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Dimensionless Model of a Thermoelectric Cooling Device Operating at Real Heat Transfer Conditions: Maximum Cooling Capacity Mode

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Real operating conditions of a thermoelectric cooling device are in the presence of thermal resistances between thermoelectric material and a heat medium or cooling object. They limit performance of a device and should be considered when modeling. Here we propose a dimensionless mathematical steady state model, which takes them into account. Analytical equations for dimensionless cooling capacity, voltage, and coefficient of performance (COP) depending on dimensionless current are given. For improved accuracy a device can be modeled with use of numerical or combined analytical-numerical methods. The results of modeling are in acceptable accordance with experimental results. The case of zero temperature difference between hot and cold heat mediums at which the maximum cooling capacity mode appears is considered in detail. Optimal device parameters for maximal cooling capacity, such as fraction of thermal conductance on the cold side y, fraction of current relative to maximal j' are estimated in range of 0.38–0.44 and 0.48–0.95, respectively, for dimensionless conductance $K' = 5$–100. Also, a method for determination of thermal resistances of a thermoelectric cooling system is proposed.

Key words: Thermoelectrics, thermoelectric theory, thermoelectricity, thermoelectric modeling, optimal design, thermoelectric module, thermoelectric cooler

INTRODUCTION

Since the development and propagation of thermoelectric generator module (TGM) applications, much attention is paid to modeling its characteristics depending on the construction of the module and parameters of thermoelectric (TE) material. As a result, a number of works have been published, including TGM modeling depending on electrical contact resistances,[1–5] thermal contact resistances,[2–6] numerical methods modeling,[3–6] transient thermal and electric processes,[7] equivalent circuits of TGMs,[8] and influence of Thomson effect.[4,8] For modeling of thermoelectric cooling modules (TCM), there are works dedicated to the influence of the Thomson effect on module characteristics,[9–12] numerical methods modeling,[12] equivalent circuits of TCMs,[13] influence of temperature dependence of material properties,[14] and optimal design of TCMs.[15–17] A review of progress in the area of thermoelectric modeling is given in Ref. 17.

It is known that thermal contact resistances strongly affect TGM[3] and TCM[15–17] characteristics. Taking into account thermal contact resistances can also explain why an electrical current is usually lower than calculated by the classical TE equations at temperature difference $\Delta T = 0$. The proposed equations are based on the classical TE equations, which are extended to a case with the presence of thermal resistance between a TE material and a heat medium. This extension provides more

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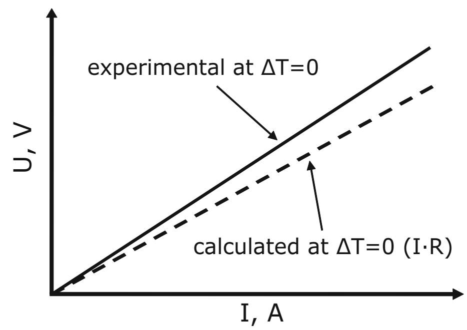

Fig. 1. Typical calculated and experimental $U(I)$ dependence of a TCM.

accurate modeling of TCM characteristics depending on the ZT parameter of a TE material, dimensionless current intensity, dimensionless thermal conductance, and its fraction on the cold and hot side.

Manufacturers and developers of TCMs experience a phenomenon of a current decline compared to calculated value, when a TCM is operating at its normal conditions. Figure 1 shows typical mismatch between experimental and calculated $U(I)$ dependence at $\Delta T = 0$. Experimental values of a current are always lower than calculated at same voltage. The decline of a current can be caused by electrical resistance of copper interconnectors and wires in the TCM and measurement system, but these contributions should be constant for different TCMs and usually are of tenths of an Ohm. In the case of a TCM with long pellets and high electrical resistance, such contributions would be negligible; however, significant decline of a current takes place in this case as well.

Usually, contact thermal resistances in a TCM are assumed to be very small and are neglected during measurement. Thus, measured ΔT is supposed to be actual ΔT in a material, but it will be shown below that even rather small thermal resistances can have a significant effect on actual ΔT in a material and, consequently, $U(I)$ and $Q_c(I)$ dependence. Also, it is shown that the Seebeck coefficient and resistance dependence on temperature have a strong effect.

Dimensionless Model

Practical calculations of TE cooling systems may be carried out by solving the classical system of heat balance equations per unit area[18,19]:

$$\begin{aligned} q_c &= \alpha_c j T_c - \frac{j^2 \rho L}{2} - (T_h - T_c)\frac{k}{L} \\ q_h &= \alpha_h j T_h + \frac{j^2 \rho L}{2} - (T_h - T_c)\frac{k}{L} \\ w &= j^2 \rho L + j(\alpha_h T_h - \alpha_c T_c) \end{aligned} \tag{1}$$

where q_c, q_h—specific heat current from cold (specific cooling capacity) and hot (specific heating capacity) side of a TE material respectively, W m^{-2}; w—specific supplied electrical power, W m^{-2}; α_c, α_h—the Seebeck coefficient on the cold and the hot side of a TE material respectively, V K^{-1}; j—electric current density, A m^{-2}; ρ—electrical resistivity of a TE material, Ohm m; k—thermal conductivity of a TE material, W m^{-1} K^{-1}; L—length of TE pellets, m; T_h, T_c—temperature on the hot and the cold sides of a TE material, respectively, K.

