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TECHNICAL ARTICLE—PEER-REVIEWED

Mixed-Mode Stress Intensity Factor Estimation of Inclined Cracks in an Unnotched Round Bar

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Abstract The extent on mixed-mode stress intensity factor (SIF) of inclined multiple cracks in an unnotched bar has been investigated numerically. Opposite cracks with various crack depth ratios (*a*/*d*), crack aspect ratios (*a*/*c*) and inclination angles (θ) were considered. The geometry correction factor (*Y*) was calculated by considering the effect of mode-II and mode-III fracture. It was observed that, for short cracks [(*a*/*d*) = 0.1], as the inclination angle increases, SIF of mode-I decreases and conversely mode-III SIF increases. For short cracks in crack-2, as the inclination angle increases, the mode-I SIF decreases similar to crack-1, but the SIF of mode-III decreases as well. The mode-III contributes to a significant amount of SIF owing to the inclination of the crack. Regardless of the inclination angle, with an increase in the aspect ratio [*a*/*c*], the mode-I and mode-III SIF for both crack-1 and crack-2 decreases.

Keywords Multiple cracks · Stress intensity factor · Mixed mode · Crack inclination · Crack aspect ratio · Crack depth ratio

Introduction

Cracks initiate on cylindrical bars due to various reasons like the effects of machining processes and load acting on them. Two symmetric elliptical-arc cracks in a round bar under constant axial loading with constant amplitude are considered. In general, surface cracks in structural components are approximated by elliptical, semi-elliptical cracks, circular or semicircular cracks. In order to accurately predict safe life of such cracked components, it is important to know the mode-I, II and mode-III stress intensity factors (SIF). Many researchers have estimated the SIF for round bars under various loading conditions, and the results are limited either to a single crack or to mode-I. Moreover, the effect of crack interaction caused by multiple cracks has been neglected. In the present work, an attempt has been made to determine the effect of crack inclination, crack depth ratio (*a*/*d*; '*d*' diameter of the bar) and aspect ratio (*a*/*c*) on stress intensity factor. Many researchers have made an attempt by experimental and numerical methods to determine the effect of mixed-mode fractures and multiple cracks. Vaziri and Nayeb-Hashemi [1] investigated the effects of mode-III fracture in circular shafts subjected to torsion by considering the crack surface in a form of a factory roof, and fracture surface interaction in circumferentially cracked shafts could result in a significant reduction in the effective mode-III stress intensity factor. Nao-Aki Noda and Yasushi Takase [2] used the singular integral equation of the body force method to calculate the stress intensity factor of all the modes for a notched round bar under tension, bending and torsion and found that if the crack depth ratio is constant, the SIF values are small for a range of notch angles. Lebahn, Heyer and Sander [3] investigated the effect of cyclic tension and bending on round bars. They concluded that if the assumptions of the LEFM are fulfilled, the cracks propagate with elliptical crack geometry and furthermore the aspect ratio and interaction angle were almost independant on the stress ratio but dependant on the crack depth.

