Relationship between Defect Size and Fatigue Life Distributions in Al-7 Pct Si-Mg Alloy Castings

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A new method for predicting the variability in fatigue life of castings was developed by combining the size distribution for the fatigue-initiating defects and a fatigue life model based on the Paris–Erdoğan law for crack propagation. Two datasets for the fatigue-initiating defects in Al-7 pct Si-Mg alloy castings, reported previously in the literature, were used to demonstrate that (1) the size of fatigue-initiating defects follow the Gumbel distribution; (2) the crack propagation model developed previously provides respectable fits to experimental data; and (3) the method developed in the present study expresses the variability in both datasets, almost as well as the lognormal distribution and better than the Weibull distribution.

DOI: 10.1007/s11661-009-9847-8

I. INTRODUCTION

STRUCTURAL defects, such as porosity and oxide inclusions, affect fatigue life of aluminum alloy castings by causing premature failure[1,2] and increasing the variability in properties. It has been of interest to engineers to model variability in fatigue life, so that parts can be designed accordingly. In a majority of the cases found in the literature, the variability in the fatigue life of aluminum alloy castings has been modeled by either the lognormal[3] or two-parameter Weibull,[2,4,5] the cumulative probability (P) functions of which are written as

$$P(N_f) = \int_0^{N_f} \frac{1}{N_f \omega \sqrt{2\pi}} \exp\left[\frac{-\left(\ln(N_f) - \theta\right)^2}{2\omega^2}\right] dN_f \quad [1]$$

$$P(N_f) = 1 - \exp\left(-\left(\frac{N_f}{N_0}\right)^q\right) \quad [2]$$

respectively. In Eq. [1], N_f is the number of fatigue cycles until final fracture, *i.e.*, fatigue life; and ω and θ are the standard deviation and average of $\ln(N_f)$, respectively. In Eq. [2], N_0 is the scale parameter and q is the shape parameter, alternatively referred to as the Weibull modulus. Although, in most cases, these two distributions provide acceptable fits to the data, both distributions disregard the main source of the scatter observed in fatigue life: the variability in the defect sizes in the specimens.

There have been several attempts to link the distribution of defect sizes with the fatigue life of aluminum castings. Casellas *et al.*[4] followed the approach taken by Jayatilaka and Trustrum[6] to fit a power equation to the upper tail of the defect size distribution, which was originally developed for ceramics.[7] Casellas *et al.* showed that the Weibull modulus for the distribution of fatigue life is a function of the power of the equation fitted to the upper tail of the defect size distribution. Although this approach can be taken as a good approximation, the size distribution of the largest defects should theoretically follow one of the extreme value distributions, and the power fit by Jayatilaka and Trustrum does not constitute an extreme value distribution. In another study, Yi *et al.*[8,9] assumed that pore size in A356 castings follows the lognormal distribution, which is consistent with their histograms as well as statistical analysis of pore sizes for Mg alloy castings.[10] The authors, using fatigue crack propagation models and maximum pore size data, attempted to estimate the distribution of fatigue life of A356 castings. Although the results were promising, cumulative probability plots indicated systematic lack of fit.

The present study addresses a gap in the literature by linking the size distribution of defects found on fracture surfaces and fatigue life distributions. A theoretical approach is first presented and then applied to two datasets from the literature.

II. THEORETICAL BACKGROUND

To establish the relationship between fatigue life and defect size distributions, one needs to (1) determine the size distribution of failure-initiating defects and (2) a crack propagation model based on fracture mechanics to estimate fatigue life. These two aspects are discussed first.

A. *Defect Size Distribution*

All fatigue models based on the microstructure need the size distribution of defects as an input. In the absence of defects, fatigue cracks will start from slip planes within primary phase grains. Fatigue life, when

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Manuscript submitted October 26, 2008.
Article published online May 21, 2009

failure is initiated from slip planes, is several orders of magnitude higher than when defects are present.[2,11] This study addresses the case when defects, such as oxides and pores, are present, which is unfortunately the norm for castings.

The selection of the appropriate distribution affects the accuracy of the model outcomes and is therefore a crucial step in the construction of a microstructure-based fatigue model. A survey of the literature by the author[12] has shown that various statistical distributions have been used for the size of the fatigue-initiating defects in metals. In a majority of the studies in the literature, the authors did not provide a goodness-of-fit test to determine whether the distribution of choice was appropriate for the data.

