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A Robust Fractional Order Fuzzy P + Fuzzy I + Fuzzy D Controller for Nonlinear and Uncertain System

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Abstract: In this paper, a robust fractional order fuzzy P + fuzzy I + fuzzy D (FOFP + FOFI + FOFD) controller is presented for a nonlinear and uncertain 2-link planar rigid manipulator. It is a nonlinear fuzzy controller with variable gains that makes it self-adjustable or adaptive in nature. The fractional order operators further make it more robust by providing additional degrees of freedom to the design engineer. The integer order counterpart, fuzzy P + fuzzy I + fuzzy D (FP + FI + FD) controller, for a comparative study, was realized by taking the integer value for the fractional order operators in FOFP + FOFI + FOFD controller. The performances of both the fuzzy controllers are evaluated for reference trajectory tracking and disturbance rejection with and without model uncertainty and measurement noise. Genetic algorithm was used to optimize the parameters of controller under study for minimum integral of absolute error. Simulation results demonstrated that FOFP + FOFI + FOFD controller show much better performance as compared to its counterpart FP + FI + FD controller in servo as well as the regulatory problem and in model uncertainty and noisy environment FOFP + FOFI + FOFD controller demonstrated more robust behavior as compared to the FP + FI + FD controller. For the developed controller bounded-input and bounded-output stability conditions are also developed using Small Gain Theorem.

Keywords: Fuzzy P + fuzzy I + fuzzy D (FP + FI + FD) controller, fractional order fuzzy P + fuzzy I + fuzzy D (FOFP + FOFI + FOFD) controller, fractional order operator, robust control, model uncertainty, noise suppression.

1 Introduction

Conventional proportional plus integral plus derivative (PID) controller has been the most popular choice among the control engineers in process industries. It is so famous due to its simple structure, cost effectiveness and simplicity in implementation. Literature survey reveals that share taken by PID controllers is approximately 90%[1−4]. But in the real world, processes are nonlinear and uncertain. Therefore, conventional PID controller usually fails to give satisfactory results. Due to this reason researchers always keep trying to find alternative solutions which can yield better outcomes for such processes. Adaptive or self-tuning controllers are the optimum way out to deal with nonlinear and uncertain processes[2, 3, 5−7].

Popularity of classical PID controller insisted researchers to club intelligence in this classical structure using intelligent techniques, such as fuzzy logic. Some of the fuzzy logic based controllers[8, 9], which preserve linear structure of PI/ PD/ PID controllers and have self-tuning capabilities along with simple analytical formulae as a final result are presented next. Ying et al.[10] proposed a nonlinear fuzzy PI controller with proportional and integral gains changing with error and rate of change of error about a set point. Simulation results showed that the performance of the fuzzy controller was almost the same as that of PI controller when first and second order linear processes were considered. Malki et al.[11] analyzed the performance of a fuzzy PD controller in comparison with the conventional PD controller. The designed fuzzy PD controller had the structure of a digital PD controller. The gains of fuzzy PD controllers were nonlinear functions of input signals and had a self-tuning capability. The performance of fuzzy PD controller was evaluated in simulation on some linear and complex nonlinear systems and it was found that it is far better than its counterpart conventional PD controller.

Further, Misir et al.[12] designed a fuzzy PID controller. It was a combination of fuzzy PI and fuzzy D controller. In fuzzy D controller derivative action is performed on process variable rather than on error signal. Through computer simulation they demonstrated the advantage of fuzzy PID controller in setpoint tracking, particularly for the case of nonlinear system. They also performed the stability analysis using small gain theorem and gave sufficient conditions for the bounded-input and bounded-output (BIBO) stability of the complete feedback loop. Sooraksa and Chen[13] developed a fuzzy $(\mathrm{PI{+}D})^2$ control scheme for set-point tracking and vibration suppression of a "shoulder-elbow-like" single flexible link robot arm model. Simulation results showed that fuzzy controller executed the desired task very well. In continuation of this, Chen[7] presented an excellent survey over the conventional and fuzzy PID controller. Carvajal et al.[14] presented a new design of fuzzy PID controller. It consisted of three inputs and an output. It was designed to control some nonlinear systems. They

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demonstrated the effectiveness of proposed fuzzy PID controller for both linear and nonlinear processes with help of computer simulations. Lu et al.[15] developed a novel real time ultrasound guided fuzzy laser control system for coagulation. They used fuzzy PD controller, which periodically adjusted the output power of a laser.

Tang et al.[16, 17] presented a fuzzy PID controller tuned with multi objective genetic algorithm (MOGA) for a solar power plant. It was a fuzzy PI + fuzzy D controller where fuzzy PI operated over error signal and fuzzy D controller over process variable. Simulation results demonstrated that controller can provide a good tracking behavior against the system variations. Kim and Oh[18] introduced a nonlinear fuzzy PID controller having time varying gains. Simulation results showed the effectiveness of fuzzy PID controller for a nonlinear and uncertain system. Lu et al.[19] proposed a predictive fuzzy PID control theory. They successfully tested the developed controller on chaotic systems. Also, they claimed that this control method provides an effective and new approach to control the uncertain and nonlinear systems. Tang et al.[20] presented a design of a fuzzy PD + I controller. The parameter of controller were optimized using MOGA. It was tested in simulation on a nonlinear system and it was found that the optimized gains made the fuzzy control robust having faster response time and less overshoot as compared to its conventional and non-optimized counterparts. Veeraiah et al.[21] proposed a fuzzy PIPD controller tuned by GA. The gains of fuzzy controllers were nonlinear function of their input signals. Simulation results showed that optimized controllers demonstrated better transient performance.

Further, Kumar and Mittal[22−24] proposed a parallel fuzzy P + fuzzy I + fuzzy D controller with analytical formulae based upon the parallel structure of a classical PID controller. It has variable gains having adaptive capability. Simulation study demonstrated the effectiveness of proposed controller in servo and regulatory problem for some complex linear, nonlinear and non-stationary systems. Further, they also performed the stability analysis using small gain theorem and established sufficient conditions for BIBO stability. Kumar et al.[25] also proposed the design, performance and stability analysis of formula-based fuzzy PI controller. Simulation results demonstrated that the formula-based fuzzy PI controller outperformed the conventional fuzzy PI controller in controlling the outlet flow concentration of a nonlinear non-thermic catalytic continuous stirred tank reactor for setpoint tracking, disturbance rejection and noise suppression. Further, Kumar et al.[26] presented a detailed survey on classical and fuzzy PID controllers. They presented the history of PID controllers and their enhancements using fuzzy logic theory.

