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Adaptive Fuzzy Sliding Mode Control of Under-actuated Nonlinear Systems

Amir Hossein Davaie Markazi Mohammad Maadani Seyed Hassan Zabihifar
Nafiseh Doost-Mohammadi

School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran

Abstract: A new extension of the conventional adaptive fuzzy sliding mode control (AFSMC) scheme, for the case of under-actuated and uncertain affine multiple-input multiple-output (MIMO) systems, is presented. In particular, the assumption for non-zero diagonal entries of the input gain matrix of the plant is relaxed. In other words, the control effect of one actuator can propagate from a subgroup of canonical state equations to the rest of equations in an indirect sense. The asymptotic stability of the proposed AFSM control method is proved using a Lyapunov-based methodology. The effectiveness of the proposed method for the case of under-actuated systems is investigated in the presence of plant uncertainties and disturbances, through simulation studies.

Keywords: Adaptive fuzzy sliding mode control (AFSMC), nonlinear systems, uncertain systems, under-actuated systems, remote environmental monitoring units (REMUS).

1 Introduction

Many of the existing methods for control of nonlinear systems need a rather accurate model of the plant. In recent decades, many control methods for less-known nonlinear systems have been developed[1,2]. The well-known, sliding mode control (SMC) which is based on the theory of variable structure systems is a powerful method for control of uncertain nonlinear systems[3,4]. In order to preserve the closed-loop stability of uncertain systems, conventional SMC methodology may potentially suffer from chattering in the control input signal, when the estimated bound of uncertainty is not small enough. Such a high-frequency chattering may damage the actuators and also excite the un-modeled high-frequency dynamics of the system which degrades the control performance and may even lead to instability[5].

Fuzzy logic control (FLC) methodology has been widely considered as another alternative for coping with nonlinearities and unknown dynamics and external disturbances[6, 7]. Conventional FLC methods suffer from lack of systematic methods for incorporation of human knowledge into the rule base of a fuzzy inference system, for guaranteeing the closed-loop stability[8, 9].

In order to exploit the best of the SMC and FLC methods, different combinations of those methods have been proposed in the literature[10, 11]. The objective of this new class of methods is to cope with uncertainty and external disturbances, and at the same time prevent the chattering phenomena, as much as possible.

One such hybrid control approach is the so called, adaptive fuzzy sliding mode control (AFSMC). AFSMC is, in particular, suitable for systems with a rather large bound of uncertainty[12,13]. In this approach, fuzzy control rules can be determined systematically and the asymptotic stability of the closed loop system can be guaranteed under certain conditions[14].

In [15], an indirect AFSM control is proposed to strengthen the tracking control performance of a certain class of multiple-input multiple-output (MIMO) nonlinear uncertain systems. This indirect approach requires the so called multiple estimation algorithms. In [16], a new AFSM controller with a model predictor is proposed for a class of uncertain nonlinear systems with unknown constant input time delay. In [17], an adaptive type-2 fuzzy sliding mode control to tolerate actuator faults of unknown nonlinear systems is proposed while two adaptive type-2 fuzzy logic systems are used to approximate the unknown functions. They also considered that the G matrix is always non-zero. In [18], a stable adaptive fuzzy sliding mode controller is investigated for a class of uncertain underactuated nonlinear dynamic systems, where the underactuated system is decoupled into two subsystems. AFSM algorithm has also been applied in various practical nonlinear control systems such as in the control of MicroElectroMechanical Systems (MEMS) resonators[19], and some other applications[20–22].

The AFSMC technique presented in this paper can be considered as an important extension of the direct AFS-

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MC method of [13] and [23], which were applicable to the affine form of unknown chaotic systems. In those papers, the closed-loop asymptotic stability was guaranteed only if the MIMO plant had a diagonal input matrix with non-zero entries. There are, however, many well-known under-actuated systems, such as robotic systems, mass-spring-damper systems, and electrical machines, to name a few. In this paper, the original AFSMC method is extended to the case of nonlinear affine systems in which some of the diagonal entries of the gain matrix could be zero.

As a case study, the control of the well-known remote environmental monitoring units (REMUS) which is autonomous underwater vehicle (AUV) of [24] is considered. REMUS is a low-cost, modular vehicle with applications in autonomous docking and long-range oceanographic survey. A 6-DOF dynamic model is used for the simulation of the motions. The results of this case study reveal the effectiveness of the new AFSMC methodology for both diving and steering modes control, despite the existence of under-actuation structure in the plant model.

The remainder of this paper is organized as follows: In Section 2, the most relevant features of the conventional SMC are introduced. The extensions of SMC and AFSMC methods to the case of under-actuated systems are elaborated in Section 3, respectively. In order to demonstrate the effectiveness of the proposed control scheme, an extensive case study on the control of REMUS AUV is presented in Section 4 and the simulation results are depicted in Section 5. Finally, conclusions are presented in Section 6.

2 Conventional SMC

Consider the class of MIMO affine nonlinear systems

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} f_1(X) \\ \vdots \\ f_m(X) \end{bmatrix} + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_{mm} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}. \quad (1)$$

Using a more compact notation, (1) can be written as

$$Y^{(R)} = F(X) + GU \quad (2)$$

where $X = [y_1, \dot{y}_1, \cdots, y_1^{(r_1-1)}, \cdots, y_m, \dot{y}_m, \cdots, y_m^{(r_m-1)}]^{\rm T}$ is the vector of states which are assumed to be measurable. Furthermore, $U = [u_1, \cdots, u_m]^{\rm T}$ is the vector of control inputs, and $F(x) = [f_1(X), \cdots, f_m(X)]^{\rm T}$ is the vector of smooth functions of X. Also, $R = [r_1, \cdots, r_m]^{\rm T}$ is the vector of relative degrees for the m subsystems, where $r_1 + \cdots + r_m = n$ and n is the overall system order. Furthermore, it is assumed that the entries of the input gain matrix, G, are not known exactly. In presence of uncertainties, the description of system 1 can be modified as

$$Y^{(R)} = F(X) + GU + D \quad (3)$$

in which $D = [d_1, \cdots, d_m]^{\rm T}$ is the vector of lumped uncertainties which are assumed to be of bounded norms, i.e., $|d_i| < \delta_i$, $i = 1, \cdots, m$.

The vector of outputs is defined as $Y = [y_1, \cdots, y_m]^{\rm T}$. Let us consider the desired trajectory vector $Y_d = [y_{1d}, \cdots, y_{md}]^{\rm T}$. The tracking error is then defined as

$$\tilde{Y} = Y_d - Y = [\tilde{y}_1, \cdots, \tilde{y}_m]^{\rm T}. \quad (4)$$

The objective is to design a control law so that $\tilde{y}_i$ converges to zero asymptotically.

2.1 Control method

For now, it is assumed that all the entries $g_{ii}, i = 1, \cdots, m$ in (1) are non-zero. This assumption will be later on relaxed in Section 3. The design of sliding mode control includes two steps: Step one is to design a sliding surface, $s(x)$, to represent the desired system dynamics, which is of a lower order than the given plant. The second step is to design a variable structure control u such that any state outside the switching surface is driven to reach the surface in finite time. On the sliding surface, the sliding mode takes place, following the desired system dynamics. In this way, the stability of trajectory on s is guaranteed[3].

The m-dimensional vector of sliding surfaces are defined as

$$S = [s_1, \cdots, s_m]^{\rm T} = [\beta_1, \cdots, \beta_m] \begin{bmatrix} \tilde{Y}_1 \\ \vdots \\ \tilde{Y}_m \end{bmatrix} \quad (5)$$

where $\beta_i = [\beta_{i1}, \cdots, \beta_{i(r_i-1)}, 1]$ is vector of a Hurwitz polynomial coefficients and $\tilde{Y}_i = [\tilde{y}_i, \cdots, \tilde{y}_i^{(r_i-1)}]^{\rm T}$ is the states tracking error of the i-th subsystem.

