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Theoretical Analysis of the Thermoelectric Generator Considering Surface to Surrounding Heat Convection and Contact Resistance

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A general theoretical model of thermoelectric generation (TEG) is proposed based on the one-dimensional steady heat transport in this paper. The effect of heat convection between the thermoelectric legs and the ambient environment, and contact resistance between the heat reservoirs and thermoelectric couple on the performance of the TEG is studied. Fundamental formulas and closed-form solutions for the output power and conversion efficiency are derived. Numerical results show that the maximum output power and maximum conversion efficiency of the TEG are lower than those of the ideal TEG when the influence of heat convection and contact resistance are taken into consideration. The heat convection has a very small effect on the maximum output power, but causes a large reduction of conversion efficiency for the TEG, and this reduction becomes more significant as the length of thermoelectric couple increases. In addition, there always exists an optimum length of thermoelectric couple for the actual TEG, so as to achieve the maximum conversion efficiency when the effects of heat convection together with contact resistance are considered. The results of this paper may help to improve the design and optimization of TEG devices.

Key words: Thermoelectric generator, heat convection, contact resistance, efficiency, output power

INTRODUCTION

A thermoelectric generator (TEG), as a solid-state energy conversion technology, can transform heat energy into electrical energy directly by employing thermoelectric materials without any moving parts.[1,2] However, both thermoelectric materials and devices design should be optimized to achieve good performance. The energy conversion performance is quantified by the dimensionless figure of merit $ZT = (\alpha^2 \sigma T)/k$, where α is the Seebeck coefficient, σ is electrical conductivity, k is thermal conductivity and T is absolute temperature.[3,4] Current efforts in enhancing $\alpha^2\sigma$ and reducing k have been devoted towards improving their figure of merit.[5–9]

So far, a number of models have been developed to study the heat transfer and performance of the TEGs based on the generalized thermoelectric energy balance equation, which involves the Fourier heat transfer, Joule heat, Seebeck effect and Peltier effect and Thomson effect.[10–13] The thermoelectric legs of the TEG are often assumed to be thermally isolated from the surroundings in the previous

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researches.[5–13] Yet, to make the best use of thermoelectric materials it is necessary to consider the effect of interaction between the constitutive laws of the TEG and the laws related to the external environment. The detailed modeling of the TEG considering the phenomenon of energy loss from thermoelectric couple legs was proposed through linearizing the nonlinear temperature distribution by Xiao et al.[14] The effect of convection heat transfer that occurred between the thermoelectric device and the ambient gas was investigated based on the finite element method by Wang et al.[15] and Rabari et al.[16] The performance of a thermal-concentrated solar TEG was studied by Chen et al.,[17] and it is found that water cooling is better than air cooling for the net output power of the TEG for the equal forced heat convection coefficients due to the higher specific heat of water.

On the other hand, the contact resistances become significantly more important and cannot be neglected when the TEG is relatively short.[4,18] A numerical method to determine the actual temperature of thermoelectric materials, and optimum working conditions for TEG by considering the influence of thermal resistances between a thermoelectric module and heat reservoirs was exploited by Gomez et al.[19] A general model to study the effect of interface layers on the performance of the annular TEG was presented by Zhang et al.,[20] and solutions for the maximum output power and maximum conversion efficiency were obtained in the closed-form. From Refs. 4, 15, 18–20 it can be found that the contact resistance plays an essential role in the relatively short TEG, especially in the TEG microdevices. In this paper, a theoretical model is proposed to characterize the influence of heat convection and contact resistance on the performance of the TEG. Closed-form solutions for output power and conversion efficiency are derived, and some numerical results are given. This work provides a basis to predict optimum working conditions for the application of the actual TEG devices.