To model the effect of structural defects on the fatigue performance of metals, two main approaches have been taken by researchers in the literature: (1) taking the entire pore size distribution into account[8,13,14] or (2) modeling the distribution of the largest defects or inclusions[4,15,16] that initiate fracture. For instance, the model of Yi *et al.*[8,9] assumes that pore size follows the lognormal distribution, which is consistent with their histograms as well as statistical analysis of pore sizes for Mg alloy castings.[10] Laz and Hillberry[13] also used the lognormal distribution for defect sizes in the 2024-T3 alloy in their model and found that crack initiating inclusions were primarily from the upper tail of the inclusion size distribution as expected. Gruenberg *et al.*[14] reached the same conclusion in their study on modeling the fatigue variability in 7075-T6 aluminum alloy by using the lognormal distribution. They suggested that an extreme value distribution be used for modeling purposes, but did not indicate which extreme value distribution should be used.

The second approach taken in microstructure-based models is to assume a size distribution for the largest pores and inclusions. Zamber and Hillberry[15] developed a model to predict fatigue lives of 2024-T3 aluminum alloy components with corrosion pits. They measured the size of crack-initiating pits and found that the Gumbel distribution, an extreme value distribution, provided excellent fits to the measured size of the crack-initiating pits. Przystupa *et al.*[16] suggested that the pores in 7050-T7451 alloy components follow a log-Gumbel distribution, and they estimated the largest pores from this distribution. The authors did not provide a reason why this distribution was selected. Moreover, the fits to measured pore sizes suggest a significant degree of lack-of-fit. For cast Al and Mg alloys, the size distribution of fatigue-initiating defects has also been reported as lognormal.[17–19]

If the largest defects (upper tail of the defect size distribution) are responsible for initiating cracks, as suggested by Murakami,[20] then their size has to follow an extreme value distribution, based on mathematical statistics.[21,22] For the largest values from *any* parent distribution, Gnedenko[23] defined three types of limiting extreme value distributions: the Gumbel distribution (type 1), the Fréchet distribution (type 2), and the Weibull distribution (type 3). Gnedenko also showed that the distribution of the largest (or smallest) values is determined by the distribution from which the sample is taken. For distributions decreasing exponentially at upper tails, such as in exponential, normal, and lognormal distributions, the distribution of the largest values will be Gumbel.

Tiryakioğlu[24] analyzed the size of fatigue-initiating defects found on the fracture surfaces of four different Al and three Mg castings from 17 datasets in the literature by using the equivalent defect diameter, d_{eq}:

$$d_{eq} = \sqrt{4A_i/\pi} \quad [3]$$

where A_i is the area of the fatigue-initiating defect on the fracture surface. By using the general extreme value distribution, the author found that 16 of the datasets followed the Gumbel distribution, for which the cumulative probability, P, can be written as

$$P = \exp\left(-\exp\left(\frac{d_{eq}-\lambda}{\delta}\right)\right) \quad [4]$$

where λ and δ are location and scale parameters, respectively. Tiryakioğlu pointed out that the results are consistent with the size of inclusion defects measured on the fracture surfaces of steel castings, which have been shown to follow the Gumbel distribution.[25–28]

B. *Modeling Crack Propagation*

For a part that contains a cracklike defect of initial length a_i, the crack length increases to a value, a, at any given number of stress cycles, N. The fatigue crack growth rate in the power-law or steady-state stage, as expressed by the Paris–Erdoğan law,[29] can be written as

$$\frac{da}{dN} = C(\Delta K_{\text{eff}})^m \quad [5]$$

where C and m are Paris–Erdoğan constants and ΔK_{eff} is the effective stress intensity factor range, which can be written as

$$\Delta K_{\text{eff}} = 2UY\sigma_a\sqrt{\pi a} \quad [6]$$

where U is the crack closure factor, Y is the compliance calibration factor, and σ_a is the alternating stress amplitude. Taking Y as independent of defect size with propagating crack introduces only a minor error to the model.[30] The term U can also be assumed to be only a function of the stress ratio, R.[5] Inserting Eq. [6] into Eq. [5], the Paris–Erdoğan equation can be integrated as

$$\int_{a_i}^{a_f} a^{-\frac{m}{2}}da = 2^m\pi^{\frac{m}{2}}CU^mY^m\sigma_a^m\int_{N_i}^{N_f} dN \quad [7]$$

where N_i is the number of cycles to initiate a fatigue crack, and a_f is the final crack length just before final fracture at N_f cycles. After integrating, we obtain

$$\frac{2}{2-m}a^{\frac{2-m}{2}}\Big|_{a_i}^{a_f} = 2^m\pi^{\frac{m}{2}}CU^mY^m\sigma_a^mN\Big|_{N_i}^{N_f} \quad [8]$$

$$\frac{2}{2-m}\left(a_f^{\frac{2-m}{2}} - a_i^{\frac{2-m}{2}}\right) = 2^m \pi^{\frac{m}{2}} C U^m Y^m \sigma_a^m \left(N_f - N_i\right) \qquad [9]$$

