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Output Regulation of Multiple Heterogeneous Switched Linear Systems

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Abstract: This paper addresses the cooperative output regulation problem of a class of multi-agent systems (MASs). Each agent is a switched linear system. We propose an agent-dependent multiple Lyapunov function (MLF) approach to design the switching law for each switched agent. The distributed controller for each agent is constructed based on a dynamic compensator. A sufficient condition for the solvability of the output regulation problem of switched agent networks is presented by distributed controllers and agent-dependent multiple Lyapunov function approach. Finally, simulation results demonstrate the effectiveness of our theory.

Keywords: Switched multi-agent systems (MASs), cooperative output regulation, switched heterogeneous linear system, agent-dependent multiple Lyapunov function (MLF), distributed control.

1 Introduction

Multi-agent systems (MASs), as a group of autonomous agents, can be used to undertake large and complicated tasks through coordination among agents. Switched MASs in this study mean that the dynamics of each agent can be described by a switched system. A switched system is composed of a family of continuous-time or discrete-time subsystems and a rule determining the switching between the subsystems[1, 2]. Many practical systems may be modeled by switched systems due to the environment or parameter changes during the process of operation of systems such as constrained robot, power systems and vehicle control systems[3−5], etc. This inspires the study of switched systems. The main concern in the study of switched systems is the issue of stability. Under an arbitrary switching law, a necessary and sufficient condition to achieve asymptotic stability for a switched system is the existence of a common Lyapunov function (CLF) for all subsystems[6, 7]. But, a CLF does not often exist or is difficult to construct. Further, multiple Lyapunov function (MLF) approach proposed in [8] is a powerful and effective tool for designing switching. Successful applications of this approach to the stability analysis, the stabilization problem and robust H_∞ control problem of switched systems can be seen in [9, 10]. Other approaches about stability analysis of switched systems such as average dwell time approach were also proposed in [11, 12].

On the other hand, distributed cooperative control for MASs is a very active research area in the system and control society. Distributed control, which applies constrained information among agents to implement control objectives, achieves better desired performance similar to centralized control as well as reduces the complexity for designing controller like decentralized control[13]. Several main issues on distributed cooperative control include consensus, coordinated controller tuning, flocking and formation[14−17], etc.

Output regulation, which involves designing of a controller to ensure the system output tracking reference signal generated by an autonomous system meanwhile realizing interference suppression, is one of the most fundamental topics in control theory. The rich results on output regulation have appeared in [18, 19]. Many approaches on solving the output regulation problem, such as state feedback control, output feedback control and internal model principle[20−23], are proposed. Furthermore, these approaches are extended to cope with the distributed output regulation problem of MASs. For non-switched agent networks, by a dynamic full information distributed control scheme, the cooperative output regulation problem can be solved as in [24]. Further, a consensus control design approach for heterogeneous MASs is proposed to ensure that the outputs of all the agents converge to the same desired output trajectory by exploiting the internal model design strategy in [25]. Xiang et al.[26] address the problem of synchronized output regulation of linear networked system where all the nodes have their outputs which track signals produced by the same exosystem and the state of exosystem is accessible only to leader nodes, while follower nodes regulate their outputs via a distributed

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synchronous protocol. Moreover, a host internal model (h-IM) approach to distributed output regulation of reaching nonlinear leader-following consensus is presented in [27]. The switching of multi-agent systems involves two cases. One case is that the dynamics of each agent are described by a switched system, e.g., this paper. The other case is that the communication interconnection of agents is switching such as [28−30]. In [28], a cooperative problem of MASs with switched jointly connected interconnection topologies is addressed and a sufficient condition to make all the agents′ states converge to a common value is given for the problem by a Lyapunov-based approach and related space decomposition technique. Under the condition of jointly connected switching interconnected topologies, the leader-following consensus can be reached by designing appropriate control law in [29]. In [30], for different network topologies, three types of event-triggered schemes (ETSs) are proposed to reach leader-following consensus. Limited literature exists on output regulation for the case where each agent is a switched system. In [31], the output regulation problem of switched linear multi-agent systems with stabilisable and unstabilisable subsystems is investigated and an agent-dependent average dwell time method is proposed. However, if the output regulation problem is solvable for none of the subsystems of each agent, the proposed approach in [31] is invalid. But, the cooperative output regulation can still be achieved for switched agent networks by using the agent-dependent multiple Lyapunov function approach proposed in this paper. Furthermore, under arbitrary switching, the output regulation problem of switched agent networks with a special form is studied by designing a distributed control law in [32].

