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Addendum to: Uniform Elongation and the Stress-Strain Flow Curve of Steels Calculated from Hardness Using Empirical Correlations

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The original study by Pavlina and Van Tyne developed correlations of hardness and strength (yield and ultimate tensile strength) (Pavlina and Van Tyne in J Mater Eng Perform 17:888–893, 2008). As an extension to this original work, a later paper developed an empirical relationship between the hardness and uniform elongation of non-austenitic hypoeutectoid steels (Pavlina and Van Tyne in J Mater Eng Perform 23:2247–2254, 2014). The empirical hardness-elongation relationship was combined with the correlations in the original study to show how a single hardness test could predict a reasonable stress-strain flow curve for a steel. The current study provides tables of parameter values for four different hardening models, based on the hardness correlations that were developed in the two previous studies. The models are the two-parameter Holloman model and the three-parameter Ludwig, Swift, and Voce models. Although they are empirical, these parameters allow the flow behavior of steels to be reasonably well characterized, based on a single hardness value. These tables should only be used when limited material is available, or when insufficient data are available for the specific grade of steel needing characterization.

Keywords constitutive models, hardness testing, mechanical static, steel

1. Introduction

Over the years, empirical relationships have been developed to relate hardness to strength for different metals (Ref 1-3). The authors of the current study have developed empirical correlations to determine reasonable values of yield strength and ultimate tensile strength of steels, based on their hardness (Ref 1). These correlations were based on a large number of experimental studies, and encompassed over 150 data points from 21 different Advanced Steel Processing and Products Research Center theses.

Both of the strength correlations showed a linear dependence of strength on hardness. Yield strength, s_{ys}, is expressed as

$$s_{ys} = -90.7 + 2.876H, \quad \text{(Eq 1)}$$

where H is the diamond pyramid hardness, or Vickers hardness, of the steel. Similarly, the ultimate tensile strength, s_{uts}, is

$$s_{uts} = -99.8 + 3.734H. \quad \text{(Eq 2)}$$

In Eq 1 and 2, strength has units of MPa and hardness has units of kgf/mm^2.

In a second study, Pavlina and Van Tyne (Ref 4) developed an empirical correlation between the hardness and the uniform elongation of steel. The uniform plastic strain, $\varepsilon_{pl\text{-}u}$, is given by

$$\varepsilon_{pl-u} = 0.463 - 0.439\left(1 - e^{-0.00742H}\right). \quad \text{(Eq 3)}$$

These three correlations were combined to show how the parameters for the stress-strain flow curve of steel can be obtained from a single hardness measurement. The stress-strain behaviors originally examined include the Holloman, Ludwig, and Voce hardening models. Although the method to obtain these parameters has been previously presented, tabulated values for the material model parameters as a function of hardness were not given. The method developed in Ref 4 has also been applied to the Swift hardening model in the present paper.

The objective of the present paper is to provide easy look-up tables that engineers can use to determine the parameters for different types of constitutive flow behavior, based on a single hardness value. These tables may be of particular interest to the simulation community, as access to material behavior parameters may be limited or extensive mechanical testing may not be possible. In these cases, using the tabular values presented in the current study may allow for a reasonably accurate prediction of the stress-strain behavior.

2. Hardening Models

In order to determine the parameters of a simple hardening model for a plastic stress-strain flow curve of a material, at least two (and often three) of the material's mechanical property values must be known. Three properties that can be used in determining these parameters are the yield strength, the ultimate tensile strength, and the uniform plastic strain. Pavlina and Van Tyne (Ref 1, 4) have provided empirical correlations of these

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Table 1 Holloman hardening parameters

