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Stability Analysis of an Underactuated Autonomous Underwater Vehicle Using Extended-Routh′s Stability Method

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Abstract: In this paper, a new approach to stability analysis of nonlinear dynamics of an underactuated autonomous underwater vehicle (AUV) is presented. AUV is a highly nonlinear robotic system whose dynamic model includes coupled terms due to the hydrodynamic damping factors. It is difficult to analyze the stability of a nonlinear dynamical system through Routh′s stability approach because it contains nonlinear dynamic parameters owing to hydrodynamic damping coefficients. It is also difficult to analyze the stability of AUVs using Lyapunov′s criterion and LaSalle′s invariance principle. In this paper, we proposed the extended-Routh′s stability approach to verify the stability of such nonlinear dynamic systems. This extended-Routh′s stability approach is much easier as compared to the other existing methods. Numerical simulations are presented to demonstrate the efficacy of the proposed stability verification of the nonlinear dynamic systems, e.g., an AUV system dynamics.

Keywords: Routh′s stability, extended-Routh′s stability, autonomous underwater vehicle (AUV), underactuated system, underwater robots.

1 Introduction

In the recent years, the research on autonomous underwater vehicle (AUV) has become promising in the field of advanced robotics due to their specific applications such as security patrols, search and rescue in hazardous environments[1−8]. In underactuated systems, a fewer number of control inputs are available than the degrees of freedom (DOF). Further, there lies difficulty in the stability proof of such system owing to the absence of some control inputs for DOF to control[9].

An AUV is an unmanned mobile robot which is deployed into motion in acoustic environment, e.g., oceans. In military applications, AUVs are also known as unmanned undersea vehicles (UUVs). AUVs constitute part of a larger group of undersea systems known as unmanned underwater vehicles. Another part which includes non-autonomous remotely operated underwater vehicles (ROVs) is controlled and powered from the surface by an operator/pilot.

During the last two and more decades, a group of AUVs forming a formation or cooperation are assigned different group tasks. In military missions, a group of autonomous vehicles are put to keep a specified formation for area coverage and reconnaissance. In small satellite clustering, formation helps to reduce the fuel consumption for propulsion and expand their sensing capabilities.

Due to the increasing demand of oil and exponential rise in oil prices, importance of exploring new energy sources has given prime importance. As the importance of energy sources increases, the area of finding the same also increases and it is extended to the deep see areas as well. Different countries strive hard for achieving the success of finding new energy resources through search operations by a group of AUVs. Exploring and developing deep sea areas necessitate different kinds of sensors and devices. Remotely operated vehicles (ROVs) and AUVs directly meet these requirements[4]. In case of exploration and exploitation of resources located at deep oceanic environment, AUVs plays an important role. The AUVs are used in risky and hazardous operations such as bathymetric surveys, oceanographic observations, recovery of lost man-made objects, ocean floor analysis, etc.

In some complex cases, the goal can be achieved by utilizing more than one vehicles simultaneously in a group[10−14]. This is because the exploitation of these complex tasks are beyond the capability of a single AUV. Multiple AUVs are employed to perform the task easily within a short period of time (Fig. 1). This group performance of the multiple AUVs are known as cooperative control. Formation and flocking control of multiple AUVs are considered as cooperative control. Cooperative control of AUVs is an important research topic. Through formation control algorithm, the cooperative motion multi-AUV systems can be achieved. The formation control deals with the problem of controlling the relative positions and orientations of AUVs moving in

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a group while allowing the group to move as a whole. The step-by-step problems of formation control are accomplished through the following distinct steps: 1) assignment of feasible formation, 2) moving in formation, 3) maintenance of formation shape, 4) switching between formations. Formation is specified in two different ways, i.e., the rigid formation[15] and flexible formation (desired configuration may vary)[16]. Flocking is the flying behaviours of a group (flock) of birds. This is applicable to control a group of multiple AUVs to perform a desired task. Flocking control of multiple AUVs is similar to that of formation control with only difference is that there are no constraints on distance among AUVs (no distance among AUVs is zero to avoid collision). In case of formation control, the distances among AUVs are always fixed.

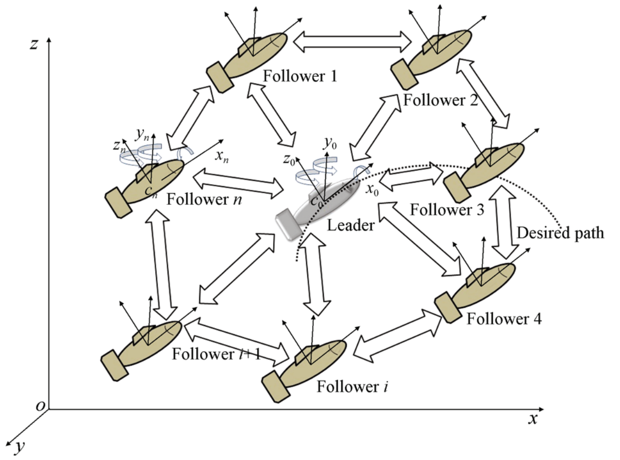

Fig. 1 Schematic representation of cooperative control of AUVs (leader-follower approach)

Keeping in view of aforesaid applications of AUVs, their control design and stability analysis is a subject of enormous importance to the control engineers. In this paper, we focus on the stability analysis of such nonlinear systems. Prior to presenting our work, we first review some of the approaches to nonlinear system stability analysis. Some of the stability analysis methods employed for nonlinear systems are as follows. The describing function (DF) method is used for stability analysis of nonlinear systems for which detailed description is available in [17]. DF method can be combined with the small gain theorem (SGT) to analyze the robust stability of nonlinear systems which has separable nonlinearities[18]. Bode envelop of linear transfer functions along with DF may be used for presenting the stability analysis of nonlinear systems[19]. In the early seventies, the stability analysis of nonlinear bounded input and bounded output (BIBO) systems was carried out by using Popov′s stability criterion[20]. The nonlinear systems whose stability probability is one as well as the p-stability of the nonlinear systems can be analyzed with the use of Popov′s stability criterion[21].