Motivated by the above considerations, this note studies the cooperative output regulation problem of MASs in which each agent dynamics are represented by a switched heterogeneous linear system. The paper has two main contributions. First, an agent-dependent multiple Lyapunov function approach to solve the output regulation problem of switched MASs is proposed. Agent-dependent means the switching law for each agent is dependent on its own state and does not rely on other agents′ states. Since each agent is a switched system, the switched MASs is naturally an overall switched system. If the classical multiple Lyapunov function approach were applicable to the whole switched MASs, the study would be trivial. Further, this agent-dependent switching approach is more flexible and practicable because the overall switched MAS is no longer taken as a full system to which the multiple Lyapunov function approach is applied. To our best knowledge, this approach has not appeared in the existing literature. The results of this paper extend the existing work for non-switched multi-agent systems[24] to switched agent network where each agent is a switched linear system. This brings more difficulties and challenges for the solvability of the output regulation problem. In addition, the output regulation problem for each agent is solvable in [24]. However, the output regulation problem is solvable for none of the subsystems of each agent. Then, the agent-dependent multiple Lyapunov function approach is proposed to achieve output regulation for switched multi-agent systems. Finally, each agent is described by a switched system considered in this paper whereas the communication interconnection of agents is considered switching in [29, 30]. Also, the switching signal satisfies dwell time. In this paper, the switching law is designed to realize the regulation goal. Secondly, owing to the constrained information exchanges, i.e., not every agent can obtain the exosystem′s information and each agent can only obtain the information of its neighboring agents, a distributed controller for each agent is designed. Then, a sufficient condition for the solvability of the output regulation problem of switched agent networks is presented by distributed controllers and agent-dependent multiple Lyapunov function approach. In addition, compared with the existing literature[32], the switched linear MASs considered have more general structure.

The remainder of the paper is organized as follows. The preliminaries and problem statement are given in Section 2. In Section 3, the main results are developed. An example is given in Section 4. Section 5 draws a conclusion.

Notations. Let M and N are two real matrices. The Kronecker product of M and N is denoted by $M \otimes N$. $M^{\rm T}$ represents the transpose of M. We use $\|\cdot\|_2$ to represent the Euclidean 2-norm of vectors.

2 Problem formulation

The information flow among agents and exosystem is described by a directed or undirected graph and the detailed content can be found in [13]. A graph is denoted by $G = (\vartheta, \varepsilon)$, where $\vartheta = \{0, 1, 2, \cdots, N\}$ is a node set and $\varepsilon \in \vartheta \times \vartheta$ is an edge set. If an edge $(i, j) \in \varepsilon$, this implies that node j is a neighbor of node i, i.e., node j can get information from node i and the term a_{ji} is a positive weight, otherwise $a_{ji} = 0$. $A = [a_{ij}], i, j = 1, 2, \cdots, N$ is an adjacency matrix of the graph G. The i-th agent′s neighbor set is denoted by $\Phi_i = \{j : (j, i) \in \varepsilon, i = 1, 2, \cdots, N\}$. Let a diagonal matrix $D = \mathrm{diag}\,\{\sum_{j=1}^N a_{1j},\ \sum_{j=1}^N a_{2j}, \cdots, \sum_{j=1}^N a_{Nj}\}$, the Laplacian matrix L will be defined as $L = D - A$. Let a_{i0} expresses the connection weight from the i-th agent to the leader, and $a_{i0} > 0$ if the i-th agent can obtain information from the leader, otherwise $a_{i0} = 0$. Set a diagonal matrix $A_0 = \mathrm{diag}\,\{a_{10}, a_{20}, \cdots, a_{N0}\}$. Let $H = L + A_0$, which describes the connectivity of the whole multi-agent systems in G. In order to solve the output regulation problem, we will introduce a lemma about the matrix H.

Lemma 1.[13] H is positive definite if and only if node 0 is globally reachable in G.

The MASs considered in our paper is composed of N switched heterogeneous agents, the dynamics of each agent are described by a switched linear system

$$\begin{aligned}\dot{x_i} &= A_{i\sigma_i(t)}x_i + B_{i\sigma_i(t)}u_i + E_{i\sigma_i(t)}\omega\\ e_i &= C_i x_i + F\omega, \quad i \in I_N = \{1, 2, \cdots, N\}\end{aligned} \tag{1}$$

where $x_i \in \mathbf{R}^{n_i}, u_i \in \mathbf{R}^{p_i}, y_i \in \mathbf{R}^q$ are the state, control input and output of the i-th agent, respectively. $\sigma_i(t)$: $[0,\infty) \rightarrow M_i = \{1, 2, \cdots, m_i\}$ is the i-th agent′s switching signal which depends on agent′s state, time or both. $A_{ij_i}, B_{ij_i}, E_{ij_i}, C_i, i \in I_N, j_i \in M_i$ are constant real matrices of suitable dimensions. e_i is the i-agent′s regulated error output. $\omega \in \mathbf{R}^q$ is the exogenous signal and is generated by the following autonomous linear system.

$$\dot{\omega} = S\omega, \quad y_0 = -F\omega \tag{2}$$

where S, F are constant real matrices and y_0 is the reference output.