By time differentiation of the sliding surface, one gets

$$\begin{aligned} \dot{s}_i = \beta_i \dot{\tilde{Y}}_i = \sum_{j=1}^{r_i-1} \beta_{ij} \tilde{y}_i^j + \tilde{y}_i^{(r_i)} = \\ \sum_{j=1}^{r_i-1} \beta_{ij} \tilde{y}_i^j + y_{di}^{(r_i)} - y_i^{(r_i)} = \\ E_{\beta i} + y_{di}^{(r_i)} - y_i^{(r_i)} = \\ E_{\beta i} + y_{di}^{(r_i)} - f_i(x) - g_{ii} u_i - d_i \end{aligned} \quad (6)$$

in which

$$E_{\beta i} = \sum_{j=1}^{r_i-1} \beta_{ij} \tilde{y}_i^j.$$

The sliding mode control law is defined as[13]

$$u_i = u_i^{eq} + u_i^{rb} \quad (7)$$

where the equivalent control law u_i^{eq} can be obtained using the condition $\dot{s}_i = 0$, i.e.,

$$\dot{s}_i = E_{\beta i} + y_{di}^{(r_i)} - f_i(x) - g_{ii}u_i^{eq} = 0 \Rightarrow \\ u_i^{eq}\left(\frac{1}{g_{ii}}\right)\left(E_{\beta i} + y_{di}^{(r_i)} - f_i(x)\right). \quad (8)$$

The robust controller u_i^{rb} is used to overcome the uncertainties of the plant, ensuring a finite time reaching towards the sliding surface, where

$$u_i^{rb} = \left(\frac{1}{g_{ii}}\right)\nu_i. \quad (9)$$

The control term ν_i is selected as

$$\nu_i = \delta_i \mathrm{sgn}(s_i) \Rightarrow \nu = [\Delta \mathrm{sgn}(S)]^{\mathrm{T}} \quad (10)$$

where $\mathrm{sgn}(S) = [\mathrm{sgn}(s_1), \cdots, \mathrm{sgn}(s_m)]^{\mathrm{T}}$ and $\mathrm{sgn}(\cdot)$ is the sign function. Also, $\Delta = [\delta_1, \cdots, \delta_m]^{\mathrm{T}}$. Substituting (7) into (6), and using (8) and (9) lead to

$$\begin{aligned}\dot{s}_i =& E_{\beta i} + y_{di}^{(r_i)} - f_i(x) - g_{ii}u_i - d_i = \\ & E_{\beta i} + y_{di}^{(r_i)} - f_i(x) - g_{ii}\left(u_i^{eq} + u_i^{rb}\right) - d_i = \\ & E_{\beta i} + y_{di}^{(r_i)} - f_i(x) - \left(E_{\beta i} + y_{di}^{(r_i)} - f_i(x) + \nu_i\right) - d_i = \\ & -d_i - \nu_i = -d_i - \delta_i \mathrm{sgn}(s_i).\end{aligned} \quad (11)$$

Now, the following Lyapunov candidate function is defined as

$$L_i = \frac{1}{2}s_i^2. \quad (12)$$

The time derivative of (12) together with (11) yields

$$\begin{aligned}\dot{L}_i =& s_i\dot{s}_i = -s_i d_i - |s_i|\,\delta_i \leq |s_i|\,|d_i| - |s_i|\,\delta_i = \\ & -|s_i|\,(\delta_i - |d_i|) \leq 0.\end{aligned} \quad (13)$$

Therefore, the sliding mode control (7) can guarantee the stability of the MIMO system of (1) in the Lyapunov sense.

3 AFSMC for under-actuated systems

The sliding mode controller outlined in the previous section is only applicable to systems where all g_{ii} in (1) are non-zero, i.e., every subsystem of (1) is directly affected by a specific control input channel. In this section, the sliding mode control is extended to the case of under-actuated systems and is combined with the adaptive fuzzy concept to drastically increase the robustness of the system in the presence of un-modeled dynamics.

3.1 Problem definition

Consider an under-actuated system with m outputs and m–1 inputs, in the form of

$$\begin{bmatrix} z_1^{(r_z)} \\ x_1^{(r_x)} \\ \hline y_3^{(r_3)} \\ \vdots \\ y_m^{r_m} \end{bmatrix} = \begin{bmatrix} f_1 + g_1 x_1 \\ f_2(Z_1, X_1) + d_x \\ \hline f_3(X) \\ \vdots \\ f_m(X) \end{bmatrix} + \left[\begin{array}{c|ccc} 0 & & & \\ g_{22} & & & \\ \hline & g_{33} & & \\ & & \ddots & \\ & & & g_{mm} \end{array}\right] \begin{bmatrix} u_2 \\ \vdots \\ u_m \end{bmatrix} \quad (14)$$

where f_1 and g_1 are smooth functions of z_1, and x_1 can be considered as the input of this subsystem. This is a modification of (1), in which $g_{11} = 0$ and z_1 and x_1 are the first and second outputs of the system, respectively. Consider the first subsystem of (4), where z_1 and its derivatives are not directly affected by the control input, and instead are indirectly controlled through the second output, x_1, justifying the name under-actuated. Here, $Z_1 = [z_1, \cdots, z_1^{(r_z-1)}]^{\mathrm{T}}$, $X_1 = [x_1, \cdots, x_1^{(r_x-1)}]^{\mathrm{T}}$, $X = \left[Z_1, X_1, y_3, \dot{y}_3, \cdots, y_3^{(r_3-1)}, \cdots, y_m, \dot{y}_m, \cdots, y_m^{(r_m-1)}\right]^{\mathrm{T}}$ and the output vector is defined as $Y = [z_1, x_1, y_3, \cdots, y_m]^{\mathrm{T}}$. Also, d_x is the lumped uncertainties of the second subsystem, which is assumed to be of bounded norms, i.e., $|d_x| < \delta_2$.

3.2 SMC control

The sliding surface for under-actuated system must be defined first. For this purpose, the output error is defined as

$$\tilde{z}_1 = z_{1d} - z_1 \quad (15)$$

where z_{1d} and $\tilde{z}_1$ are the desired values for z_1 and its tracking error, respectively.

The primary sliding surface is constructed for the first part of state variables as

$$s_z = \lambda_z \tilde{Z}_1 \quad (16)$$

where $\lambda_z = \left[\lambda_{z1}, \cdots, \lambda_{z(r_z-1)}, 1\right]$ is a vector of Hurwitz polynomial coefficients and $\tilde{Z}_1 = [\tilde{z}_1, \cdots, \tilde{z}_1^{(r_z-1)}]^{\mathrm{T}}$ is the vector of states tracking errors. Define, $Z_{1d} = [z_{1d}, \cdots, z_{1d}^{(r_z-1)}]^{\mathrm{T}}$ as the desired trajectory vector for Z_1. Suppose the first subsystem of (14) can be written in the following form:

$$z_1^{(r_z)} = f_1^* + g_1\tilde{x}_1 \quad (17)$$

where f_1^* is smooth function of z_1, and $\tilde{x}_1$ can be considered as the input of this subsystem. Suppose the system of (17) can be stabilized by a smooth state feedback control law $\tilde{x}_1 = \varphi_1(Z_1)$, with $\varphi_1(Z_{1d}) = 0$, i.e., the origin of

$$z_1^{(r_z)} = f_1^* + g_1\varphi_1(Z_1) \quad (18)$$

is asymptotically stable. Suppose that we know a smooth and positive definite Lyapunov function $V_1(Z_1)$ that satisfies the inequality.

$$\frac{\partial V_1}{\partial z_1}[f(z_1) + g(z_1)\phi(z_1)] \leq -W(z_1). \quad (19)$$

Inspired by the well-known back-stepping technique, (18) is re-written in the tracking error form. By adding and subtracting $g_1\varphi_1(Z_1)$ on the right-hand side of (17), one can obtain the equivalent representation

$$z_1^{(r_z)} = f_1^* + g_1\varphi_1(Z_1) + g_1(\tilde{x}_1 - \varphi_1(Z_1)). \quad (20)$$

To backstep, the following change of variables

$$e_1 = \tilde{x}_1 - \varphi_1(Z_1) \quad (21)$$

results in the system of

$$z_1^{(r_z)} = f_1^* + g_1\varphi_1(Z_1) + g_1 e_1. \quad (22)$$

In (22), e_1 can be viewed as the input of system and the output has an asymptotically stable origin when the input is zero. The derivative of e_1 can be written as

$$\dot{e}_1 = \tilde{x}_1^{(1)} - \dot{\varphi}_1(Z_1). \quad (23)$$

In (23), $\tilde{x}_1^{(1)}$ acts as a virtual input for state variable e_1.