The stability of the nonlinear system can be analyzed using different new and efficient methods. These methods include Lyapunov stability criteria[22, 23], *D*-stabilization using the Lyapunov′s method[24], LaSalles invariance principle[25], etc. Asymptotical stabilization of a feedback linearized controller is analyzed based on potential functions in [26]. The stability of the controller for path following of underactuated AUV has been analyzed by employing direct Lyapunov candidate function in [27]. This Lyapunov′s stability criterion is employed for the stability analysis of formation controllers of fully actuated AUVs in [3, 28]. Lyapunov′s method along with LaSalles invariance principle both have been applied in combination for stability analysis of consensus based flocking controller of mobile robots whose dynamics is nonlinear in [29]. These are generally complex methods because in case of Lyapunov approach, it is difficult to choose the Lyapunov′s function and further Lasalles invariance principle also depends upon Lyapunov approach. The stability of the nonlinear systems is analyzed in the discrete time domain[30−32].

When considering the kinematics and dynamics of the AUV, different nonlinear and complex are taken to be consideration. These are, the inter-coupled mass matrix which is the inertia matrix contains added masses, the centripetal and Coriolis forces and torques, hydrodynamic damping factors, the lifting and gravitational effects, etc. These factors make the dynamics of the AUV are highly nonlinear and coupled. However, the stability of the AUV systems should be analyzed and ensured once the controller is to be applied in real-time. Hence, the stability analysis of the AUVs and the controllers are important as well as difficult tasks. For simplifying this task, a new approach using extended-Routh′s Stability criterion is presented in this paper. The contribution of this paper lies in the development of an innovative idea for the proof of stability of the dynamics of a nonlinear system particularly underactuated AUVs. The stability is proved based on extended-Routh′s stability criterion. It is a simpler approach to analyze the stability of a dynamic system in comparison to the other methods described in previous paragraphs.

The rest of the paper is organized as follows. The stability analysis of nonlinear systems in generalized form with existing techniques are briefly explained in Section 2. Extended-Rouths stability method is then presented in Section 3. In Section 4, AUV kinematics and dynamics are described. Stability analysis of the dynamic system is demonstrated in Section 5. The stability of the AUV system is proved through a simulation study and discussions are presented in Section 6. Section 7 presents the conclusions of the paper.

2 Existing methods of stability analysis of nonlinear systems

There are many techniques exist in literature which are directly or indirectly applicable to analyses the stability of the nonlinear system. But out of these, in this section only important existing techniques utilized for stability analysis of nonlinear systems are briefly explained.

2.1 Jacobian eigenvalue theorem

Consider a nonlinear system which is presented by

$$\dot{x}(t) = f(x, y) \tag{1a}$$

$$\dot{y}(t) = g(x, y) \tag{1b}$$

where f and g are mathematical functions which are differentiable with continuous partial derivatives. These functions are vanishes at the point (x_0, y_0). To analyze the functional value at this point, let be the Jacobian matrix at this point (x_0, y_0) and this matrix is presented by

$$J = \begin{bmatrix} f_x(x_0, y_0) & f_y(x_0, y_0) \\ g_x(x_0, y_0) & g_y(x_0, y_0) \end{bmatrix}. \tag{2}$$

A system is said to be stable at (x_0, y_0) if all the eigenvalues of possesses have negative real parts. And if one or more than eigenvalues have negative real parts, then the system is said to be unstable at the point (x_0, y_0).

From this it is observed that, this theorem is not suitable to verify the stability of a system when the eigenvalue of the Jacobian possesses zero real parts.

2.2 Linearization method

Most of the nonlinear systems are mathematically represented by nonlinear differential equations. Linearization of a nonlinear differential equation generally produces a time varying linear system. Since stability is the local property any system, after linearization one can easily expect that whether the system is stable or not. An n order nonlinear system may be presented by only one nonlinear n order equation or by a set of n different first order equations. This is resented below[33].

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \cdots, x_n) \\ \dot{x}_2 &= f_2(x_1, x_2, \cdots, x_n) \\ &\cdots \\ \dot{x}_n &= f_n(x_1, x_2, \cdots, x_n). \end{aligned} \tag{3}$$

In matrix form, this can be presented by

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}). \tag{4}$$

The solution of equation (3) is given by a phase plane trajectory in an n dimensional state space.

At equilibrium points, the rate of change of the states are zeros, i.e., $\dot{x}_1 = \dot{x}_2 = \cdots = \dot{x}_n = 0$, hence the points are singular points where $f_1(\boldsymbol{x}) = f_2(\boldsymbol{x}) = \cdots = f_n(\boldsymbol{x}) = 0$. Considering this, system (3) is represented by a linear system where $\boldsymbol{f}(\boldsymbol{x})$ is a linear function of $\boldsymbol{x}$ and can be presented by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with det $(\boldsymbol{A}) \neq 0$. A nonlinear system may possess more than one equilibrium points as $\boldsymbol{f}(\boldsymbol{x})$ has more than one solutions. These equilibrium points may be stable or unstable. The stability of these points depends upon the phase-plane trajectory. These are stable in the phase-plane, if the trajectory approaches the equilibrium points as t tends to infinity and these points are unstable if the trajectory move away from the equilibrium points in the phase-plane.