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Paluszny and Zimmerman [4] presented a method for growing crack surfaces using a set of stress intensity factor-dependant constraints. By using NURBS fracture surface, they explained their method which does not require fitting of the fracture tip, restricted node and edge locations in meshes which over-constrain the domain to be discretized. Kotousov et al. [5] investigated the coupled shear and anti-plane fracture modes which, according to the authors, are neglected and implicitly assumed to have less intensities compared with that of the primary modes (mode-I, mode-II and mode-III). The authors concluded that in the case of very thick or very thin plates, the intensity of coupled modes grows or decays as a power function of the plate thickness. Further, they proposed that the future work has to be directed to the experimental confirmation of the theoretical tendencies and effects. Ilhyun Kim et al. [6] investigated the effects of various misalignments of circular notched bar specimens on the fatigue crack propagation behavior of pipe grade polyethylene. The authors used finite element analysis and concluded that in the case of concentric misalignment, the asymmetric crack growth accelerated which reduced the time taken to reach the critical SIF. Similarly, in the case of angular misalignments, the time taken to reach the critical SIF decreased with an increase in the misalignments. Bovsunovsky [7] experimentally investigated steel R2 M cylindrical specimens with a surface transverse and slant crack at torsional vibrations. They concluded that the energy dissipation in the transverse non-propagating crack at torsional vibration is caused mainly by the plastic zone along the crack front. The level of energy dissipation was described as a function of equivalent SIF. Also the energy dissipation in surface slant cracks is more intensive than in transverse one. Carpinteri et al. [8] analyzed the surface cracks in notched round bars under cyclic tension and bending. They calculated the SIF values for different crack configurations and found that the notch effect on the SIF is significant for any size and shape. Further, they declared that fatigue life for a notched bar is shorter than that for an unnotched bar in the case of tension, while the opposite occurs in the case of bending. Tanaka [9] investigated the circumferentially notched bars of austenitic stainless steel, SUS316L and carbon steel, SGV410 under fatigued cyclic torsion with and without static tension and made number of conclusions. They stated that the fatigue life of circumferentially notched SUS316L steel was longer than that of smooth bars. The static tension on cyclic torsion of SUS316L reduced the retardation of crack propagation; however they do not seem to have any effect on SGV410. They also concluded that the path of small crack propagation near notch root was as a function of stress amplitude, notch acuity and material deformation characteristics. Toribio et al. [10] proposed a numerical procedure for calculating the crack shape evolution in semi-elliptical surface flaws in round bars under tensile loading with different aspect ratios and relative crack depths. The Paris-Erdogan law was used, and the effect of quasi-static, quasi-circular initial crack geometries and deep cracks has been discussed. It was concluded that, in the case of free sample ends and very deep cracks, the dimensionless stress intensity factor is higher than twice the corresponding one for constrained sample ends. Many researchers have studied the effect of crack depth ratio on fracture parameters for a single crack in a round bar. Available SIF solutions are limited to single crack subjected to mode-I loading condition, and the influence of mode-II and mode-III has been neglected. The main objectives of the present work are:

1. To investigate the effect of multiple cracks on mixed-mode SIF
2. To determine the influence of individual fracture modes on SIF
3. To understand the effect of crack depth (a/d) and aspect ratios (a/c) on mixed-mode SIF

In the present work, an attempt has been made to determine the effect of angle of inclination on elliptical cracked round bars. Stainless steel was considered as bar material. Two opposite cracks were considered. Both these cracks have any set of crack parameters mentioned below at a given time. Crack depth ratio of 0.1 and 0.4 and crack aspect ratio of 0.2 and 1.0 have been considered. Angle of inclinations of 10°, 20° and 30° was considered for the crack. The top surface is constrained, and far-field tensile load is applied at the bottom surface (Fig. 1).

Description of the Finite Element Model

SIF for opposite cracks in a round bar is estimated using ABAQUS finite element software. A 3-D FE model of the round bar of 40 mm length and 10 mm radius with steel material was considered (Fig. 2). Multiple straight opposite surface cracks are introduced at the mid-region of the

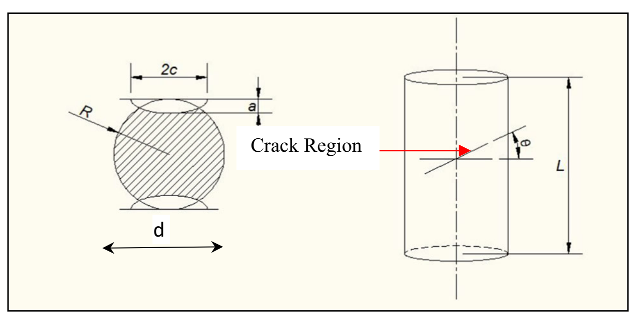
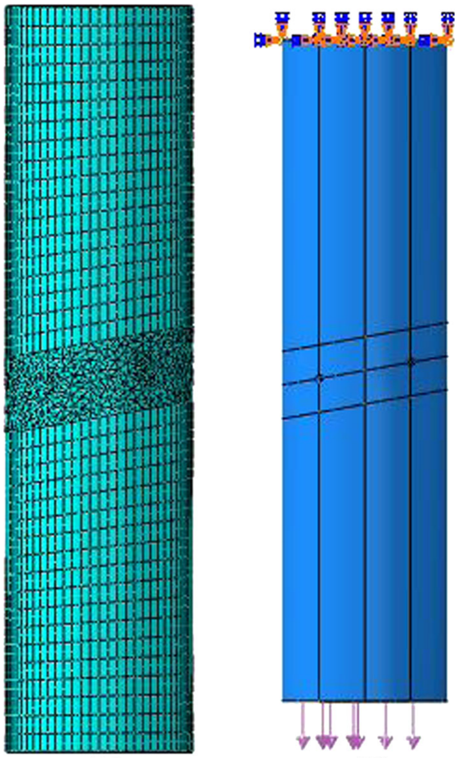


