

# Simulation of Sidewall Failure in Coal Mine Roadways Using an Extended Continuous Joint Model

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**Abstract** Coal is well developed with beddings, and the beddings play an important role in the stability of tunnel stability of coal mine. This study focused on jointed rock mass with two sets of joint and proposed an extended ubiquitous model and associated numerical implementation accounting for joint spacing to represent geometric features of coal. The failure criteria of two sets of joint proposed in this paper not only accounted for joint spacing, but also contained the ubiquitous joint model. This extended theory was validated against the discontinuous deformation analysis method, the plane of weakness theory and experimental results for jointed coal with different joint fabric. The model is also used to analyze tunnel stability of roadways in coal mine with different joint spacing. The ubiquitous joint model could simulate jointed coal with small joint spacing, but for that with larger joint spacing, the ubiquitous joint model is not applicable and the joint model containing joint

spacing proposed by this paper should be used. A series of comparisons demonstrated that the proposed model was capable of considering the influence of joint spacing on jointed coal tunnel with two sets of joint.

**Keywords** Coal mine · Joint spacing · Ubiquitous joint model · Jointed coal tunnel · Tunnel stability

## Introduction

The stability of a jointed coal tunnel, as geological material containing several sets of discontinuities (Fig. 1), has a close relationship with joint configuration (that is, joint orientation and spacing). Joint spacing plays an important role in determining the strength and deformation of a coal tunnel. If the joint spacing is too small, collapse may happen (Fig. 2).

Several methods have been developed to evaluate the influence of joint spacing on the stability of jointed tunnel. Empirical methods, in which joint spacing (joint frequency) is considered in the rock classify system, include RQD [10], Rock Mass Rating (RMR) [4], Q [3], geological strength index (GSI) [20, 19], and RMI systems [26]. The International Society for Rock Mechanics classifies joint spacing into four grades, specifically, 6, 20, 60, and 200 cm [5].

Laboratory testing has also been used to evaluate the influence of joint spacing, and various researchers such as Brown [6], Goldstein et al. [12], Singh et al. [32], Hayashi [14], Einstein and Hirschfeld [11], and Ramamurthy and Arora [28] have experimented on artificial specimens made by blocks.

Two groups of constitutive models are used to simulate joint spacing: discrete models and continuum models. The distinct element method [9] and block theory [30] fall into

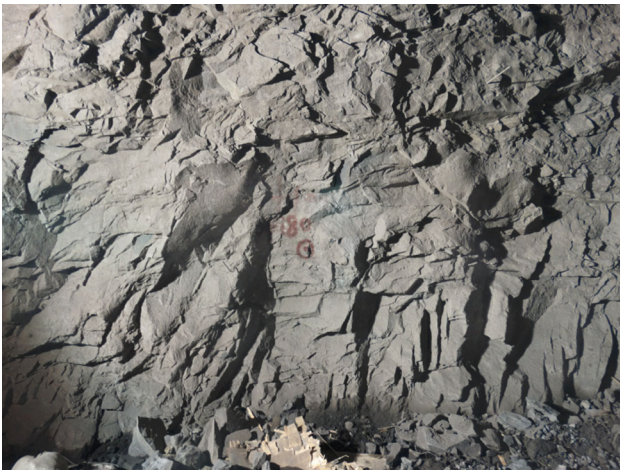
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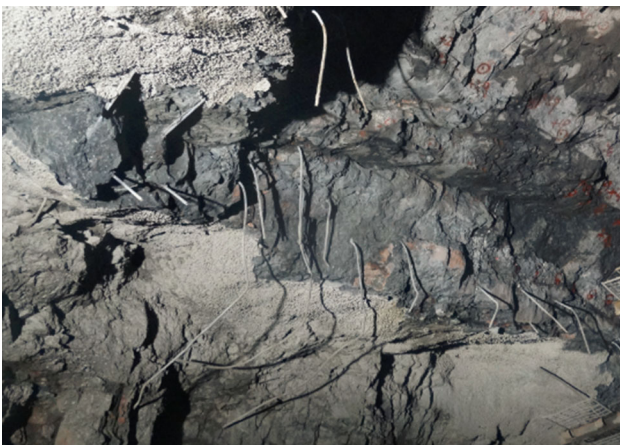
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**Fig. 1** Jointed rock mass distribution in tunnel



**Fig. 2** Collapse of jointed rock mass in a tunnel

the former category, while the joint element method [13] and the practical equivalent continuum models developed by Singh [31], Sitharam et al. [33], and Morland [25] belong to the latter group.