For stability analysis of the nonlinear system given in (3), it should be linearized first in the neighborhood of the equilibrium points and then the stability of the whole system should be checked. In the neighborhood of the equilibrium points, the given nonlinear system is act as the linearized system if the linearization is possible.

The function of the nonlinear equation (4), i.e., $f_i(x_1, x_2, \cdots, x_n)$ with $i = 1, 2, 3, \cdots, n$, can be expanded in the Taylers series in the neighborhood of each singular point and this can be represented by

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_i - x_{i0}) = \left(\frac{\partial f_i}{\partial x_1}\right)(x_1 - x_{10}) + \left(\frac{\partial f_i}{\partial x_2}\right)(x_2 - x_{20}) + \cdots + \left(\frac{\partial f_i}{\partial x_n}\right)(x_n - x_{n0}). \tag{5}$$

Equation (5) in matrix form may be presented by

$$\frac{\mathrm{d}}{\mathrm{d}t}(x - x_0) = J(x_0)(x - x_0) \tag{6}$$

with

$$J(x_0) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \tag{7}$$

$J(x_0)$ is the Jacobian matrix. The elements of this matrix are the partial derivatives of $f_i(x_1, x_2, \cdots, x_n)$ with the numerical values corresponding to the singular points. Equation (6) is the linearized form of the nonlinear system presented in (1).

Remark 1. This linearization method of stability analysis of the nonlinear systems is one of the easiest methods of stability analysis of the nonlinear system with some demerits. It does not give complete information about the stability of the nonlinear system. It follows the necessary and sufficient condition that for the system to be stable, all the roots of the characteristic equation should be lies in the left half of the s-plane. If any one or more than one roots of that characteristic equation lie in the right half of the s-plane, the system becomes unstable. It does not tell about stability of the system when the root lies on the imaginary axis of the s-plane.

Linearization method provides the idea which is adapted by the Lyapunov and is used in the stability analysis of nonlinear systems by using the Lyapunov′s indirect method. In order to prepare the ground for the Lyapunov′s indirect method of stability analysis, the linearization of the nonlinear differential equations must be done.

2.3 Lyapunov′s direct method

Stability analysis using the Lyapunov′s direct method is the powerful and common procedure in recent era. The analysis of this method is presented here.

Consider the Lyapunov candidate function for the system presented by (4) is $V(\boldsymbol{x})$. This function is a scalar function and possesses positive definite value. Here $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ is $n \times 1$ state vector whose elements are the state variables of the n order nonlinear system. The origin, i.e., $\boldsymbol{x}=0$ where all the state variables have zero values $x_1=x_2=\cdots=x_n=0$ in the state space are assume to be an equilibrium solution. To analyze the stability of the system, the first derivative of the Lyapunov function is mandatory to analyze first. The first derivative of this function is given as

$$\dot{V}=\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\partial V}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}t}+\frac{\partial V}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}t}+\cdots+\frac{\partial V}{\partial x_n}\frac{\mathrm{d}x_n}{\mathrm{d}t}. \tag{8}$$

By assuming the Lyapunov candidate function $V(\boldsymbol{x})$ is an energy function, the stability can be analyzed. $V(\boldsymbol{x})$ is a positive definite and gradually decreases with time so that it vanishes at the origin and the system became stable. Hence first derivative of the Lyapunov candidate function is negative definite or semi-definite.

The stability of the system can be analyzed by examining the Lyapunov candidate function along with the first derivative of this function. The sufficient conditions for different stability conditions are mathematically presented as follows.

Consider the Lyapunov candidate function $V(\boldsymbol{x})$ such that

$V(\boldsymbol{x}):\mathbf{R}^n\rightarrow\mathbf{R}$.

$V(\boldsymbol{x})\geq 0$, if and only if $\boldsymbol{x}=0$ (positive definite).

Then the different conditions of stability are as follows.

$$\begin{cases}\dot{V}(\boldsymbol{x})=\dfrac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{x})<0, & \text{system is GAS}\\ \dot{V}(\boldsymbol{x})=\dfrac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{x})\leq 0, & \text{system is GS}\\ \dot{V}(\boldsymbol{x})=\dfrac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{x})\geq 0, & \text{system is unstable.}\end{cases} \tag{9}$$

GAS: Globally asymptotically stable

GS: Globally stable.

If $V(\boldsymbol{x})$ is indefinite, then it is not possible to decide about the stability of the system.

Remark 2. Construction of the Lyapunov candidate function is a very difficult task, it is because there no definite procedure to develop or construct the Lyapunov function. This function is created only by assumptions and trial and error method.

Remark 3. There is only sufficient condition of the stability analysis, but there are no necessary conditions. In case of failure of the Lyapunov candidate function in testing for the stability or asymptotically, it does not give any guarantee about the origin is not stable.

2.4 Popov criterion

It is one of the popular methods of analysis of the stability of the nonlinear systems. In some attributes, this method can be compared with the Nyquist stability criteria for linear systems.