HV	n_H	K_H
130	0.191	640.7
135	0.185	664.9
140	0.179	688.7
145	0.174	712.1
150	0.168	735.1
155	0.163	757.7
160	0.158	780.0
165	0.153	801.9
170	0.148	823.6
175	0.144	844.9
180	0.139	866.0
185	0.135	886.8
190	0.131	907.4
195	0.127	927.7
200	0.124	947.9
205	0.120	967.8
210	0.116	987.6
215	0.113	1007.1
220	0.110	1026.5
225	0.107	1045.8
230	0.104	1064.9
235	0.101	1083.9
240	0.098	1102.8
245	0.095	1121.6
250	0.093	1140.3
255	0.090	1158.8
260	0.088	1177.3
265	0.085	1195.8
270	0.083	1214.1
275	0.081	1232.4
280	0.079	1250.6
285	0.077	1268.8
290	0.075	1287.0
295	0.073	1305.1
300	0.071	1323.2
305	0.070	1341.2
310	0.068	1359.3
315	0.066	1377.3
320	0.065	1395.3
325	0.063	1413.3
330	0.062	1431.3
335	0.061	1449.3
340	0.059	1467.3
345	0.058	1485.3
350	0.057	1503.3
355	0.056	1521.3
360	0.054	1539.4
365	0.053	1557.4
370	0.052	1575.5
375	0.051	1593.6
380	0.050	1611.7
385	0.049	1629.8
390	0.048	1648.0
395	0.047	1666.2
400	0.047	1684.4
405	0.046	1702.7
410	0.045	1720.9
415	0.044	1739.3
420	0.043	1757.6
425	0.043	1776.0
430	0.042	1794.4
435	0.041	1812.9
440	0.041	1831.4
445	0.040	1849.9
450	0.040	1868.5
455	0.039	1887.1
460	0.038	1905.7
465	0.038	1924.4
470	0.037	1943.1
475	0.037	1961.9
480	0.036	1980.7
485	0.036	1999.5
490	0.036	2018.4
495	0.035	2037.3
500	0.035	2056.2
505	0.034	2075.2
510	0.034	2094.3
515	0.034	2113.3
520	0.033	2132.4
525	0.033	2151.6
530	0.033	2170.7
535	0.032	2189.9
540	0.032	2209.2
545	0.032	2228.5
550	0.031	2247.8
555	0.031	2267.2
560	0.031	2286.5
565	0.031	2306.0
570	0.030	2325.4
575	0.030	2344.9

three mechanical properties to hardness for a wide range of steels. These equations are given as Eq 1 to 3. A method to determine the parameters for three different hardening models has also been developed (Ref 4). The methodology can be found in the original paper. The goal of the present work is to use that methodology to provide a set of tables that can be used to determine the parameters for four different stress-strain hardening models from a single hardness test. The four hardening models are presented below.

2.1 Holloman Hardening

The Holloman power-law hardening model is given by

$$\sigma = K_H \varepsilon_{pl}^{n_H}, \qquad \text{(Eq 4)}$$

where σ is the true stress, ε_{pl} is the plastic strain component, K_H is the strength coefficient, and n_H is the strain hardening exponent. The two parameters in Eq 4 are K_H and n_H. Table 1 uses the method of Pavlina and Van Tyne (Ref 3) to provide specific values of K_H and n_H as a function of hardness. As a result, from a single hardness measurement, the two parameters can be found in the table and can then be used to quantify the stress-strain behavior by the Holloman hardening model.

2.2 Ludwig Hardening

The Ludwig hardening model provides a modification of the Holloman equation and incorporates the yield strength into the simple power-law hardening model. The Ludwig power-law hardening model is expressed as