In the 1960s, Jaeger proposed the plane of weakness theory (PW), which suggests that the strength of an anisotropic rock is related to the orientation of the weakness planes [21, 22, 27]. Since then, many researchers have developed the model to make it more complete and practical. In 1988, the PW model was expanded by Amadei [1] to account for intermediate principle stress. In addition, the constitutive model of ubiquitous joints in the Fast Lagrangian Analysis of Continua (FLAC) was also based on this theory, as was the ubiquitous joint rock mass (UJRM) model [2, 8, 24, 29]. Many papers have focused on the influence of joint spacing on deformation behavior through modifying the PW model [17, 18, 20, 23, 31, 33–35]. Huang et al. [20] proposed a model in which the influence of joint spacing on elastic modulus was properly considered to predict the stress–strain curve for a mass. Wang and Huang [35]

established a model that determined the failure mode and simulated the complete pre- and post-deformation of rock mass with multi-sets of joints. In 2010, an extension of the plane of weakness theory accounting for nonlinear Barton–Bandis shear strength and rock bridges was proposed by Halakatevakis and Sofianos [15, 16]. However, none of these models consider the effect of joint spacing on failure criterion. As Halakatevakis stated: “...the extended theory of one plane of weakness does not take into account the spacing of the discontinuities...”.

From the above, joint spacing has obviously not been included in the failure criteria of existing models with continuum approaches. However, given the importance of joint spacing on the stability of rock masses, it cannot be neglected. To provide a more general model for jointed coal tunnel, it is essential to consider joint spacing. Since the jointed rock mass with one set of joint has been well studied, and that with three or more sets of joint could be viewed as isotropic rock mass, two sets of joint with strong anisotropic characteristics are focused in this paper.

In the present paper, the model proposed by Jaeger is extended by considering joint spacing to solve actual engineering problems of jointed coal tunnel. In the second section of this paper, the joint spacing theory is validated against discontinuous deformation analysis method (DDA). The third section implemented the proposed theory into finite difference methods for engineering purposes. The model is validated against the plane of weakness theory (PW) and experimental results for jointed rock masses with different joint fabric. Finally, a jointed coal tunnel with different joint spacing is analyzed with the extended model to examine performance, especially the influence of joint spacing on tunnel stability of jointed coal tunnel. The obtained results were compared with in situ test data.

## Verification the Theory of Joint Spacing by the DDA Method

### Verification of the Magnitude of Joint Spacing

A two-dimensional section of unit thickness perpendicular to the tunnel axis shown in Fig. 3 was analyzed. The height and width of the model were 200 m. The applied geo-stress was 18.2–20.6 MPa. The tunnel had a 10-m-wide flat invert, 9-m-high vertical side walls, and a 6.2-m-diameter semicircular roof. The joint sets were nearly orthogonal, with dip angles of 0° and 90°. The excavation was removed at once in the simulations.

The input data for the rocks and joints are listed in Table 1. To determine the effect of joint spacing on the stability of jointed rock mass, the same joint mechanical parameters are adopted to every joint set.

The three numerical models in Fig. 4 differed only in the joint spacing magnitude, that is, 1, 2, and 8 m. The relationship between joint spacing magnitude and tunnel stability is shown in Fig. 4. With the increase in joint spacing magnitude, the tunnel is more stability, which is also obtained by Shi [30] and Ramamurthy and Arora [28].