Usually, integer order controllers are used in the control system to obtain a desired response. The emergence of fraction calculus offered a liberty to have non-integer orders of integration and differentiation operators. Therefore, capability of conventional PID controller can further be enhanced to yield the better performance in servo as well as in regulatory mode by using the non-integer orders of integration and differentiation operators. It offers an additional degree of freedom in terms of two more variables to the design engineer[27, 28]. Normally, fuzzy PID controllers are implemented with error and rate of change of error signal as its inputs and an output in an increment form. To achieve this task a derivative operation on error is required at the input to obtain the rate of change of error signal and an integration operation is required at output to get the controller output from the incremental form. Various applications of fractional order control have been reported in literature, such as, robotic manipulator control[29, 30], automatic voltage regulator[31−33], coupled tank system[34].

Literature survey presented above clearly indicates that the use of fractional order control for integer order process can provide greater robustness and the performance of the fractional order controller can be further enhanced by coupling self-tuning or adaptive capability with it. In the present work a robust fractional order fuzzy P + fuzzy I + fuzzy D controller (FOFP + FOFI + FOFD) is proposed. It is simply derived by replacing the integer order derivative operator and integration operator with non-integer order derivative and integration from the parallel fuzzy P + fuzzy I + fuzzy D (FP + FI + FD) controller. The performance of proposed controller is evaluated in simulation on a 2-link planar rigid manipulator for setpoint tracking, disturbance rejection, noise suppression and under model uncertainties.

This paper is organized as follows: after a detailed literature review in the first Section, the brief mathematical model of a 2-link planar rigid manipulator is presented in Section 2. In Section 3, the design of FOFP + FOFI + FOFD and FP + FI + FD controllers have been presented along with the used tuning criteria for both the controllers. Also, implementation aspects of the fractional order operators have been presented in this Section. In Section 4, the performance comparison of FOFP + FOFI + FOFD and FP + FI + FD controllers in terms of trajectory tracking, model uncertainties, disturbance rejection and noise suppression has been carried out and has been presented. In Section 5, sufficient BIBO stability conditions are presented for the developed fuzzy controllers using small gain theorem. Finally, the conclusions of the proposed work are drawn in Section 6.

2 Dynamic model of 2-link planar rigid manipulator

A 2-link planar rigid manipulator is presented in Fig. 1. The dynamic model of 2-link planar rigid manipulator was described in [35]. This dynamic model of 2-link manipulator has been utilized in this work.

The mathematical model of the 2-link planar rigid manipulator is as follows:

$$\tau_1 = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2(2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \tag{1}$$

$$\tau_2 = m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 l_2 g c_{12} + m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) \tag{2}$$

where $s_2 = \sin(\theta_2)$, $c_1 = \cos(\theta_1)$, $c_2 = \cos(\theta_2)$, $c_{12} = \cos(\theta_1+\theta_2)$, l_1 and l_2 are the lengths (m), m_1 and m_2 are the uniformly distributed masses (kg), θ_1 and θ_2 are the angular positions (rad) of link-1 and link-2 respectively, and τ_1 and τ_2 are the torques (N·m) required to rotate the both links of robotic manipulator. The manipulator has two degrees of freedom. Equations (1) and (2) gives the torque at the actuators as a function of joints angular position, velocity, and acceleration. The parameters of the manipulator are listed in Table 1.

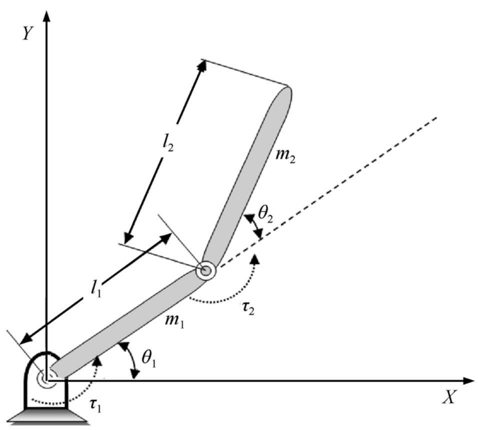

Fig. 1 A 2-link planar rigid manipulator

Table 1 Parameter′s values for the 2-link planar rigid robotic manipulator and its simulation configuration parameters

Parameters	Link-1	Link-2
Mass	0.1 kg	0.1 kg
Length	0.8 m	0.4 m
Acceleration due to gravity (g)	9.81 m/s^2	9.81 m/s^2
ODE solver	Runge Kutta 4	
Step size	0.001 s	

3 Design and implementation of FOFP + FOFI + FOFD controller

In this section, design and implementation of FOFP + FOFI + FOFD controller is presented. It is an enhancement of parallel FP + FI + FD controller[22]. The main difference is that FOFP + FOFI + FOFD controller is realized with non-integer differentiation and integration operators whereas FP + FI + FD controller with integer order differentiation and integration operators. Parallel FP+FI+FD controller design and performance analysis has been discussed in detail in [22]. In this work, same notations are used as in [22] to describe and analyze this FOFP + FOFI + FOFD controller.

3.1 FOFP+FOFI+FOFD controller

The control structure of FOFP + FOFI + FOFD controller is shown in Fig. 2. It is clear from Fig. 2 that input to the fuzzy P/I/D components are error "e" and rate of change of error "r" while a direct output "u_p" is obtained from the fuzzy P component and incremental output "Δu_I and Δu_D" are attained from fuzzy I and fuzzy D components, respectively. Also, K_p^1, K_p^2, K_i^1, K_i^2, K_d^1, K_d^2, K_{UP}, K_{UI} and K_{UD} are the corresponding scaling factors and μ_p, μ_i, μ_d, μ_i and λ_d are λ_d, the fractional order power parameters of the fuzzy P/I/D components, respectively. For input variables, two triangular membership functions, while in case of output variable three singleton memberships were considered. The input and output membership functions for fuzzy P/I/D components are shown in Figs. 3 and 4, where L_p, L_i and L_d are the adjustable constants for fuzzy P/I/D components, respectively. For each fuzzy component, rule base with the four control rules, max-min inference mechanism and center of mass for defuzzification method are considered. For each fuzzy component, the entire two dimensional input space formed by e and r is divided in 12 input combination (IC) regions as shown in Fig. 5. Depending upon the input point location, i.e., (e, r), the formulas assigned for each IC regions, as tabulated in Table 2, calculate the controller output for each fuzzy component. The output of the FOFP + FOFI + FOFD controller is algebraic sum of control actions of three fuzzy components, i.e., fuzzy P, fuzzy I and fuzzy D, and can be defined as

$$u_{FOFP+FOFI+FOFD} = K_{UP} u_P + K_{UI}\left(\frac{\mathrm{d}^{-\lambda_i}}{\mathrm{d}t^{-\lambda_i}}(\triangle u_I)\right) + K_{UD}\left(\frac{\mathrm{d}^{-\lambda_d}}{\mathrm{d}t^{-\lambda_d}}(\triangle u_D)\right) \tag{3}$$

where $u_P, \Delta\mu_I$ and Δu_D are the outputs of fuzzy P/I/D components and K_{UP}, K_{UI} and K_{UD} are the scaling factors of corresponding outputs of fuzzy components. λ_i and λ_d are the non-integer order of the integration used to get the control action from the incremental output from the fuzzy I and fuzzy D components.