Table 2 Ludwig hardening parameters

HV	σ_0	n_L	K_L
130	283.2	0.486	410.6
135	297.6	0.477	422.3
140	311.9	0.468	433.5
145	326.3	0.458	444.1
150	340.7	0.449	454.2
155	355.1	0.440	463.9
160	369.5	0.431	473.0
165	383.8	0.423	481.7
170	398.2	0.414	489.9
175	412.6	0.406	497.7
180	427.0	0.397	505.2
185	441.4	0.389	512.3
190	455.7	0.381	519.0
195	470.1	0.373	525.4
200	484.5	0.365	531.5
205	498.9	0.358	537.2
210	513.3	0.350	542.8
215	527.6	0.343	548.0
220	542.0	0.336	553.0
225	556.4	0.329	557.8
230	570.8	0.322	562.4
235	585.2	0.315	566.8
240	599.5	0.309	571.1
245	613.9	0.302	575.1
250	628.3	0.296	579.1
255	642.7	0.290	582.8
260	657.1	0.284	586.5
265	671.4	0.278	590.1
270	685.8	0.273	593.5
275	700.2	0.267	596.9
280	714.6	0.262	600.2
285	729.0	0.256	603.4
290	743.3	0.251	606.6
295	757.7	0.246	609.7
300	772.1	0.242	612.7
305	786.5	0.237	615.8
310	800.9	0.232	618.8
315	815.2	0.228	621.8
320	829.6	0.224	624.7
325	844.0	0.219	627.7
330	858.4	0.215	630.7
335	872.8	0.211	633.6
340	887.1	0.208	636.6
345	901.5	0.204	639.6
350	915.9	0.200	642.6
355	930.3	0.197	645.6
360	944.7	0.193	648.7
365	959.0	0.190	651.7
370	973.4	0.187	654.8
375	987.8	0.184	657.9
380	1002.2	0.181	661.1
385	1016.6	0.178	664.3
390	1030.9	0.175	667.6
395	1045.3	0.172	670.8
400	1059.7	0.170	674.2
405	1074.1	0.167	677.5
410	1088.5	0.165	680.9
415	1102.8	0.162	684.4
420	1117.2	0.160	687.9
425	1131.6	0.158	691.5
430	1146.0	0.156	695.1
435	1160.4	0.154	698.8
440	1174.7	0.151	702.5
445	1189.1	0.150	706.2
450	1203.5	0.148	710.0
455	1217.9	0.146	713.9
460	1232.3	0.144	717.8
465	1246.6	0.142	721.8
470	1261.0	0.141	725.8
475	1275.4	0.139	729.9
480	1289.8	0.137	734.0
485	1304.2	0.136	738.2
490	1318.5	0.135	742.5
495	1332.9	0.133	746.7
500	1347.3	0.132	751.1
505	1361.7	0.130	755.5
510	1376.1	0.129	759.9
515	1390.4	0.128	764.4
520	1404.8	0.127	768.9
525	1419.2	0.126	773.5
530	1433.6	0.125	778.2
535	1448.0	0.124	782.8
540	1462.3	0.123	787.6
545	1476.7	0.122	792.4
550	1491.1	0.121	797.2
555	1505.5	0.120	802.0
560	1519.9	0.119	807.0
565	1534.2	0.118	811.9
570	1548.6	0.117	816.9
575	1563.0	0.116	822.0

$$\sigma = \sigma_0 + K_L \varepsilon_{pl}^{n_L}, \qquad \text{(Eq 5)}$$

where σ_0 is the pre-stress (or yield strength) of the material and K_L and n_L are the strength coefficient and strain hardening exponent, respectively. The three parameters in Eq 5 are σ_0, K_L, and n_L. These values are different from the values found in the Holloman equation. Table 2 provides specific values for σ_0, K_L, and n_L as a function of hardness.

2.3 Swift Hardening

The Swift hardening model provides a second modification of the Holloman equation and introduces a strain offset, ε_0, to account for the initial flow strength of the material. The Swift model is given by

$$\sigma = K_S\left(\varepsilon_0 + \varepsilon_{pl}\right)^{n_S}, \qquad \text{(Eq 6)}$$

where K_s and n_S are the strength coefficient and strain hardening exponent, respectively. Because the pre-strain term is positive, the flow curve defined by the Swift model is simply the Holloman flow curve shifted in strain space. The three parameters in Eq 6 are ε_0, K_s, and n_S. These values are different from the values for the other two power-law hardening models (i.e., the Holloman and Ludwig models). Table 3 provides specific values for ε_0, K_s, and n_S as a function of hardness.

2.4 Voce Hardening

The Voce hardening law represents exponential hardening behavior, and is given by

$$\sigma = \sigma_\infty - (\sigma_\infty - \sigma_o)e^{-B\varepsilon_{pl}}, \qquad \text{(Eq 7)}$$