Verification of Joint Spacing Ratio

The joint spacing ratio also influences the tunnel stability for two sets of joint. The three numerical models in Fig. 5 differed only in the joint spacing ratios, that is, 0.5, 1, and 2. Under the same joint spacing magnitude (block area), the relationship between joint spacing ratio and tunnel stability

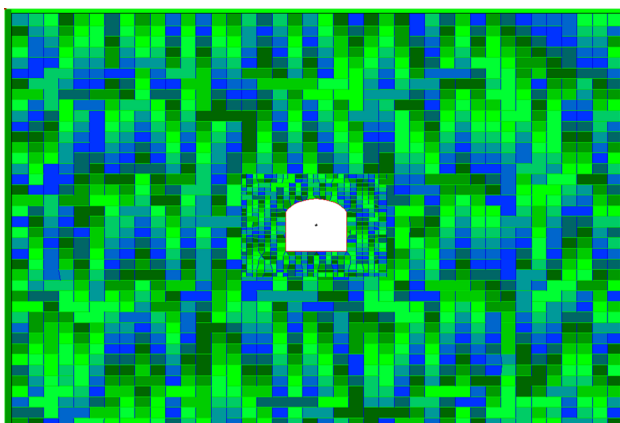


Fig. 3 Numerical simulation mesh model

is shown in Fig. 5. With the increase in joint spacing ratio, the tunnel is more stability, which is not noticed before.

Complement and Verification of Extended Ubiquitous Model

The stress–strain relationship of the extended model is established through the theory of the second section. Additionally, a new failure criterion is added in of the extended model, to replace that of the ubiquitous model for two sets of joint.

Prior to failure, the existing stress  $\sigma$  is updated by adding  $d\sigma$  and the updated stress state is checked to determine whether it reached the failure criterion of intact rock and joints. The extended model accounting for joint spacing was coded in C++ and implemented using the FLAC UDM option with a dynamic-link library file.

The extended model is adopted to analyze the effect of joint spacing against the plane of weakness theory and experiments to verify its correctness.

Verification Against the Plane of Weakness Theory (PW)

Single Joint

According to the Jaeger theory, the variation in rock mass anisotropy strength with joint dip is U-shaped for single joints. If the joint spacing in the model proposed in this

Table 1 Mechanical parameters of rock and joints

Materials	Density (g/cm <sup>3</sup> )	Elastic modulus (GPa)	Poisson's ratio	Internal frictional angle (°)	Cohesion (MPa)	Tension strength (MPa)
Rock	2.65	20	0.25			
Joint set 1#				26.2	0.260	0.2
Joint set 2#				26.2	0.260	0.2

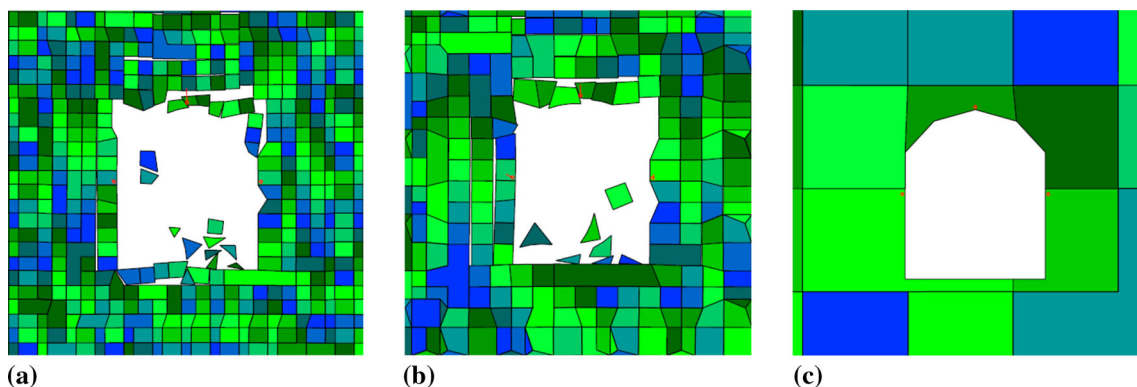
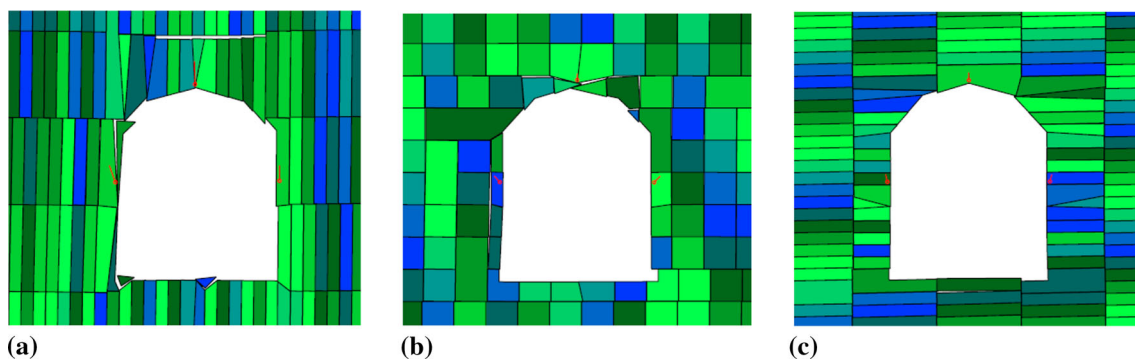