We assume that each agent′s state is measurable, but each agent does not obtain the information of the exosystem. Instead, each agent can only obtain the information of its neighboring agents. Since the information flow among agents is constrained, each agent′s controller is designed relying on all of these available information. Here, a distributed controller is proposed similar to that in [24].

$$u_i = K_{i\sigma_i(t)}x_i + G_{i\sigma_i(t)}\eta_i \tag{3}$$

$$\dot{\eta_i} = S\eta_i + \sum_{j\in\Phi_i} a_{ij}(\eta_j - \eta_i) + a_{i0}(\omega - \eta_i) \tag{4}$$

where $K_{ij_i} \in \mathbf{R}^{p_i\times n_i}, G_{ij_i} \in \mathbf{R}^{p_i\times q},\ i \in I_N, j_i \in M_i$ are gain matrices to be determined later and a_{ij}, a_{i0}, which are introduced in Section 2, are entries of the adjacency matrix A and degree matrix D, respectively.

Remark 1. As in [24], the dynamical system (4) is called a dynamic compensator (or a distributed observer) and can be viewed as an estimate of the exosystem state ω for the i-th agent.

The control objective is to design distributed controllers (3)−(4) and switching laws $\sigma_i(t)$ to solve the cooperative output regulation problem of switched MASs (1) and the exosystem (2). The details are stated below.

The cooperative output regulation problem of switched MASs. Given systems (1) and (2), if possible, find the control law (3)−(4) and switching signals $\sigma_i(t)$ such that

1) The origin of the switched MASs (1) with controllers (3) and (4) are asymptotically stable under the designed switching law $\sigma_i(t)$ when $\omega = 0$.

2) The tracking error e_i satisfies

$$\lim_{t\rightarrow\infty} e_i(t) = 0 \tag{5}$$

for any initial condition $x_i(0), \eta_i(0)$ and $\omega(0), i \in I_N$.

Throughout this paper, Assumptions 1–4 and Lemma 2 are needed.

Assumption 1. All the eigenvalues of S are semi-simple with zero real part.

Remark 2. This assumption relaxes Assumption 1 in [24] in order to simplify the calculation. If this assumption was replaced by Assumption 1 in [24], the output regulation problem for switched multi-agent systems would also be solved by the same technique under the controller in [24].

Assumption 2. For $i \in I_N, j_i \in M_i$, the linear matrix equations

$$\begin{aligned}X_i S &= A_{ij_i}X_i + B_{ij_i}U_{ij_i} + E_{ij_i}\\ 0 &= C_i X_i + F\end{aligned} \tag{6}$$

have the solution pairs (X_i, U_{ij_i}), respectively.

Assumption 3. $\sigma_i(t), i = 1, 2, \cdots, N$, has finite number of switchings in any finite interval of time.

Remark 3. Assumption 2 is a standard assumption in the literature to solve the output regulation for non-switched linear multi-agent systems[24]. Assumption 4 is a standard assumption in the literature to rule out Zeno behavior for all types of switching[1].

Let G be a graph describing the connection among agents in company with the exosystem (2).

Assumption 4. The graph G includes a spanning tree with root on node 0 (the exosystem (2)).

Lemma 2.[31] Consider the switched linear time-varying system

$$\begin{aligned}\dot{x_c} &= A_{c\sigma_c(t)}x_c + B_{c\sigma_c(t)}v\\ \dot{v} &= \Gamma v\\ e &= C_c x_c + D_c v\end{aligned} \tag{7}$$

where $x_c \in \mathbf{R}^{n_c}, v \in \mathbf{R}^{q_c}$ are states. $\sigma_c(t) : [0,\infty) \rightarrow \{1, 2, \cdots, \rho\}$ is the switching signal. $A_{ci}, B_{ci}, i = 1, 2, \cdots, \rho$ and C_c, D_c are constant matrices. Suppose that the switched linear system $\dot{x_c} = A_{c\sigma_c(t)}x_c$ is asymptotically stable under the switching signal $\sigma_c(t)$ and there exists a set of constant matries X_c that satisfies the following linear matrix equations:

$$\begin{aligned}X_c\Gamma &= A_{ci}X_c + B_{ci}\\ 0 &= C_c X_c + D_c, i = 1, 2, \cdots, \rho\end{aligned} \tag{8}$$

then, we have

$$\lim_{t\rightarrow\infty} e(t) = 0. \tag{9}$$

3 Main result

In this section, a sufficient condition for the solvability of output regulation problem of switched agent networks is given via the agent-dependent multiple Lyapunov function approach and designing distributed controllers.