Suppose the system of (23) can be stabilized by a smooth state feedback control law $\tilde{x}_1^{(1)} = \varphi_2(Z_1, X_1)$, with $\varphi_2(Z_{1d}, X_{1d}) = 0$, i.e., the origin of

$$\dot{e}_1 = \varphi_2(Z_1, X_1) - \dot{\varphi}_1(Z_1) \quad (24)$$

is asymptotically stable. Also, $X_{1d} = [x_{1d}, \cdots, x_{1d}^{(r_x-1)}]^{\mathrm{T}}$ is the desired trajectory vector for X_1. In order to preserve the closed-loop stability, the function $\varphi_2(Z_1, X_1)$ must be selected such that the rate of change of the following Lyapunov function remains negative. Using $V_{e_1} = V_1 + \frac{1}{2}e_1^2$ as a Lyapanov function candidate, we obtain

$$\begin{aligned}
V_{e_1} &= V_1 + \frac{1}{2}(e_1)^2 \\
\dot{V}_{e_1} &= \dot{V}_1 + e_1\dot{e}_1 = \\
&\dot{V}_1 + e_1(\tilde{x}_1^{(1)} - \dot{\varphi}_1(Z_1)) = \\
&\frac{\partial V_1}{\partial {z_1}^{(r_z-1)}}(z_1)^{(r_z)} + e_1(\varphi_2) - e_1\dot{\varphi}_1(Z_1) = \\
&\frac{\partial V_1}{\partial {z_1}^{(r_z-1)}}(f_1^* + g_1\varphi_1(z_1) + g_1 e_1) + e_1(\varphi_2) - e_1\dot{\varphi}_1(Z_1) \leq \\
&-W(z_1) + \frac{\partial V_1}{\partial {z_1}^{(r_z-1)}}(g_1 e_1) + e_1(\varphi_2) - e_1\dot{\varphi}_1(Z_1).
\end{aligned} \quad (25)$$

By having $\frac{\partial V_1}{\partial {z_1}^{(r_z-1)}}(g_1) + (\varphi_2) - \dot{\varphi}_1(Z_1) = -K_1 e_1$, we obtain

$$V_{e_1} \leq -W(z_1) + -K_1(e_1)^2 \quad (26)$$

in which K_1 is a positive constant. By choosing φ_2 as follows, the rate of change of the Lyapunov function remains negative.

$$\varphi_2 = -\frac{\partial V_1}{\partial z_1(r_z - 1)}(g_1) + \dot{\varphi}_1(Z_1) - K_1 e_1. \quad (27)$$

Similarly, by adding and subtracting φ_2 on the right-hand side of (13), one can obtain the equivalent representation

$$\dot{e}_1 = (\tilde{x}_1^{(1)} - \varphi_2) + \varphi_2 - \dot{\varphi}_1(Z_1). \quad (28)$$

The change of variables

$$e_2 = \tilde{x}_1^{(1)} - \varphi_2(Z_1, X_1) \quad (29)$$

results in the system of

$$\dot{e}_1 = e_2 + \varphi_2(Z_1, X_1) - \dot{\varphi}_1(Z_1). \quad (30)$$

In (30), e_2 can be viewed as the input of system and the output has an asymptotically stable origin when the input is zero. This procedure will be continued for other state variable $x_1^{(j)}$ up to $j = r_x - 1$. Eventually, the last subsystem change of variable, $e_{(r_x)}$ will actually define the sliding surface for the given subsystem, namely,

$$s_1 = \tilde{x}_1^{(r_x-1)} - \varphi_{(r_x)}(Z_1, X_1). \quad (31)$$

By differentiating s_1, one gets

$$\begin{aligned}
\dot{s}_1 =& \tilde{x}_1^{(r_x)} - \frac{\partial\varphi_{(r_x)}}{\partial X_1}(\dot{X}_1) - \frac{\partial\varphi_{(r_x)}}{\partial Z_1}(\dot{Z}_1) = \\
& x_{1d}^{(r_x)} - x_1^{(r_x)} - \frac{\partial\varphi_{1(r_x)}}{\partial X_1}(\dot{X}_1) - \frac{\partial\varphi_{1(r_x)}}{\partial Z_1}(\dot{Z}_1) = \\
& x_{1d}^{(r_x)} - x_1^{(r_x)} - E_x - E_z = \\
& x_{1d}^{(r_x)} - f_2(Z_1, X_1) - g_{22}u_2 - d_x - E_x - E_z
\end{aligned} \quad (32)$$

where $E_x = \frac{\partial\varphi_{(r_x)}}{\partial X_1}(\dot{X}_1)$ and $E_z = \frac{\partial\varphi_{(r_x)}}{\partial Z_1}(\dot{Z}_1)$. The equivalent control law, u_2^{eq}, can then be obtained from $\dot{s}_1 = 0$, as

$$u_2^{eq} = (g_{22})^{-1}\left(x_{1d}^{(r_x)} - E_x - E_z - f_2(Z_1, X_1)\right). \quad (33)$$

The sliding mode control law is defined as

$$u_2 = u_2^{eq} + u_2^{rb}. \quad (34)$$

The robust controller u_2^{rb} is used to overcome the uncertainties of the plant, ensuring a finite time reaching towards the sliding surface, where

$$u_2^{rb} = g_{22}^{-1}\nu_2. \tag{35}$$

The control term ν_2 is selected as

$$\nu_2 = \delta_2 \mathrm{sgn}(s_1). \tag{36}$$

This leads to

$$\dot{s}_1 = -d_x - \nu_2 = -d_x - \delta_2 \mathrm{sgn}(s_1). \tag{37}$$

Now, the following Lyapunov candidate function is defined:

$$L_1 = \frac{1}{2}s_1^2. \tag{38}$$

The time derivative of (38) together with (37) yields

$$\dot{L}_1 = s_1\dot{s}_1 = -s_1 d_x - |s_1|\,\delta_2 \le |s_1|\,|d_x| - |s_1|\,\delta_2 = \\ -|s_1|\,(\delta_2 - |d_x|) \le 0. \tag{39}$$

Therefore, the sliding mode control (34) can guarantee the stability of the first subsystem of (14) in the Lyapunov sense.

In the remainder of this paper, $S = [s_1, \cdots, s_m]^{\mathrm{T}}$ is supposed to be the vector of sliding surface for system of (14).

3.3 AFSMC control

By combining the fuzzy and SMC approaches, the control effort can be defined to be a nonlinear function of the deviations from the sliding surface. The advantage is that, unlike the conventional SMC approach, the control input is not computed from the plant dynamic equations directly. The fuzzy sliding mode controller is actually a fuzzy logic controller, for which the inputs are the deviations from the sliding surface and its time derivative and the output is the control command.