3.2 FP+FI+FD controller

The structure of FP + FI + FD controller is quite similar to the FOFP + FOFI + FOFD controller. The configuration of FP + FI + FD controller was achieved by putting the values of μ_p, μ_i, μ_d, λ_i and λ_d as unity in FOFP + FOFI + FOFD controller. The control action of FP + FI + FD controller can also be described as

$$u_{FP+FI+FD} = K_{UP} u_P + K_{UI}\left(\frac{\mathrm{d}^{-1}}{\mathrm{d}t^{-1}}(\triangle u_I)\right) + K_{UD}\left(\frac{\mathrm{d}^{-1}}{\mathrm{d}t^{-1}}(\triangle u_D)\right) \tag{4}$$

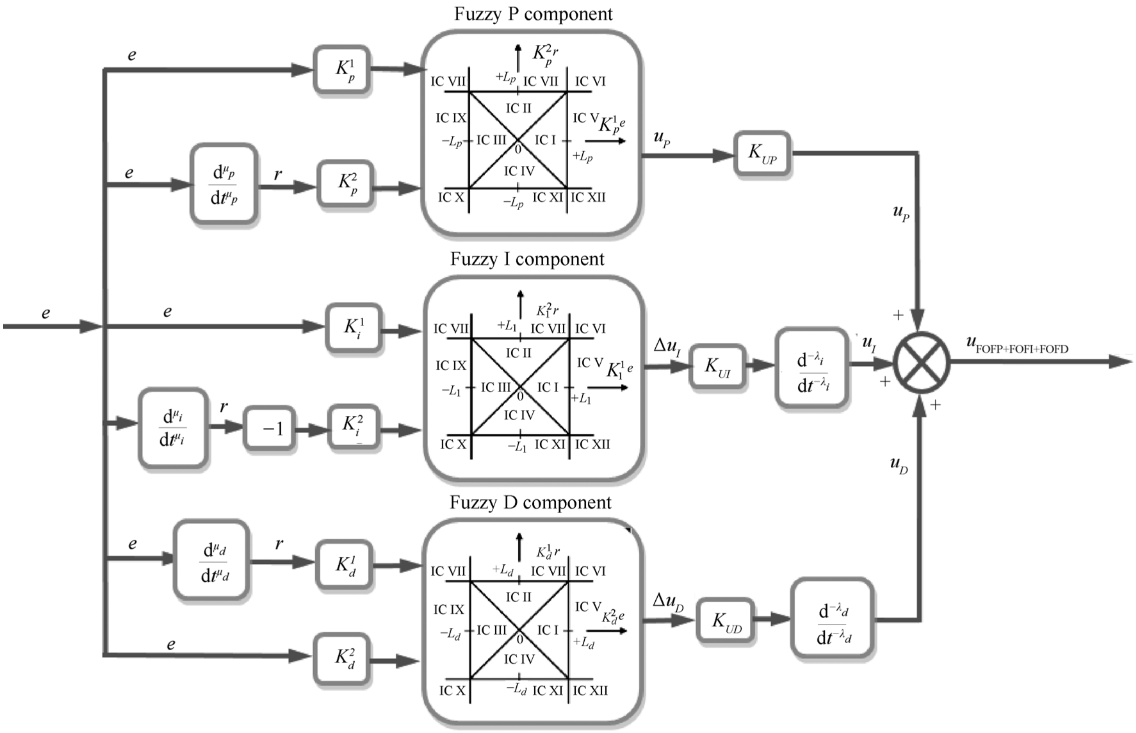

Fig. 2 Structure of FOFP + FOFI + FOFD controller

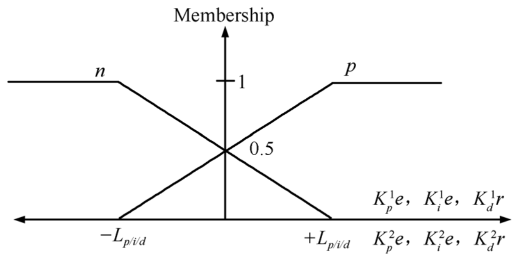

Fig. 3 Input membership function for fuzzy P/I/D components

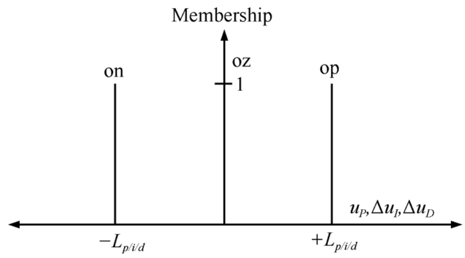

Fig. 4 Output membership function for fuzzy P/I/D components

Table 2 Analytical formulae for the 12 IC regions for fuzzy P/I/D components

IC#	Fuzzy P controller output "u_P"	Fuzzy I controller output "Δu_I"	Fuzzy D controller output "Δu_D"
IC I & IC III	$\frac{L_p[K_p^2r - K_p^1e]}{2[2L_p - \lvert K_p^1e\rvert]}$	$\frac{L_i[K_i^1e + K_i^2r]}{2[2L_i - \lvert K_i^1e\rvert]}$	$\frac{L_d[K_d^2e - K_d^1r]}{2[2L_d - \lvert K_d^2e\rvert]}$
IC II & IC IV	$\frac{L_p[K_p^2r - K_p^1e]}{2[2L_p - \lvert K_p^2r\rvert]}$	$\frac{L_i[K_i^1e + K_i^2r]}{2[2L_i - \lvert K_i^2r\rvert]}$	$\frac{L_d[K_d^2e - K_d^1r]}{2[2L_d - \lvert K_d^1r\rvert]}$
IC V	$\frac{1}{2}[-L_p + K_p^2r]$	$\frac{1}{2}[L_i + K_i^2r]$	$\frac{1}{2}[L_d - K_d^1r]$
IC VI	0	L_i	0
IC VII	$\frac{1}{2}[L_p - K_p^1e]$	$\frac{1}{2}[L_i + K_i^1e]$	$\frac{1}{2}[-L_d + K_d^2e]$
IC VIII	L_p	0	$-L_d$
IC IX	$\frac{1}{2}[L_p + K_p^2r]$	$\frac{1}{2}[-L_i + K_i^2r]$	$\frac{1}{2}[-L_d - K_d^1r]$
IC X	0	$-L_i$	0
IC XI	$\frac{1}{2}[-L_p - K_p^1e]$	$\frac{1}{2}[-L_i + K_i^1e]$	$\frac{1}{2}[L_d + K_d^2e]$
IC XII	$-L_p$	0	L_d