It is easy to know that the agent′s switching results in the switching of the overall MAS. Here, the merging signal technique in [32] is used to describe the switching signal of the whole switched MAS. This emerging switching signal of the whole switched agent networks is denoted by $\sigma(t) = (\sigma_1(t), \sigma_1(t), \cdots, \sigma_1(t)) : [0,\infty) \rightarrow M_1 \times M_2 \times \cdots \times M_N$, where $\sigma_i(t), i \in I_N$ is the i-th agent′s switching signal. The merging switching behavior means that the whole switched multi-agent system has $M_1 \times M_2 \times \cdots \times M_N$ subsystems and the set of switching times for $\sigma(t)$ is the union of the sets of switching times for $\sigma_i(t), i \in I_N$.

Suppose that the gain matrices are $K_{ij_i}, i \in I_N, j_i \in M_i$, which will be given later. Then, let

$$G_{ij_i} = U_{ij_i} - K_{ij_i} X_i \tag{10}$$

where X_i, U_{ij_i} are the solutions of (6).

Let

$$\begin{aligned}
&A_j = \text{block diag}(A_{1j_1}, A_{2j_2}, \cdots, A_{Nj_N})\\
&B_j = \text{block diag}(B_{1j_1}, B_{2j_2}, \cdots, B_{Nj_N})\\
&E_j = \text{block diag}(E_{1j_1}, E_{2j_2}, \cdots, E_{Nj_N})\\
&K_j = \text{block diag}(K_{1j_1}, K_{2j_2}, \cdots, K_{Nj_N})\\
&G_j = \text{block diag}(G_{1j_1}, G_{2j_2}, \cdots, G_{Nj_N})\\
&C = \text{block diag}(C_1, C_2, \cdots, C_N)\\
&X = \text{block diag}(X_1, X_2, \cdots, X_N)\\
&i \in I_N, j_i \in M_i, j \in M_1 \times M_2 \times \cdots \times M_N.
\end{aligned}$$

By (6) and (10), we have

$$\begin{aligned}
&X(I_N \otimes S) = (A_j + B_j K_j)X + (E_j + B_j G_j)\\
&0 = CX + (I_N \otimes F).
\end{aligned} \tag{11}$$

We now give the solvability condition of cooperative output regulation for switched agent networks.

Theorem 1. Let Assumptions 1−3 hold. Suppose that there exists symmetric and positive-definite matrices P_{ij_i} and constants $\beta_{ij_ij_s} > 0, i \in I_N, j_i, j_s \in M_i$ such that

$$\begin{aligned}
&(A_{ij_i} + B_{ij_i} K_{ij_i})^{\mathrm{T}} P_{ij_i} + P_{ij_i}(A_{ij_i} + B_{ij_i} K_{ij_i}) + \\
&\quad \gamma_{01} I_{n_i} + \sum_{j_s=1}^{m_i} \beta_{ij_ij_s}(P_{ij_s} - P_{ij_i}) < 0.
\end{aligned} \tag{12}$$

Then, the output regulation problem for the system (1) and exosystem (2) is solved by the controller (3) and (4) under the following agent-dependent switching law

$$\sigma_i(t) = \arg\min_{1 \le j_i \le m_i} \{x_i^{\mathrm{T}} P_{ij_i} x_i\}. \tag{13}$$

Proof. Under the distributed control laws (3) and (4), the closed-loop system of i-th agent is

$$\begin{aligned}
&\dot{x_i} = (A_{ij_i} + B_{ij_i} K_{ij_i})x_i + B_{ij_i} G_{ij_i} \eta_i + E_{ij_i}\omega\\
&\dot{\eta_i} = S\eta_i + \sum_{j \in \Phi_i} a_{ij}(\eta_j - \eta_i) + a_{i0}(\omega - \eta_i)\\
&\dot{\omega} = S\omega\\
&e_i = C_i x_i + F\omega.
\end{aligned} \tag{14}$$

Let $x = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_N^{\mathrm{T}}), \eta = (\eta_1^{\mathrm{T}}, \eta_2^{\mathrm{T}}, \cdots, \eta_N), e = (e_1, e_2, \cdots, e_N)$ and $\tilde{\omega} = 1_N \otimes \omega$. Thus, the overall closed-loop system of the switched MAS can be rewritten as

$$\begin{aligned}
&\dot{x} = (A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)})x + B_{\sigma(t)} G_{\sigma(t)}\eta + E_{\sigma(t)}\tilde{\omega}\\
&\dot{\eta} = ((I_N \otimes S) - (H \otimes I_q))\eta + (H \otimes I_q)\tilde{\omega}\\
&\dot{\tilde{\omega}} = (I_N \otimes S)\tilde{\omega}\\
&e = Cx + (I_N \otimes F)\tilde{\omega}.
\end{aligned} \tag{15}$$

where $\sigma(t)$ is the merging switching signal of the the whole MAS, while the switching signal of the i-th agent is still $\sigma_i(t)$.