Because $a_f \gg a_i$, Eq. [9] can be simplified as

$$a_i^{\frac{2-m}{2}} = (m-2)2^{m-1}\pi^{\frac{m}{2}} C U^m Y^m \sigma_a^m \left(N_f - N_i\right) \qquad [10]$$

Let us now assume that the projected area of the defect, A_i, can be written as βa_i^2, where β is a coefficient determined by the geometry of the defect. Consequently, Eq. [10] can now be written as

$$A_i^{\frac{2-m}{4}} = (m-2)2^{m-1}\beta^{\frac{2-m}{4}}\pi^{\frac{m}{2}} C U^m Y^m \sigma_a^m \left(N_f - N_i\right) \qquad [11]$$

After rearranging and further simplification, we obtain

$$N_f = N_i + B\sigma_a^{-m} A_i^{\frac{2-m}{4}} \qquad [12]$$

where

$$B = \frac{2^{1-m}}{m-2}\beta^{\frac{m-2}{4}}\pi^{\frac{-m}{2}} C^{-1} U^{-m} Y^{-m} \qquad [13]$$

In several studies, cracks were observed to grow from structural defects at or shortly after the first stress cycle.[31–33] Therefore, N_i can be taken as zero in aluminum castings.

Equation [12] is a simplistic but useful tool to estimate fatigue life based on stress levels and defect size distributions. The validity of Eq. [12] (with $N_i = 0$) was verified by Davidson *et al.*[34] and Wang *et al.*[35] In these studies, the authors provided A_i *vs* N_f plots and found that the slope of $\log(A_i)$ *vs* $\log(N_f)$ is approximately –0.5, indicating that (1) N_f is related to $\sqrt{A_i}$ in cast Al-Si alloys and (2) $m \approx 4$. Similarly, Murakami and Endo[36,37] used $\sqrt{A_i}$ as a parameter to model the fatigue limit of surface and internal defects in steels. Moreover, they showed that the fatigue strength is governed by one critical inclusion, which usually has the largest size, not by the presence of many inclusions.

C. *Evaluating Statistical Fits*

The most common goodness-of-fit test is the use of a probability plot in which the axes are transformed to provide a linear relationship if the data indeed come from the distribution being tested. The probability plot is the only technique that has been employed for the size of fatigue-initiating defects to test whether they follow the lognormal,[17,18] Gumbel,[24] and Weibull distributions.[2,38] The use of a probability plot is subjective and insufficient to test the hypothesis that the data come from the particular distribution. That is why it is strongly recommended that probability plots be always augmented by formal goodness-of-fit hypothesis tests.[39]

A statistic measuring the difference between the fitted distribution and the data is called an empirical distribution function (EDF) statistic. There are two types of EDF statistics:[40] supremum and quadratic. Supremum statistics measure the maximum discrepancy between the fit and the data, whereas quadratic statistics measure the discrepancy for all data. The Anderson–Darling goodness-of-fit test statistic,[41] a quadratic test, is known for its sensitivity to the tails of the distribution and was shown[42,43] to be superior to a majority of other goodness-of-fit tests for a variety of distributions. The test statistic, A^2, is written as

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}[(2i-1)\ln(P(x_i)) + (2n+1-2i)\ln(1-P(x_i))] \qquad [14]$$

where n is the sample size, i is the rank in ascending order, and $P(x_i)$ is the cumulative probability for each data point, calculated with the estimated distribution parameters. The lower the value of A^2, the higher the confidence that data follow the hypothesized distribution. The hypothesis is rejected when the p value is less than a specified value for type I error (α), which is typically prescribed as 0.05.