Consider a nonlinear system having a transfer function which contains the linear part $G(s)$ and a nonlinear part. This is globally asymptotically stable if there exists a real number q having any value (positive, negative or zero) for every value of ω such that it satisfies the following inequality.

$$\mathrm{Re}\left[(1+\mathrm{j}\omega q)\,G(\mathrm{j}\omega)\right]+\frac{1}{k}>0 \tag{10}$$

where k is the slope. In more appropriate manner, the Popov criterion can be applied in the $G(\mathrm{j}\omega)$ plane graphically. A modified frequency response function $G^*(\mathrm{j}\omega)$ which is the Popov locus can be applied on it and can be defined as

$$G^*(\mathrm{j}\omega)=\mathrm{Re}\left[G(\mathrm{j}\omega)\right]+\mathrm{j}\omega\mathrm{Im}\left[G(\mathrm{j}\omega)\right]. \tag{11}$$

Hence,

$$\mathrm{Re}\left[G^*(\mathrm{j}\omega)\right]=\mathrm{Re}\left[G(\mathrm{j}\omega)\right] \tag{12a}$$

$$\mathrm{Im}\left[G^*(\mathrm{j}\omega)\right]=\omega\mathrm{Im}\left[G(\mathrm{j}\omega)\right] \tag{12b}$$

with every value of $\omega\geq 0$. For this nonlinear system to be globally asymptotically stable, the graphical interpretation is necessary. For a system to be GAS, the sufficient condition is that, the plot of the $G^*(\mathrm{j}\omega)$ should lie fully to the right of the Popov line. The Popov line crosses the real axis at $-\frac{1}{k}$ with the slope $-\frac{1}{q}$, here q is a real number having any vale.

2.5 Lasalles invariance principle

LaSalle′s invariance principle is also known as the invariance principle[34] and Barbashin-Krasovskii-LaSalle principle[35]. It is the criterion for the asymptotic stability analysis of nonlinear autonomous dynamical system.

Let us consider an n order nonlinear system, whose state vector is presented by $\boldsymbol{x}=[x_1,x_2,\cdots,x_n]$ and the elements are the state variables of the system.

The dynamics of the system is presented as follows[36].

The origin $\boldsymbol{x}=\mathbf{0}$, i.e., all the state variables have zero values $(x_1=x_2=\cdots=x_n=0)$ in the state space, which is assume to be an equilibrium solution.

$$\boldsymbol{f}(\mathbf{0})=\mathbf{0},\quad \text{at origin} \tag{13a}$$

$$(\dot{\boldsymbol{x}})=\boldsymbol{f}(x). \tag{13b}$$

Let $\Omega\subset D\subset\mathbf{R}^n$ be a compact positively invariant set with respect to the system defined in (13). Let $V:D\rightarrow\mathbf{R}$ is the continuously differentiable function which satisfies the condition $\dot{V}(x)\leq 0$ in Ω. Again, let $E\subset\Omega$ be the set of all points in Ω where $\dot{V}(x)\leq 0$. Let E, $M\subset E$ be the largest invariant set. Then, every solution starting in the set Ω must approaches M at $t\rightarrow\infty$. That is

$$\lim_{t\rightarrow\infty}\left(\underbrace{\inf_{z\in M}\|x_i(t)-z\|}_{dist(x_i(t)M)}\right)=0,\quad i=1,\,2,\,3,\cdots,n. \tag{14}$$

It should be noted that the inclusion of the sets in the LaSalles theorem is $M\subset E\subset\Omega\subset D\subset\mathbf{R}^n$.

3 Extended-routh′s stability method

In this section extended-Routh′s stability criterion (ERSC) for analysis of nonlinear systems is described briefly.

Consider a nonlinear dynamic system given as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t, \boldsymbol{u}),\ t \in [t_0, \infty),\quad \boldsymbol{x} \in \mathbf{R}^n,\ \boldsymbol{u} \in \mathbf{R}^m,\quad m \le n \tag{15a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, t, \boldsymbol{u}),\quad \boldsymbol{y} \in \mathbf{R}^l,\ 1 \le l \le n \tag{15b}$$

where $\boldsymbol{x} = \boldsymbol{x}(t)$ is the state vector of the system, $\boldsymbol{u} = \boldsymbol{u}(t)$ is the control input, $\boldsymbol{f}$ is the continuously differentiable nonlinear function. $\boldsymbol{x}(t_0) = \boldsymbol{x}_0,\ t_0 > 0$ is the initial condition, $\boldsymbol{y} = \boldsymbol{y}(t)$ is the system output, $\boldsymbol{h}$ is a continuously differentiable nonlinear function.

The state-space model of the nonlinear dynamic (15) may also be described as

$$\dot{x}(t) = \boldsymbol{A}(\boldsymbol{x}, t)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{16a}$$

$$y(t) = \boldsymbol{C}(\boldsymbol{x}, t)\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \tag{16b}$$

where $\boldsymbol{A} = \boldsymbol{A}(x, t)$ is the state or system matrix and $\boldsymbol{A} \in \mathbf{R}^{n\times n}$ whose elements are state dependent, $\boldsymbol{B}$ is the input matrix, $\boldsymbol{B} \in \mathbf{R}^{n\times m}$, $\boldsymbol{C}$ is the output matrix, $\boldsymbol{C} \in \mathbf{R}^{l\times n}$, $\boldsymbol{D}$ is the feed forward matrix, $\boldsymbol{D} \in \mathbf{R}^{l\times m}$. The coefficients of state matrix $\boldsymbol{A}$ are state dependent and the states are input dependent both in frequency and amplitude.

The Routh′s stability criterion was developed for linear time invariant systems, and therefore is not directly applicable to nonlinear systems given in (16a) and (16b). For stability analysis of nonlinear dynamic systems, a new plane named as $g(t)$-plane is introduced here. The conventional Routh′s stability method is extended to extended-Routh′s stability method in $g(t)$-plane for the stability analysis of nonlinear time varying systems.

Let us define the $g(t)$-plane as

$$g(t) = \sigma(t) + \mathrm{j}\omega(t) \tag{17}$$

where $g(t)$ is differential operator $\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)$ and a time varying complex variable. $\sigma(t)$ and $\omega(t)$ are real numbers. As a whole, in the $g(t)$-plane, $\sigma(t)$ represents the real part and $\omega(t)$ represents the imaginary part. j is the imaginary unit[37−39]. Here the dynamic equation is nonlinear, it is considered in a $g(t)$-plane instead of the s-plane which is used for analysis of linear time invariant systems. For linear time invariant system $s = \sigma + \mathrm{j}\omega$, the $g(t)$-plane is shown in Fig. 2.