For this purpose, for the i-th subsystem of (14), a Takagi-Sugeno (TS) fuzzy system with the output u_i^{fuz} and the fuzzy IF-THEN rules are considered, with,

Rule r: If s_i is A_i^r, then $u_i^{fuz} = b_i^r$, $r = 1, \cdots, n_r$, where b_i^r is the fuzzy singleton for the output of the r-th rule, and A_i^r is a fuzzy set characterized by a Gaussian membership function as

$$\mu_{A_i^r}(s_i) = \exp\left[-\left(\frac{s_i - c_i^r}{\sigma_i^r}\right)^2\right]. \tag{40}$$

Here, c_i^r and σ_i^r are the center and width of the membership functions, respectively. Using singleton fuzzifier, product inference, and center average defuzzifier, the output of the fuzzy system is obtained as

$$u_i^{fuz} = \frac{\sum\limits_{r=1}^{n_r} b_i^r \mu_{A_i^r}(s_i)}{\sum\limits_{r=1}^{n_r} \mu_{A_i^r}(s_i)}. \tag{41}$$

By defining the firing strength of the r-th rule as

$$w_i^r = \frac{\mu_{A_i^r}(s_i)}{\sum\limits_{r=1}^{n_r} \mu_{A_i^r}(s_i)}, \qquad r = 1, \cdots, n_r \tag{42}$$

the output of fuzzy system can be written as

$$u_i^{fuz}(s_i, b_i) = b_i^{\mathrm{T}} w_i \tag{43}$$

where

$$w_i = \left[w_i^1, \cdots, w_i^{n_r}\right]^{\mathrm{T}}, \qquad b_i = \left[b_i^1, \cdots, b_i^{n_r}\right]^{\mathrm{T}}.$$

For nominal case, i.e., when accurate mathematical model of the system is available, the output of fuzzy controller for a system with m inputs $S = [s_1, \cdots, s_m]^{\mathrm{T}}$ and m outputs $u_1^{fuz}, \cdots, u_m^{fuz}$ is denoted as

$$u^{fuz*} = \left[u_1^{fuz*}(s_1, b_1^*), \cdots, u_m^{fuz*}(s_m, b_m^*)\right]^{\mathrm{T}}. \tag{44}$$

The ideal controller is obtained as

$$u^* = u^{fuz*}(S, B^*) + \Xi = \mathrm{diag}(B^{*T}W) + \Xi \tag{45}$$

where

$$W = [w_1, \cdots, w_m]^{\mathrm{T}}, \qquad B^{*\mathrm{T}} = [b_1^*, \cdots, b_m^*]^{\mathrm{T}}$$

and $\Xi = [\xi_1, \cdots, \xi_m]^{\mathrm{T}}$ is the approximation error or the uncertainty which is assumed to be bounded, $|\xi_i| < \kappa_i$. Also B^* is the optimal parameter vector

$$B^* \triangleq \arg\min_B \left\{\left|\mathrm{diag}(B^{\mathrm{T}}W) - u^*\right|\right\}. \tag{46}$$

In practice, the entries of the optimal parameter vector b_i^* and the uncertainty or approximation error bounds $K = [\kappa_1, \cdots, \kappa_m]^{\mathrm{T}}$ may be unknown. Denoting the estimation of this uncertainty bounds as $\hat{K}$, the estimation error is defined as

$$\tilde{K}(t) = K - \hat{K}(t). \tag{47}$$

The output of fuzzy system to approximate the ideal controller can be rewritten as

$$\hat{u}_i^{fuz}(s_i, \hat{b}_i) = \hat{b}_i^{\mathrm{T}} w_i, \qquad k = 1, 2, \cdots, m \tag{48}$$

Here, $\hat{b}_i$ is the estimation of b_i^*, thus the control law can be represented as

$$u_i = \hat{u}_i^{fuz}(s_i, \hat{b}_i) + u_i^{rb}(s_i), \qquad k = 1, 2, \cdots, m \tag{49}$$

where u_i^{rb} is employed to compensate the difference between the fuzzy controller and the ideal one. By substituting the matrix form of (49) into (1), one gets

$$y^{(r)} = F(x) + G\left[\hat{u}^{fuz} + u^{rb}\right]. \tag{50}$$

By defining the approximation errors as

$$\tilde{u}^{fuz} = u^* - \hat{u}^{fuz}, \;\; \tilde{B} = B^* - \hat{B} \tag{51}$$

and by using (45), (48) and (51), one gets

$$\tilde{u}^{fuz} = \mathrm{diag}(\tilde{B}^{\mathrm{T}} W) + \Xi. \tag{52}$$

Multiplying (8) with g_{ii}, added to (50) and using (6), it turns out for the i-th subsystem that

$$\dot{s}_i = g_{ii}\left(u_i^* - \hat{u}_i^{fuz} - {u_i}^{rb}\right) \rightarrow$$
$$\dot{S} = G\left[\tilde{u}^{fuz} - u^{rb}\right] = G\left(\mathrm{diag}(\tilde{B}^{\mathrm{T}} W) + \Xi - u^{rb}\right). \tag{53}$$

In the case of underactuated variable in first subsystem of (14), multiplying (33) with g_{22}, added to (50) and using (32), provide the same results as (53).

The closed-loop stability of the AFSM controlled system is proved in the sequel.

Theorem 1. Consider system of (1) and the control law given by (49), where

1) The fuzzy controller is tuned by the adaptive law

$$\dot{\hat{B}} = -\dot{\tilde{B}} = \alpha_1 W(S) \tag{54}$$

2) The switching part of the control input is obtained from

$$u^{rb} = \mathrm{diag}(\hat{K})\mathrm{sgn}(G)\mathrm{sgn}(S(t)) \tag{55}$$

3) The estimated value of uncertainty bound is adaptively tuned according to

$$\dot{\hat{K}} = -\dot{\tilde{K}} = \alpha_2 \mathrm{sgn}(G)\left|S(t)\right|. \tag{56}$$

α_1 and α_2 are pre-selected positive values for adaption rates. Then, the tracking error converges to zero asymptotically.

Proof. By choosing the following Lyapunov candidate function

$$V = \sum_{i=1}^{m} V_i$$
$$V_i(s_i, \tilde{b}_i, \tilde{\kappa}_i) = \frac{1}{2}s_i^2 + \frac{1}{2\alpha_1}g_{ii}\tilde{b}_i^{\mathrm{T}}\tilde{b}_i + \frac{1}{2\alpha_2}g_{ii}(\tilde{\kappa}_i)(\tilde{\kappa}_i) \tag{57}$$

and by differentiating (57) with respect to time, and employing (53)–(56), it can be shown that

$$\begin{aligned}
&\dot{V}_i(s_i, \tilde{b}_i, \tilde{\kappa}_i) = s_i\dot{s}_i + \frac{1}{\alpha_1}g_{ii}\tilde{b}_i^{\mathrm{T}}\dot{\tilde{b}}_i + \frac{1}{\alpha_2}g_{ii}(\tilde{\kappa}_i)(\dot{\tilde{\kappa}}_i) = \\
&s_i g_{ii}\left(\tilde{b}_i^{\mathrm{T}} w_i + \xi_i - {u_i}^{rb}\right) + \frac{1}{\alpha_1}g_{ii}\tilde{b}_i^{\mathrm{T}}\dot{\tilde{b}}_i + \frac{1}{\alpha_2}g_{ii}(\tilde{\kappa}_i)(\dot{\tilde{\kappa}}_i) = \\
&g_{ii}\tilde{b}_i^{\mathrm{T}}\left(s_i w_i + \frac{\dot{\tilde{b}}_i}{\alpha_1}\right) + s_i g_{ii}\left(\xi_i - u_i^{rb}\right) + \frac{1}{\alpha_2}g_{ii}(\tilde{\kappa}_i)(\dot{\tilde{\kappa}}_i) = \\
&s_i g_{ii}\xi_i - s_i g_{ii}\hat{\kappa}_i \mathrm{sgn}\left(g_{ii}\right)\mathrm{sgn}\left(s_i\right) - \frac{1}{\alpha_2}\left|g_{ii}\right|(\tilde{\kappa}_i)\alpha_2\left|s_i\right| = \\
&s_i g_{ii}\xi_i - \left|s_i\right|\left|g_{ii}\right|(\hat{\kappa}_i + \tilde{\kappa}_i) = s_i g_{ii}\xi_i - \left|s_i\right|\left|g_{ii}\right|(\kappa_i) = \\
&s_i g_{ii}\xi_i - \left|s_i\right|\left|g_{ii}\right|(\kappa_i) \leq (\left|s_i\right|\left|g_{ii}\right|\xi_i - \left|s_i\right|\left|g_{ii}\right|(\kappa_i)) = \\
&-(\left|s_i\right|\left|g_{ii}\right|(\kappa_i - \left|\xi_i\right|)) \leq 0.
\end{aligned} \tag{58}$$