III. ESTIMATING THE FATIGUE LIFE DISTRIBUTION FROM THE DEFECT SIZE DISTRIBUTION

Let us assume that the equivalent diameter of fatigue-initiating defects follows the Gumbel distribution (the validity of this assumption will be tested later). Equation [4] can be rearranged to obtain the inverse function of the Gumbel distribution for equivalent defect diameter as

$$d_{eq} = \lambda + \delta(-\ln(-\ln(P))) \qquad [15]$$

Hence, A_i can be calculated as

$$A_i = \frac{\pi(\lambda + \delta(-\ln(-\ln(P))))^2}{4} \qquad [16]$$

Inserting into Eq. [12] and solving for P, we obtain

$$P = \exp\left(-\exp\left(\frac{\lambda}{\delta} - \frac{2}{\delta\sqrt{\pi}}\left(\frac{N_f - N_i}{B\sigma_a^{-m}}\right)^{\frac{2}{2-m}}\right)\right) \qquad [17]$$

In Eq. [17], P represents the exceedance probability because a low defect size implies long fatigue life. Hence, the cumulative statistical distribution of fatigue life, $P(N_f)$, is found by subtracting Eq. [17] from unity:

$$P\left(N_f\right) = 1 - \exp\left(-\exp\left(\frac{\lambda}{\delta} - \frac{2}{\delta\sqrt{\pi}}\left(\frac{N_f - N_i}{B\sigma_a^{-m}}\right)^{\frac{2}{2-m}}\right)\right) \qquad [18]$$

The validity of Eq. [18] was assessed by using two datasets from the literature and goodness-of-fit statistics.

IV. DATA ANALYSIS, RESULTS, AND DISCUSSION

Two datasets were used to examine the relationship between defect size and fatigue life distributions:

(1) A356 aluminum alloy castings with high (0.20 to 0.23 ppm) hydrogen levels by Yi *et al.*[8,9,44,45] The specimens were tested at a stress ratio (R) of 0.1 and two maximum stress levels: 120 and 150 MPa (σ_a being 54 and 68 MPa, respectively). A total of 56 specimens were tested.
(2) Al-7 wt pct Si-0.6 wt pct Mg-0.11 wt pct Fe castings by Davidson *et al.*[34] The specimens were tested at $R = 0$ and three maximum stress levels: 170, 190, and 210 MPa (σ_a being 85, 95, and 105 MPa, respectively). A total of 64 specimens were tested.

A. *Defect Size Distribution*

In both studies, fatigue-initiating defect sizes on fracture surfaces were measured. Gumbel probability plots for the two datasets are shown in Figure 1, where cumulative probability, P, was assigned to each data point using the following probability estimator:

$$P = \frac{i - 0.5}{n} \qquad [19]$$

The Gumbel parameters estimated by the maximum likelihood method are presented in Table I, which also shows the results of the goodness-of-fit hypothesis test using the Anderson–Darling test statistic, A^2. Because p values for both cases are in excess of 0.05, the hypotheses that the datasets follow the Gumbel distribution could not be rejected. These results are in agreement with previous findings.[24]

B. *Effect of Defect Size on Fatigue Life*

After assuming that $N_i = 0$, as reported in earlier studies,[31–33] Eq. [12] can be written as

$$\log(N_f) = \log(B) - m\log(\sigma_a) + \frac{2-m}{4}\log(A_i) \qquad [12a]$$

Equation [12a] suggests a linear relationship between $\log(N_f)$ and $\log(A_i)$ with a slope of $(2 - m)/4$ and an intercept of $(\log(B) - m\log(\sigma_a))$. The fatigue life of castings tested at a stress level of $\sigma_{a(1)}$ can be adjusted to an equivalent fatigue life, $N_{f(eq)}$, at a different stress level, $\sigma_{a(2)}$, by Eq. [12]:

$$N_{f(eq)} = N_f\left(\frac{\sigma_{a(1)}}{\sigma_{a(2)}}\right)^m \qquad [12b]$$

The data collected by Yi *et al.* at a maximum stress of 150 MPa were adjusted by using Eq. [12b] for equivalent fatigue lives at a maximum stress of 120 MPa. The values of B and m were adjusted by using the Newton–Raphson method so that yield of a linear relationship between $\log(N_f)$ and $\log(A_i)$ with the smallest error was obtained while adjusting the stress level simultaneously using Eq. [12b]. The same approach was followed with the data of Davidson *et al.*, where equivalent fatigue lives at a maximum stress of 170 MPa were determined. The results are presented in Figure 2, which shows reasonable agreement between the data and Eq. [12a], with B and m values listed in Table II. Note that m values for both datasets are very close to each other and fall within the values reported previously[46] for Al-7 pct Si-0.6Mg (D357) alloy aerospace castings.