The characteristic equation of system (16) may be written as

$$|gI - A| = 0. \tag{18}$$

This is a nonlinear differential equation of order n as $\dim(A(\cdot)) = n \times n$. The coefficients of this nonlinear equation contain the state dependent terms as the elements of the matrix A are also state dependent. Hence this nonlinear differential equation may be presented in the form of

$$a_n g^n + a_{n-1} g^{n-1} + \cdots + a_1 g + a_0 = 0 \tag{19}$$

where a_is are the coefficients of g^i and contain the state dependent nonlinear terms, $i = 0, 1, 2, 3, \cdots, n$. The roots of (19) are the poles as these are state dependent and vary with time, hence these poles are called as dynamic poles.

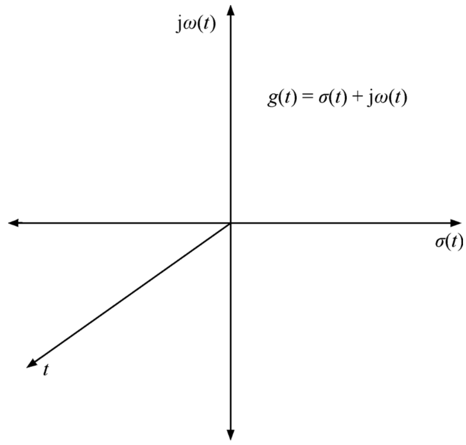

Fig. 2 Presentation of g-plane

Theorem 1.[39] The necessary condition for the nonlinear system to be stable is that, all the elements of the first column of dynamic-Routh′s array must have positive values.

$$\begin{array}{llll} g^n & a_n & a_{n-2}a_{n-4} & a_{n-6}\cdots \\ g^{n-1} & a_{n-1} & a_{n-3}a_{n-5} & a_{n-7}\cdots \\ g^{n-2} & b_1 & b_2 b_3 & \cdots\cdots \\ g^{n-3} & c_1 & c_2 c_3 & \cdots\cdots \\ g^{n-4} & & \vdots & \\ \vdots & & \vdots & \\ g^1 & & \vdots & \\ g^0 & & \vdots & \end{array} \tag{20}$$

where the polynomials b_j and c_j can be determined as follows, $j = 1, 2, 3, \cdots$.

$$\begin{aligned} b_1 &= \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} = \frac{-1}{a_{n-1}}(a_n a_{n-3} - a_{n-1}a_{n-2}) \\ b_2 &= \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} = \frac{-1}{a_{n-1}}(a_n a_{n-5} - a_{n-1}a_{n-4}) \\ b_3 &= \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix} = \frac{-1}{a_{n-1}}(a_n a_{n-7} - a_{n-1}a_{n-6}) \\ &\vdots \\ c_1 &= \frac{-1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix} = \frac{-1}{b_1}(a_{n-1}b_2 - b_1 a_{n-3}) \\ c_2 &= \frac{-1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix} = \frac{-1}{b_1}(a_{n-1}b_3 - b_1 a_{n-5}) \\ &\vdots \end{aligned} \tag{21}$$

This theorem is used to analyze the stability of the AUV in Section 4.

Here an example is presented to demonstrate the role of extended-Routh′s stability method as a mean to show the stability of the nonlinear systems. Let us consider a simple nonlinear system presented by[40]

$$\begin{aligned}\dot{x} &= y + x(x^2 + y^4)\\ \dot{y} &= -x + y(x^2 + y^4).\end{aligned} \tag{22}$$

Considering $\alpha = (x^2 + y^4)$, in state space form, (22) may be presented as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}\alpha & 1\\ -1 & \alpha\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}. \tag{23}$$

The dynamic characteristic equation is $|gI - A| = 0$, with

$$A = \begin{bmatrix}\alpha & 1\\ -1 & \alpha\end{bmatrix}$$

or

$$g^2 - 2\alpha g + (\alpha^2 + 1) = 0. \tag{24}$$

The stability of the system can be checked through dynamic-Routh′s stability array as

$$\begin{aligned}&g^2|1 \ \ (\alpha^2+1)\\ &g^1| - 2\alpha\\ &g^0|(\alpha^2+1).\end{aligned} \tag{25}$$

As $\alpha = (x^2 + y^4)$, so α is always positive. So all the elements of the first column do not have positive values. So the system is unstable. As the number of sign changes of the elements of the first column of the extended-Routh′s stability array is two, so there are two dynamic poles on the right hand side of the g-plane.

4 AUV kinematics and dynamics

The kinematic and dynamic equations of an AUV are presented here. There are two types of frames of references considered, i.e., body fixed frame of reference $\{B\}$ and earth fixed frame of reference $\{I\}$. The later is known as inertial frame of reference. The origin of B is coinciding with the center of mass of the vehicle. The schematic presentation of an AUV is presented in Fig. 3. In Fig. 3, the parameters X_e, Y_e and Z_e present the X, Y and Z axis coordinates of the earth-fixed frame of reference, and O_e presents its origin. In the similar manner, the parameters X_b, Y_b and Z_b present the X, Y and Z axis coordinates of the body-fixed frame of reference and O_b presents its origin. The motion of an AUV in six degrees of freedom (DOF) can be described by the following vectors[41]:

$$\begin{aligned}\eta &= [x, y, z, \varphi, \theta, \psi]^{\mathrm{T}}\\ \nu &= [u, v, w, p, q, r]^{\mathrm{T}}\\ \tau &= [X, Y, Z, K, M, P]^{\mathrm{T}}\end{aligned} \tag{26}$$

where η denotes the position and orientation vector of AUV in the inertial frame. x, y, z are the coordinates of position and φ, θ, ψ are orientation coordinates along longitudinal, transversal and vertical axes respectively. ν is the velocity vector within the body-fixed frame. u, v, w denote linear velocities, p, q, r are angular velocities. X, Y, Z are forces, K, M, P denote moments. The nonlinear dynamic and kinematic equations of motion can be expressed as

$$\begin{aligned}&M\dot{\nu} + C(\nu)\dot{\nu} + D(\nu)\dot{\nu} + g(\eta) = \tau\\ &\dot{\eta} = J(\eta)\,\nu\end{aligned} \tag{27}$$

where M is the inertia matrix including added mass, $C(\nu)$ is the matrix of Coriolis and centripetal terms including added mass. $D(\nu)$ denotes hydrodynamic damping and lift matrix and $g(\eta)$ is the vector of gravitational forces and moments. τ is the vector of forces and moments acting on the AUV in the body-fixed frame. $J(\eta)$ is the velocity transformation matrix between AUV frame and earth fixed frame.

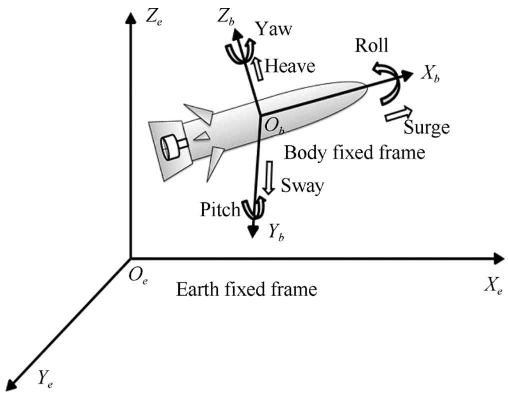

Fig. 3 Schematic representation of an AUV showing forces and torques

This transformation matrix can be represented as

$$J(\eta) = \begin{bmatrix}J_1(\eta) & 0_{3\times3}\\ 0_{3\times3} & J_2(\eta)\end{bmatrix} \tag{28}$$

where

$$J_1(\eta) = \begin{bmatrix}\cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\theta)\sin(\phi) & \sin(\psi)\sin(\phi) + \cos(\psi)\cos(\phi)\sin(\theta)\\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi) + \sin(\phi)\sin(\theta)\cos(\psi) & -\cos(\psi)\sin(\phi) + \sin(\theta)\sin(\psi)\cos(\phi)\\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\theta)\cos(\phi)\end{bmatrix} \tag{29}$$

$$J_2(\eta) = \begin{bmatrix}1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta)\\ 0 & \cos(\phi) & -\sin(\phi)\\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)}\end{bmatrix}. \tag{30}$$

Here we have described the AUV kinematics and dynamics in two degrees of freedom as an example for simpler analysis. We considered the underactuated AUV is moving only in horizontal plane, i.e., yaw plane. The origin of body fixed frame of reference B is coinciding with the center of mass of the vehicle. For a simpler analysis, the following assumptions are made, which are as follows. Center of mass (CM) and center of buoyancy (CB) coincide with each other. Mass distribution all over the body is homogeneous. The hydrodynamic terms of higher order as well as heave, pitch and roll motions are negligible.

The kinematic equation of the vehicle in the horizontal plane is given by[27]

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}\cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix}. \quad (31)$$

The dynamic equations of the AUV are given by

$$\begin{aligned}\dot{u} &= \frac{m_{22}}{m_{11}}vr - \frac{X_u}{m_{11}}u - \frac{X_{u|u|}}{m_{11}}u\,|u| + \frac{1}{m_{11}}F_u\\ \dot{v} &= -\frac{m_{11}}{m_{22}}ur - \frac{Y_v}{m_{22}}v - \frac{Y_{v|v|}}{m_{22}}v\,|v|\\ \dot{r} &= \frac{m_{11}-m_{22}}{m_{33}}uv - \frac{N_r}{m_{33}}r - \frac{N_{r|r|}}{m_{33}}r\,|r| + \frac{1}{m_{33}}F_r. \end{aligned} \quad (32)$$

Here F_u = force applied to the body for linear motion, F_r = force applied to the body for angular motion, m_{11}, m_{22} = combined rigid-body and added mass terms, m_{11}= combined rigid-body and added moment of inertia about the Z_b axis, X_u, Y_v and N_r are the linear and quadratic drag terms coefficients, $X_{u|u|}$ is the non-linear axial drag coefficient, $Y_{v|v|}$ and $N_{r|r|}$ are the nonlinear cross-flow drag coefficients.