GENERAL MODEL OF THE TEG CONSIDERING CONVECTION

A general TEG device is formed from several thermoelectric elements, connected thermally in parallel and electrically in series. A TEG element is extracted and shown in Fig. 1 with consideration of periodicity, p-type and n-type semiconductor legs connected by electrically conductive metal strips to form a thermoelectric couple, and sandwiched between two electric insulating but thermally conducting ceramic plates. Each leg has a cross-sectional area of $A = L_1 \times L_2$ and a length of H. It is assumed that the high- and low-temperature heat reservoirs are maintained at constant values of T_h and T_c, respectively, and all the material properties, such as electrical conductivity σ_p and σ_n, thermal conductivity k_p and k_n, and Seebeck coefficient α_p and α_n are independent of temperature. The temperature of TEG is generally higher than that of the ambient environment during power generation operation, and the heat will be transferred from the TEG to the ambient air by convection and radiation. However, the amount of radiation heat loss is very small in contrast to heat convection loss in low-temperature conditions. Therefore, the influence of radiation heat loss is neglected in the paper.

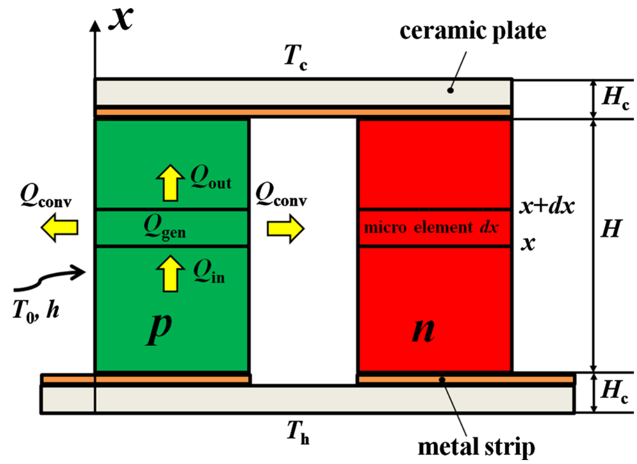

Fig. 1. Schematic diagram of a thermoelectric generator.

The first law of thermodynamics is applied to an infinitesimal thermoelectric element with length dx as shown in Fig. 1, and it leads to $Q_{\text{in}} - Q_{\text{out}} + Q_{\text{gen}} - Q_{\text{conv}} = 0$, where $Q_{\text{in}} = q(x)A$, $Q_{\text{out}} = q(x + \mathrm{d}x)A$, $Q_{\text{gen}} = j^2 A\mathrm{d}x/\sigma$ and $Q_{\text{conv}} = h(T - T_0)P\mathrm{d}x$ are the Fourier heat input, Fourier heat output, Joule heat and convection heat loss in this infinitesimal element, respectively. T_0 is the ambient temperature, $P = 2(L_1 + L_2)$ is the circumference of the p-type or n-type thermoelectric leg, q is heat flow density and j is the electric current density. Using the Fourier heat transfer law $q = -k\mathrm{d}T/\mathrm{d}x$, the energy conservation equation of thermoelectric material for the steady-state can be written as

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} - \frac{2hP}{KH}(T - T_0) + \frac{I^2R}{KH^2} = 0, \tag{1}$$

where I and h are the electric current and heat transfer coefficient between the thermoelectric leg and ambient environment, respectively, $h = 0\ \mathrm{W/(m^2K)}$ denotes an adiabatic boundary condition, and $K = K_p + K_n = k_pA/H + k_nA/H$ and $R = R_p + R_n = H/(\sigma_pA) + H/(\sigma_nA)$. According to the assumption, the boundary conditions of temperature are $T_p(0) = T_h$ and $T_p(H) = T_c$, the temperature distribution in the thermoelectric leg can be obtained as

$$T_p(x) = B_0 + B_1\exp(\omega x) + B_2\exp(-\omega x), \tag{2}$$

where

$$\omega = \sqrt{2hP/(KH)}$$

$$B_0 = T_0 + \frac{I^2R}{2hPH}$$

$$B_1 = \frac{T_c - T_h\exp(-\omega H) - [1-\exp(-\omega H)][T_0 + I^2R/(2hPH)]}{\exp(\omega H) - \exp(-\omega H)}$$

$$B_2 = -\frac{T_c - T_h\exp(\omega H) - [1-\exp(\omega H)][T_0 + I^2R/(2hPH)]}{\exp(\omega H) - \exp(-\omega H)}.$$