Huang et al.[11] and Chen et al.[12] showed that the Thomson effect can increase the cooling capacity when the Thomson coefficient is positive. Nevertheless, for now, the Thomson effect appears negligible compared to total cooling capacity in typical commercial materials.[10] In addition, the full analytic equation for cooling capacity taking into account the Thomson effect given in Ref. 11 is too complex for further transformations. For these reasons it is not considered in this work, although the temperature dependence of the Seebeck coefficient is considered anyway, when TCMs are calculated by a numerical method described below. In the classical model a decline of a current is connected with this difference $\alpha_h T_h - \alpha_c T_c$, which can be neglected if a material has a linear regression of the Seebeck coefficient with temperature ($\alpha_h/\alpha_c = T_c/T_h$).

Maximum cooling capacity mode in the classical model is described by an equation derived from Eq. 1 at $T_h = T_c = T$, $\alpha_h = \alpha_c = \alpha$, and $j_{0\max} = \alpha T/\rho/L$[18,19]:

$$q_{0\max} = \frac{j_{0\max}^2 \rho L}{2} = \frac{\alpha^2 T^2}{2\rho L} \tag{2}$$

Equations 1 and 2 can be used only for calculations of properties of a material, not devices, because, in practice, there is always a non-zero thermal resistance between a TE material and a heat transfer medium. Air, water, or a solid block could be a heat medium depending on construction of a device or a measurement system. In order to take into account all thermal resistances, equivalent thermal conductance (ETC) K_c, K_h , or equivalent thermal resistance R_c, R_h, the sums of all resistances between a material and cold and hot heat medium, respectively, can be used as shown on Fig. 2. This approach was used in Refs. 15–17 as well. Even a complex measurement system or TE device can be simply considered and calculated with such scheme with addition of thermal resistance of corresponding construction elements.

The presence of thermal resistance/conductance between a material and a heat medium complements Eq. 1 with terms:

$$q_c = \frac{K_c(T_{cm} - T_c)}{nS}; \quad q_h = \frac{K_h(T_h - T_{hm})}{nS}, \tag{3}$$

where K_c—ETC on the cold side, W K^{-1}; K_h—ETC on the hot side, W K^{-1}; T_{cm}—temperature of cooled

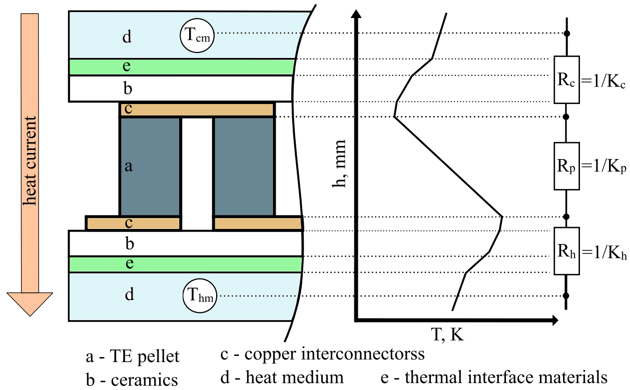

Fig. 2. Schematic thermal interface, temperature profile and equivalent thermal resistance/conductance scheme of a TCM.

heat medium, K; T_{hm}—temperature of heated heat medium, K; n—number of TE pellets connected electrically in series and thermally in parallel in a TE device; S—cross-sectional area of a TE pellet, m^2.

When considering a complex TE cooling device operating at real conditions (cooling one heat medium and heating another) the maximum cooling capacity mode corresponds to zero temperature difference of heat mediums $\Delta T_m = T_{hm} - T_{cm} = 0$, in contrast to the theoretical mode corresponding to material temperature difference $\Delta T = T_h - T_c = 0$. Thus, hereinafter we consider $T_{hm} = T_{cm} = T$. Complemented with Eq. 3 and temperature dependence of parameters of a TE material, Eq. 1 can be written as follows:

$$\begin{aligned} q_c &= \frac{(T - T_c)K_c}{nS} = \alpha(T_c) j T_c - \frac{j^2 \rho(T_c, T_h) L}{2} - \frac{(T_h - T_c)k}{L} \\ q_h &= \frac{(T_h - T)K_h}{nS} = \alpha(T_h) j T_h + \frac{j^2 \rho(T_c, T_h) L}{2} - \frac{(T_h - T_c)k}{L} \\ w &= \frac{(T_h - T)K_h - (T - T_c)K_c}{nS} \\ &= j^2 \rho(T_c, T_h) L + j(\alpha(T_h)T_h - \alpha(T_c)T_c) \end{aligned} \tag{4}$$

The thermal conductivity $k(T)$ dependence leads to a non-linear temperature gradient in a material, which complicates the determination of other characteristics, thus, $k(T)$ is not taken into account here.

It is reasonable to approximate material characteristics dependence with polynomials or linear dependence and solve a system of Eq. 4 relative to T_h and T_c. The result will be very complex in this case, and it would not make sense for an analytical solution, but it can be solved numerically and used as a reference to control the analytical solution, as demonstrated below in this work. For an analytical solution we replace dependence $\alpha(T_c)$, $\alpha(T_h)$, $\rho(T_c, T_h)$ with constants α, ρ, respectively.