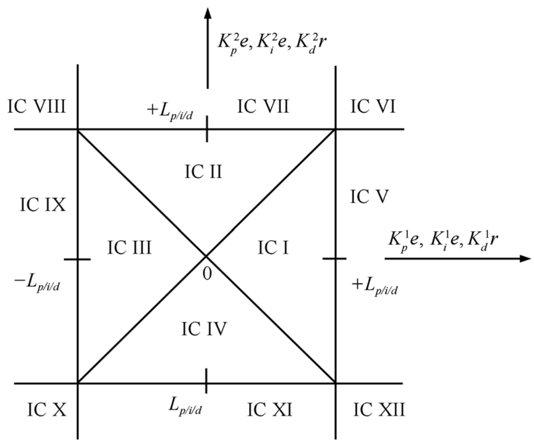

Fig. 5 Regions of fuzzy P/I/D component input-combinations

3.3 Fractional order operators

For the past decade, the fractional calculus has been widely used in the control engineering applications. Fractional calculus can be defined as the generalization of classical calculus to orders of integration and differentiation not necessarily an integer.

In literature, different definitions of fractional order differentiator as well as integrator are reported. In the present work, fraction order operator, i.e., differentiator $\left(\frac{\mathrm{d}^{\mu}}{\mathrm{d}t^{\mu}}\right)$ and integrator $\left(\frac{\mathrm{d}^{-\lambda}}{\mathrm{d}t^{-\lambda}}\right)$ was realized using Gründwald-Letnikov (G-L) fractional derivative definition which is given as follows[36]:

$$_{\alpha}D_t^{\beta}g(t) = \lim_{h\to 0}\frac{1}{h^{\beta}}\sum_{i=0}^{\frac{(t-\alpha)}{h}}(-1)^i\binom{\beta}{i}g(t-ih) \tag{5}$$

where t and a are the limits, β is the order of the operation.

3.4 Optimization of fuzzy controllers

Tuning of a controller plays a significant role in performance of the controller in closed loop control system. There is no specific method to tune the nonlinear and intelligent controller, such as fuzzy controller. Also, the design engineer must have some freedom to choose the performance indices and tune the controller for specific needs as per the requirement of process under consideration. The modern day optimization tool, such as GA has ability to deal with large number of decision variables while tuning the controller. GA is a standard optimization technique inspired by natural evolution. Therefore in the present work, GA has been considered to optimize the controller′s parameters having integral of absolute error (IAE) as a performance index. The population size was considered to be 20 and the tolerance level was kept as 10^{-6} and the maximum numbers of iterations were kept as 100. Due to the capability of GA, different adjustable constants, such as L_p, L_i and L_d are considered for fuzzy P/I/D components respectively in contrast to single adjustable constant L in [22]. The objective function is defined as

$$J = \int\{(w_1 * |e_1(t)|) + (w_2 * |e_2(t)|)\}\mathrm{d}t \tag{6}$$

and the corresponding fitness function becomes as

$$F = \frac{1}{J + 0.001} \tag{7}$$

where $e_1(t)$ and $e_2(t)$ are the tracking errors for link-1 and link-2 respectively. Also, w_1 and w_2 are the weights assigned to each link.

In the present work, weights, i.e., $w_1 = w_2 = 0.5$ were assigned to IAE while optimizing the parameters of FOFP + FOFI + FOFD and FP + FI + FD controllers. For tuning purpose, the calculation of IAE was done for a period of 3.2 s. For simulation study, following reference trajectories are considered for link-1 and link-2 angular positions θ_1 and θ_2, respectively.

$$\theta_{rt_1} = \frac{5\pi}{6}\sin(2t) \tag{8}$$

$$\theta_{rt_2} = \frac{5\pi}{6}\cos(2t). \tag{9}$$

The cost function vs. iteration curve for both the controllers is shown in Fig. 6. The values of cost function and IAE for both the links are further listed in Table 3. The optimized parameters for both the controllers are tabulated in Table 4. The reference trajectory tracking response of both controllers are shown in Fig. 7 and the corresponding variation in control actions and errors are presented in Figs. 8 and 9. Also, the variation of end point positions with respect to time are depicted in Fig. 10 and XY curve is shown in Fig. 11. It can be clearly seen from the Fig. 7 that both the controllers show good trajectory tracking performance. But FOFP + FOFI + FOFD controller is able to reduce the cost function 2.5 times as compared to FP + FI + FD controller as shown in Fig. 12.

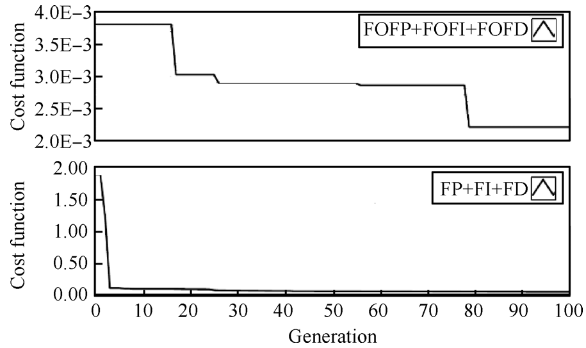

Fig. 6 Cost function versus generation curve

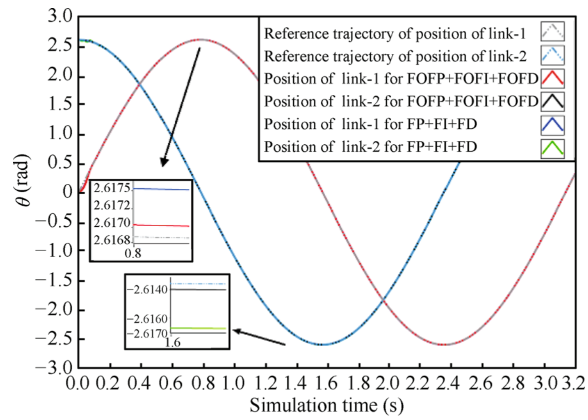

Fig. 7 Trajectory tracking performance

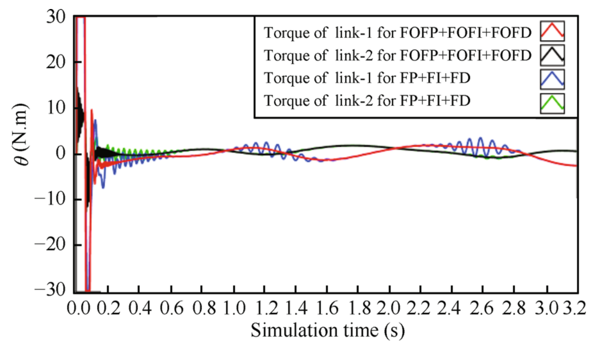

Fig. 8 Corresponding control action

Table 3 Performance index comparison for trajectory tracking

Controller	Cost function (J)	IAE Link-1	IAE Link-2
FOFP+FOFI+FOFD	0.003 259 66	0.005 704 53	0.000 814 791
FP+FI+FD	0.008 318 61	0.009 976 18	0.006 661 050