Fig. 4 Tunnel stability with different joint spacing magnitudes (a) 1 m, (b) 2 m and (c) 8 m



**Fig. 5** Tunnel stability with different joint spacing ratios (a) 0.5, (b) 1, and (c) 2

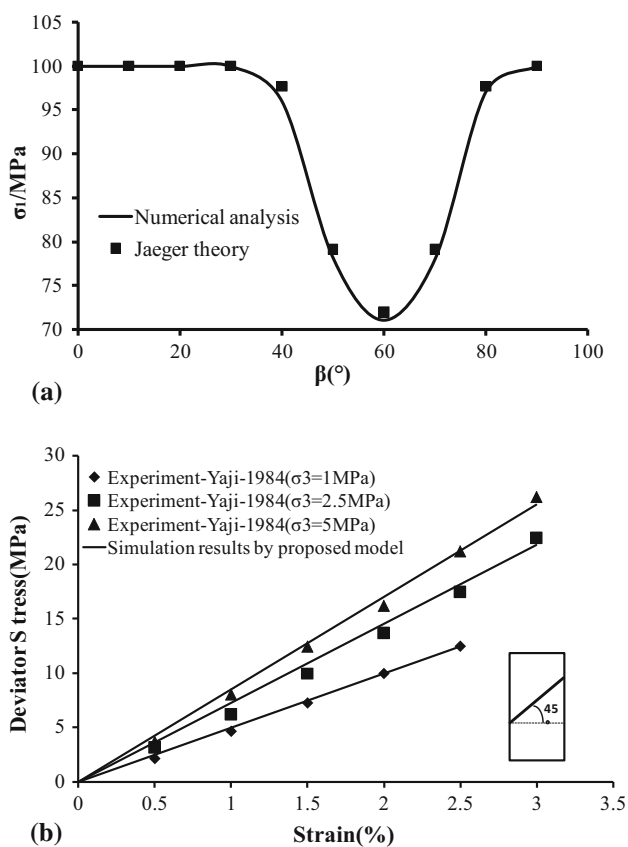
paper was set as 0, the model could be degenerated as a ubiquitous joint model. The present model is used to analyze the variation law of anisotropy strength with joint dip and was compared with the Jaeger theory to verify the correctness of the model. The results are shown in Fig. 6a, which demonstrates that the numerical simulation results were consistent with the theoretical analyses of Jaeger.

To verify the correctness of the model, we also compared the model with laboratory tests conducted by Yaji [36]. The single joint experiments conducted by Yaji produced a pre-deformation stress–strain curve (Fig. 6b). As shown, the obtained results had a good consistency with the experimental results.

### Two Sets of Joint

With the Jaeger theory, every joint is considered independently for two sets of joints and rock mass strength is minimum value of all the joint strength. According to the Jaeger theory, when a rock mass contains two sets of joint planes, the variation of rock mass anisotropy strength with joint dip exhibits a wavy shape. The rock mass strengths calculated by the model proposed in this paper also displayed a wavy shape, but the strengths were greater than the values obtained by the Jaeger theory as shown in Fig. 7a. This is because the Jaeger theory considers every joint as ubiquitous and rock mass strength is minimum value of all the joint strength, while the model containing joint spacing considers two sets of joints together and rock mass strength is affected by all joints. Thus, the value calculated by the Jaeger theory is less than the model proposed by this paper.