Dimensionless analysis is very convenient since it is not connected with the dimensions of a particular system or a device. For its realization we propose following dimensionless parameters:

$T'_c = \frac{T_c}{T}; T'_h = \frac{T_h}{T}$ —dimensionless temperature on the cold and the hot side of a material, respectively

$j' = \frac{j}{j_{0max}} = \frac{j\rho L}{\alpha T}$—dimensionless current density

$K = K_c + K_h$—sum of ETCs on the cold and hot sides, W K^{-1}

$y = \frac{K_c}{K}; 1 - y = \frac{K_h}{K}$—fractions of corresponding ETC relative to sum of ETCs

$K' = \frac{KL}{knS}$—dimensionless thermal conductance

$F = \frac{\alpha^2 T}{\rho k}$—$ZT$ parameter of a material at temperature T

$q'_c = \frac{q_c}{q_{0max}}$; $q'_h = \frac{q_h}{q_{0max}}$; $w' = \frac{w}{q_{0max}}$—dimensionless cooling capacity, heating capacity, and supplied power, respectively.

Note that K' is a ratio between a thermal conductance of all intermediate layers between a material and heat mediums and thermal conductance of TE pellets in a device, including the geometrical shape factor of TE pellets. Higher K' means better heat transfer. Fraction y is the proportion between conductance on the cold and the hot sides. Variation of y preserving the same K' is associated with variation of dimensions of hot and cold heat sinks preserving the same total mass at other things being equal; greater y means larger heat sink on the cold side, smaller on the hot side. Thermal conductivity of a material k is embedded in two dimensionless parameters: F and K'. Dimensionless cooling capacity, heating capacity and supplied power indicate fraction of corresponding power density relative to maximal specific cooling capacity q_{0max} at j_{0max}, calculated by classical equations. If $K' \to \infty$ and $j' \to 1$, q'_c, q'_h, w' reach 1, 3, 2, respectively. Label F for ZT parameter is used here just for better visibility.

After substituting dimensionless parameters into Eq. 4 we get a system of equations:

$$\begin{aligned} q'_c &= \frac{2(1 - T'_c)yK'}{F} = 2j'T'_c - j'^2 - \frac{2(T'_h - T'_c)}{F} \\ q'_h &= \frac{2(T'_h - 1)(1 - y)K'}{F} = 2j'T'_h + j'^2 - \frac{2(T'_h - T'_c)}{F} \quad , \\ w' &= \frac{2K'}{F}\left(T'_h - 1 - y(T'_h - T'_c)\right) = 2j'\left(T'_h - T'_c\right) + 2j'^2 \end{aligned} \tag{5}$$

which can be solved for $T'_c(y, K', j', F), T'_h(y, K', j', F)$.

As one can see, all variables in the model are now dimensionless. The model allows to calculate temperatures and other parameters depending only on four variables which greatly facilitates the calculations. Compared to a model proposed by Lee[17] the main difference is that specific power densities

q'_c, q'_h, w' are calculated not relative to convection conductance of medium on the hot side, which means that in this case it should be initially provided, but relative to classical specific maximum cooling capacity, determined only by parameters of a material. Also, this principle we found more easy to perceive.

After determination of $T'_c(y, K', j', F), T'_h(y, K', j', F)$ equations for q'_c, q'_h will take a form:

$$q'_c = \frac{2 - j' - \frac{2j'}{K' - yK' - j'F}}{\frac{1}{j'} + \frac{F}{yK'} + \frac{1}{yj'(K' - yK' - j'F)}};$$
$$q'_h = \frac{(y-1)\left(2 + j' - \frac{K'(2+j')+2j'}{K' - yK' - j'F}\right)}{\frac{y}{j'} + \frac{F}{K'} + \frac{1}{j'(K' - yK' - j'F)}} \quad (6)$$

If a system/device is thermally symmetric ($y = 0.5$), then:

$$q'_c = \frac{\frac{1}{j'} - \frac{1}{2} - \frac{F(2-j')+2}{K'}}{\frac{1}{2j'^2} - \frac{2F^2}{K'^2} + \frac{2}{K'j'^2}}; \quad q'_h = \frac{\frac{1}{j'} + \frac{1}{2} + \frac{F(2+j')+2}{K'}}{\frac{1}{2j'^2} - \frac{2F^2}{K'^2} + \frac{2}{K'j'^2}} \quad (7)$$

COP, calculated as $\mathrm{COP} = q'_c/w'$ and dimensionless voltage, calculated as $u' = w'/j'$ are determined as:

$$\mathrm{COP} = \frac{\frac{Fj'}{K'} + \frac{1}{j'}}{1 + \frac{4F+4}{K'}} - \frac{1}{2}; \quad u' = \frac{1 + \frac{4F+4}{K'}}{\frac{1}{2j'} - \frac{2j'F^2}{K'^2} + \frac{2}{K'j'}} \quad (8)$$

If $K' \to \infty$ Eqs. 7 and 8 take a form of:

$$q'_c = 2j' - j'^2; \quad q'_h = 2j' + j'^2; \quad \mathrm{COP} = \frac{1}{j'} - \frac{1}{2}; \quad u' = 2j' \quad (9)$$

Equation 9 were used to compare the model with the classical theory.