First, we will prove the origin of the closed-loop system

$$\begin{aligned}
&\dot{x} = (A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)})x + B_{\sigma(t)} G_{\sigma(t)}\eta\\
&\dot{\eta} = [(I_N \otimes S) - (H \otimes I_q)]\eta
\end{aligned} \tag{16}$$

without the disturbance input (i.e., $\omega = 0$) is asymptotically stable. Obviously, according to Assumptions 1 and 3, $[(I_N \otimes S) - (H \otimes I_q)]$ is Hurwitz. Therefore, there exist a symmetric and positive-define matrix Q and a positive constant γ_{02} such that

$$\begin{aligned}
&[(I_N \otimes S) - (H \otimes I_q)]^{\mathrm{T}} Q + Q[(I_N \otimes S) - (H \otimes I_q)] + \\
&\quad \gamma_{02} I_{Nq} < 0.
\end{aligned} \tag{17}$$

Let $V_{ij_i}, i \in I_N, j_i \in M_i$ be Lyapunov candidate functions for each subsystem of the i-th agent, then, the i-th agent′s Lyapunov function is denoted as

$$V_i = \sum_{j_i=1}^{m_i} \iota_{ij_i} V_{ij_i}$$

where ι_{ij_i} is $Z^+ \to \{0, 1\}$ and $\sum_{j_i=1}^{m_i} \iota_{ij_i} = 1$, which indicates that $\iota_{ij_i} = 1$ when the j_i-th subsystem of the i-th agent is active, otherwise $\iota_{ij_i} = 0$.

For the closed-loop system (16), we define the following Lyapunov function

$$\begin{aligned}
V(x, \eta) = &\sum_{i=1}^{N} \sum_{j_i=1}^{m_i} \iota_{ij_i} V_{ij_i}(x_i) + kV(\eta) = \\
&\sum_{i=1}^{N} \sum_{j_i=1}^{m_i} \iota_{ij_i} x_i^{\mathrm{T}} P_{ij_i} x_i + k\eta^{\mathrm{T}} Q\eta
\end{aligned} \tag{18}$$

where $x = (x_1, x_2, \cdots, x_N)^{\mathrm{T}}$ and k is a positive constant which will be determined later.

Without loss of generality, when the j_i-th subsystem of the i-th agent is active, by (12) and (17), we can derive that

$$\begin{aligned}
\dot{V}(x, \eta) = &\sum_{i=1}^{N} (\dot{x}_i^{\mathrm{T}} P_{ij_i} x_i + x_i^{\mathrm{T}} P_{ij_i} \dot{x}_i) + \\
&k(\dot{\eta}^{\mathrm{T}} Q\eta + \eta^{\mathrm{T}} Q\dot{\eta}) = \\
&\sum_{i=1}^{N} \{x_i^{\mathrm{T}}[(A_{ij_i} + B_{ij_i} K_{ij_i})^{\mathrm{T}} P_{ij_i} + \\
&P_{ij_i}(A_{ij_i} + B_{ij_i} K_{ij_i})]x_i + 2x_i^{\mathrm{T}} P_{ij_i} B_{ij_i} G_{ij_i} \eta_i\} + \\
&k\eta^{\mathrm{T}}\{[(I_N \otimes S) - (H \otimes I_q)]^{\mathrm{T}} Q + \\
&Q[(I_N \otimes S) - (H \otimes I_q)]\}\eta < \\
&\sum_{i=1}^{N} \{-\sum_{j_s=1}^{m_i} [\beta_{ij_ij_s} {x_i}^{\mathrm{T}}(P_{ij_s} - P_{ij_i})x_i] - \gamma_{01}\|x_i\|_2^2 + \\
&2x_i^{\mathrm{T}} P_{ij_i} B_{ij_i} G_{ij_i} \eta_i\} - k\gamma_{02}\|\eta\|_2^2, (x, \eta)^{\mathrm{T}} \ne 0.
\end{aligned} \tag{19}$$

We know that there exist constants $\alpha_i > 0, \delta_i > 0, i \in I_N, j_i \in M_i$, such that

$$\|x_i^{\mathrm{T}} P_{ij_i}\|_2 \le \alpha_i \|x_i\|_2, \quad \|B_{ij_i} G_{ij_i} \eta_i\|_2 \le \delta_i \|\eta_i\|_2.$$

Let $\theta = \max\{\alpha_i\delta_i, i \in I_N\}$, we have

$$\dot{V}(x,\eta) < \sum_{i=1}^{N}\{-\sum_{j_s=1}^{m_i}[\beta_{ij_ij_s}{x_i}^{\mathrm{T}}(P_{ij_s}-P_{ij_i})x_i]- \sum_{i=1}^{N}\gamma_{01}(\|x_i\|_2-\frac{\theta}{\gamma_{01}\parallel\eta_i\parallel_2})^2\}+ \sum_{i=1}^{N}\frac{\theta^2}{\gamma_{01}^2}\|\eta_i\|_2^2-k\gamma_{02}\|\eta\|_2^2< \sum_{i=1}^{N}\left\{-\sum_{j_s=1}^{m_i}\left[\beta_{ij_ij_s}{x_i}^{\mathrm{T}}(P_{ij_s}-P_{ij_i})x_i\right]\right\}- \frac{k\gamma_{02}-\theta^2\gamma_{01}^2}{\|\eta\|_2^2},\quad (x,\eta)^{\mathrm{T}}\neq 0. \tag{20}$$