We also compared the model with laboratory tests conducted by Brown and Trollope [7]. The experiments with two sets of joint conducted by Brown produced a complete stress–strain curve (Fig. 7b). As shown, the obtained results had a good consistency with the experimental result.

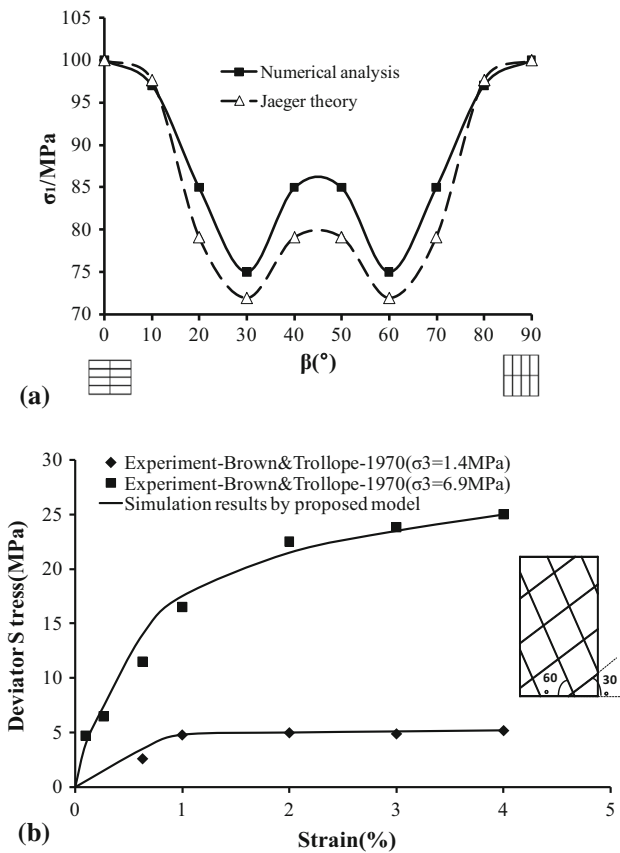


**Fig. 6** Comparison of strength, stress–strain curves for numerical simulation results by proposed model and Jaeger theory as well as experiment results for single joint. (a) Comparison of Jaeger theory and numerical model proposed by this paper, single joint. (b) Comparison of stress–strain curves by proposed model and experiments, single joint

### Case Study

#### General Project Information

The coal mine chosen as the research area is the NO. 1 underground coal mine in the Pingshuo mine region of China Coal Group, Shuozhou, Shanxi Province. The first



**Fig. 7** Comparison of stress–strain curves for numerical simulation by proposed model and experiments for two sets of joint. (a) Comparison of Jaeger theory and numerical model proposed by this paper, two sets of joint (b) Comparison of stress–strain curves by proposed model and experiments, two sets of joint

and fifth working faces are 4106 and, 4110 respectively, both of drive with wind tunnels, as shown in Fig. 8. The wind tunnel is distributed in jointed rock mass of two sets of joint, however, with different joint spacing, as shown in Fig. 8. In 4106 working face, the joint spacing is 0.4 and 0.8 m, and in 4110 working face, it is 0.1 and 0.5 m.

**Field Monitoring**

The acoustic instrument was adopted in this monitoring test. At the test site, a single borehole transducer was placed on the bottom of a borehole and a bottom-up approach was used.

The depths of the excavation damaged zone of the wind tunnel excavated from a jointed rock mass in 4106 and 4110 working face were monitored. The monitoring results are shown in Table 2.

The excavation damaged zone of the wind tunnel in 4106 working face is smaller than that of 4110 working face, although both of them are composed of slate, which have same mechanical properties through triaxial

compression test. It indicates that the tunnel stability of 4110 working face is better than those of 4106, which may be induced by the different joint spacing of them. However, the ubiquitous joint model cannot consider the joint spacing. Therefore, it was necessary to adopt the joint model containing joint spacing to analyze the stability of these two tunnels.

