can see that the reliability, which considered the nonlinear strength degradation process and the Poisson process, decreases with the increment of time. Within 0–400 h, the reliability is close to 1, during 400–1000 h, it decreases rapidly, and after 1000 h, the reliability of the load-sharing parallel system is almost zero. The model proposed in this paper provides an analytical method for reliability analysis considering nonlinear strength degradation and Poisson process over time.

Case 3: Reliability with changed working load.

As long as we know the change law of stress, the reliability under application of random load times and considered Poisson process over time will be got.

In this paper, we study the situation that the working load linearly increases with time, which can be seen frequently in an actual engineering, and the rate of this linear function is v. Under this approximation, the working load increment process can be expressed as $W(t) = vt$. But there is always fluctuation in the increment process of working load. Candidate stochastic processes (Gamma process, Compound Poisson process, Inverse Gaussian distribution) are suitable to describe the working load increment process [25]. In this paper, Possion process is used to describe the variation of the number of random loads over time. So in this case, compound Poisson process is choose to describe the working load increment process.

The Compound Poisson Process If the rate of working load application is not high, we may consider the compound Poisson process to describe the process with increased working load. The expression of compound Poisson process can be seen as:

$$W(t) = \sum_{i=1}^{N(t)} W_i$$

where $\{N(t); t \geq 0\}$ is a non-homogeneous Poisson process with rate function $\lambda(t)$, and W_i is the working load with each arrival, which are i.i.d. The mean of $W(t)$ is given by $\lambda(t)t \cdot E(W_i)$.

The working load increment process of the compound Poisson process can be seen as in Fig. 5.

In this figure, the rate function of Poisson process is $\lambda(t) = 0.6\,\mathrm{h}^{-1}$ and the workloads associated with each arrival follows exponential distribution. The rate parameter of exponential distribution is $\frac{1}{\mu}$, which means $W_i \sim \Gamma(1, \frac{1}{\mu})$. The increment of stress is 5 MPa per 100 h.

In this case, the torsional spring is used as an example. The length, width and height of each thin plate are 306 mm, 21 mm and 2 mm, respectively. The material is made up of 60Si2Mn. The shear modulus $G = 79.9 \times 10^3$ MPa, the Poisson ratio $\lambda = 0.29$. According to the *S–N* curve of 60Si2Mn, the material parameters are $m = 3$, $a = 1$ and $C = 10^{13}$ MPa^2. After torqueing 90°, the maximum shear stress is 820 MPa. The mean and standard deviation values of the stress and initial strength are listed in Table 2.

The torsional spring fails without fracture of components, and each component degrades almost in the same path. The reliability of the torsional spring system can be expressed by the reliability of each component. So the reliability comparison of torsional spring system with no increased working load and increased working load can be seen in Fig. 6. In Fig. 6, the process of increment working load is the same with the process shown in Fig. 5.

In this figure, we can see that the reliability decreases with the increment of time. The reliability with increased working load decreases more rapidly than the reliability with no increased working load, which means that when the working load increases, the system will fail easily.

Table 2 Value of mean and standard deviation of stress and initial strength

$\mu(r_0)$ (MPa)	$\sigma(r_0)$ (MPa)	$\mu(s)$ (MPa)	$\sigma(s)$ (MPa)
900	10	820	10

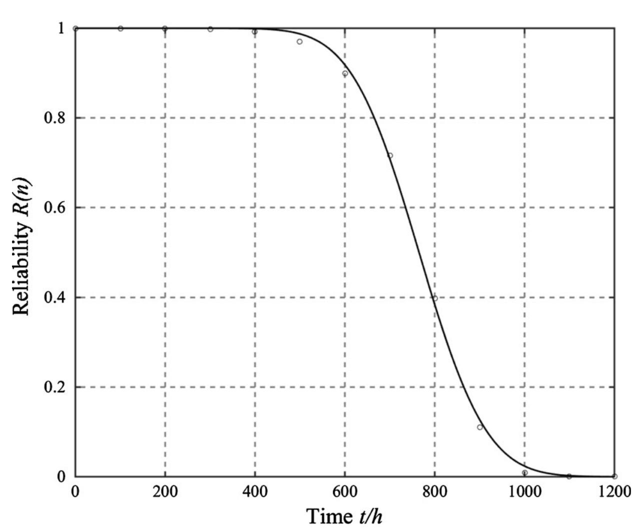

Fig. 4 Reliability with time of load

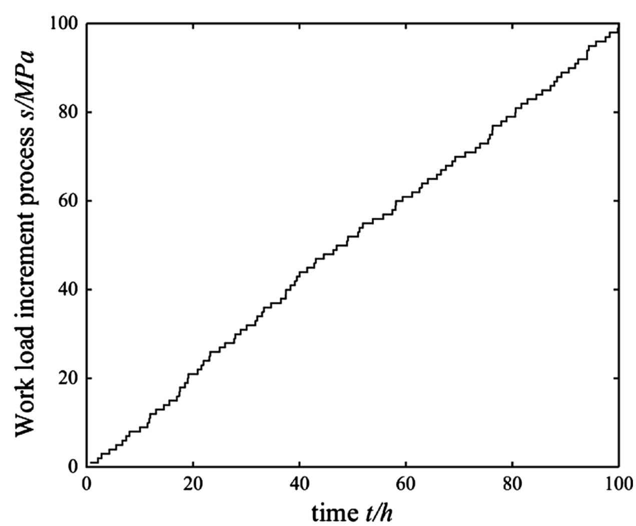

Fig. 5 Working load increment process of compound Poisson process

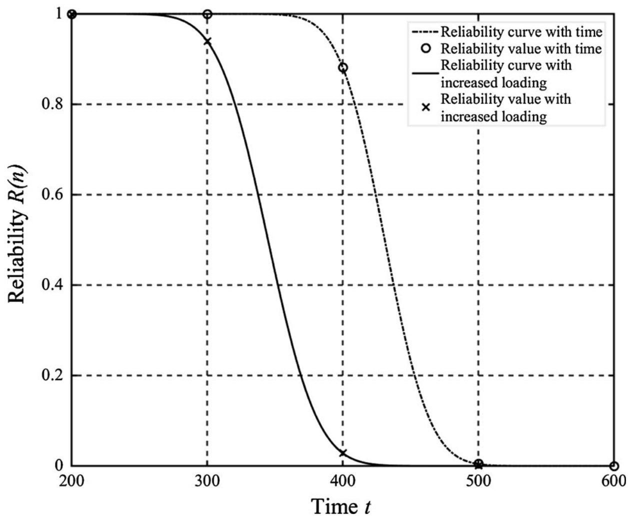

Fig. 6 Reliability of torsional springs system with time *t*

Conclusions

In the working process of the load-sharing parallel system, there are some situations that the components of the system have no failure. For this condition, the reliability calculation of the system can be simplified to the reliability calculation of each component. Three cases are used to illustrate the different conditions.

1. With the increment of load application times, the strength of components will degrade. The Miner rule is widely used to describe the strength degradation of mechanical components, but the Miner rules have no consideration of the effects of loading sequence, load interaction and the damage contribution induced by those stresses below the fatigue limit. Based on nonlinear strength degradation, the relationship between dynamic reliability model of load-sharing parallel system with no failure components and load application times is developed in this paper.
2. With the consideration of the Poisson process, the reliability model of the system over time is also studied.
3. When mechanical components works, the working loads are not always constant but changed. The model of this situation is also studied. The situation with gradually increased working load is studied in this paper. Compound Poisson process is used to describe the working load increment process. The larger the working load, the faster the strength degradation process, and the reliability of load-sharing parallel system decreases more rapidly.

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