Choose

$$k > \frac{\theta^2}{\gamma_{02}\gamma_{01}^2}. \tag{21}$$

We have

$$\dot{V}(x,\eta) < -\sum_{i=1}^{N}\sum_{j_s=1}^{m_i}{x_i}^{\mathrm{T}}[\beta_{ij_ij_s}(P_{ij_s}-P_{ij_i})]x_i, \quad (x,\eta)^{\mathrm{T}}\neq 0. \tag{22}$$

Under the switching law (13), we obtain that $\dot{V}(x,\eta) < 0, (x,\eta)^{\mathrm{T}} \neq 0$. Thus, the origin of the switched MASs (1) with controllers (3) and (4) are asymptotically stable under the designed switching law $\sigma_i(t)$ when $\omega = 0$.

Then, we will prove the second condition of the cooperative output regulation problem of switched MASs. Rewrite the closed-loop system (15) as

$$\begin{aligned}\dot{x_c} &= A_{cj}x_c + E_{cj}\tilde{\omega}\\ \dot{\tilde{\omega}} &= (I_N\otimes S)\tilde{\omega}\\ e &= C_cx_c + F_c\tilde{\omega}\end{aligned} \tag{23}$$

where

$$x_c = \begin{pmatrix} x\\ \eta\end{pmatrix}$$

$$A_{cj} = \begin{pmatrix} A_j+B_jK_j & B_jG_j\\ 0 & (I_N\otimes S)-(H\otimes I_q)\end{pmatrix}$$

$$E_{cj} = \begin{pmatrix} E_j\\ H\otimes I_q\end{pmatrix}, C_c = \begin{pmatrix} C & 0\end{pmatrix}$$

$$F_c = I_N\otimes F, j\in M_1\times M_2\times\cdots M_N.$$

By (11), we have

$$A_{cj}X_c + E_{cj} = \begin{pmatrix} A_j+B_jK_j & B_jG_j\\ 0 & (I_N\otimes S)-(H\otimes I_q)\end{pmatrix} \begin{pmatrix} X\\ I_{qN}\end{pmatrix} + \begin{pmatrix} E_j\\ H\otimes I_q\end{pmatrix} = \begin{pmatrix} (A_j+B_jK_j)X+B_jG_j+E_j\\ I_N\otimes S\end{pmatrix} = \begin{pmatrix} X(I_N\otimes S)\\ I_N\otimes S\end{pmatrix}\begin{pmatrix} X\\ I_{qN}\end{pmatrix}(I_N\otimes S)$$

and $C_cX_c+F_c = \begin{pmatrix} C & 0\end{pmatrix}\begin{pmatrix} X\\ I_{qN}\end{pmatrix} + I_N\otimes F = C_cX_c + I_N\otimes F = 0$.

Using the result of Lemma 2, we obtain

$$\lim_{t\to\infty} e(t) = 0. \tag{24}$$

□

Remark 4. According to Theorem 1, even though the output regulation problem is solvable for none of the subsystems of each agent, the output regulation can still be achieved for the switched agent networks under the designed switching law. This certainly increases the possibility of the solvability of the output regulation problem for switched multi-agent systems.

Remark 5. Applying the Schur complement lemma, we can convert (12) to the linear matrix inequalities (LMIs)

$$\begin{pmatrix} \begin{matrix}X_{ij_i}^{\mathrm{T}}A_{ij_i}^{\mathrm{T}}+\\ R_{ij_i}^{\mathrm{T}}B_{ij_i}^{\mathrm{T}}+\\ A_{ij_i}X_{ij_i}+\\ B_{ij_i}R_{ij_i}+\\ \sum_{j_s=1}^{m_i}\beta_{ij_ij_s}X_{ij_i}\end{matrix} & \sqrt{\beta_{ij_i1}}X_{ij_i} & \sqrt{\beta_{ij_i2}}X_{ij_i} & \cdots & \sqrt{\beta_{ij_im_i}}X_{ij_i}\\ \sqrt{\beta_{ij_i1}}X_{ij_i} & -X_{i1} & 0 & \cdots & 0\\ \sqrt{\beta_{ij_i2}}X_{ij_i} & 0 & -X_{i2} & \cdots & 0\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ \sqrt{\beta_{ij_im_i}}X_{ij_i} & 0 & 0 & \cdots & -X_{im_i}\end{pmatrix} < 0$$

where $P_{ij_i}^{-1} = X_{ij_i}, K_{ij_i}P_{ij_i}^{-1} = R_{ij_i}$, $i = 1,2,\cdots,N; j_i = 1,2,\cdots,m_i$, and $\beta_{ij_ij_s}, i = 1,2,\cdots,N; j_i, j_s = 1,2,\cdots,m_i$ are given.