Fig. 1 Crack nomenclature. a—crack length (mm), c—semi-major axis of the ellipse (mm), L—length of the bar (mm), d—diameter of the bar (mm), θ—angle of inclination of the crack, (a/d)—crack depth ratio, (a/c)—crack aspect ratio



(A) (B)

Fig. 2 FE model of the round bar

Table 1 Mechanical properties of the round bar

Object	Material	Young's modulus (GPa)	Poisson's ratio	Yield strength (MPa)
Round bar	Stainless steel (AISI 304)	200	0.29	215

round bar in such a way that crack plane is inclined to the loading direction to facilitate mixed-mode fracture. Mixed-mode fracture is introduced in the bar due to uneven crack opening. The top face of the bar is fixed completely arresting all possible motions. Far-field tensile load was applied to the bottom face to determine SIF along the crack front. Table 1 describes the mechanical properties considered for the material.

Crack Modeling on Unnotched Round Bar

Meshed 3-D model of the round bar is shown in Fig. 2A. Two surface cracks were introduced at the center of the bar opposite to each other. The region around the crack front was partitioned from the remaining portion to apply fine mesh around it. Semi-minor and major axes '*a*' and '*c*' correspond to the crack depth and transverse length of the crack, and θ corresponds to the angle of inclination of the crack plane.

To simulate the theoretical inverse square root singularity of stresses and strains near the crack front region, singular elements were arranged around the crack front. If all the midface nodes of the brick elements are moved to their quarter points closest to the crack line, the variations in the local stress and strain fields can be reduced. Due to 3-D nature of the crack advancement, crack propagation direction cannot be predicted and hence 'crack plane normal' approach was used to define the crack propagation direction.

Estimation of Mixed-Mode SIF

When the round bar is mechanically loaded, the cracks may simultaneously open and slide relative to each other. The mixed-mode fracture is formed in a bolted joint due to complex loading condition or crack location. When the load reaches a critical value, the crack starts to grow and usually kinks into a new direction. The different modes of fracture for a growing crack are mode-I, mode-II and mode-III. In mode-I fracture, the crack surfaces separate directly apart from each other and therefore it is designated as opening-mode fracture. Mode-II fracture causes the crack surfaces to slide over one another perpendicular to the leading edge of the crack and designated as edge-sliding mode. In mode-III fracture, the crack surfaces slide with respect to each other parallel to the leading edge of the crack and therefore are designated as tearing mode.

In the present work, mixed-mode SIF is calculated for opposite cracks located in an unnotched round bar using FEM. Transverse cracks are introduced at the centerline of the bar opposite to each other. The cracks in the bar experience mixed-mode fracture due to the inclination of the crack plane to the applied load. Far-field tensile load is applied to the bottom face which causes the cracks to open, and SIF values are calculated along the crack front. Currently, the geometric correction factor (Y) is calculated from mixed-mode fracture which includes the additional effect of mode-II and mode-III fracture. The mixed-mode SIF is calculated from the following relation.

$$K_{mix}^2 = K_I^2 + K_{II}^2 + \frac{K_{III}^2}{1 - \vartheta} \quad \text{(Eq 1)}$$

where K_I, K_{II} and K_{III} are mode-I, mode-II and mode-III stress intensity factors, respectively.

The geometry correction factor (*Y*) under mixed-mode condition is determined from the following equation.

$$K_{mix} = Y\sigma\sqrt{\pi a}$$
$$Y = \frac{K_{mix}}{\sigma\sqrt{\pi a}} \quad \text{(Eq 2)}$$

where Y—geometry correction factor, a—crack length (mm) and σ—far-field stress (MPa).

The fatigue crack growth rate and residual life of the bar can be calculated from Paris' equation

Results and Discussion

The present work brings out the influence of inclination of crack planes on SIF of a round bar subjected to far-field loading. Cracked round bar with different crack depth (a/d) and aspect ratios (a/c) was modeled and analyzed. Multiple-crack SIF was compared under different inclinations of the crack plane. Normalized coordinate system was used to represent the points along the crack front.