Table 4 Optimized fuzzy controller parameters

Parameters	FOFP+FOFI+FOFD Link-1	FOFP+FOFI+FOFD Link-2	FP+FI+FD Link-1	FP+FI+FD Link-2
K_p^1	37.240 9	15.308 7	2.081 66	0.020 60
K_p^2	99.831 7	1.008 19	33.971 6	37.783 2
K_{UP}	79.171 6	83.829 5	4.192 52	14.485 5
L_p	10.294 2	20.299 7	89.193 9	5.431 65
K_i^1	36.551 2	97.419 3	38.287 5	34.948 9
K_i^2	−68.799 9	−77.989 9	−83.334 1	−32.817 8
K_{UI}	87.732 8	65.934 2	44.800 2	43.316 2
L_i	71.979 0	45.400 8	22.342 3	99.681 6
K_d^1	5.968 04	14.531 6	49.652 0	47.190 8
K_d^2	38.986 2	89.856 7	19.348 3	32.441 5
K_{UD}	56.058 1	76.589 8	30.013 0	8.298 83
L_d	85.135 0	70.132 5	87.836 5	3.204 57
μ_p	0.569 686	0.342 043	1	1
μ_i	0.440 188	0.934 686	1	1
λ_i	0.010 133	0.343 934	1	1
μ_d	0.133 765	0.855 618	1	1
λ_p	0.247 253	0.802 352	1	1

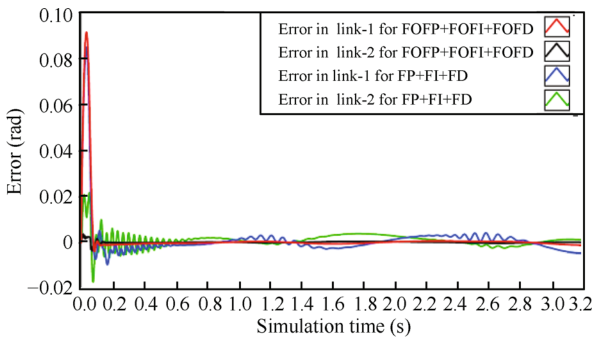

Fig. 9 Error signal

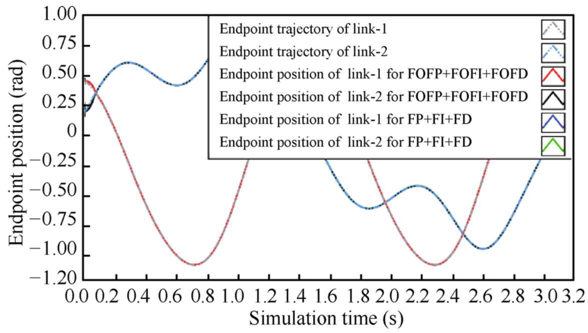

Fig. 10 Endpoint position versus time variations

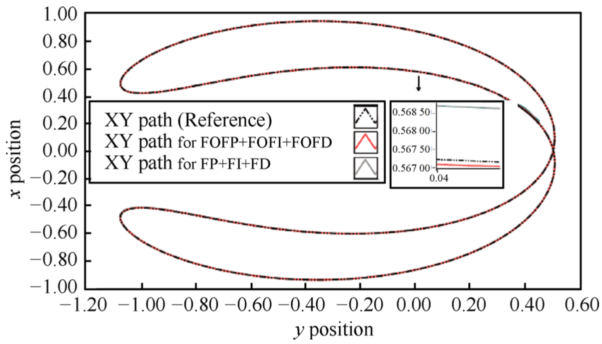

Fig. 11 XY curve for trajectory tracking

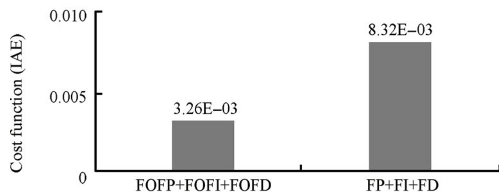

Fig. 12 Comparison of cost function for setpoint tracking

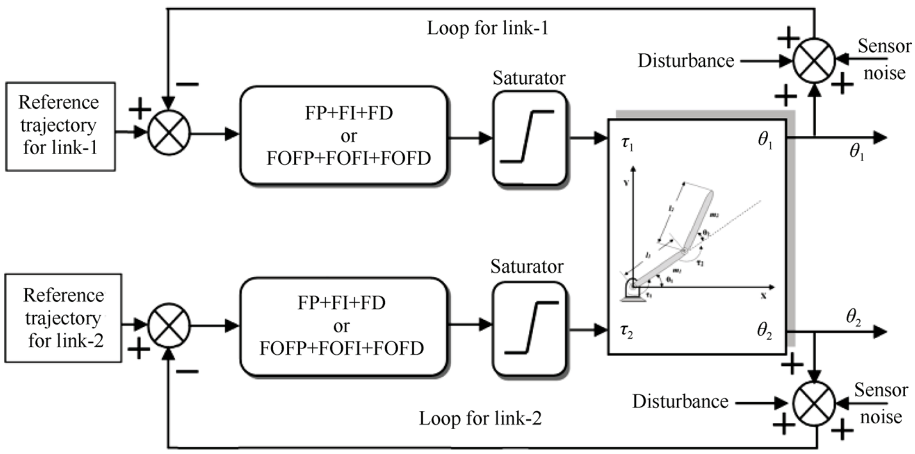

Fig. 13 Block diagram of feedback control loop

4 Performance evaluation

An intensive number of simulations were performed to critically evaluate the performances of both the fuzzy controllers under study. The controllers are tested for disturbance rejections, model uncertainties, and measurement noise suppression. A comparative study is performed in this regard and results are presented in the subsequent subsections. The block diagram of feedback control loop is shown in Fig. 13. The optimized parameters of controllers were kept unchanged throughout this study. The torque constraint for links was taken as [−30, 30] N·m during the simulation. Simulations were performed using National Instrument^{TM} software, LabVIEW^{TM} 8.5 and its add-ons "Simulation and Control Design Toolkit". In the simulation loop, Runge-Kutta 4th order ordinary differential equation solver with a fixed step size of 1 ms was used.