To determine maximal dimensionless cooling capacity q'_{max} for thermally symmetric device we need to take the derivative $\frac{dq'_c}{dj'}$ and to equate it to zero, but it is too complex for Eq. 7, so we need to simplify it. When $K' \gg F$ term $2F^2/K'^2 \to 0$, and simplified equation for dimensionless maximal current intensity j'^*_{max} takes a form:

$$j'^*_{max} = \frac{4F + K' + 4 - \sqrt{(4F + K' + 4)^2 - 12FK'}}{6F} \quad (10)$$

Substituting Eq. 10 into Eq. 7, one can define a dimensionless maximum cooling capacity.

It is easy to receive actual cooling capacity Q_c, and voltage U from dimensionless q'_c, u':

$$Q_c = q'_c \frac{n\alpha^2 T^2 S}{2\rho L}; \quad U = u' \frac{n\alpha T}{2}. \quad (11)$$

METHODS

Validation of the Model

To validate the model, we tested four different types of TCMs at a quantity of three each. The results showed acceptable agreement with the model. Here we present typical results for one type of most common geometry (type A) and one with large number of TE pellets, large electrical and thermal resistance (type B), and, consequently, affected by minimal influence of contact electrical

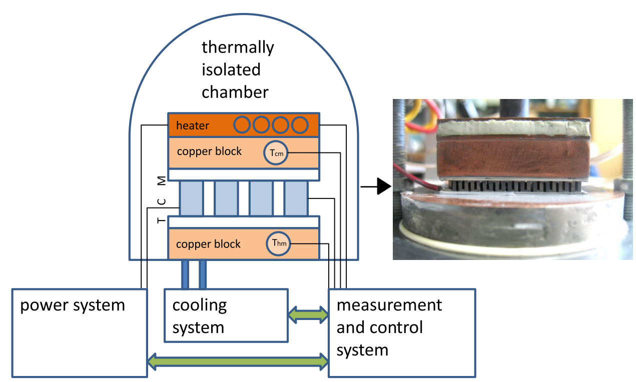

Fig. 3. Scheme of compensation method measurement of $U(I)$, $Q_c(I)$ of a TCM.

Table I. TCM initial parameters for simulations

	Actual parameters	
	Type A	**Type B**
Number of TE pellets, n	254	482
Cross-sectional area of a TE pellet, S; mm^2	1.96	1.0
Height of TE pellets, L; mm	1.6	2.5
Total electrical resistance of TE pellets at 300 K, R; Ohm	2.33	14.2
Total thermal resistance of TE pellets, R_p; K W^{-1}	1.945	2.777
Seebeck coefficient at 300 K, α; μV K^{-1}	226.0	239.7
	Dimensionless parameters	
ZT parameter of a TCM, F	0.825	0.783
Dimensionless conductance, K'	64.08	35.54

resistance and other inaccuracies. Initial TCM parameters are presented in Table I.

ZT parameter and electrical resistance *R* of modules were determined with use of a commercially available Z-meter (DX 4065, RMT Ltd). Measurement of $Q_c(I)$ and $U(I)$ was performed by widely used compensational method, where thermoelectric module is located and tested between a cooling substrate and a heater block, like shown on Fig. 3. Detailed description of an example of such measurement system could be found in Ref. 20. Temperature sensors are embedded into cooled and heated copper blocks. Hot medium temperature T_{hm} is fixed at 300 K with use of proportional-integral-derivative (PID)-controlled cooling system. Cold medium temperature T_{cm} is changed with variation of heat load, which is determined by electrical power of the heater. The TCMs and the heater were powered by lab direct current (DC) power supplies (HY5005E and HY3030E, Mastech), values of voltage and current were measured by high accuracy bench type multimeters (UT804, Uni-T) when $T_{cm} = T_{hm}$.

Determination of a dimensionless thermal conductance appeared to be the most challenging task. The following is the algorithm of the measurement is proposed. After conducting a measurement of *Z* and *R*, a TCM is mounted in the setup, as shown on Fig. 3, but not powered from the supply. When a heat load *H* is applied, and temperatures of the hot and the cold mediums are stabilized, the measurements of open circuit voltage U_{oc} and difference of temperatures of mediums ΔT_m are conducted. The measurements are repeated at different values of a heat load. Total thermal resistance of a system R_s, including thermal resistance of TE pellets, thermal interface materials, ceramics, and copper blocks, can be determined by a tangent of an angle of linear $\Delta T_m(H)$ dependence: $R_s = d\Delta T_m/dH$. In accordance with the model described above (Fig. 2), this thermal resistance for thermally symmetric system ($R_c = R_h$) can be represented as $R_s = R_p + 2R_a$, where R_p is total thermal resistance of TE pellets, R_a is thermal resistance between a TE material and a heat medium. If there were no parasitic thermal resistances ($R_a = 0$), the Seebeck coefficient could be