Let us define

$$\Gamma(t) = \sum_{i=1}^{m}(\left|s_i\right|\left|g_{ii}\right|(\kappa_i - \left|\xi_i\right|)) \leq -\dot{V}. \tag{59}$$

Integration of this equation leads to

$$\int_0^t \Gamma(\tau)\mathrm{d}\tau \leq V(S(0), \tilde{B}, \tilde{K}) - V(S(t), \tilde{B}, \tilde{K}) \tag{60}$$

where $V(S(0), \tilde{B}, \tilde{K})$ is bounded and $V(S(t), \tilde{B}, \tilde{K})$ is at least non-increasing. Therefore,

$$\int_0^t \Gamma(\tau)\mathrm{d}\tau \leq \infty. \tag{61}$$

By exploiting the semi-negativeness of $\dot{V}$ from (58) and by considering the fact that, absolute functions are uniformly continuous, one concludes, from Barbalat Lemma, that

$$\lim_{t\rightarrow\infty}\Gamma(t) = 0. \tag{62}$$

As a result, when $t \rightarrow \infty$, the sliding surface $S(t) \rightarrow 0$ uniformly, and hence the asymptotic stability is guaranteed.

4 Case study (AUV)

In order to demonstrate the feasibility of the proposed approach in previous section, AFSMC is applied to the plant of an autonomous underwater vehicle (AUV). The 6 degree of freedom (DOF) dynamic equations of motion of an AUV can be written as

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau. \tag{63}$$

Here,

$$\begin{aligned}
&\eta = [x, y, z, \phi, \theta, \psi]^{\mathrm{T}} \\
&\nu = [u, v, w, p, q, r]^{\mathrm{T}} \\
&\tau = [X, Y, Z, K, M, N]^{\mathrm{T}} \\
&M = M_{RB} + M_A \\
&\mathrm{C}(\nu) = \mathrm{C_{RB}}(\nu) + \mathrm{C_A}(\nu)
\end{aligned}$$

where η denotes the position and orientation of the vehicle with respect to the inertial or earth-fixed reference frame, ν is the translational and rotational velocities of the vehicle with respect to the body-fixed reference frame, and τ includes the total forces and moments acting on the vehicle with respect to the body-fixed reference frame. τ contains the propulsion forces and moments, which can be expressed in the same way as environmental forces acting on the vehicle. Also, M_{RB} is the rigid body inertia matrix and M_A is the added inertia matrix, $C_{RB}(\nu)$ is the rigid body Coriolis and centripetal matrix, also $C_A(\nu)$ is added hydrodynamic Coriolis and centripetal matrix, $D(\nu)$ contains all the hydrodynamic damping forces acting on the ocean vehicle throughout its mission and $g(\eta)$ illustrates gravitational and buoyant forces and moments. In hydrodynamics terminology, the gravitational and buoyant forces are called restoring forces. The position and orientation of the vehicle should be described relative to inertial reference frame while the linear and angular velocities of the vehicle should be expressed in the body-fixed coordinate system[25].

By retaining the acceleration terms of (63) on the left-hand side, one gets

$$\begin{aligned} & M\dot{\nu} = -C(\nu)\nu - D(\nu)\nu - g(\eta) + \tau \\ & \dot{\nu} = M^{-1}\left[-C(\nu)\nu - D(\nu)\nu - g(\eta)\right] + M^{-1}\tau. \end{aligned} \tag{64}$$

Based on the vehicle equations of motion presented in Appendix 7, the corresponding terms in τ vector of (64) can be written as

$$\tau = \begin{bmatrix} X_{prop} \\ Y_{uu\delta r}{u_0}^2\delta_r \\ Z_{uu\delta_s}{u_0}^2\delta_s \\ K_{prop} \\ M_{uu\delta_s}{u_0}^2\delta_s \\ N_{uu\delta_r}{u_0}^2\delta_r \end{bmatrix} =$$

$$\begin{bmatrix} X_{prop} \\ 0 \\ 0 \\ K_{prop} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ Y_{uu\delta r}{u_0}^2 & 0 \\ 0 & Z_{uu\delta_s}{u_0}^2 \\ 0 & 0 \\ 0 & M_{uu\delta_s}{u_0}^2 \\ N_{uu\delta_r}{u_0}^2 & 0 \end{bmatrix} \times \begin{bmatrix} \delta_r \\ \delta_s \end{bmatrix} \tag{65}$$

where δ_r and δ_s are control actuators of the vehicle and u_0 is the constant surge velocity. In compact form, (64) can be written as

$$\dot{\nu} = f + G\begin{bmatrix} \delta_r \\ \delta_s \end{bmatrix}. \tag{66}$$

The constant coefficient matrix of G and F are

$$G = M^{-1}\begin{bmatrix} 0 & 0 \\ Y_{uu\delta r}{u_0}^2 & 0 \\ 0 & Z_{uu\delta_s}{u_0}^2 \\ 0 & 0 \\ 0 & M_{uu\delta_s}{u_0}^2 \\ N_{uu\delta_r}{u_0}^2 & 0 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \\ g_{41} & g_{42} \\ g_{51} & g_{52} \\ g_{61} & g_{62} \end{bmatrix} \tag{67}$$

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} =$$

$$M^{-1}\left(-C(\nu)\nu - D(\nu)\nu - g(\eta)\right) + M^{-1}\begin{bmatrix} X_{prop} \\ 0 \\ 0 \\ K_{prop} \\ 0 \\ 0 \end{bmatrix}. \tag{68}$$

4.1 Pitch mode

Pitch mode is actually the diving motion of vehicle in vertical plane. In a pure vertical motion, one could eliminate and nullify some unrelated state variables, i.e., roll angle (ϕ), yaw angle (ψ), and sway velocity (v). Therefore, depth (z), pitch angle (θ), and pitch angular velocity (q), would be as the main state variables for this mode. The equations of diving motion can then be written as

$$\begin{aligned} & \dot{z} = (-\sin\theta)\,u_0 + (\cos\theta)\,w \\ & \dot{\theta} = q \\ & \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} f_3 \\ f_5 \end{bmatrix} + \begin{bmatrix} g_{32} \\ g_{52} \end{bmatrix}[\delta_s]. \end{aligned} \tag{69}$$

In this subsystem, depth z, is the output.

4.2 Yaw mode

Yaw mode describes the motion of vehicle in the horizontal plane. Again, for the pure horizontal motion, some of the state variables, such as depth (z), pitch angle (θ), heave velocity (w) and pitch angular velocity (q), can be ignored. The remaining states would then be, lateral absolute position (y), yaw angle (ψ), sway velocity (v), and yaw angular velocity (r), being the main state variables for this mode. Based on these assumptions, the equation of motion for yaw mode would be

$$\dot{y} = (\sin\psi)\, u_0 + (\cos\psi)\, v$$

$$\dot{\psi} = r$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} + \begin{bmatrix} g_{21} \\ g_{61} \end{bmatrix} [\delta_r]. \tag{70}$$

In this subsystem, the yaw angle, ψ, is the output.