5 Stability analysis of the AUV dynamics

This section presents the exploration of the soul goal of the paper, i.e., the stability analysis of the underactuated AUV whose kinematics and dynamics are explained in the Section 4. For stability analysis of the dynamic system provided in (32), extended-Routh′s stability criterion[37−39] are used. Equation (32) is a highly nonlinear and coupled equation. Rearranging (32) and presenting in matrix form one can get as follows:

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-\frac{X_u}{m_{11}} - \frac{X_{u|u|}}{m_{11}}|u| & \frac{m_{22}}{m_{11}}r & 0\\ -\frac{m_{11}}{m_{22}}r & -\frac{Y_v}{m_{22}} - \frac{Y_{v|v|}}{m_{22}}|v| & 0\\ \frac{m_{11}-m_{22}}{m_{33}}v & 0 & -\frac{N_r}{m_{33}} - \frac{N_{r|r|}}{m_{33}}|r|\end{bmatrix} \times \begin{bmatrix}u\\ v\\ r\end{bmatrix} + \begin{bmatrix}\frac{1}{m_{11}} & 0\\ 0 & 0\\ 0 & \frac{1}{m_{33}}\end{bmatrix}\begin{bmatrix}F_u\\ F_r\end{bmatrix}. \quad (33)$$

Equation (33) is in the state-space form and can be written as

$$\dot{X} = AX + BU \quad (34)$$

where $X = \begin{bmatrix}u\\ v\\ r\end{bmatrix}$

with

$$A = \begin{bmatrix}-\frac{X_u}{m_{11}} - \frac{X_{u|u|}}{m_{11}}|u| & \frac{m_{22}}{m_{11}}r & 0\\ -\frac{m_{11}}{m_{22}}r & -\frac{Y_v}{m_{22}} - \frac{Y_{v|v|}}{m_{22}}|v| & 0\\ \frac{m_{11}-m_{22}}{m_{33}}v & 0 & -\frac{N_r}{m_{33}} - \frac{N_{r|r|}}{m_{33}}|r|\end{bmatrix}$$

and

$$B = \begin{bmatrix}\frac{1}{m_{11}} & 0\\ 0 & 0\\ 0 & \frac{1}{m_{33}}\end{bmatrix}, \quad U = \begin{bmatrix}F_u\\ F_r\end{bmatrix}. \quad (35)$$

Here, the state or system matrix and the state dependent elements are coupled and nonlinear input matrix B is the function of the combined rigid-body and added mass terms. Substituting the numerical values of the parameters of (33) from Table 1, one can get (33) in approximate form as

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-0.3-0.5\,|u| & 1.25r & 0\\ -0.8r & -0.4-0.8\,|v| & 0\\ -0.6v & 0 & -0.6-1.25\,|r|\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix} + \begin{bmatrix}\frac{1}{215} & 0\\ 0 & 0\\ 0 & \frac{1}{80}\end{bmatrix}\begin{bmatrix}F_u\\ F_r\end{bmatrix}. \quad (36)$$

Some parameters are given below and the other parameters of the AUV are given in Table 1[27].

$$\begin{aligned} m_{11} &= m - X_{\dot{u}} = 215\,\text{kg}\\ m_{22} &= m - Y_{\dot{v}} = 265\,\text{kg}\\ m_{33} &= I_z - N_{\dot{r}} = 80\,\text{kg}\cdot\text{m}^2\end{aligned}$$

Table 1 Hydrodynamic parameters of the AUV[27]

Parameter	Symbol	Value	Unit
Mass	m	185	kg
Rotational mass	I_Z	50	kg·m^2
Added mass	$X_{\dot{u}}$	−30	kg
Added mass	$Y_{\dot{v}}$	−80	kg
Added mass	$N_{\dot{r}}$	−30	kg·m^2
Surge linear drag	X_u	70	kg/s
Surge quadratic drag	$X_{u\|u\|}$	100	kg/m
Sway linear drag	Y_v	100	kg/s
Sway quadratic drag	$Y_{v\|v\|}$	200	kg/m
Yaw linear drag	N_r	50	kg·m^2/s
Yaw quadratic drag	$N_{r\|r\|}$	100	kg·m^2

The stability analysis of the AUV is presented using the extended-Routh′s stability criteria in the following section. Consider the dynamic system (36) where the mass matrix can be rewritten as

$$A=\begin{bmatrix}-0.3-0.5\,|u| & 1.25r & 0\\ -0.8r & -0.4-0.8\,|v| & 0\\ -0.6v & 0 & -0.6-1.25\,|r|\end{bmatrix} \tag{37}$$

or

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{32}\end{bmatrix} \tag{38}$$

where

$$\begin{aligned}&a_{11}=-0.3-0.5\,|u|\,,\ a_{12}=1.25r,\ a_{13}=0\\ &a_{21}=-0.8r,\ a_{22}=-0.4-0.8\,|v|\,,\ a_{23}=0\\ &a_{31}=-0.6v,\ a_{32}=0,\ a_{33}=-0.6-1.25\,|r|\,.\end{aligned} \tag{39}$$

All the elements of (37) and (38) have one to one correspondences. Hence, the characteristic equation in g-plane may be presented by

$$|gI-A|=0. \tag{40}$$

Solving (40),

$$\begin{aligned}&g^3+(-a_{11}-a_{22}-a_{33})\,g^2+\\ &\quad(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}-a_{12}a_{21})\,g+\\ &\quad(a_{12}a_{21}a_{33}-a_{11}a_{22}a_{33})=0\end{aligned} \tag{41a}$$

$$b_3g^3+b_2g^2+b_1g+b_0=0 \tag{41b}$$

$$\begin{aligned}&b_3=1\\ &b_2=-a_{11}-a_{22}-a_{33}\\ &b_1=a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}-a_{12}a_{21}\\ &b_0=a_{12}a_{21}a_{33}-a_{11}a_{22}a_{33}.\end{aligned} \tag{41c}$$

Using extended-Routh′s stability criterion

$$\begin{array}{lll} g^3 & b_3 & b_1\\ g^2 & b_2 & b_0\\ g^1 & \dfrac{b_2b_1-b_3b_0}{b_2} & 0\\ g^0 & b_0 & 0.\end{array} \tag{42}$$

For a system to be stable, all the coefficients of first column of the matrix given in (42) should be positive. From Table 1, for stability, we have

$$\begin{aligned}&b_3=1>0\\ &b_2=-a_{11}-a_{22}-a_{33}=\\ &\qquad 1.3+0.5\,|u|+0.8\,|v|+1.25\,|r|\,.\end{aligned} \tag{43}$$