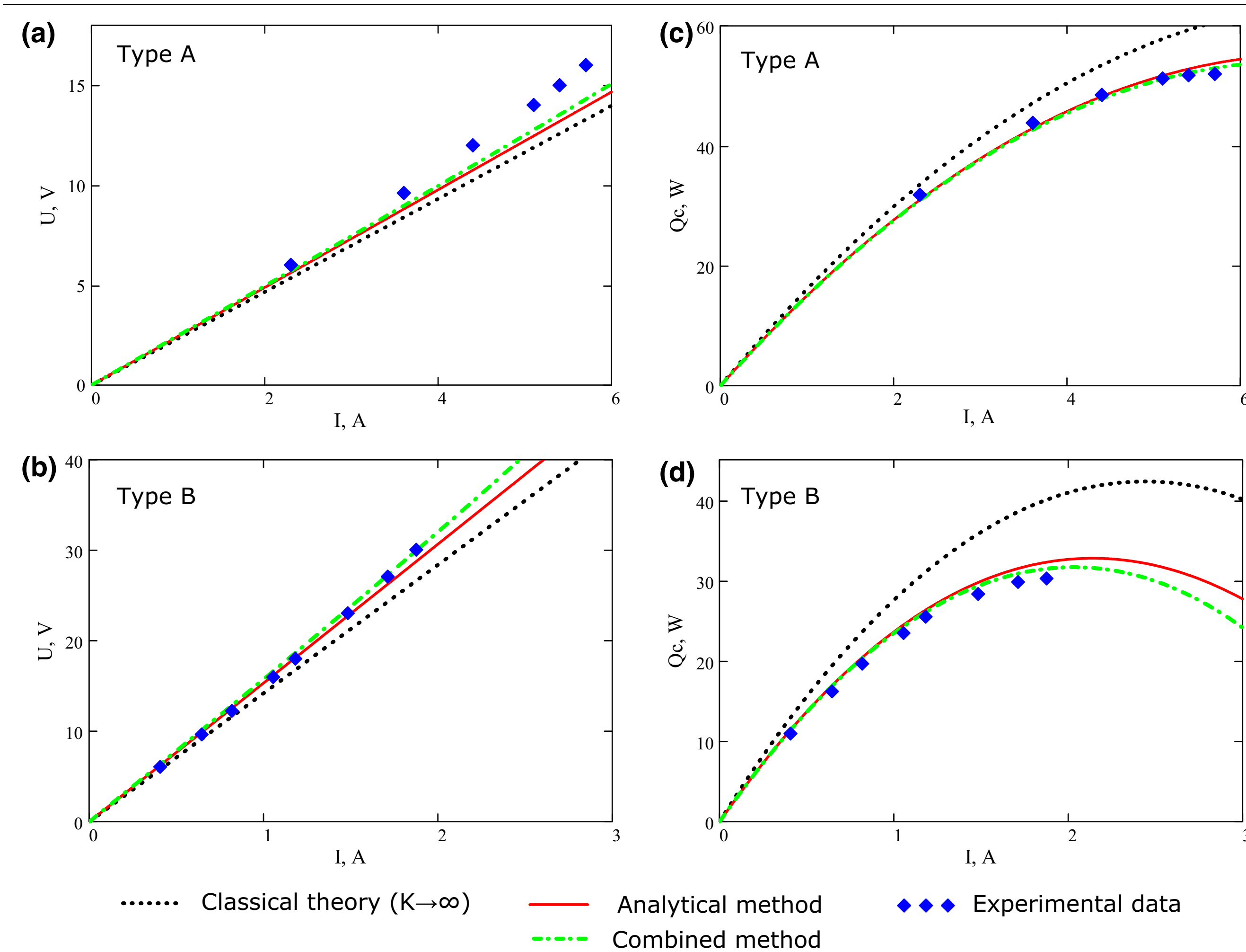

Fig. 4. Simulated and experimental $U(I)$ (a, b) and $Q_c(I)$ (c, d) dependence for type A and type B modules, respectively.

easily calculated as $\alpha = dU_{oc}/d\Delta T_m$. Otherwise, it is found to be defined as:

$$\alpha = \frac{ZR}{nR_s}\frac{d\Delta T_m}{dU_{oc}} \tag{12}$$

where $d\Delta T_m/dU_{oc}$ is a tangent of an angle of linear $\Delta T_m(U_{oc})$ dependence.

After determining α, R_p is calculated as $R_p = ZR/n^2/\alpha^2$ and, finally, dimensionless thermal conductance K' is calculated as $K' = 4R_p/(R_s - R_p)$.

Dimensionless $q'_c(j'), u'(j')$ dependences were simulated and converted into actual $Q_c(I)$ and $U(I)$ dependence with use of analytical and combined method (described below) and then compared with experimental data.