Effect of Crack Plane Inclination Angle on Short-Crack [(a/d) = 0.1] SIF

Figure 3A and B shows the effect of crack plane inclination angle on SIF of short cracks in a round bar under far-field tensile load. The crack plane angle is measured between the crack plane and the plane normal to the tensile load applied.

In both the cracks, dominant mode of failure is mode-I and the influence decreases as the angle is increased. Symmetric SIF is observed along the crack front. This is essentially so because the increased inclination facilitates more room for mode-III failure compared to mode-I failure. This can be readily observed from the mode-III plots

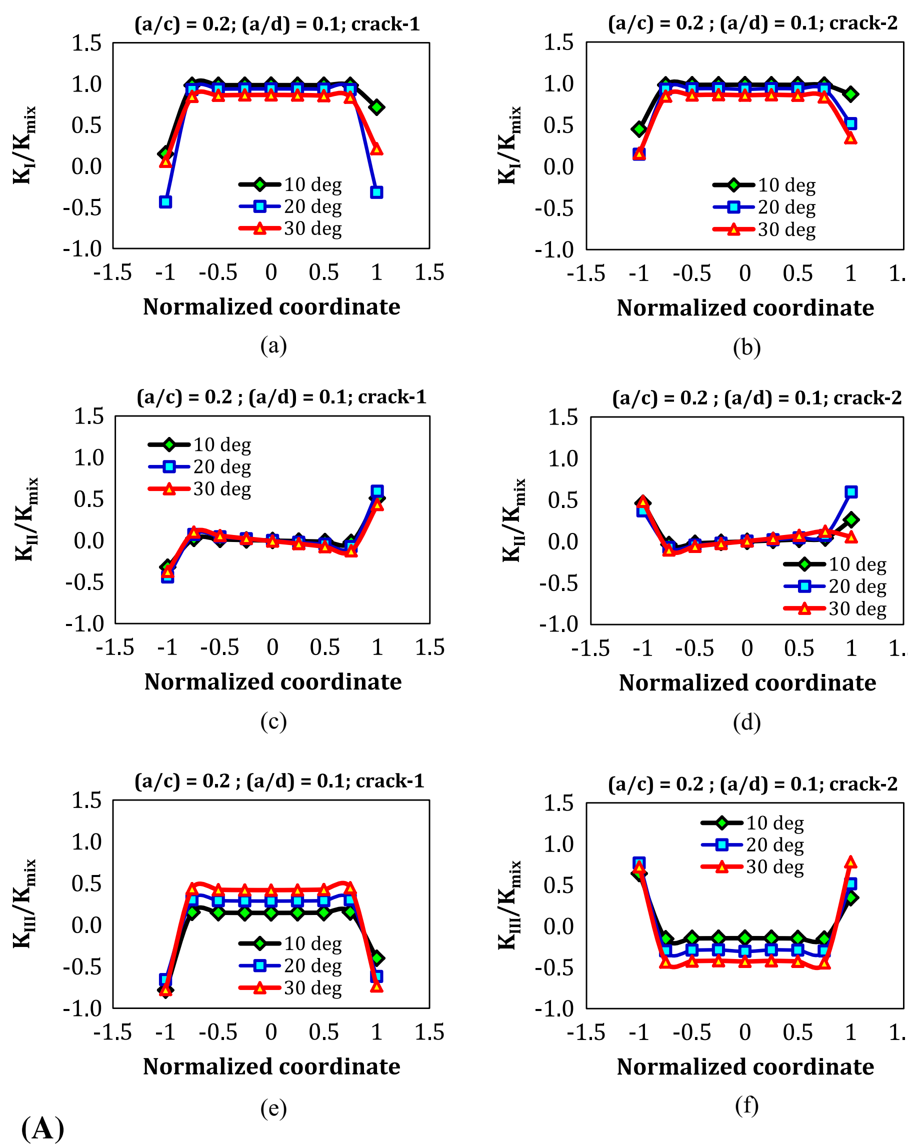
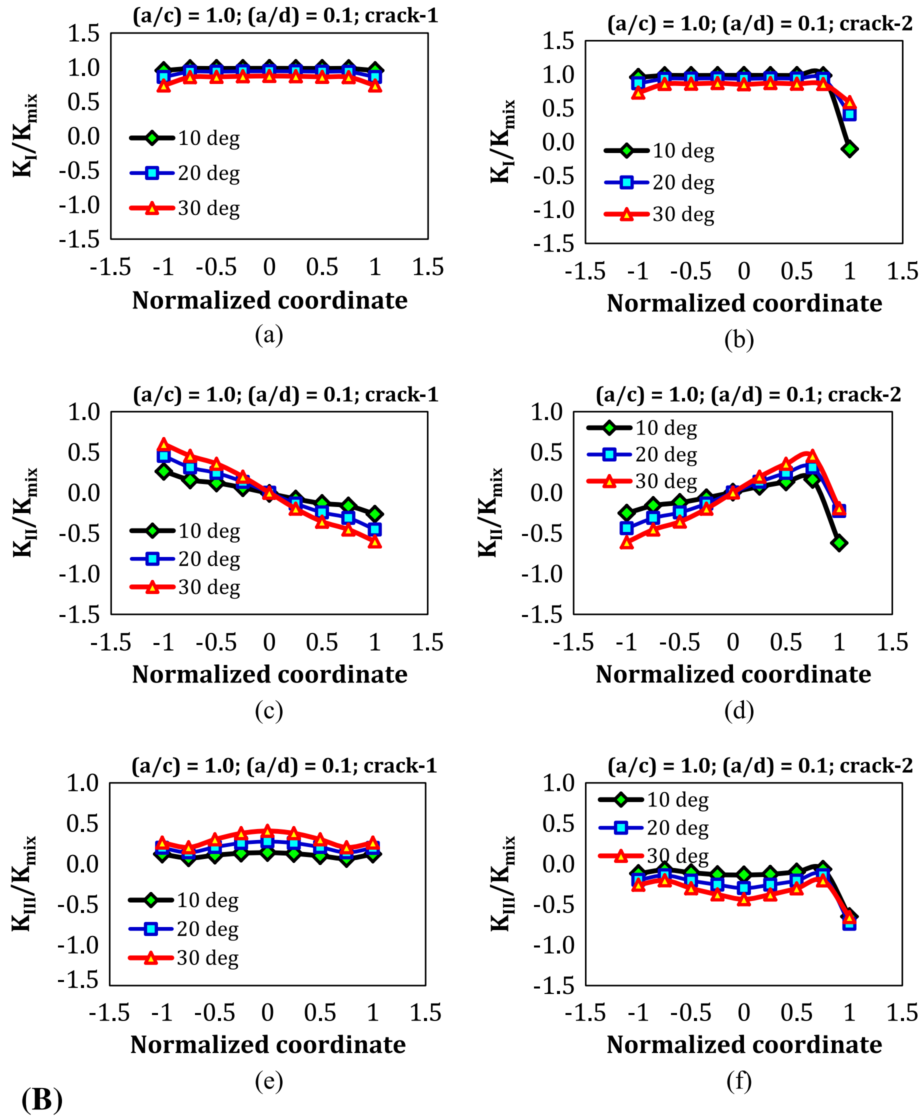