The heat flow rate along x through the p-type leg of TEG can be expressed as

$$Q_p(x) = -k_pA\frac{\mathrm{d}T(x)}{\mathrm{d}x} + \alpha_p IT(x), \tag{3}$$

where the first term and second term on the right-hand side of Eq. 3 represent Fourier heat transfer and the Peltier effect, respectively. Inserting Eq. 2 into Eq. 3, we have

$$Q_p(0) = -K_pH\omega(B_1 - B_2) + \alpha_p IT_h, \tag{4}$$

$$Q_p(H) = -K_pH\omega[B_1\exp(\omega H) - B_2\exp(-\omega H)] + \alpha_p IT_c. \tag{5}$$

The rate of heat dissipation between the p-type leg and the ambient air has the form of

$$Q_{p\mathrm{conv}} = \frac{hP}{\omega}\{B_1[\exp(\omega H) - 1] - B_2[\exp(-\omega H) - 1]\} + I^2R_p. \tag{6}$$

The same boundary conditions and the same method can be used to derive the expression of the heat flow rate Q_n and heat convection loss $Q_{n\mathrm{conv}}$ in the same form for the n-type thermoelectric leg. Consequently, the heat absorbed at the hot junction $Q(0)$ and heat rejected at the cold junction $Q(H)$ for a single-element TEG are

$$Q(0) = Q_p(0) + Q_n(0) = -KH\omega(B_1 - B_2) + \alpha IT_h, \tag{7}$$

$$\begin{aligned} Q(H) &= Q_p(H) + Q_n(H) \\ &= -KH\omega[B_1\exp(\omega H) - B_2\exp(-\omega H)] + \alpha IT_c, \end{aligned} \tag{8}$$

where $\alpha = \alpha_p - \alpha_n$. Similarly, the heat flow rate of TEG by convection can be expressed as

$$\begin{aligned} Q_{\mathrm{conv}} &= Q_{p\mathrm{conv}} + Q_{n\mathrm{conv}} \\ &= \frac{2hP}{\omega}\{B_1[\exp(\omega H) - 1] - B_2[\exp(-\omega H) - 1]\} \\ &\quad + I^2R. \end{aligned} \tag{9}$$

The output power P_{out} and conversion efficiency η are used to evaluate the performance of TEG, and they have the form of

$$P_{\mathrm{out}} = Q(0) - Q(H) - Q_{\mathrm{conv}} = \alpha I(T_h - T_c) - I^2R, \tag{10}$$

$$\eta = \frac{P_{\mathrm{out}}}{Q(0)} = \frac{\alpha I(T_h - T_c) - I^2R}{\frac{\omega H}{\sinh(\omega H)}\left\{K(T_h - T_c) - \frac{[\cosh(\omega H)-1]}{(\omega H)^2}I^2R + K[\cosh(\omega H) - 1](T_h - T_0)\right\} + \alpha IT_h}. \tag{11}$$

The electric current I can be obtained as $I = \alpha(T_h - T_c)/(R + R_L)$, if the external imposed resistance R_L is placed, and the output power is expressed as $P_{\mathrm{out}} = I^2R_L$ by substituting I into Eq. 10. It is found that the maximum output power $P_{\mathrm{out}}^{\max} = \alpha^2(T_h - T_c)^2/(4R)$ attains when $I = \alpha(T_h - T_c)/(2R)$, this is corresponding to the