Modeling

Analytical calculations of $q'_c(j'), u'(j')$ by Eqs. 7 and 8 should be accurate at low current intensity and small difference between K_h, K_c, as the

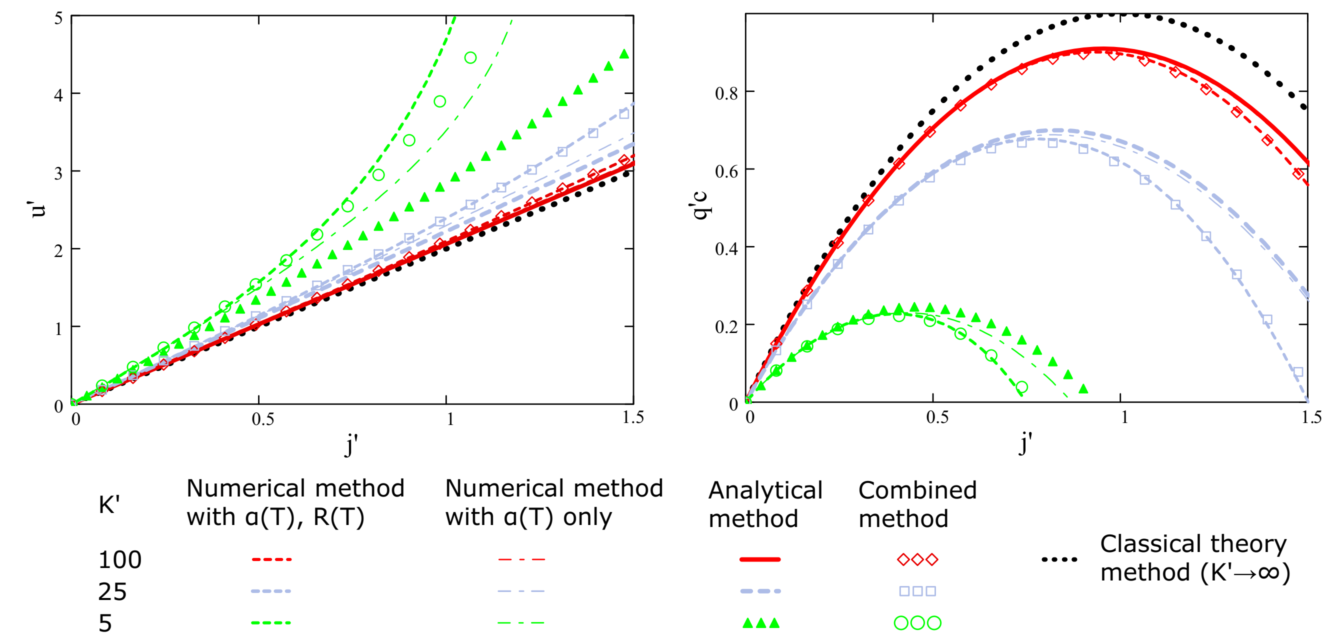

Fig. 5. $u'(j')$ and $q'_c(j')$ dependencies of TCM simulated by different methods.

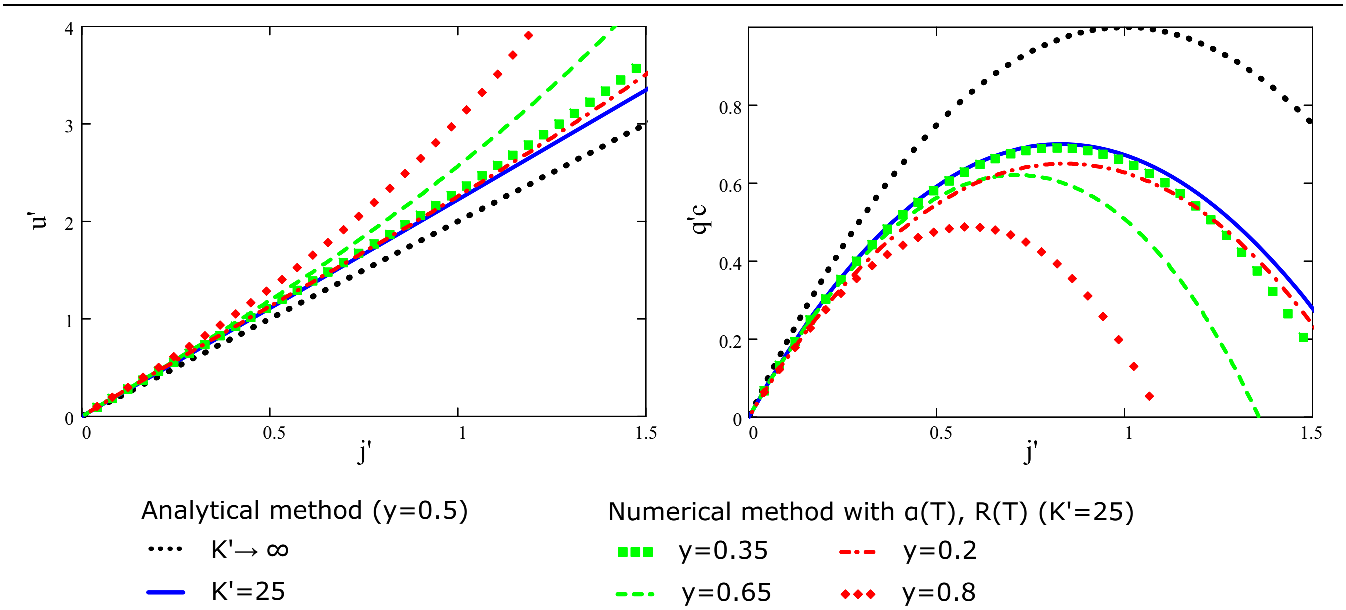

Fig. 6. $u'(j')$ and $q'_c(j')$ dependence in accordance with thermal conductance ratio y.