Fig. 3 (**A**) Effect of inclination angle on SIF—short cracks [(a/d) = 0.1)] of elliptical profile [(a/c) = 0.2]. (**B**) Effect of inclination angle on SIF—short cracks [(a/d) = 0.1)] of circular profile [(a/c) = 1.0]

Fig. 3 continued



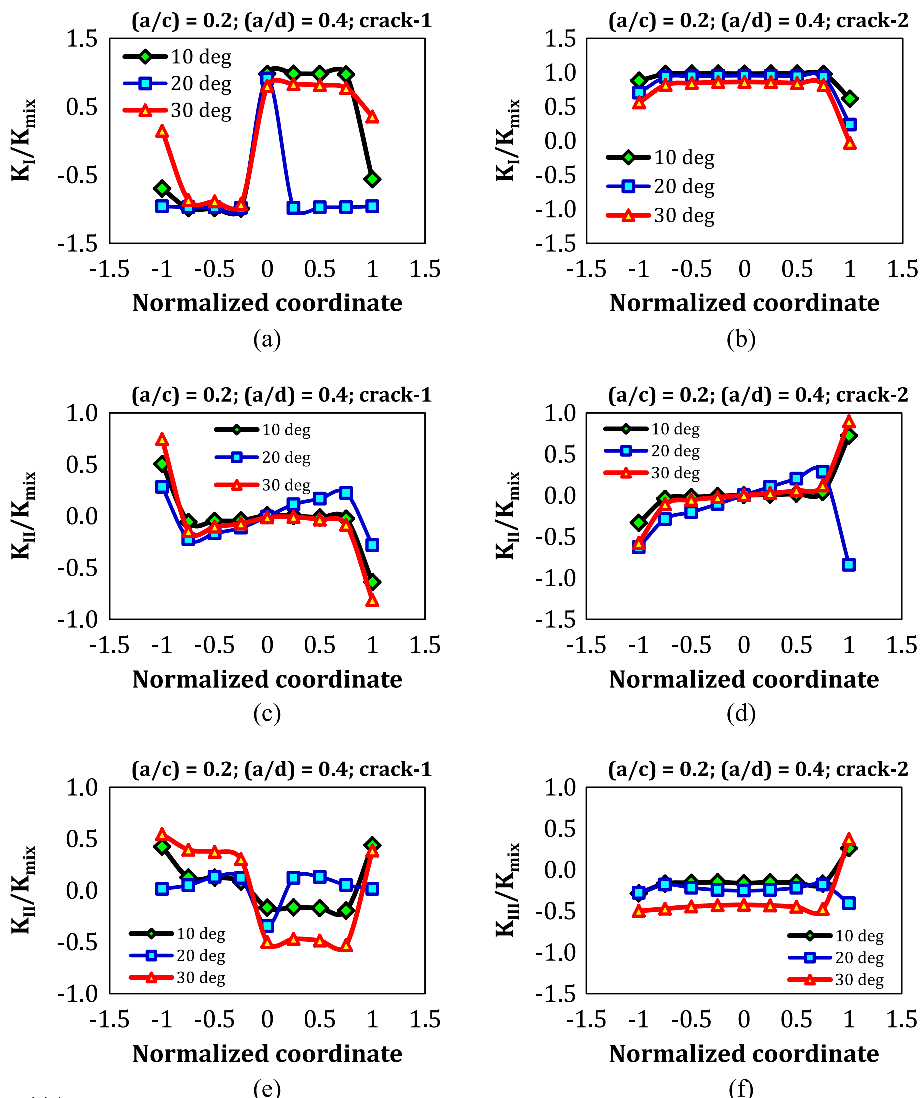
shown above. As the inclination increases, mode-III failure increases, but on the contrary in crack-2, the SIF decreases. This is due to the fact that the crack-2 is opposite in direction to the crack-1, and hence, the influence decreases as the inclination increases. SIF of short cracks [(*a*/*d*) = 0.1] located at the front and back side of the bar is compared and shows a variation in the distribution under mode-III. This is primarily due to the inclination being opposite to one other.

Figure 3B shows the effect of inclination angle on short cracks [(*a*/*d*) = 0.1] of circular profile [(*a*/*c*) = 1.0] with far-field tensile load. The plots look generally similar to that of elliptical cracks [(*a*/*c*) = 0.2]. Hence, it can be inferred that compared with elliptical cracks, the crack profile does not have much influence in altering the distribution of SIF.

Effect of Crack Plane Inclination Angle on Deep-Crack [(*a*/*d*) = 0.4] SIF

Figure 4A and B shows the effect of crack plane inclination angle on SIF of deep cracks in a round bar under far-field tensile load. SIF of deep cracks decreases from crack surface region and increases toward the center of the crack front. It may be due to the mode-I being predominant toward the center of the crack front. SIF values of deep cracks are found to be lower than that of the short cracks.

Variation of deep crack [(*a*/*d*) = 0.4] of elliptical profile [(*a*/*c*) = 0.2] SIF at the crack-1 and crack-2 of the bar is shown in Fig. 4A. It can be inferred that in crack-1, SIF values are less than in crack-2. The mode-I failure is found to be predominant owing to the tensile load, and a



(A)

Fig. 4 (**A**) Effect of inclination angle on SIF—deep cracks [(*a*/*d*) = 0.4)] of elliptical profile [(*a*/*c*) = 0.2]. (**B**) Effect of inclination angle on SIF—deep cracks [(*a*/*d*) = 0.4)] of circular profile [(*a*/*c*) = 1.0]

significant difference in values of SIF due to mode-III is also observed owing to the inclination of the crack plane.