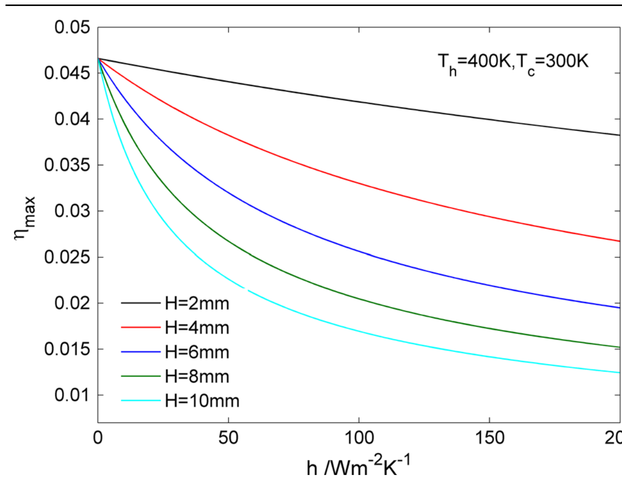

Fig. 2. Variation of heat convection coefficient and leg's length on the maximum conversion efficiency of the TEG.

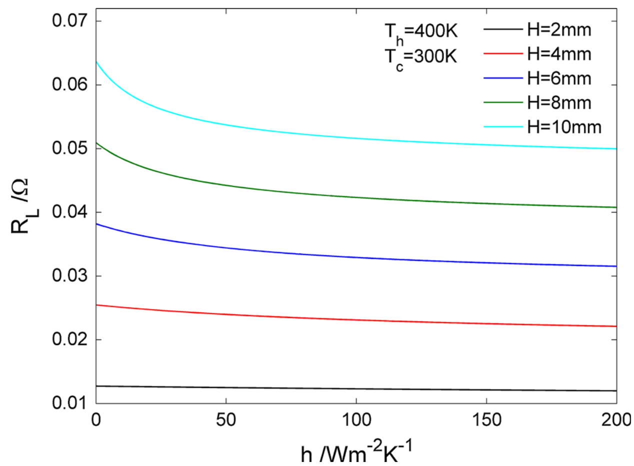

Fig. 3. Variation of optimum imposed resistance corresponding to the maximum conversion efficiency with heat convection coefficient and leg's length.

case of $R = R_L$. For the purpose of studying the influence of heat convection between the thermoelectric legs and the ambient environment on the performance of TEG, numerical results of the TEG subjected to the temperature loadings $T_h = 400$ K and $T_c = 300$ K are shown in Figs. 2 and 3. The considered thermoelectric material is Bi_2Te_3, and its material properties are $\alpha_p = -\alpha_n = 2 \times 10^{-4}$ V/K, $\sigma_p = \sigma_n = 1.1 \times 10^5$ S/m and $k_p = k_n = 1.6$ W/(mK), the side length of cross-section for the thermoelectric leg is $L_1 = L_2 = 2$ mm,[21] and the ambient temperature T_0 is assumed to be T_c. It can be seen that the existence of heat convection reduces the maximum thermoelectric conversion efficiency $\eta_{\max}$ largely from Fig. 2, and this decrease in $\eta_{\max}$ is very significant as the length of thermoelectric leg H and heat convection coefficient h increase. In Fig. 3, we show a plot of optimum imposed resistance R_L corresponding to $\eta_{\max}$ against h, and the value of R_L is smaller than that of ideal TEG [$h = 0$ W/(m^{2}K)] due to the effect of convection. Moreover, the reduction of $\eta_{\max}$ and R_L are very large when the heat convection coefficient h is within the range of natural convection [0 W/(m^{2}K) to $h = 20$ W/(m^{2}K)].

EFFECT OF CONVECTION AND CONTACT RESISTANCE

The effect of thermal contact resistances of ceramic plates and electrical contact resistances between the metallic strip and thermoelectric legs is often neglected, which is reasonable for relatively long thermoelectric legs. However, the contact effect should be considered when the thermoelectric legs are relative short.[4] The temperature difference ΔT_1 and ΔT_2 are introduced to reflect the temperature drops across the ceramic plates, and the actual temperature difference ΔT_0 across the thermoelectric couple can be expressed as $\Delta T_0 = \Delta T - \Delta T_1 - \Delta T_2$, with $\Delta T = T_h - T_c$. The electric current does not go through the ceramic plates and the heat convection loss is neglected for the ceramic plates, so that only the heat transfer needs to be considered. Equation (1) is reduced to $\mathrm{d}^2T/\mathrm{d}x^2 = 0$, and we have