It is noteworthy that very respectable fits are obtained in Figure 2 with the assumption that fatigue starts on the first cycle. The fact that fatigue often starts from the first cycle or very early cycle can be taken as evidence that the material is precracked, so that no initiation is required. The obvious precrack is a pore or a bifilm, both of which originate from external, entrainment events.[47]

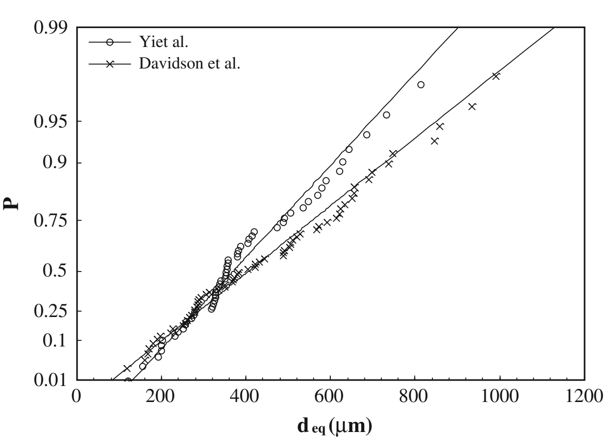


Fig. 1—Gumbel probability plots for the size data of fatigue initiating defects.

Table I. Gumbel Fits to d_{eq} Data from the Two Datasets

	λ (μm)	δ (μm)	A^2	p Value
Yi *et al.*	323.2	125.5	0.502	0.212
Davidson *et al.*	343.7	170.4	0.379	>0.250

C. *Modeling Variability in Fatigue Life of Al-7 Pct Si-Mg Alloy Castings*

The cumulative probability plots for the two datasets and the fit by Eq. [18] with parameters listed in Tables I and II are shown in Figure 3. The data were assigned probabilities using Eq. [19]. Note in Figure 3 that there is excellent agreement between the data and the fatigue life distribution (Eq. [18] developed in this study). The goodness of fit of Eq. [18] was evaluated using A^2 statistics. The results are presented in Table III, with p values for both cases being in excess of 0.250, based on the values listed by Stephens.[40] Hence, the hypothesis that fatigue life follows the statistical distribution in Eq. [18] cannot be rejected for either dataset.

For comparison purposes, lognormal and two-parameter Weibull distributions were also fitted to the fatigue life data by the maximum likelihood method. The fits are presented in Figure 4 and the estimated parameters as well as the results of goodness-of-fit tests are given in Table IV. For both datasets, the hypothesis that the data follow the lognormal distribution could not be rejected. The two-parameter Weibull distribution, however, did not provide acceptable fits to either dataset.

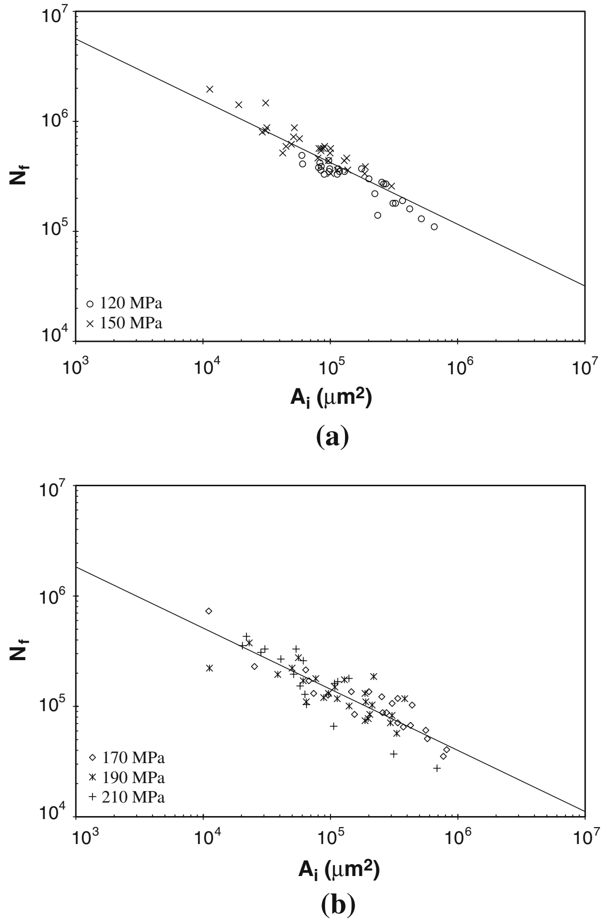


Fig. 2—Fatigue life data of (*a*) Yi *et al.* adjusted for a maximum stress of 120 MPa and (*b*) Davidson *et al.* adjusted for a maximum stress of 170 MPa, plotted as a function of the area of fatigue-initiating defect. Data were categorized based on the initial maximum stress level to expose any systematic errors. For both cases, there is no systematic error after stress adjustment.