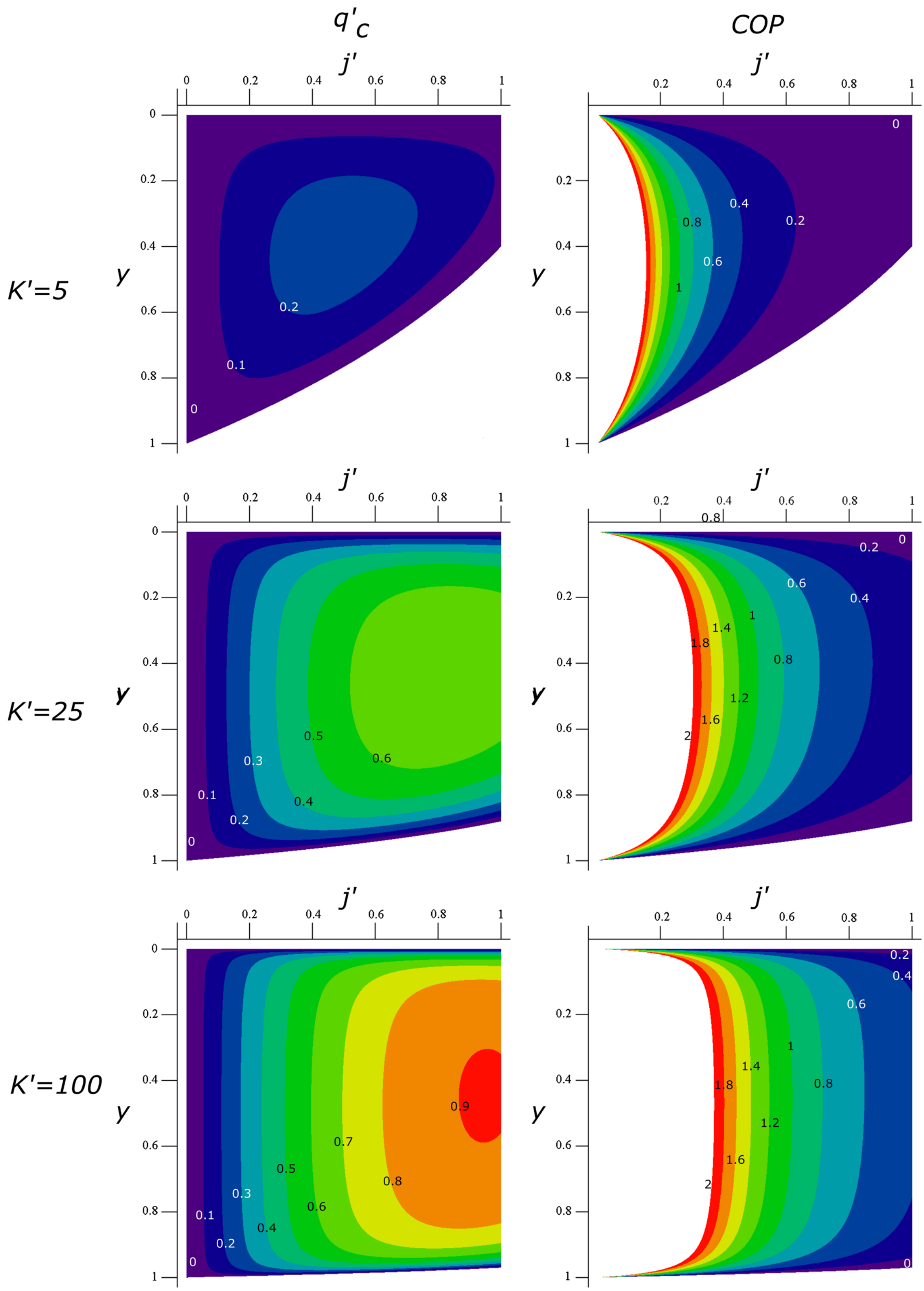

Fig. 7. $q'_c(y, j')$, COP(y, j') dependence for different K' at $ZT = 1$.

temperature dependencies are not considered. These calculations are called further as the analytical method. The most reliable simulation corresponds to solving Eq. 4 numerically with temperature dependence of $\alpha(T_c)$, $\alpha(T_h)$, $\rho(T_c, T_h)$ approximated with polynomials. This method was

Table II. Optimal parameters for maximal cooling capacity simulated for different K' and $ZT = 1$