Table 3 Swift hardening parameters

HV	ε_0	n_s	K_s
130	0.01974	0.211	648.4
135	0.01811	0.203	672.7
140	0.01659	0.196	696.5
145	0.01517	0.189	719.8
150	0.01385	0.182	742.7
155	0.01263	0.176	765.2
160	0.01150	0.169	787.3
165	0.01046	0.164	809.0
170	0.00950	0.158	830.5
175	0.00862	0.152	851.6
180	0.00782	0.147	872.4
185	0.00708	0.142	893.0
190	0.00640	0.138	913.3
195	0.00578	0.133	933.3
200	0.00522	0.129	953.2
205	0.00470	0.125	972.8
210	0.00424	0.121	992.3
215	0.00381	0.117	1011.6
220	0.00343	0.113	1030.8
225	0.00308	0.110	1049.8
230	0.00276	0.106	1068.6
235	0.00247	0.103	1087.4
240	0.00222	0.100	1106.0
245	0.00198	0.097	1124.6
250	0.00177	0.094	1143.0
255	0.00158	0.092	1161.4
260	0.00141	0.089	1179.7
265	0.00126	0.087	1197.9
270	0.00112	0.084	1216.1
275	0.00100	0.082	1234.3
280	0.00089	0.080	1252.3
285	0.00079	0.078	1270.4
290	0.00070	0.076	1288.4
295	0.00062	0.074	1306.4
300	0.00055	0.072	1324.4
305	0.00049	0.070	1342.3
310	0.00043	0.068	1360.3
315	0.00039	0.067	1378.2
320	0.00034	0.065	1396.1
325	0.00030	0.064	1414.0
330	0.00027	0.062	1432.0
335	0.00024	0.061	1449.9
340	0.00021	0.059	1467.8
345	0.00018	0.058	1485.8
350	0.00016	0.057	1503.8
355	0.000145	0.056	1521.7
360	0.000128	0.054	1539.7
365	0.000113	0.053	1557.7
370	0.000100	0.052	1575.8
375	0.000088	0.051	1593.9
380	0.000078	0.050	1611.9
385	0.000069	0.049	1630.1
390	0.000067	0.049	1633.7
395	0.000066	0.049	1637.3
400	0.000064	0.049	1640.9
405	0.000063	0.049	1644.6
410	0.000061	0.048	1648.2
415	0.000054	0.047	1666.4
420	0.000048	0.047	1684.6
425	0.000043	0.046	1702.8
430	0.000038	0.045	1721.1
435	0.000034	0.044	1739.4
440	0.000030	0.043	1757.7
445	0.000026	0.043	1776.1
450	0.000024	0.042	1794.5
455	0.000021	0.041	1813.0
460	0.000019	0.041	1831.4
465	0.000017	0.040	1850.0
470	0.000015	0.040	1868.5
475	0.000013	0.039	1887.1
480	0.000012	0.038	1905.8
485	0.000011	0.038	1924.4
490	0.000010	0.037	1943.2
495	0.000009	0.037	1961.9
500	0.000008	0.036	1980.7
505	0.000007	0.036	1999.5
510	0.000006	0.036	2018.4
515	0.000006	0.035	2037.3
520	0.000005	0.035	2056.3
525	0.000005	0.034	2075.2
530	0.000004	0.034	2094.3
535	0.000004	0.034	2113.3
540	0.000004	0.033	2132.4
545	0.000003	0.033	2151.6
550	0.000003	0.033	2170.7
555	0.000003	0.032	2190.0
560	0.000003	0.032	2209.2
565	0.000002	0.032	2228.5
570	0.000002	0.031	2247.8
575	0.000002	0.031	2267.2

where σ_∞ is the saturation stress, σ_0 is the yield strength, and B is related to the decay constant of the system. The three parameters are σ_∞, σ_0, and B. Table 4 gives values for these three parameters as a function of hardness.

3. Discussion

In Tables 1 to 4, the first column is the hardness in HV and the other columns provide parameter values for the specific hardening model. The tables should be used as follows: (1) a hardness test is performed on the steel of interest; (2) a hardening model is chosen to characterize the flow behavior of the steel; and (3) the appropriate table is consulted, so that the values of the parameters in the model are obtained.

Because these tables are based on empirical correlations, there is no guarantee that they will provide a perfect fit for the steel of interest. Their use becomes important when a flow model is needed, especially in simulation work and in situations where only a limited amount of physical material is available. By performing the hardness test and using these tables, the engineer can make a reasonable approximation for the actual behavior, which is based on the averaged behavior of many steels, as determined by the empirical correlations.

3.1 Remarks on Uncertainty

There is a range of uncertainty associated with the three hardness correlations. The nature of this type of empirical engineering analysis will always have a relatively high degree