Table II. Values of *m* and *B* in Equation [12] Estimated for Both Datasets

	m	B
Yi *et al.*	4.25	6.16×10^{15}
Davidson *et al.*	4.21	1.14×10^{16}

It is noteworthy that the two-parameter Weibull distribution is rejected for both datasets. The Weibull distribution is based on the weakest link theory and largest defects in castings are taken as the weakest links. The results for both datasets emphasize that the two-parameter Weibull distribution should be used with caution to model the variability in the fatigue life of Al-7 pct Si-Mg alloy castings. It is also advised that formal hypothesis for goodness of fit be conducted when a particular distribution is selected to model the variability in mechanical properties of aluminum castings.

A comparison of Tables III and IV shows that fits provided by Eq. [18] are much better than those by the two-parameter Weibull distribution and almost as good

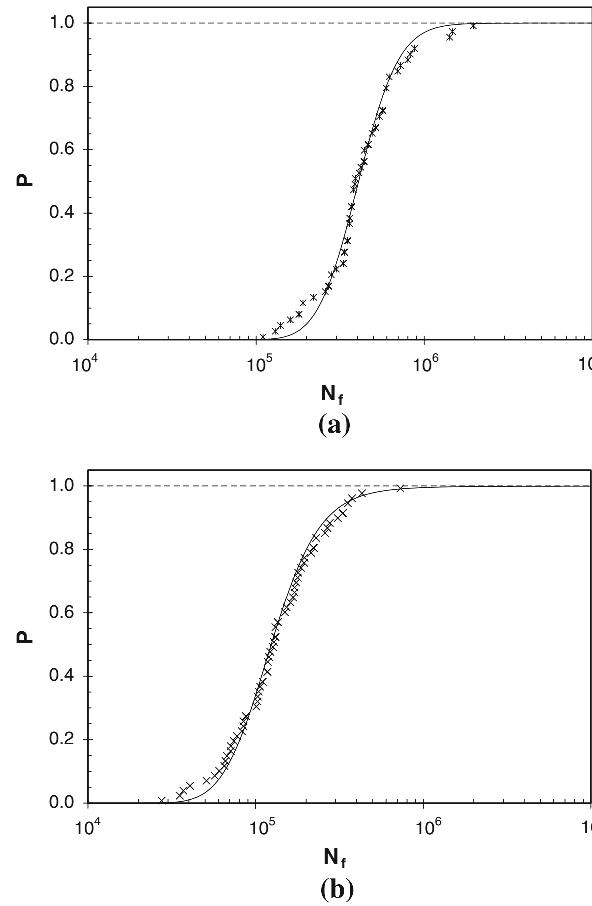


Fig. 3—Cumulative probability plot for fatigue life data of (*a*) Yi *et al.* and (*b*) Davidson *et al.* The fits obtained by using Eq. [18] are indicated by the solid curves.

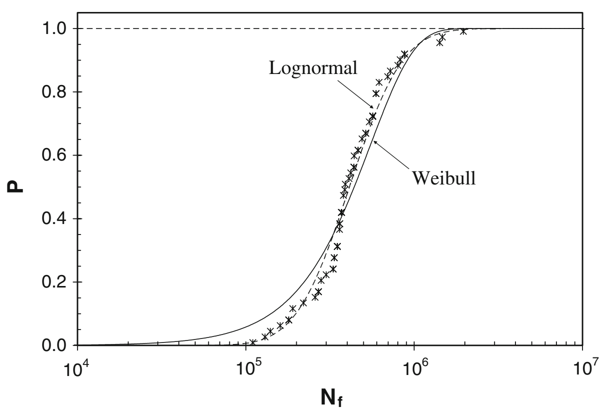
Table III. Goodness-of-Fit Results for Equation [18] to the Two Datasets

	A^2	p Value
Yi *et al.*	1.091	>0.250
Davidson *et al.*	0.678	>0.250

as those by the lognormal distribution. Because Eq. [18] incorporates the size distribution of fatigue-initiating defects and the mechanics of fatigue failure, it is recommended that Eq. [18] be used to characterize the distribution of fatigue life of Al-7 pct Si-Mg alloy castings.