This is obtained by substituting the value of (39) in (41c). As all the terms of b_2 of (43) are positive and absolute values, hence $b_2>0$. Also the term $\frac{b_2b_1-b_3b_0}{b_2}>0$, as $b_3=1$. Again substituting the values of parameters such as a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32} and a_{32} provided in (39) in (41c), it is obtained as

$$\begin{aligned}b_2b_1-b_0=\ &0.63+0.8\,|u|+1.224\,|v|+1.662\,|r|+\\ &1.04\,|u|\,|v|+2.6\,|v|\,|r|+1.624\,|u|\,|r|+0.7r^2+\\ &|u|\,|v|\,|r|+0.25|u|^2+0.576|v|^2+1.09|r|^2+\\ &0.2|u|^2\,|v|+0.313|u|^2\,|r|+0.3\,|u|\,|v|^2+\\ &0.8|v|^2\,|r|+1.25\,|v|\,|r|^2+0.78\,|u|\,|r|^2+\\ &0.5\,|u|\,r^2+0.8\,|v|\,r^2.\end{aligned} \tag{44}$$

As all the terms of (44) are positive hence $(b_2b_1-b_0)>0$. Substituting the values of parameters from (39) in (41c), b_0 can be found out as

$$\begin{aligned}b_0=\ &0.072+0.12\,|u|+0.144\,|v|+0.15\,|r|+0.24\,|u|\,|v|+\\ &0.3\,|v|\,|r|+0.25\,|u|\,|r|+0.6r^2+1.25r^2\,|r|+\\ &0.5\,|u|\,|v|\,|r|\,.\end{aligned} \tag{45}$$

Similarly, all the terms of (45) are of positive signs $b_0>0$.

Hence, all the elements of the first column of the matrix given in (42) are positive. This implies that all the poles of the dynamic system are on the left half of the g-plane. Thus, AUV dynamics are stable.

6 Results and discussion

An AUV may be used for path following, path planning, obstacle avoidance, etc. And a group of multiple AUVs is used for cooperative control including formation and flocking control. In the case of complex and tedious tasks which are not possibly by the use of a single AUV, multiple AUVs are deployed. In each of these motion tasks, advanced controllers such as adaptive, sliding mode, etc., including intelligent controller should be used instead of traditional controllers. It is because of the fact that the dynamics as well as the kinematics of the AUV are highly nonlinear and coupled. Before applying the controllers to the AUVs directly, it is necessary to observe the stability of the AUV by checking the response to the basic test signals. In this work, the step signals are considered as test signals. The stability analysis of AUVs by using the existing methods mentioned in Sections 1 and 2 are more complicated[27−29]. In this paper, the stability of an AUV is analyzed using the extended-Routh′s method. In the first step, the stability is analyzed mathematically and then the simulation is carried out to verify the mathematical results.

To ensure the stable control dynamics of the underactuated AUV, the simulation is carried out using Matlab/Simulink environment. The Simulink model of the dynamics of the AUV is created and simulated by using parameters of the AUV given in Table 1. Here the inputs are taken as step inputs having highest amplitude values. The numerical parameters used for simulation are provided in Table 1.

Fig. 4 presents the response of the AUV dynamics when the step signal is applied. It is observed that the dynamic stable condition of a highly nonlinear underactuated AUV is established when the desired inputs are applied. Equation (24) presents the dynamic model consisting as the output states and as well as as the control inputs which are step signals having magnitude 1. From Fig. 4, it is also observed that the control inputs and the output states of the AUV match each other without any damping or oscillations. This shows the stable condition of the system.

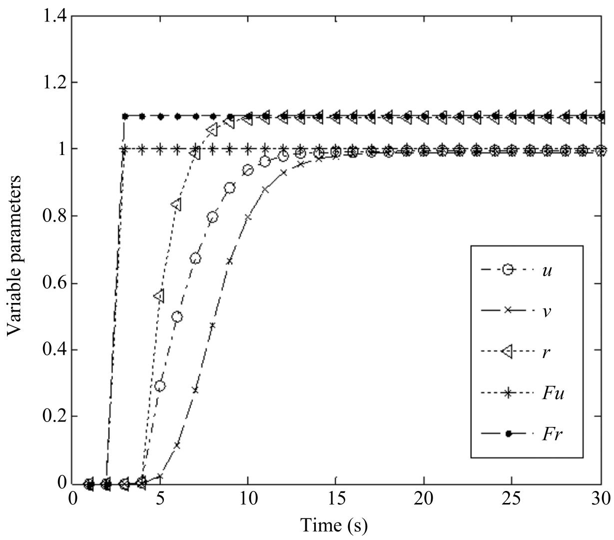

Fig. 4 Dynamic response to the step input

7 Conclusions

A brief discussion on AUV and the cooperative control of a group of multiple AUVs are presented. The important and essential applications of single and multiple AUVs are explained. The stability analysis of nonlinear systems using different existing techniques are presented. Also the stability analysis of nonlinear systems through extended-Routh′s approach is developed. The stability of the highly nonlinear underactuated robotic system, i.e., AUV, is presented using extended-Routh′s stability approach. The dynamics of the AUV are considered here including the hydrodynamic damping factors. To verify the efficacy of the stability analysis through the extended-Routh′s stability approach, a nonlinear dynamic system, e.g., AUV dynamics, is considered in this paper, and numerical simulation is carried out. From the obtained results, it is clear that the dynamic system considered is stable. In the future work, the stability analysis of the different controllers are to be presented with the use of the extended-Routh′s stability criterion.

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