Remark 6. The conventional state-dependent switching law may lead to Zeno effect, which can degrade and even damage the performance of the systems. Hysteresis switching approach can be adopted to avoid Zeno behavior, e.g., see [33, 34] and the references therein.

Remark 7. According to the proof of Theorem 1, we know that the cooperative output regulation problem of the switched MASs is solved when k is a sufficiently large constant by using the agent-dependent multiple Lyapunov function approach.

4 Example

In what follows, we present a numerical example to illustrate the effectiveness of our result. Consider the switched MASs described by

$$\begin{aligned}\dot{x_i} &= A_{ij_i}x_i + B_{ij_i}u_i + E_{ij_i}\omega\\ e_i &= C_ix_i + F\omega, i = 1,2,\ j_1 = 1,2,\ j_2 = 1,2.\end{aligned} \tag{25}$$

The exosystem is given by

$$\dot{\omega} = S\omega \tag{26}$$

where

$$A_{11} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}, A_{12} = \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix}, B_{11} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$B_{12} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, E_{11} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, E_{12} = \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, A_{22} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, B_{21} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$B_{22} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, E_{21} = \begin{pmatrix} 2 & 0 \\ -1 & 0 \end{pmatrix}, E_{22} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, C_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, F = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

By Assumption 3, we give the matrix H describing the information exchange of the agent networks as

$$H = D - A + A_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

Solving the linear matrix (6), we obtain

$$X_1 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, U_{11} = \begin{pmatrix} -2 & -6 \end{pmatrix}$$

$$U_{12} = \begin{pmatrix} 5 & -2 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, U_{21} = \begin{pmatrix} 2 & 0 \end{pmatrix}$$

$$U_{22} = \begin{pmatrix} -1 & 0 \end{pmatrix}.$$

Let $\beta_{111} = 0.25$, $\beta_{112} = 0.09$, $\beta_{121} = 0.16$, $\beta_{122} = 0.36$, $\beta_{211} = 0.04$, $\beta_{212} = 0.16$, $\beta_{221} = 0.09$, $\beta_{222} = 0.16$, $\gamma_{01} = 1.1$, $\gamma_{02} = 2.1$, according to the inequalities (12), we have

$$P_{11} = \begin{pmatrix} 0.013\,3 & 0.006\,3 \\ 0.006\,3 & 0.007\,9 \end{pmatrix}$$

$$P_{12} = \begin{pmatrix} 0.008\,1 & 0.012\,3 \\ 0.012\,3 & 0.037\,6 \end{pmatrix}$$

$$P_{21} = \begin{pmatrix} 0.012\,9 & 0.002\,2 \\ 0.002\,2 & 0.000\,8 \end{pmatrix}$$

$$P_{22} = \begin{pmatrix} 0.001\,4 & -0.000\,6 \\ -0.000\,6 & 0.001\,6 \end{pmatrix}$$

$$K_{11} = \begin{pmatrix} -7.217\,6 & -1.982\,4 \end{pmatrix}$$

$$K_{12} = \begin{pmatrix} -8.935\,2 & -19.187\,8 \end{pmatrix}$$

$$K_{21} = \begin{pmatrix} -16.631\,4 & -3.464\,9 \end{pmatrix}$$

$$K_{22} = \begin{pmatrix} -1.875\,4 & 0.173\,0 \end{pmatrix}.$$

Choosing $x_{11}(0) = 0.03, x_{12}(0) = -0.12, x_{21}(0) = -0.08, x_{22}(0) = 0.32, \eta_{11}(0) = 0.24, \eta_{12}(0) = -0.01, \eta_{21}(0) = -0.13$, $\eta_{22}(0) = 0.21$ as the initial conditions and applying distributed switched dynamic feedback control law (3) and (4), we obtain the simulation results in Figs.1−5. These verify the validity of our theory in this paper.

5 Conclusions

The cooperative output regulation problem of switched heterogeneous linear agent networks has been investigated. The switching dynamics of each agent make the cooperative output regulation problem more difficult for switched MASs. Owing to the limited information exchanges among

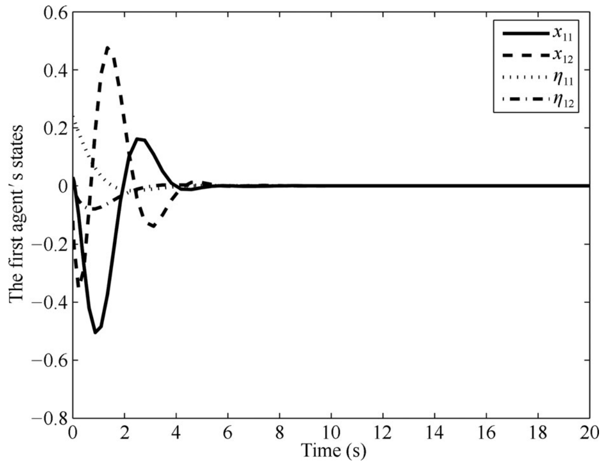

Fig. 1 The first agent′s states response of close-loop systems without the disturbance input