Table 4 Voce hardening parameters

HV	σ_0	σ_∞	*B*
130	283.2	535.5	6.81
135	297.6	554.1	7.21
140	311.9	572.4	7.62
145	326.3	590.6	8.06
150	340.7	608.6	8.51
155	355.1	626.5	8.99
160	369.5	644.2	9.48
165	383.8	661.9	10.00
170	398.2	679.4	10.54
175	412.6	696.9	11.10
180	427.0	714.3	11.69
185	441.4	731.6	12.29
190	455.7	748.9	12.93
195	470.1	766.2	13.58
200	484.5	783.4	14.26
205	498.9	800.6	14.97
210	513.3	817.8	15.70
215	527.6	835.0	16.45
220	542.0	852.2	17.24
225	556.4	869.3	18.05
230	570.8	886.5	18.88
235	585.2	903.7	19.74
240	599.5	920.9	20.63
245	613.9	938.1	21.55
250	628.3	955.3	22.49
255	642.7	972.6	23.46
260	657.1	989.8	24.46
265	671.4	1007.1	25.49
270	685.8	1024.4	26.54
275	700.2	1041.7	27.62
280	714.6	1059.1	28.73
285	729.0	1076.4	29.87
290	743.3	1093.8	31.03
295	757.7	1111.2	32.22
300	772.1	1128.7	33.43
305	786.5	1146.2	34.67
310	800.9	1163.7	35.93
315	815.2	1181.2	37.22
320	829.6	1198.8	38.54
325	844.0	1216.4	39.87
330	858.4	1234.0	41.23
335	872.8	1251.6	42.61
340	887.1	1269.3	44.01
345	901.5	1287.0	45.44
350	915.9	1304.8	46.88
355	930.3	1322.5	48.34
360	944.7	1340.3	49.81
365	959.0	1358.2	51.31
370	973.4	1376.0	52.81
375	987.8	1393.9	54.34
380	1002.2	1411.8	55.87
385	1016.6	1429.8	57.42
390	1030.9	1447.7	58.97
395	1045.3	1465.7	60.54
400	1059.7	1483.7	62.11
405	1074.1	1501.8	63.69
410	1088.5	1519.9	65.28
415	1102.8	1538.0	66.87
420	1117.2	1556.1	68.46
425	1131.6	1574.3	70.06
430	1146.0	1592.4	71.65
435	1160.4	1610.6	73.25
440	1174.7	1628.9	74.84
445	1189.1	1647.1	76.43
450	1203.5	1665.4	78.01
455	1217.9	1683.7	79.59
460	1232.3	1702.0	81.16
465	1246.6	1720.3	82.72
470	1261.0	1738.7	84.27
475	1275.4	1757.1	85.81
480	1289.8	1775.5	87.34
485	1304.2	1793.9	88.86
490	1318.5	1812.3	90.37
495	1332.9	1830.8	91.86
500	1347.3	1849.3	93.33
505	1361.7	1867.8	94.79
510	1376.1	1886.3	96.23
515	1390.4	1904.8	97.65
520	1404.8	1923.4	99.06
525	1419.2	1942.0	100.44
530	1433.6	1960.6	101.81
535	1448.0	1979.2	103.15
540	1462.3	1997.8	104.48
545	1476.7	2016.4	105.78
550	1491.1	2035.1	107.06
555	1505.5	2053.8	108.32
560	1519.9	2072.4	109.56
565	1534.2	2091.1	110.77
570	1548.6	2109.9	111.96
575	1563.0	2128.6	113.13

of uncertainty. The data used to develop the correlations have come from steels with large variations in chemical composition, grain size, phase or constituent fractions, and processing. Each of these variations affects the strain hardening behavior of the steel—and likewise, these variations will influence the measured hardness. Thus, the correlations based on hardness will have an uncertainty that is larger than that of carefully controlled experiments. Nevertheless, when no or little mechanical data exist for a steel, the tables developed in the current study provide a reasonable means to quantify the expected approximate stress-strain value of a steel.

The tables developed in the current study provide a prediction of the average properties or the average mechanical response of steel from a single hardness measurement. Although these tables should not replace actual experimental stress-strain curves, in situations where experiments or material is limited, the tables provide a good engineering method to quantify the hardening response of steels based on a single hardness value. These tables should not be used if sufficient material is available for appropriate testing; but they do allow for the use of flow models for steels, especially in simulations where only hardness values are available. In sheet metal forming simulations, there is often only limited material available to the person performing the computer modeling. Although they are not perfect, the tabular values developed in the current study can give an analyst confidence that the simulations represent the reasonable response of the steel, based upon averaged values that have been empirically correlated over a wide range of steel grades.

4. Summary

Correlations for yield strength, ultimate tensile strength, and uniform elongation with respect to hardness were developed for steels over a wide range of compositions. These correlations were combined to predict the stress-strain flow curve from a single hardness test for four different hardening models: Holloman, Ludwig, Swift, and Voce. Tables for the parameters for these four hardening models were calculated and are presented. These tables allow an engineer to obtain a reasonable set of parameters for one of the hardening models, based on a single value of hardness.

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