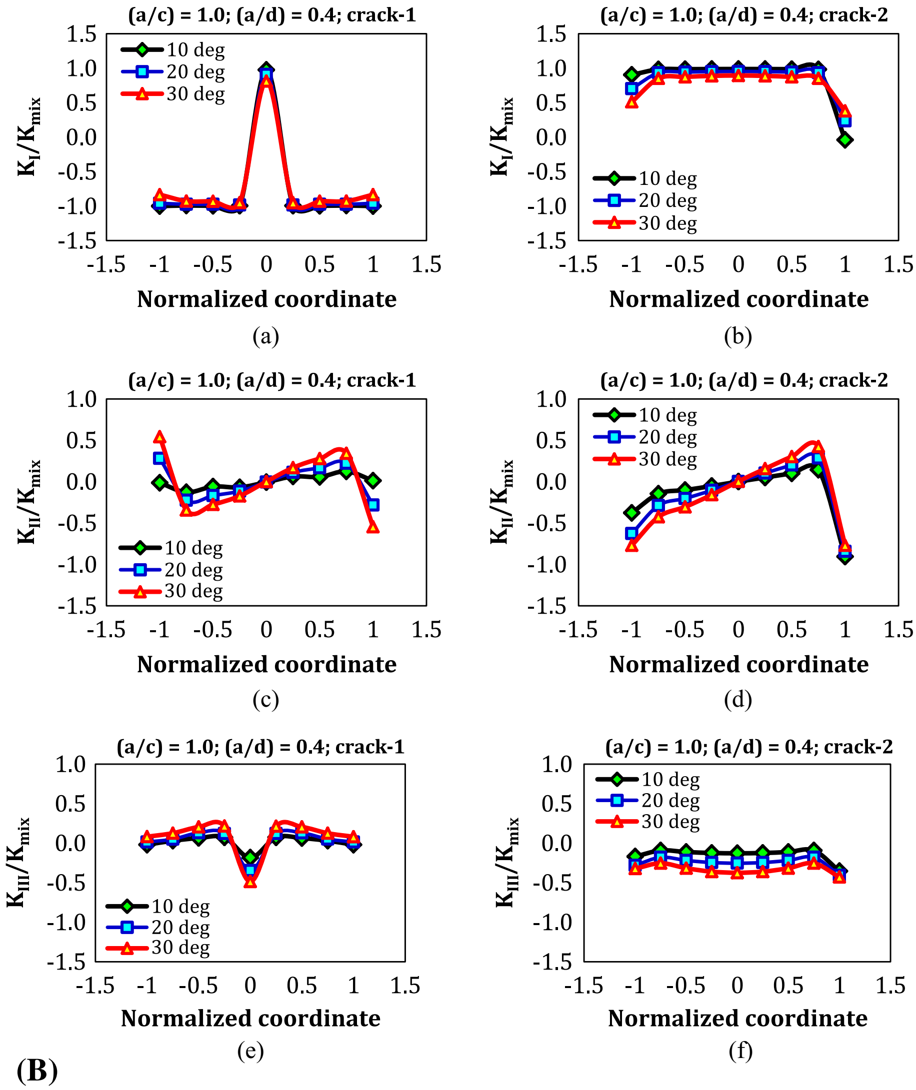
Figure 3B shows the effect of inclination angle on long cracks [(*a*/*d*) = 0.4] of circular profile [(*a*/*c*) = 1.0] with far-field tensile load. The SIF values of circular profile cracks are found to be higher than the ones of elliptical profile [(*a*/*c*) = 0.2]. It can be observed that as the crack depth increases, the SIF increases.

Conclusions

Mixed-mode SIF of multiple cracks in an unnotched round bar has been estimated numerically for various inclination angles of crack planes, and the following conclusions are obtained.

1. The mode-I SIF for the cracks with elliptical profile was observed to be lower than that for the cracks with circular profile. On contrary, mode-III SIF for elliptical cracks is larger than for circular cracks.
2. Compared to short cracks, mode-I SIF for long cracks is lower due to gradual relieving of internal stresses as the crack depth ratio increases.
3. Mode-II and mode-III SIF cannot be neglected while calculating the correction factor for short cracks as their influence is significant.
4. As the crack plane inclination increases, SIF for short cracks reduces considerably. It may be due to higher compressive stress which might have formed during the extrusion process.
5. Mode-I SIF of crack-1 is almost similar and equal to that of crack-2 owing to the nature of the load

Fig. 4 continued



applied—a far-field tensile load. Mode-III SIF of crack-1 is almost equal but opposite in direction to that of crack-2. It may be due to the direction of one crack being opposite to the other.

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