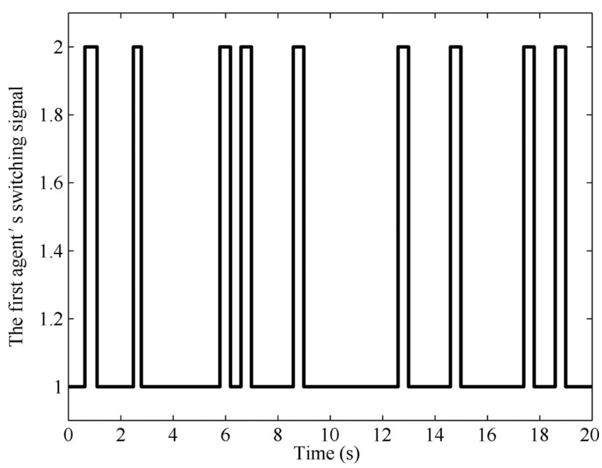

Fig. 2 The first agent′s switching signal

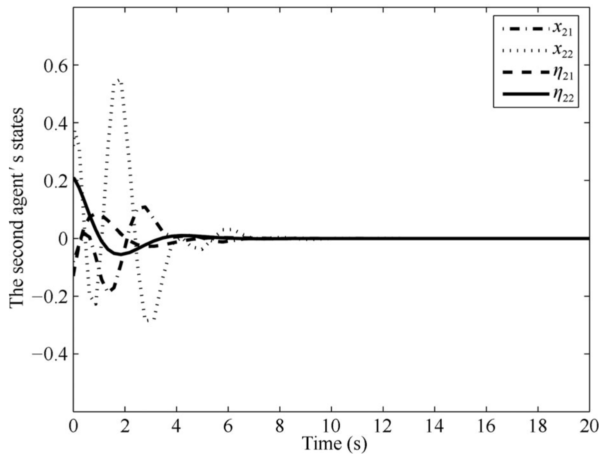

Fig. 3 The second agent′s states response of close-loop systems without the disturbance input

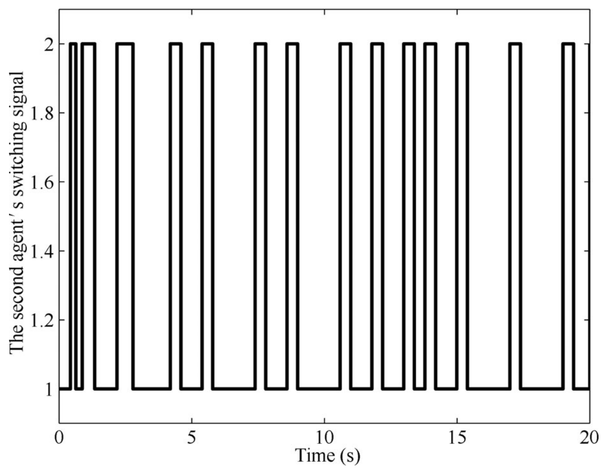

Fig. 4 he second agent′s switching signal

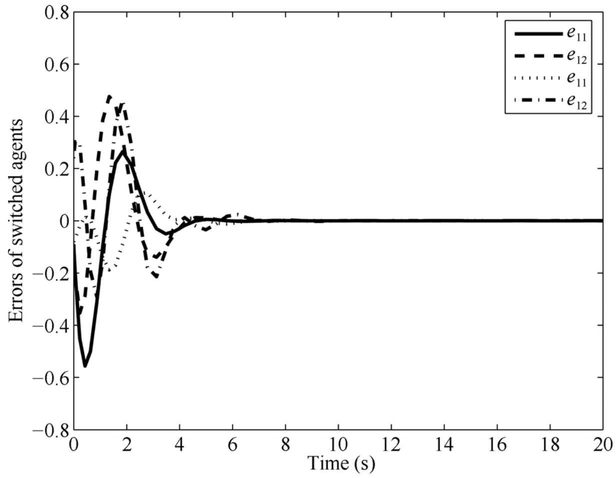

Fig. 5 Errors of switched agents

agents, a distributed controller for each agent is established based on a distributed dynamic compensator. On the basis of the traditional multiple Lyapunov function approach, a new agent-dependent multiple Lyapunov function approach is presented in order to solve the output regulation problem. Further work will be focused on the output regulation problem of MASs in which each agent dynamics are described by a switched nonlinear system.

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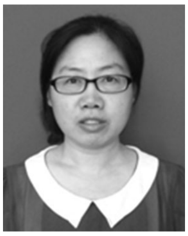

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