4.1 Disturbance rejection

In any real-time control system, there is always chance that some unavoidable disturbance can shift the response from reference path. In order to study the disturbance rejection capability of the proposed fuzzy controllers during the run time, step disturbances injections were considered. First the disturbance was introduced at $t = 1$ s at the output of link-1 then in link-2 and finally it was injected in both links simultaneously and the corresponding cost functions were calculated from $t = 0$ to $t = 3.2$ s. The obtained cost functions for disturbance rejection response for both the fuzzy controllers for different cases were tabulated in Table 5. It can be observed that FOFP + FOFI + FOFD controller attenuates injected disturbance much nicely as compared to FP + FI + FD controller in all cases. Fig. 14 shows the disturbance rejection response for step magnitude of 0.25 rad injected in angular position of both links. The corresponding control signals and error signals are shown in Figs. 15 and 16. Endpoint position versus the time variations and the XY curve are shown in Figs. 17 and 18, which clearly depicts the supremacy of FOFP + FOFI + FOFD controller over FP + FI + FD controller in disturbance rejection.

Table 5 Comparison of cost function for step disturbance rejection

Disturbance (rad)		Cost function	
Link	Magnitude	FOFP + FOFI + FOFD	FP + FI + FD
Link-1	0.1	0.004 181 44	0.016 659 8
	0.2	0.005 786 73	0.018 489 7
	0.3	0.008 062 06	0.026 554 3
	0.4	0.010 923 10	19.664 900
Link-2	0.1	0.003 802 98	0.014 394 5
	0.2	0.004 540 75	0.459 454 0
	0.3	0.008 982 59	0.458 471 0
	0.4	0.012 893 30	0.438 495 0
Both links	0.05	0.003 776 01	0.014 547 2
	0.10	0.004 744 57	0.015 621 5
	0.15	0.005 941 07	0.575 656 0
	0.20	0.007 380 44	0.730 864 0
	0.25	0.012 254 50	0.741 836 0

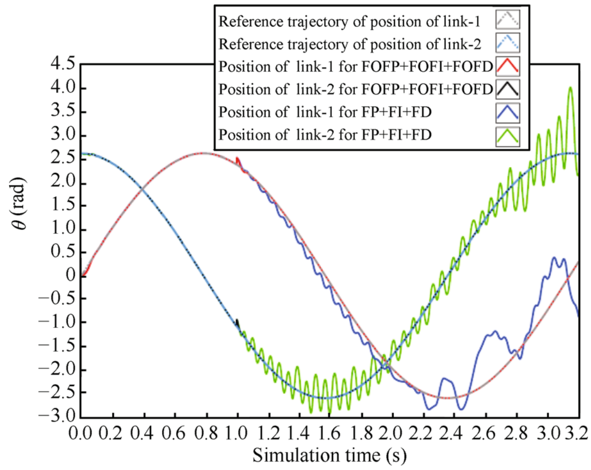

Fig. 14 Response for step disturbance of 0.25 rad in both links

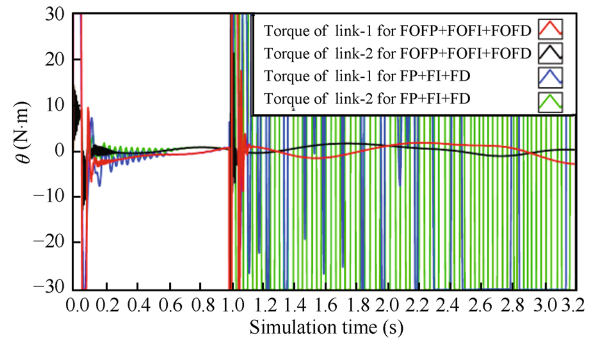

Fig. 15 Control signal for disturbance of 0.25 rad in both links

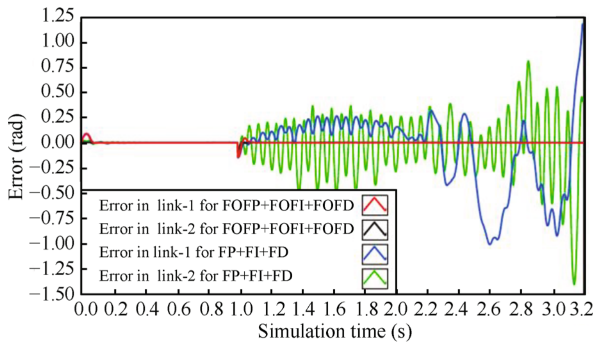

Fig. 16 Error signal for disturbance of 0.25 rad in both links

4.2 Model uncertainty

It is very difficult to get the exact mathematical model of any system. There might be always some uncertainties in the estimation of parameters of the system. These uncertainties may vary from parameter to parameter in a system and also in magnitude. Therefore, model uncertainty plays a significant role in the robustness testing of any controller.

In the present work, uncertainty in the parameters of 2-link rigid manipulator, such as length of the links (l_1& l_2) and mass of the links (m_1& m_2) are considered for study. It is assumed that there is a uniform mass distribution in each link and both the links have a definite ratio of mass and length. For link-1 it is 0.125 and for link-2 it is 0.25. So, if there is a change in the length of any link the corresponding mass of link will also vary. Therefore, two cases were considered for uncertainty study. In the first case length of the link was increased and in second case it was decreased.

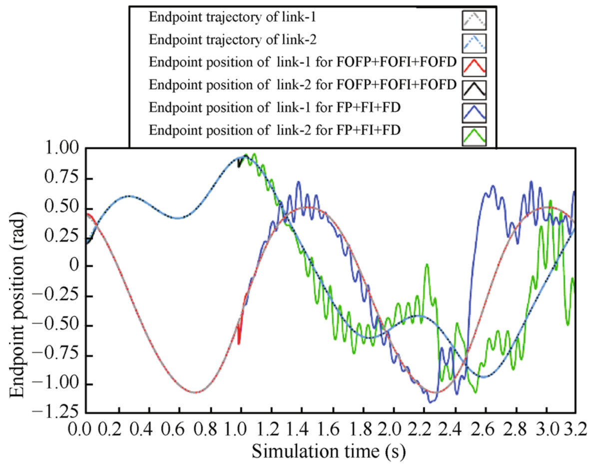

Fig. 17 Endpoint position versus time variations for disturbance of 0.25 rad in both links

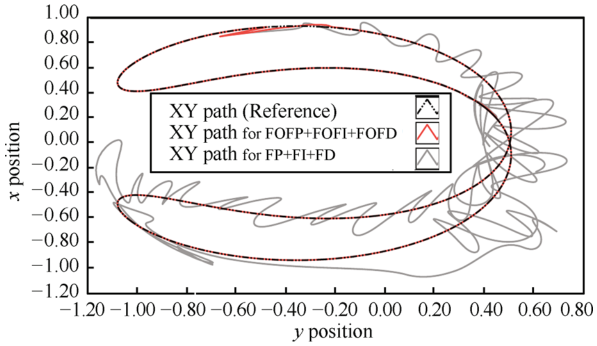

Fig. 18 XY curve for disturbance of 0.25 rad in both links

Tuned gains of fuzzy controllers without model uncertainty are tabulated in Table 6. Also, the normal parameters of 2-link manipulator are listed in Table 1. The effect of uncertainty in the parameters of 2-link manipulator are discussed in more detail in subsequent subsection.