$$\Delta T_1 = \frac{Q_p(0)}{K_c} \tag{12}$$

$$\Delta T_2 = \frac{Q_p(H)}{K_c}, \tag{13}$$

where $K_c = (k_c A)/H_c$, k_c and H_c are the heat conductivity and thickness of ceramic plate, respectively. For the p-type thermoelectric leg, the form of temperature distribution is similar as Eq. 2, and the heat flow rate $Q_p(x)$ at $x = 0$ and $x = H$ are derived as

$$Q_p(0) = -K_p H\omega(B_3 - B_4) + \alpha_p I(T_h - \Delta T_1), \tag{14}$$

$$Q_p(H) = -K_p H\omega[B_3\exp(\omega H) - B_4\exp(-\omega H)] + \alpha_p I(T_c + \Delta T_2), \tag{15}$$

with

$$B_3 = \frac{(T_c + \Delta T_2) - (T_h - \Delta T_1)\exp(-\omega H) - [1 - \exp(-\omega H)]\left[T_0 + I^2R/(2hPH)\right]}{\exp(\omega H) - \exp(-\omega H)}$$

$$B_4 = -\frac{(T_c + \Delta T_2) - (T_h - \Delta T_1)\exp(\omega H) - [1 - \exp(\omega H)]\left[T_0 + I^2R/(2hPH)\right]}{\exp(\omega H) - \exp(-\omega H)}.$$

When the contribution of electrical contact resistance is considered, and the current I through the TEG is expressed as

$$I = \frac{\alpha\Delta T_0}{R + R_L + 4R_c}, \tag{16}$$

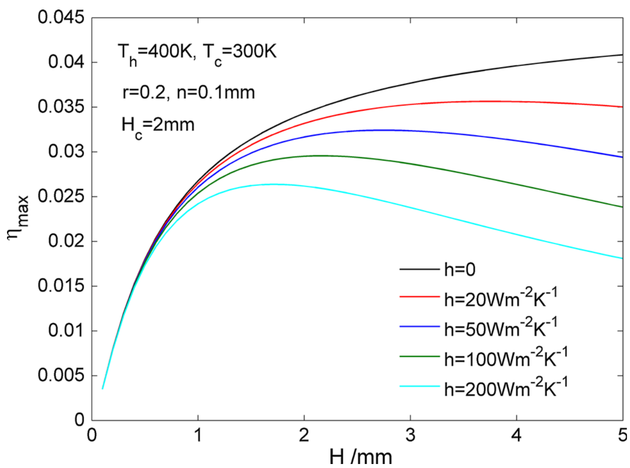

Fig. 4. Variation of heat convection coefficient and leg's length on the maximum conversion efficiency of the TEG by considering contact resistance.

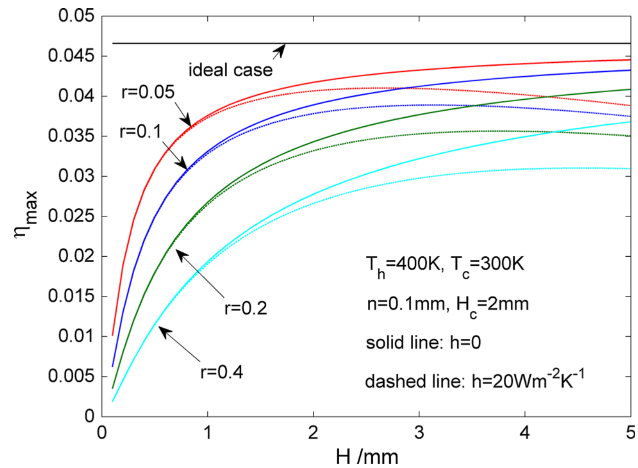

Fig. 5. Variation of contact resistance and leg's length on the maximum conversion efficiency of the TEG.