**Model Setup**

Based on the extended model, the stability of the jointed rock mass in two wind tunnels is analyzed, in particular the influence of joint spacing.

The geometry of the two-dimensional numerical simulation model is as the same as shown in Fig. 8. The joint sets in 4106 working face were nearly orthogonal, with dip angles of  $18^\circ$  and  $68^\circ$ , and that in 4110 working face are  $5^\circ$  and  $82^\circ$ . As the depth of the excavation damaged zone is affected by the grid size, mesh refinement within two times of tunnel diameter used for raising precision and speed of operation is adopted in this simulation. 1 m grid size, which is much smaller than the depth of the excavation damaged zone, is used.

**Parameter Values**

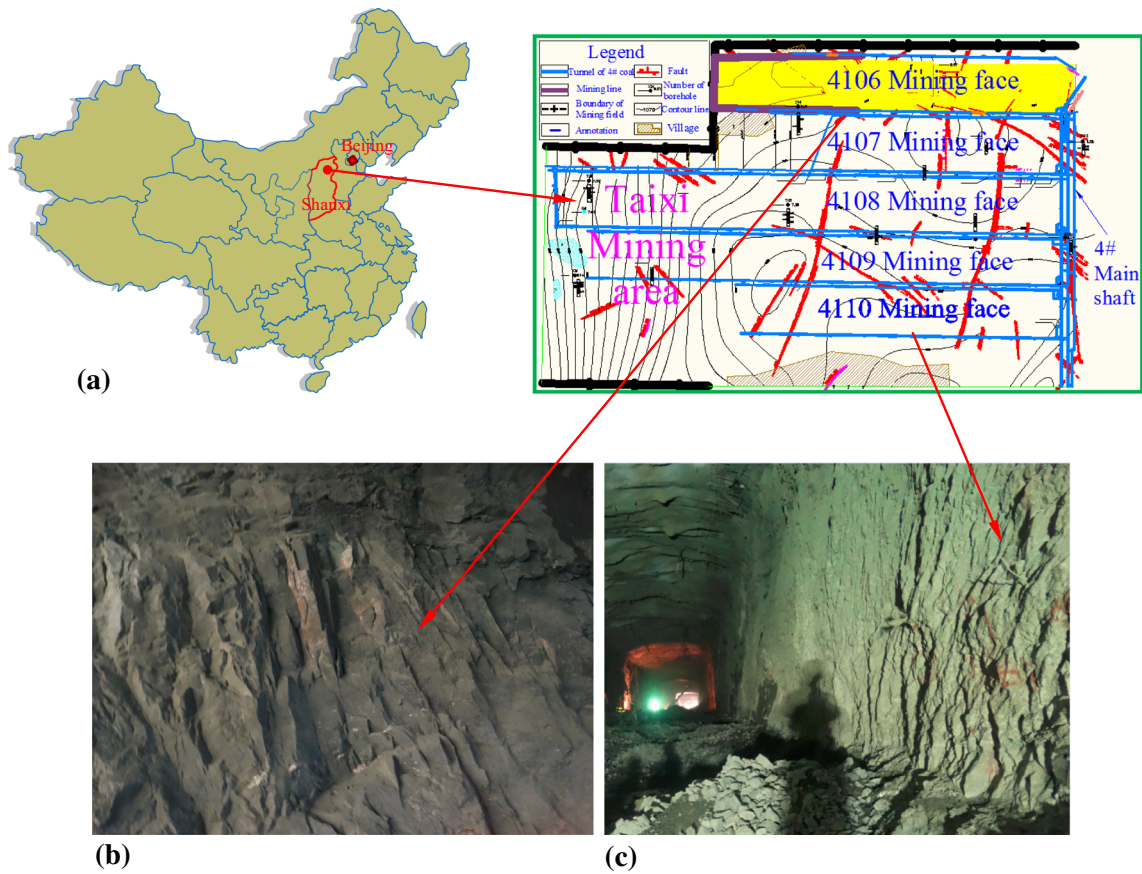
Some parameters, such as shear modulus, were obtained by in situ testing, and some, such as cohesion and friction angle of intact rock, were acquired through library testing. The obtained parameter values are shown in Table 3.