4.2.1 Uncertainty: Increase in the length of links

For robustness testing of fuzzy controllers, length of the links were increased from 5% to 25% of the actual size. The corresponding masses of links were also changed. The cost functions of set point tracking responses of uncertain link lengths were calculated and compared in Table 6. It can be observed from the Table 6 that FOFP +FOFI + FOFD controller outperformed the FP +FI + FD controller in all cases under study and demonstrated very robust behavior by effectively handling the uncertainty in length of any one or both the links together. It can be also noted from Table 6 that FOFP +FOFI + FOFD controller can comfortably track the trajectory upto the 25% increase in l_1 or l_2 and l_1 & l_2 simultaneously. A nominal change in the cost function was observed in case of FOFP +FOFI + FOFD controller during the variations in the lengths of links. However, FP +FI + FD controller was able to track upto 10% change in length l_2 and upto 15% in case of change in l_1 & l_2 simultaneously and beyond this it failed to track the trajectory and becomes unstable. Also, in case of l_1 it is able to track upto 25% change in link length.

The trajectory tracking responses for 20% increase in the lengths of both links of manipulator, i.e., from 0.8 m and 0.4 m to 0.96 m and 0.48, respectively, are shown in Fig. 19. The corresponding control actions and errors are depicted in Figs. 20 and 21. Variations of endpoint positions with respect to time and XY curve are shown in Figs. 22 and 23. It again demonstrates the robust behaviour of FOFP + FOFI + FOFD controller over FP + FI + FD controller.

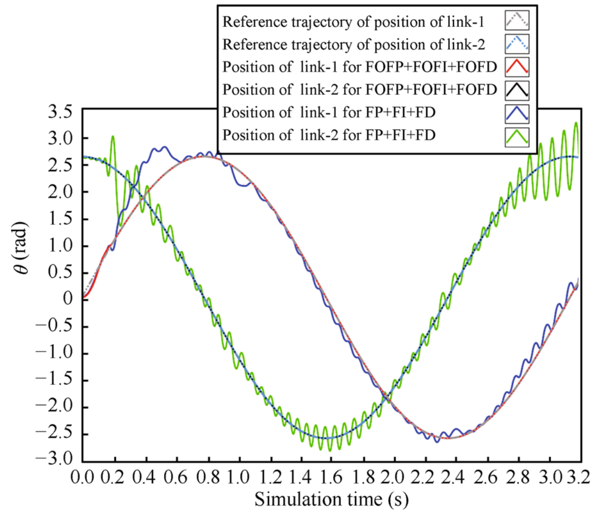

Fig. 19 Response for 20% increase in length of both links

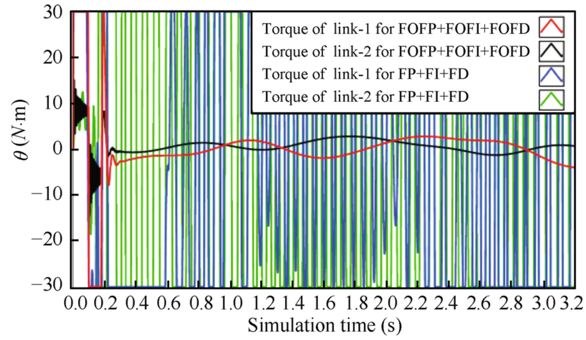

Fig. 20 Control action for 20% increase in length of both links

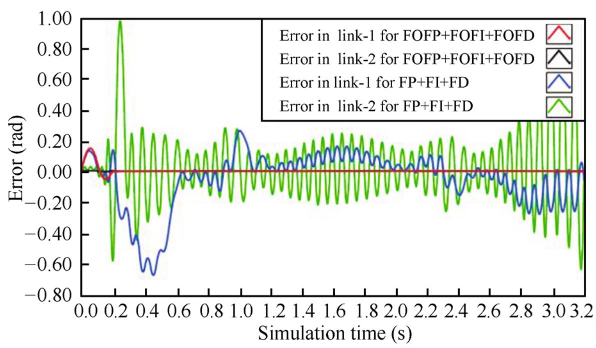

Fig. 21 Error signal for 20% increase in length of both links

Table 6 Comparison of cost function for increase in link lengths

Link	%	Link-1		Link-2		Cost function	
		Length (m)	Mass (kg)	Length (m)	Mass (kg)	FOFP + FOFI + FOFD	FP + FI + FD
Link-1	5	0.84	0.105	0.40	0.10	0.003 782 08	0.008 913 71
	10	0.88	0.110	0.40	0.10	0.004 407 21	0.009 707 11
	15	0.92	0.115	0.40	0.10	0.005 148 94	0.010 697 80
	20	0.96	0.120	0.40	0.10	0.006 031 49	0.011 800 70
	25	1.00	0.125	0.40	0.10	0.007 072 14	0.012 972 60
Link-2	5	0.80	0.10	0.42	0.105	0.004 135 65	0.009 217 35
	10	0.80	0.10	0.44	0.110	0.007 730 94	0.010 367 60
	15	0.80	0.10	0.46	0.115	0.008 485 90	0.655 589 00
	20	0.80	0.10	0.48	0.120	0.008 842 96	0.616 202 00
	25	0.80	0.10	0.50	0.125	0.008 762 71	37.721 600 0
Link-1 & Link-2	5	0.84	0.105	0.42	0.105	0.004 195 04	0.009 749 75
	10	0.88	0.110	0.44	0.110	0.005 372 62	0.011 419 60
	15	0.92	0.115	0.46	0.115	0.006 814 13	0.013 631 30
	20	0.96	0.120	0.48	0.120	0.008 722 70	0.455 040 00
	25	1.00	0.125	0.50	0.125	0.011 204 10	0.386 796 00

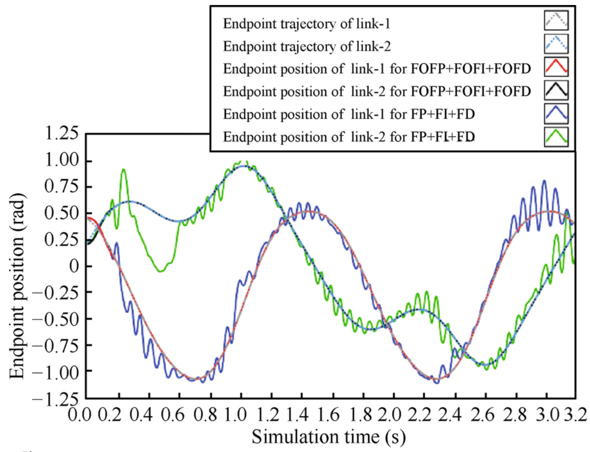

Fig. 22 Endpoint position versus time variation for 20% increase in length of both links