where $R_c = 1/(\sigma_c A)$, σ_c is the electrical conductivity of the interface layer between the metallic strip and the thermoelectric leg. The rate of heat dissipation between the p-type leg and the ambient environment in this case can be obtained replacing B_1 and B_2 in Eq. 9 by B_3 and B_4, respectively. Combining Eqs. 10, and 12–16, the equation of current I is given as

$$D_3 I^3 + D_2 I^2 + D_1 I + D_0 = 0, \tag{17}$$

where

$$D_3 = \frac{\alpha K R \omega[\cosh(\omega H) - 1]}{2hPK_c \sinh(\omega H)} - \frac{\alpha(R + R_L + 4R_c)}{4K_c}$$

$$D_2 = -\frac{\alpha^2(T_h - T_c)}{4K_c}$$

$$\begin{aligned} D_1 = {} & \frac{\alpha(T_h - T_c)}{2} - \frac{\alpha K H \omega (T_h + T_c - 2T_0)[\cosh(\omega H) - 1]}{4K_c \sinh(\omega H)} \\ & + \frac{K_c(R + R_L + 4R_c)}{\alpha}\left\{1 + \frac{KH\omega[\cosh(\omega H) + 1]}{2K_c \sinh(\omega H)}\right\} \\ & \left\{1 + \frac{KH\omega[\cosh(\omega H) - 1]}{2K_c \sinh(\omega H)}\right\} \end{aligned}$$

$$\begin{aligned} D_0 = {} & \frac{KH\omega(T_h - T_c)[\cosh(\omega H) + 1]}{2\sinh(\omega H)} \\ & \left\{1 + \frac{KH\omega[\cosh(\omega H) - 1]}{2K_c \sinh(\omega H)}\right\} \\ & - K_c(T_h - T_c)\left\{1 + \frac{KH\omega[\cosh(\omega H) + 1]}{2K_c \sinh(\omega H)}\right\} \\ & \left\{1 + \frac{KH\omega[\cosh(\omega H) - 1]}{2K_c \sinh(\omega H)}\right\} \end{aligned}.$$

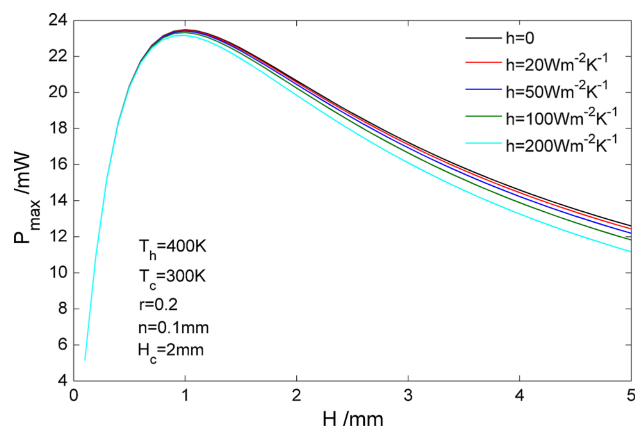

Fig. 6. Variation of heat convection coefficient and leg's length on the maximum output power of the TEG.

The physically acceptable solution of Eq. 17 has the following form

$$I = -\frac{D_2}{3D_3} + s^2 \cdot \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + s \cdot \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}, \tag{18}$$

with $s = \frac{-1+\sqrt{3}i}{2}$, $p = \frac{D_1}{D_3} - \frac{D_2^2}{3D_3^2}$ and $q = \frac{D_0}{D_3} + \frac{2D_2^3}{27D_3^3} - \frac{D_2 D_1}{3D_3^2}$.