**Simulation Results**

The predicted depths of the excavation damaged zone by the extended model with different joint spacing were close to the corresponding measurements (Fig. 9), which indicated that the extended model containing joint spacing was capable of simulating rock mass with joint spacing.

**Discussion**

The variation of the depth of the excavation damaged zone surrounding a tunnel with different joint spacing and same joint mechanic properties is shown in Fig. 10. For a rock mass containing no joint set, the depth of the EDZ is smallest with best tunnel stability. The wind tunnel of 4106 working face with larger joint spacing has a smaller depth of EDZ, resulting in better tunnel stability, followed by the wind tunnel of 4110 working face, which exhibits the worse tunnel stability and larger depth of EDZ. And the ubiquitous joint model exhibits the poorest tunnel stability and the largest depth of EDZ.



**Fig. 8** Arrangement of wind tunnel of 4106 and 4110 working faces (a). Wind tunnel of jointed rock mass with two joint sets in (b) 4106 working face with joint spacing 0.4 and 0.8 m as well as (c) 4110 working face with joint spacing 0.1 and 0.5 m

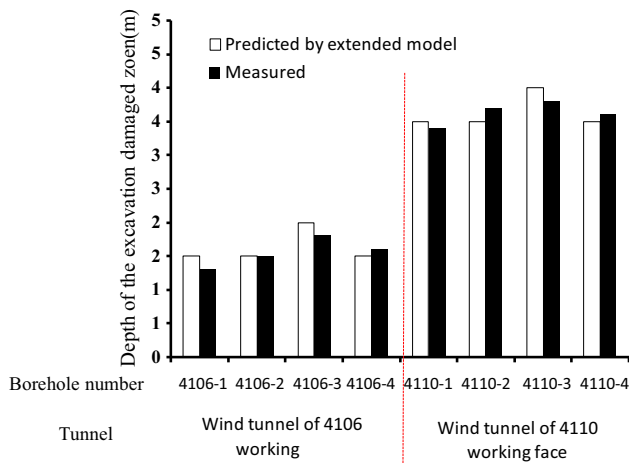
**Table 2** Monitoring results of the failure zone depth by acoustic waves

Number of boreholes	Depth of failure zone (m)	Height from tunnel bottom (m)	Location	Part of tunnel	Test date	Type of rock mass
4106-1	1.3	2	Wind tunnel of 4106 working face	Sidewall	2013.12.3	Jointed rock mass with joint spacing 0.4, 0.8 m
4106-2	1.5	2				
4106-3	1.8	5				
4106-4	1.6	5				
4110-1	3.4	2	Wind tunnel of 4110 working face	Sidewall	2015.3.10	Jointed rock mass with joint spacing 0.1, 0.5 m
4110-2	3.7	2				
4110-3	3.8	5				
4110-4	3.6	5				

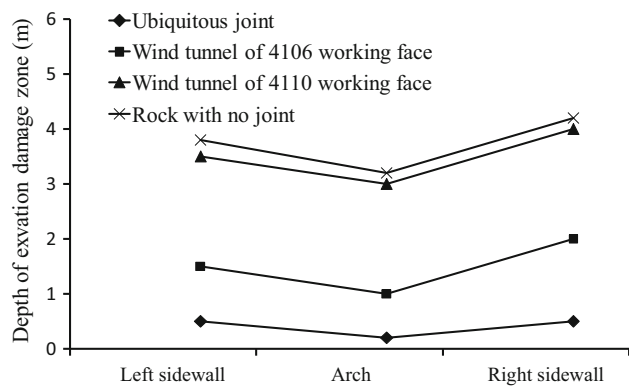
**Table 3** Mechanical parameters of rocks and joints

	1st joint set	2nd joint set	Intact rock
Internal friction angle (°)	27.3	24.5	40
Cohesion (MPa)	0.26	1.19	5
Tensile strength (MPa)	0.2	0.5	2
Elasticity modulus (GPa)	...	...	10
Poisson ratio	...	...	0.26

As the ubiquitous joint model of FLAC does not consider joint spacing, it can be applied when joints are so well developed that the joint spacing can be assumed to be zero, such as the wind tunnel of 4110 working face shown in Fig. 10. However, if the joint spacing is larger, the ubiquitous joint model is no longer applicable, such as for the wind tunnel of 4106 working face shown in Fig. 10.