Both pitch and yaw mode subsystems can be further simplified, by assuming small perturbations in θ. In the following section, AFSM controllers are designed for diving and steering mode subsystems.

4.3 Pitch mode sliding surface

For simplicity, it is assumed that the heave velocity (w) is small during the diving phase[26].

The equations of diving mode then are reduced to

$$\begin{aligned} \dot{z} &= -(\theta)u_0 \\ \dot{\theta} &= q \\ \dot{q} &= f_5 + (g_{52})\,[\delta_s]. \end{aligned} \tag{71}$$

It is noted that in the above equations, z is indirectly controlled through the variable θ, which is directly actuated by the input δ_s. The method proposed in Section 3 is used for defining the sliding surfaces. The relative degree of this subsystem is 2 and the tracking errors are defined as

$$\begin{aligned} \tilde{z} &= z_d - z \\ \tilde{\theta} &= \theta_d - \theta \\ \tilde{q} &= q_d - q. \end{aligned} \tag{72}$$

In the case of REMUS AUV example, z_d would be a positive constant scalar, $\theta_d = 0$ and $q_d = 0$. Based on the methodology described in Section 3.2, $\tilde{\theta} = \varphi_1(z)$ is chosen such that the time derivative of the following Lyapunov function would be negative definite

$$V_z = \frac{1}{2}s_z^2 = \frac{1}{2}\tilde{z}^2 \tag{73}$$

i.e.,

$$\dot{V}_z = \tilde{z}\,(\dot{z}_d - \dot{z}) = \tilde{z}(\dot{z}_d + u_0\theta) = \tilde{z}(\dot{z}_d + u_0\,(\theta_d - \varphi_1)). \tag{74}$$

Since $\dot{z}_d = 0$ and $\theta_d = 0$, by defining $\varphi_1(z) = k_1\tilde{z}$, where $k_1 > 0$, the derivative of the Lyapunov function would be negative. Consider the first dynamic error as

$$e_1 = \tilde{\theta} - \varphi_1(z). \tag{75}$$

Again, the function $\tilde{q} = \varphi_2(z, \theta)$ is chosen such that the time derivative of the following Lyapunov function would be negative definite

$$V_{e1} = \frac{1}{2}e_1^2 \tag{76}$$

i.e.,

$$\dot{V}_{e1} = e_1(\dot{e}_1) \tag{77}$$

and

$$\dot{e}_1 = \dot{\tilde{\theta}} - k_1\dot{\tilde{z}} = \tilde{q} - k_1\dot{\tilde{z}}. \tag{78}$$

In order to make the Lyapunov function negative, the function of $\varphi_2\,(z, \theta)$ is determined as

$$\varphi_2\,(z, \theta) = -k_2 e_1 + k_1\dot{\tilde{z}} \tag{79}$$

where $k_2 > 0$.

The final dynamic error is the sliding surface of the underactuated variable configuration which must be adopted for control of diving mode:

$$\begin{aligned} s_p = e_2 &= \tilde{q} - \varphi_2\,(z, \theta) = \\ &\tilde{q} + k_2 e_1 - k_1\dot{\tilde{z}} = \\ &\tilde{q} + k_2\tilde{\theta} - k_2 k_1\tilde{z} - k_1\dot{\tilde{z}} = \\ &\tilde{q} + k_2\tilde{\theta} - k_2 k_1\tilde{z} - k_1 u_0\theta = \\ &\tilde{q} + k_2\tilde{\theta} - k_2 k_1\tilde{z} + k_1 u_0\tilde{\theta} = \\ &\tilde{q} + (k_2 + k_1 u_0)\,\tilde{\theta} - k_2 k_1\tilde{z} \end{aligned} \tag{80}$$

where s_p is the sliding surface used in control of depth in pitch mode.

4.4 Yaw mode sliding surface

In this mode, it can be assumed that $\dot{y}$ during the steering motion is small. Therefore, the steering equations of motion of (70) can be written as

$$\dot{\psi} = r$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} + \begin{bmatrix} g_{21} \\ g_{61} \end{bmatrix} [\delta_r]. \tag{81}$$

By combining these two equations, one gets

$$\begin{bmatrix} \dot{v} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} + \begin{bmatrix} g_{21} \\ g_{61} \end{bmatrix} [\delta_r]. \tag{82}$$

If the distance between center of pressure of control fin and the vehicle's center of gravity is denoted by d_0, the relation between the sway and heading angle rate can be approximately written as

$$v = d_0\dot{\psi} \tag{83}$$

i.e., (82) can be re-written as

$$\begin{aligned} d_0\ddot{\psi} &= f_2 + (g_{21})\delta_r \\ \ddot{\psi} &= f_6 + (g_{61})\delta_r. \end{aligned} \tag{84}$$

Therefore,

$$\ddot{\psi} = \frac{(f_2+f_6)}{(d_0+1)} + \frac{(g_{21}+g_{61})}{(d_0+1)}\delta_r. \tag{85}$$

This final equation is a single-input single-output equation, and the conventional sliding surface, s_y, for this mode is

$$s_y = \begin{bmatrix} \lambda & 1 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \dot{\tilde{\psi}} \end{bmatrix} = \begin{bmatrix} \lambda & 1 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \tilde{r} \end{bmatrix} = \lambda\tilde{\psi} + \tilde{r}. \tag{86}$$

where s_y is the sliding surface used in control of vehicle in yaw mode.

5 Simulation studies

In this section, the effectiveness of the proposed AFSMC method is assessed in controlling the 6 DOF dynamic model of the REMUS AUV. Technical details and performance testing results of REMUS are introduced in [24]. The dynamic equations of REMUS are used to study the performance of the proposed AFSMC algorithm in diving and steering modes of motion. In the first part of this section, disturbances are not taken into account but in the second part, they are considered in the form of ocean currents. Furthermore, the effect of un-modeled dynamics on the closed-loop performance of the system is studied.

In order to demonstrate the statistic and accidental characteristics of the ocean currents, a first order Gaussian-Markov process is considered, which could appear in any direction. In particular, the following equation is considered:

$$\dot{V}_c(t) + \mu_0 V_c(t) = \varpi(t) \tag{87}$$

where V_c is the earth-referenced velocity of the current and ϖ is the vector of a white noise signal. In order to confine the magnitude of the ocean currents, a saturation block is used, as in Fig. 1.

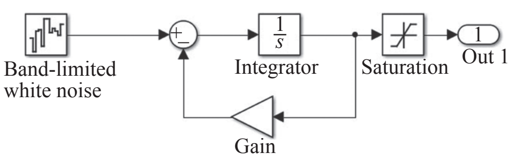

Fig. 1 Implementation of Gaussian-Markov[26] to model ocean currents

Ocean currents are defined in the earth-based coordinate and could be transformed to the body fixed coordinates. One example of the induced disturbance on the system is shown in Fig. 2.