Dimensionless thermal conductance, K'	Fraction of thermal conductance on the cold side, y	Dimensionless current intensity, j'	Dimensionless maximal cooling capacity, q'_{max}	Coefficient of performance, COP
1	0.34	0.15	0.026	0.324
2	0.36	0.26	0.078	0.348
5	0.38	0.48	0.245	0.370
25	0.42	0.84	0.694	0.434
100	0.44	0.95	0.908	0.485
1000	0.45	1.00	0.990	0.493

performed on PTC Mathcad 15.0 software and is called here the numerical method. It was also performed with $\alpha(T_c)$, $\alpha(T_h)$ dependence only to separate the effects of the Seebeck coefficient and resistivity dependence. Calculation of T'_h, T'_c from Eq. 5 with constant parameters, then determining $\alpha(T_c)$, $\alpha(T_h)$, $\rho(T_c, T_h)$ and calculating T_h, T_c by Eq. 4 once again substituting $\alpha(T_c)$, $\alpha(T_h)$, $\rho(T_c, T_h)$, obtained previously, is called the combined method. Classical theory method is used in the calculation by Eq. 1 for actual parameters or Eq. 9 for dimensionless parameters, coinciding with the analytical method when $K' \rightarrow \infty$. Temperature of heat mediums in all simulations is 300 K.

RESULTS AND DISCUSSION

Validation of the Model

Figure 4 depicts simulated and experimental results for TCMs with parameters represented in Table I. A value of dimensionless thermal conductance between mediums and TE material K' was determined as 64.08 for the type A module and 35.54 for the type B module using the method described above. Experimental $U(I)$ and $Q_c(I)$ dependence for the type B module are in good accordance with the numerical (not shown on the figure) and the combined method of the simulation, based on the model. This dependence behavior is close to the theory and was found for other TCMs with a large number of TE pellets. In TCMs with small amount of pellets and thermal resistance R_p $U(I)$ dependence were shifted to higher U values, as shown for the type A module. This effect can be connected with higher inaccuracy in determination of α, K' of a module due to its small electrical and/or thermal resistance, and influence of a nonlinear temperature gradient, found in different studies of TCMs[10–12] and not taken into account here.

Modeling

Influence of thermal conductance and temperature depended material properties on TCM characteristics is shown in Fig. 5. Dependencies are calculated for thermally symmetric system ($y = 0.5$). When dimensionless conductance is large ($K' = 100$), dependencies calculated by different methods almost merge, nevertheless $q'_c(j')$ is about 10% less compared to the classical theory. With decrease of K', u' rises, q'_c reduces rapidly, and the point of maximum cooling capacity shifts to lower current values. q'_c calculated by the analytical method demonstrates good match with the numerical method in a region of the current less than j'_{max}. The combined method demonstrates excellent coincidence with the numerical methods over the entire region. Based on numerical modeling, $\alpha(T)$ and $\rho(T)$ have roughly the same effect on $u'(j')$, shifting it to higher u' values compare to the classical model, but $\rho(T)$ has greater effect on $q'_c(j')$, especially with greater current values.

On Fig. 6 an effect of ratio of thermal conductance on the cold and the hot sides y is demonstrated. Characteristics of TCM are calculated by the numerical method in accordance with different ratio y preserving same K'. They are compared with the analytical method, when equal thermal conductances $K_h = K_c$ ($y = 0.5$) are assumed. It seems that the analytical method simulates characteristics of a device unreliably at high current intensities when K_c is much higher than K_h ($y > 0.65$). It is noticeable that q'_c has smaller values at $y = 0.5 + x$ rather than $y = 0.5 - x$, thus reducing thermal resistance on the hot side of a device is more preferred than that on the cold side.

Optimal y and j' for maximal q'_c in Eq. 6 could be defined for different K' or ZT-parameter (F). Actual optimal parameters are expected to be lower as Eq. 6 does not include temperature dependence. Contour plots of $q'_c(y,j')$ and COP(y, j') for $K' = 5\ldots100$ and $ZT = 1$ are shown on Fig. 7. Optimal y, j' and corresponding maximal q'_c and COP values for $K' = 1\ldots1000$ are summarized in Table II. Maximal cooling capacity is observed at $j' \rightarrow 1$ and COP $\rightarrow 0.5$ at higher K' values with respect to the classical theory. At lower K' values optimal j' and y are shifted to lower values. It is noticeable that even when thermal conductance on the sides of a device is

100 times higher than thermal conductance of TE pellets ($K' = 100$), which seems a fairly good condition for heat transfer, there is still a decrease of 9.2% in maximal cooling capacity compared to calculated by the classical theory. For $K' = 1000$ the decrease is of 1%. Thus, we can conclude that taking into account thermal resistance between a material and an environment or cooling object is of great importance.

CONCLUSIONS

In this work a dimensionless model for simulation of parameters of thermoelectric cooling device taking into account thermal resistance between a material and a heat medium is proposed. For calculation of the basic device parameters simple analytical equations are derived. The model can be used for determination of optimal design parameters for maximal cooling capacity of a device. The applicability of the analytical method is estimated in area of relatively low thermal resistance values ($K' \geq 5$) and close to thermally symmetric systems ($y = 0.2\ldots0.65$). The numerical and the combined methods are suggested for improved simulation throughout the whole range of y, K' values. It is shown that even rather small thermal resistance of components of a thermoelectric system can affect greatly on $U(I)$ and $Q_c(I)$ dependence. Thermal resistance on the hot side reduces cooling capacity more significantly than that on the cold side. A method for determining equivalent thermal resistances/conductances of thermally symmetric thermoelectric cooling system is proposed.

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