It should be noted that two simplifying assumptions were made in the development of the method in the present article: (1) $a_f \gg a_i$, leading to Eq. [10]; and (2) $N_i = 0$. Both assumptions introduce some error to the model. For the first assumption, the magnitude of error increases with a_i/a_f. Taking $m = 4.2$, the error is calculated as 8 and 17 pct when $a_i/a_f = 0.1$ and 0.2, respectively. These error contributions are at an acceptable level. The second assumption, $N_i = 0$, is reasonable, based on the observations reported in the literature. This assumption, however, may not be valid if (1) casting quality is high so that there are fewer (or no) defects and their sizes are smaller than the those in the two datasets used in this study, and (2) maximum stress achieved during the test is at such a low level that it may take a significant number of cycles to initiate a fatigue crack. Hence, this assumption may need to be abandoned, based on the dataset. Note that, when $N_i > 0$, the $\log(N_f)$-$\log(A_i)$ relationship is not linear, with data deviating from the line progressively more at high values of A_i. Therefore, the need to discard the assumption that $N_i = 0$ can be detected by trends in the data.

Turning our attention back to the variability in the fatigue life, it is of interest to plot probability density functions, f, of distributions for comparison purposes. The probability density function for Eq. [18] can be found by differentiating $P(N_f)$ with respect to N_f:



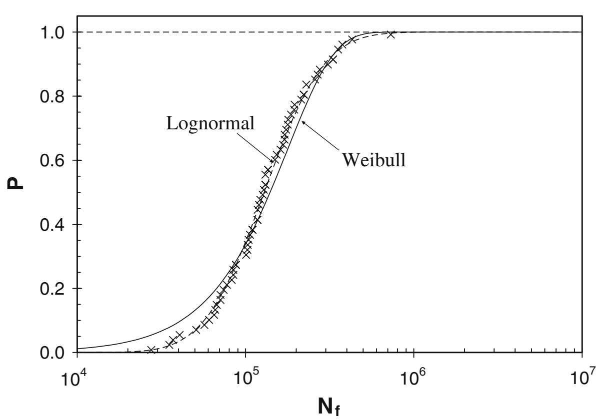


Fig. 4—Cumulative probability plots for the lognormal and Weibull fits to data of (*a*) Yi *et al.* and (*b*) Davidson *et al.*

$$f(N_f) = \frac{4B^{\frac{2}{m-2}}\sigma_a^{\frac{2m}{2-m}}}{(m-2)\delta\sqrt{\pi}}(N_f - N_i)^{\frac{m}{2-m}} \times \exp\left(\frac{\lambda}{\delta} - \frac{2}{\delta\sqrt{\pi}}\left(\frac{N_f - N_i}{B\sigma_a^{-m}}\right)^{\frac{2}{2-m}}\right) \times \exp\left(-\exp\left(\frac{\lambda}{\delta} - \frac{2}{\delta\sqrt{\pi}}\left(\frac{N_f - N_i}{B\sigma_a^{-m}}\right)^{\frac{2}{2-m}}\right)\right) \quad [20]$$

The probability density functions for the two datasets with the adjusted maximum stress levels, as described previously, are presented in Figure 5. Note that the total area under the two curves is 1.0, but the curve for Yi *et al.* appears smaller because that x-axis is plotted logarithmically. Equation [20] allows one to plot the effects of defect size parameters and test conditions, such as stress levels, on the fatigue life distribution. For instance, the fatigue life distribution for the three maximum stress levels used by Davidson *et al.* along with corresponding values from Tables I and II were used to plot the curves in Figure 6. With increasing maximum stress, the distribution is shifted to lower values of N_f, as expected.