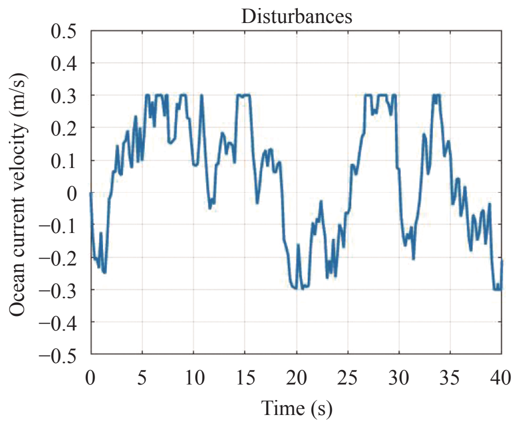

Fig. 2 Typical ocean current velocity disturbance

5.1 Nominal system performance

The initial conditions for roll and surge are taken as

$$\phi_0 = 5 \text{ deg}$$

$$u_0 = 1.54 \text{ m/s}$$

and other initial conditions are considered to be zero. The desired depth and heading angle are

$$z_d = 10 \text{ m}$$

$$\psi_d = 30 \text{ deg}.$$

Adaptation rates for diving AFSM controller are selected as $\alpha_1 = 10$ and $\alpha_2 = 8$ and for steering controller as $\alpha_1 = 9$ and $\alpha_2 = 8$.

The initial values for the output membership functions, for both controllers are arbitrarily selected as

$$\hat{B}_i = [-0.5; -0.25; -0.05; 0.05; 0.25; 0.5], i = 1, \cdots, 7.$$

In Figs. 3 and 4, the performance of the vehicle in vertical and horizontal modes is depicted.

The movements of control fins are shown in Figs. 5 and 6.

5.2 Effects of disturbances and un-modeled dynamics

To examine the effect of un-modeled dynamics, hydrodynamic coefficients of control fins are increased by 50 percent. The desired values for depth and heading angle are set as before, i.e.,

$$z_d = 10 \text{ m}$$

$$\psi_d = 30 \text{ deg}.$$

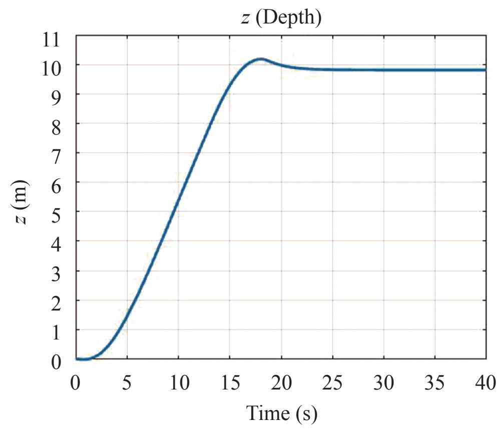

Fig. 3 AFSM control of depth–nominal case

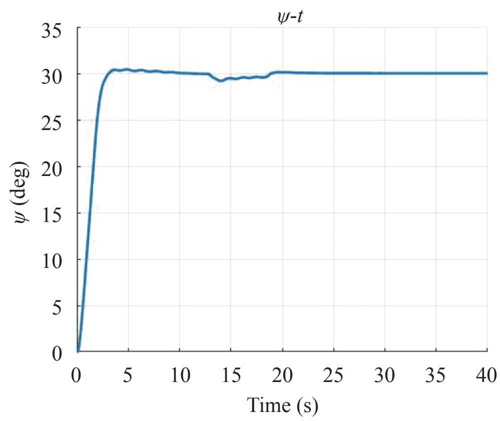

Fig. 4 AFSM control of heading angle–nominal case

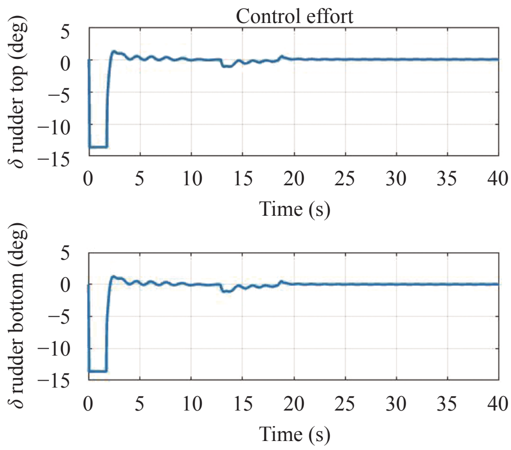

Fig. 5 Movements of top and bottom control fins–nominal case

In Figs. 7 and 8, the performance of the vehicle under AFSM control in the presence of un-modeled dynamics and random disturbances described by (87) is depicted, for vertical and horizontal modes, respectively.

The movements of the control fins under disturbance and un-modeled dynamics are shown in Figs. 9 and 10, respectively.

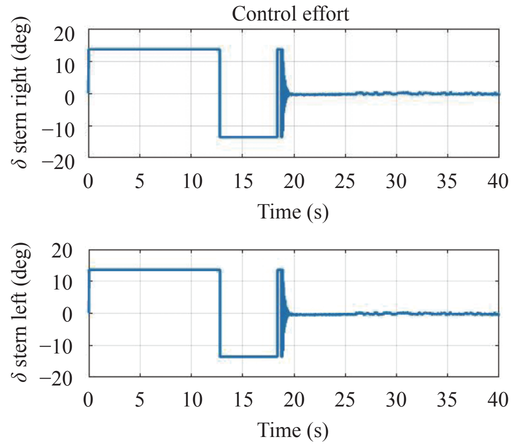

Fig. 6 Movements of right and left control fins–nominal case

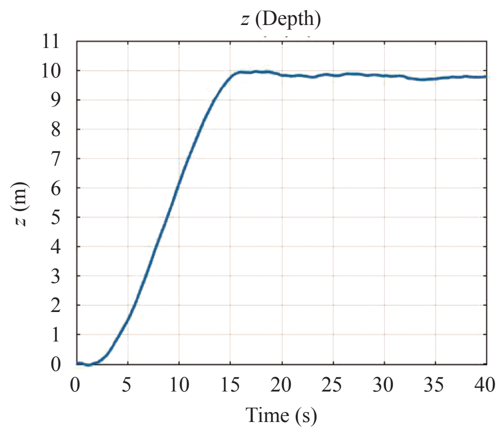

Fig. 7 AFSM control of depth under disturbance and un-modeled dynamics

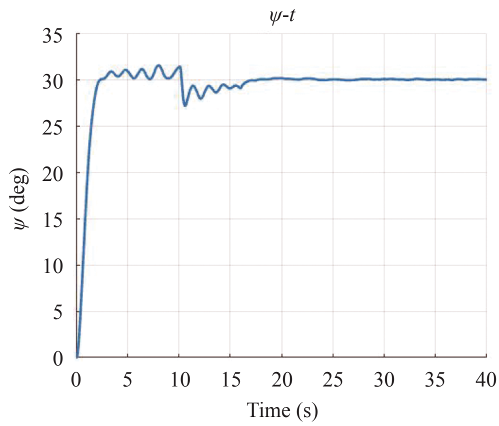

Fig. 8 AFSM control of heading angle with disturbances and un-modeled dynamics

It is clearly seen that the vehicle has been tracking the desired output variables very well. Figs. 7 and 8 show that there is no significant chattering after the vehicle reaches to the neighborhood of the desired values. However, Figs. 3 and 7 depict that the vehicle experi-

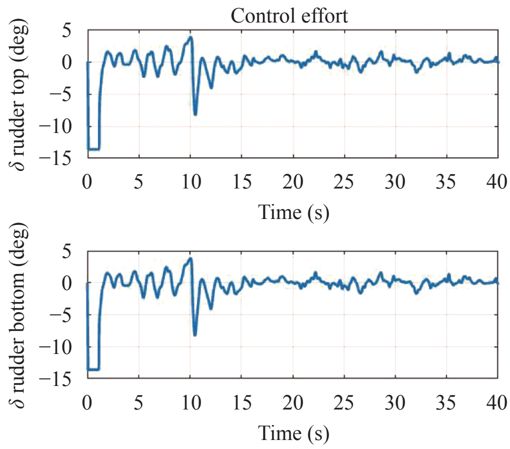

Fig. 9 Movements of top and bottom control fins with disturbances and un-modeled dynamics

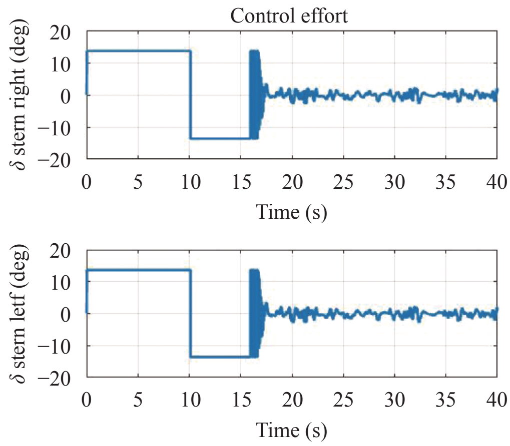

Fig. 10 Movements of right and left control fins with disturbances and un-modeled dynamics

ences a small steady state error in the control of depth. This phenomenon can be attributed to the cross-coupling between yaw and pitch modes, which was ignored at the design stage.