**Fig. 9** Comparison of the depths of excavation damaged zone measured by in situ testing and simulated results by the extended model



**Fig. 10** Comparison of the depth of excavation damaged zones surrounding a wind tunnel for rock masses containing different types of joints

However, how small this spacing remains unclear for now. The simulation results of Figs. 7 and 10 provide an answer to this problem. With joint spacing 0.1 and 0.2 m, the stress–strain curve is nearly the same as ubiquitous joint, which indicated that if the joint spacing is smaller than 0.2 m, the ubiquitous joint model could be used. For example, with joint spacing of 0.1 m, the wind tunnel of 4110 working face had nearly equal depth of EDZ compared with the ubiquitous joint model. Thus, the ubiquitous joint model could simulate the wind tunnel of 4110 working face. But for the wind tunnel of 4106 working face with larger joint spacing, which had much smaller depth of EDZ, the ubiquitous joint model was not applicable.

If joint spacing is infinitively small, the extended model can be degenerated into the ubiquitous joint model. The extended model proposed in this paper is more extensive and broadly applicable because it not only accounts for joint spacing, but also contains the ubiquitous joint model.

The extended model proposed in this paper is deduced by analyzing the stability of a block with actual joint spacing, rather than a conventional element in the finite element method. Therefore, it is more suited for the heavily jointed rock mass.

This paper was based on the failure criteria proposed by Jaeger, and the contribution of intermediate stress was not considered in the extended model. Amadei [1] proposed, however, because a jointed rock mass consists of both intact rock and joints, its strength should always depend on all three applied stress components. Thus, the influence of intermediate stress on the strength of jointed rock masses should be taken into account in future research.

Not only that, only two sets of joint are analyzed in this paper. However, how three or more sets of joint considered are need to solve in the future. Moreover, the developmental of joints also can affect their mechanical properties. This paper is mainly to determine the effect of joint spacing on stability of jointed coal mine; therefore, same mechanical parameters are adopted in this paper. The effect of joint spacing on mechanical properties should be considered in future work.

### Conclusions

The ubiquitous model is established for a single joint set, in which joint spacing is not included. Unless the joint spacing is infinitesimally small, the failure criteria could be used.

Since joint spacing is not included in the failure criteria of existing models with continuum approaches, this paper proposed an extended ubiquitous model and associated numerical implementation accounting for joint spacing for two sets of joint to represent coal tunnel geometric features.

This extended theory was validated against the discrete element method, the plane of weakness theory and experimental results for jointed rock masses with different joint fabric. For single joints, the numerical simulation results were consistent with the theoretical analyses of Jaeger. However, for two sets of joints, the rock mass strength calculated with the model proposed by this paper was greater than that obtained by the Jaeger theory. Because Jaegers theory did not consider joint spacing and the influence of multiple joints was handled independently, the corresponding rock mass strength was the minimum value of multiple sets of joints strength, which was lower than the actual strength.

The extended model is also adopted to analyze the effect of joint spacing on the stress–strain curve. The numerical simulation results showed that when joint spacing was less than 0.2 m, the ubiquitous joint model could be used,

which is consistent with the rock mass classification criteria proposed by Hoek.

Lastly, the extended model is used to analyze tunnel stability after excavation in a jointed rock masses with different joint spacing. The numerical results are consistent with the in situ test results. The results show that the ubiquitous joint model well simulates the wind tunnel of 4110 working face, but is not applicable for the wind tunnel of 4106 working face, which has larger joint spacing. The joint model containing joint spacing for two sets of joint proposed in this paper could be used for the wind tunnel of 4106 working face.

The extended model proposed in this paper is more extensive and broadly applicable as it not only accounts for joint spacing, but also contains the ubiquitous model.

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