The findings presented in this study are an important first step for modeling variability due to casting defects. To the author's knowledge, this study is the first time that variability in fatigue life is formalized by a distribution function derived from the defect size distribution and a fatigue crack propagation model. The effect of other factors, such as matrix strength and microstructure, is implicit in the constants B and m. With the data available on crack propagation rates in

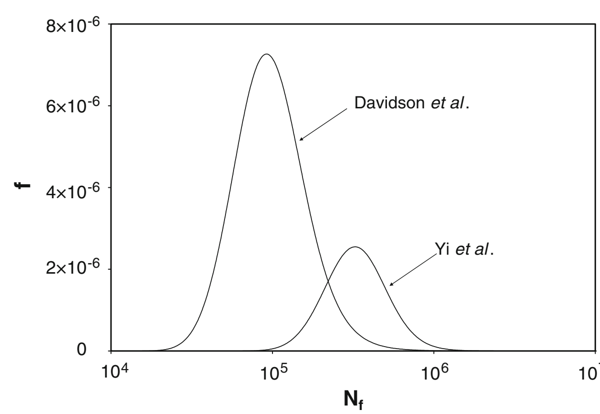


Fig. 5—Probability density functions (Eq. [20]) plotted for the two datasets after adjusting the maximum stress levels, as described previously.

Table IV. Lognormal and Weibull Fits to N_f Data from the Two Datasets

	Lognormal				Two-Parameter Weibull			
	θ	ω	A^2	p Value	N_0	q	A^2	p Value
Yi *et al.*	12.93	0.572	0.606	0.110	5.49×10^5	1.653	1.862	<0.010
Davidson *et al.*	11.77	0.641	0.148	0.963	1.77×10^5	1.550	1.050	<0.010

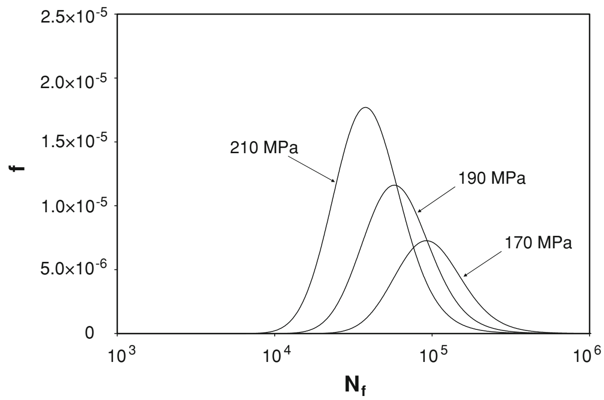


Fig. 6—Effect of maximum stress level on fatigue life distribution for the data of Davidson *et al.*

Al-7 pct Si-Mg alloys and sophisticated computer modeling software for fatigue, it is possible to determine reasonable estimates of the parameters B and m.

In several studies,[2,35,48] multiple defect distributions (multiple fatigue initiators) were reported for cast Al-7 pct Si-Mg alloy castings. The approach taken by Johnson[49] to account for different defect distributions, to model fracture strength with the Weibull distribution, can be applied to the new distribution developed in this study when different types of fatigue initiators, including slip planes, need to be considered.

It is of course more desirable to estimate the fatigue performance of aluminum castings without having to measure fatigue-initiating defects on fracture surfaces. Such an attempt has been made by Wang and Jones[50] in 319 alloy castings. The authors tried to estimate the largest defects by measuring the maximum size of pores on polished planes of castings and combining the results statistically by using the Gumbel distribution. However, pores found on fracture surfaces were larger than the largest pores found on polished sections by an order of magnitude. Similar results were found by Yi *et al.*[9] in A356 castings and Przystupa *et al.*[16] in 7050-thick plate. Hence, more research is needed before the size of fatigue-initiating (largest) pores can be estimated in a nondestructive manner.

V. CONCLUSIONS

1. The size distribution for fatigue-initiating defects in Al-7 pct Si-Mg alloys is Gumbel.
2. The fatigue life model based on the Paris–Erdoğan equation with no crack initiation stage provides respectable fits to the two datasets from the literature.
3. A new distribution function combining the size distribution of fatigue-initiating defects and fatigue life model provides fits to the two datasets that could not be rejected by goodness-of-fit tests.
4. Goodness-of-fit hypothesis tests showed that the two-parameter Weibull did not provide an acceptable fit to the fatigue life data, whereas the lognormal distribution did.
5. Because the new distribution function incorporates the defect size distribution and fracture mechanics, it is recommended that it be used to characterize the statistical distribution of fatigue life of cast Al alloys. The simplifying assumption that $N_i = 0$ can be abandoned if deemed necessary, based on the trends in the data.

ACKNOWLEDGMENTS

The author thanks Professor Peter D. Lee (Imperial College) for sharing his data, Dr. Qigui Wang (General Motors), Professor John Campbell (University of Birmingham, Birmingham, United Kingdom), and Professor Anthony Rollett (Carnegie Mellon University) for their comments on the manuscript.

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