By observing Figs. 4 and 8, in the very initial seconds, the adaptive control tries to recognize the system and determine sliding control bounds. Hence the system is not completely stable and then after it becomes stable.

For better comprehension of the vehicle behavior, a 3D graph of vehicle trajectory is depicted in Fig. 11.

6 Conclusions

The AFSMC strategy reduces the dependency of the control approach to the plant model and is robust under plant uncertainties and exogenous disturbances. Such advantages are obtained by using a central fuzzy controller, which is continuously tuned by observing the deviations from the sliding surface, and also by continuously estimating the bound of uncertainties by an adaptive approach. The concept of AFSMC was extended to the case of under-actuated systems, by successive application of a particular back-stepping approach. As a challenging example, a highly nonlinear, under-actuated and multi-mode control of the REMUS AUV was considered. Control design was performed by decoupling the vertical and horizontal motions. Simulation studies revealed, the exceptional performance of the proposed AFSMC under the effects of disturbance and un-modeled dynamics.

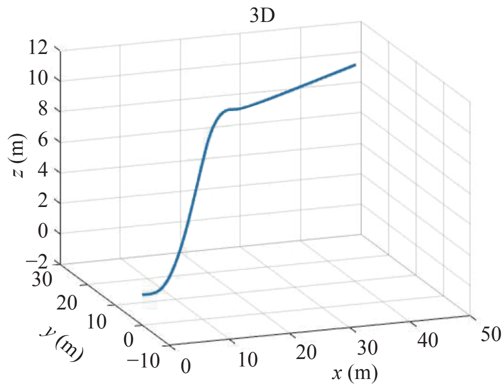

Fig. 11 Vehicle trajectory in 3D view

Two separate controllers were designed for vehicle's diving (pitch) and steering (yaw) modes of motion. It was shown that by decoupling the vertical and horizontal equations, only a small amount of error appeared in the control of depth for both nominal and perturbed systems.

7 REMUS equations and parameters

The REMUS nonlinear dynamic equations of motion in six degrees of freedom are as follows:

$$\begin{aligned}&(m-X_{\dot{u}})\dot{u}+mz_g\dot{q}-my_g\dot{r}=\\&\quad X_{HS}+X_{u|u|}u\,|u|+(X_{wq}-m)wq+\\&\quad (X_{qq}+mx_g)q^2+(X_{vr}+m)vr+(X_{rr}+mx_g)r^2-\\&\quad my_gpq-mz_gpr+X_{prop}\end{aligned}\tag{88}$$

$$\begin{aligned}&m-Y_{\dot{v}})\dot{v}-mz_g\dot{p}+(mx_g-Y_{\dot{r}})\dot{r}=\\&\quad Y_{HS}+Y_{v|v|}v\,|v|+Y_{r|r|}r\,|r|+my_gr^2+\\&\quad (Y_{ur}-m)ur+(Y_{wp}+m)wp+\\&\quad (Y_{pq}-mx_g)pq+Y_{uv}uv+my_gp^2+\\&\quad mz_gqr+Y_{uu\delta r}u^2\delta r\end{aligned}\tag{89}$$

$$\begin{aligned}&m-Z_{\dot{w}})\dot{w}+my_g\dot{p}-(mx_g+Z_{\dot{q}})\dot{q}=\\&\quad Z_{HS}+Z_{w|w|}w\,|w|+Z_{q|q|}q\,|q|+(Z_{uq}+m)uq+\\&\quad (Z_{vp}-m)vp+(Z_{rp}-mx_g)rp+Z_{uw}uw+\\&\quad mz_g(p^2+q^2)-my_gqr+Z_{uu\delta_s}u^2\delta_s\end{aligned}\tag{90}$$

$$\begin{aligned}&\text{-}\ mz_g\dot{v}+my_g\dot{w}+(I_{xx}-K_{\dot{p}})\dot{p}=\\&\quad K_{HS}+K_{p|p|}p\,|p|-(I_{zz}-I_{yy})qr+m(uq-vp)-\\&\quad mz_g(wp-ur)+K_{prop}\end{aligned}\tag{91}$$

$$\begin{aligned} & mz_g\dot{u} - (mx_g + M_{\dot{w}})\dot{w} + (I_{yy} - M_{\dot{q}})\dot{q} = \\ & M_{HS} + M_{w|w|}w\,|w| + M_{q|q|}q\,|q| + \\ & (M_{uq} - mx_g)uq + (M_{vp} + mx_g)vp + \\ & [M_{rp} - (I_{xx} - I_{zz})]rp + mz_g(vr - wq) + \\ & M_{uw}uw + M_{uu\delta_s}u^2\delta_s \end{aligned} \quad (92)$$

$$\begin{aligned} & -my_g\dot{u} + (mx_g - N_{\dot{v}})\dot{v} + (I_{zz} - N_{\dot{r}})\dot{r} = \\ & N_{HS} + N_{v|v|}v\,|v| + N_{r|r|}r\,|r| + (N_{ur} - mx_g)ur + \\ & (N_{wp} + mx_g)wp + [N_{pq} - (I_{yy} - I_{xx})]pq - \\ & my_g(vr - wq) + N_{uv}uv + N_{uu\delta_r}u^2\delta_r. \end{aligned} \quad (93)$$

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Amir Hossein Davaie Markazi received the Ph.D. degree in mechanical engineering from Mechanical Engineering Department, McGill university, Canada in 1995. He is a professor at Iran University of Since and Technology Iran. He is the Editor-in-Chief of *International Journal of Engineering Science*.

His research interests include adaptive fuzzy siding mode control (AFSMC), control of chaotic systems, control of parallel robots, networked control systems and wavelet pattern recognition.

E-mail: markazi@iust.ac.ir

ORCID iD: 0000-0003-4077-2614

Mohammad Maadani received the B. Sc. degree in mechanical engineering from Amirkabir University of Technology (Tehran Polytechnic), Iran in 2009, the M. Sc. degree in mechanical engineering from Iran University of Science and Technology, Iran in 2011.

His research interests include adaptive control, nonlinear dynamics and control.

E-mail: mohammad.maadani1@gmail.com

Seyed Hassan Zabihifar received the B. Sc. and M. Sc. degrees in mechanical engineering and mechatronic respectively from Iran University of Since and Technology, Iran in 2012. He is currently a Ph.D. degree candidate in mechatronic and robotic systems in Bauman Moscow State Technical University (BMSTU), Russia.

His research interests include nonlinear control, adaptive fuzzy sliding mode, adaptive neural network, and optimal control.

Email: zabihifar@student.bmstu.ru (Corresponding author)

ORCID iD: 0000-0002-4808-478X

Nafiseh Doost-Mohammadi received the M. Sc. degree in mechanical engineering from Iran University of Since and Technology, Iran in 2011.

Her research interests include nonlinear control, adaptive fuzzy sliding mode and fuzzy control systems.

E-mail: nafise_doostmohamadi@yahoo.com