From Eqs. 12–18, we can see that the effects of the contact resistance and heat convection on the performance of the TEG are coupled with each other, and it is very different from the models traditionally used to only simulate the contact resistance.[4,12,18] The output power P_{out} and conversion efficiency η can be calculated by $P_{out} = I^2 R_L$ and $\eta = P_{out}/Q(0)$, and the optimized external resistance corresponding to the maximum output power and maximum conversion efficiency can be obtained

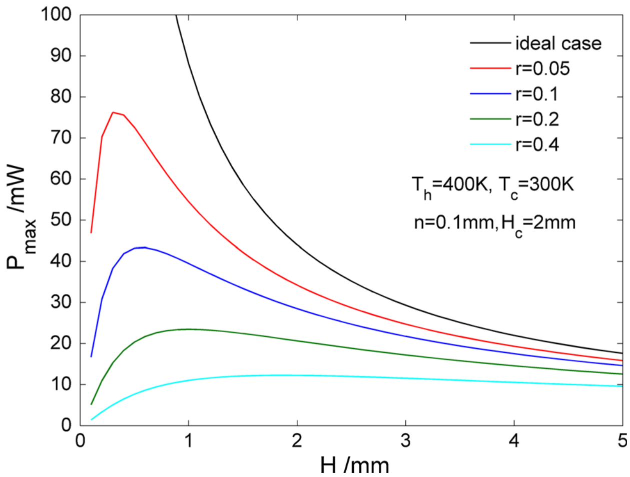

Fig. 7. Variation of contact resistance and leg's length on the maximum output power of the TEG.

by $\mathrm{d}P_{\mathrm{out}}/\mathrm{d}R_L = 0$ and $\mathrm{d}\eta/\mathrm{d}R_L = 0$, respectively. However, it can be seen that the mathematic derivation in the form of a formula is very difficult from Eqs. 16–18, therefore, the effect of heat convection and contact resistance on the performance of TEG is studied based on the following numerical results. The thickness of the ceramic plate is assumed to be $H_c = 2$ mm, $r = k_p/k_c$ and $n = \sigma_c/\sigma_p$. Figures 4 and 5 show variation of the maximum conversion efficiency $\eta_{\max}$ with length of thermoelectric legs H with the different heat convection coefficient h or different thermal contact property k_c. Obviously, the maximum conversion efficiency is affected by the length of the thermoelectric legs, which reflects the dependence of the maximum conversion efficiency of the TEG on the heat convection and properties of the thermal and electrical contacts. Figure 4 illustrates the $\eta_{\max}$ increases with the increasing value of H for the ideal case of $h = 0\ \mathrm{W/(m^2K)}$, and there always exists a maximum value for $\eta_{\max}$ as H varies when the effect of heat convection is taken into consideration. It can also be seen that the $\eta_{\max}$ is significantly affected by the contact resistance from Fig. 5, especially for the TEG with the short thermoelectric legs. Figures 6 and 7 show that the maximum output power of the TEG as a function of the leg's length with various contact resistance and heat convection coefficient. It is seen that h has a very small effect on the $P_{\max}$, and the effect of h is elevated gradually as the length of thermoelectric leg increases, and higher h leads to a reduced maximum output power. In addition, the maximum output power is reduced monotonously as the length of thermoelectric legs increased for the ideal case, and there is no maximum value of $P_{\max}$. However, the $P_{\max}$ reaches its peak value at a certain H and then monotonically drops as the H increases further. It is also noted that the $P_{\max}$ decreases as contact resistance increased.

CONCLUSION

A general model of the TEG was developed in this paper based on the basic theories of thermoelectric power generation and thermal science. The effect of heat convection between the thermoelectric couple and the ambient environment and contact resistance is considered. Closed-form solutions for output power and conversion efficiency are derived and some numerical results are also presented. It is found that the heat convection coefficient has a very small effect on the maximum output power. However, such effect can be neglected for the TEG with the short thermoelectric legs. The existence of the heat convection process does cause a large loss of conversion efficiency. The maximum conversion efficiency decreases as value of the leg's length increases. It is also found that the performance of TEG is strongly dependent on the contact resistance, especially for the very short TEG. The influence of heat convection and contact resistance should be taken into consideration in optimization and applications of TEG.

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