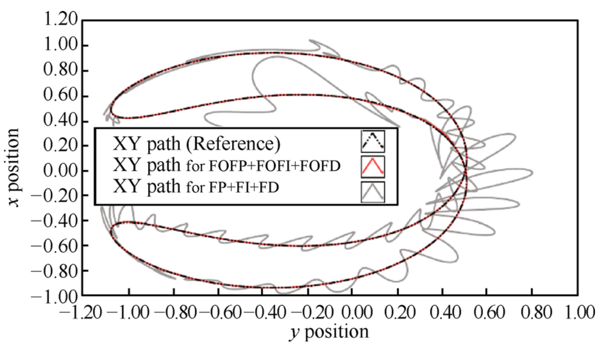

Fig. 23 XY curve for 20% increase in length of both links

4.2.2 Uncertainty: Decrease in the length of links

Further, the lengths of the links were decreased to the range from 5% to 25% of its design values. The mass of link was also changed accordingly. The cost functions of reference trajectory tracking responses of uncertain links′ length were calculated and compared in Table 7. It has been perceived from Table 7 that performance of FOFP +FOFI + FOFD controller was significantly better than FP +FI + FD controller in all cases under study and the results clearly show the robustness feature of FOFP +FOFI + FOFD controller by efficiently handling the uncertainties in the length of links of manipulator. FOFP +FOFI + FOFD controller effectively tracks the reference trajectory upto 25% decrease in l_1 or l_2 and l_1& l_2 simultaneously, and a minor change in the cost function was noted during the variation in the length of links.

Though, FP + FI + FD controller was able to track upto 15% change in length l_1 and l_2 and upto 20% in case of change in l_1 & l_2 simultaneously, and beyond this it failed to track the reference trajectory and becomes unstable.

The reference trajectory tracking response for 25% decrease in the length of both links of manipulator, i.e., from 0.8 m and 0.4 m to 0.6 m and 0.3 m, respectively, are shown in Fig. 24. The corresponding control signals and errors are depicted in Figs. 25 and 26. Variations of endpoint positions with respect to time and XY curve are shown in Figs. 27 and 28. These results again show supremacy of FOFP + FOFI + FOFD controller over FP + FI + FD controller in terms of the robustness behaviour.

4.3 Noise suppression

In control system, there is always some amount of measurement noise. Generally, this noise is random in nature. The level of the measurement noise depends on the working environment. Also, sometimes it is too difficult to avoid the noise from happening. Therefore an effective controller is also required to suppress the effects of such noise introduced in the control loop to an acceptable degree. To test the measurement noise suppression ability of the controllers under study, measurement noise was introduced in the angular positions of links as follows.

Table 7 Comparison of cost function for decrease in link lengths

Link	%	Link-1		Link-2		Cost function	
		Length (m)	Mass (kg)	Length (m)	Mass (kg)	FOFP + FOFI + FOFD	FP + FI + FD
Link-1	5	0.76	0.095	0.40	0.10	0.006 396 89	0.016 163 4
	10	0.72	0.090	0.40	0.10	0.008 764 27	0.022 834 8
	15	0.68	0.085	0.40	0.10	0.010 921 00	0.031 172 2
	20	0.64	0.080	0.40	0.10	0.015 542 70	0.043 189 5
	25	0.62	0.0775	0.40	0.10	0.019 328 70	12.529 200
	5	0.80	0.10	0.38	0.095	0.003 094 97	0.012 918 2
	10	0.80	0.10	0.36	0.090	0.002 995 18	0.020 475 0
	15	0.80	0.10	0.34	0.085	0.002 621 78	0.026 195 9
	20	0.80	0.10	0.32	0.080	0.002 385 40	0.031 880 1
	25	0.80	0.10	0.30	0.075	0.002 218 93	0.038 609 2
Link-1 & Link-2	5	0.76	0.095	0.38	0.095	0.006 357 45	0.022 058 3
	10	0.72	0.090	0.36	0.090	0.007 048 46	0.034 042 2
	15	0.68	0.085	0.34	0.085	0.008 156 74	0.046 648 7
	20	0.64	0.080	0.32	0.080	0.007 329 20	0.066 818 5
	25	0.60	0.075	0.30	0.075	0.009 445 81	31.277 000

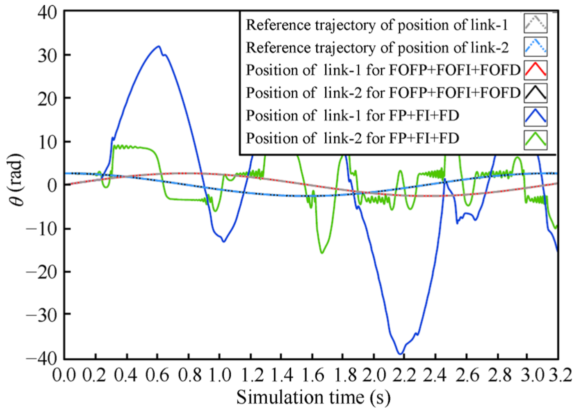

Fig. 24 Response for 25% decrease in length of both links

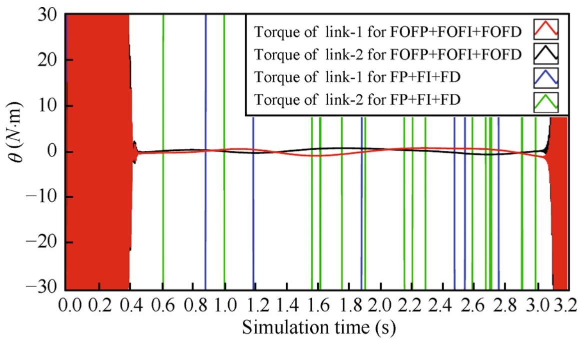

Fig. 25 Control signal for 25% decrease in length of both links

4.3.1 Trajectory tracking in presence of noise

Measurement noise (in %) was introduced in link-1 then in link-2 and finally in both links simultaneously. Noise amplitude was varied from 0.1% to 3% in link-1 and 0.01% to 0.30% in link-2. In all the cases, amplitude of noise was varied and corresponding cost functions for reference trajectory tracking responses of controllers were noted and tabulated in Table 8. It can be noted that FOFP + FOFI + FOFD controller effectively suppresses the noise and does not let the response deviate from the reference trajectory. However, FP + FI + FD controller fails to suppress noise and its response significantly deviates from the reference trajectory. From the Table 8 it is clear that FP + FI + FD controller fails to even cater the noise of magnitude 0.1% in link-1, 0.04% in link-2 and 0.2% and 0.02% in both links simultaneously, and the response drastically deviates from the reference trajectory.

A reference path tracking performance of controllers for measurement noise of amplitude 0.4% in link-1 and 0.04% in link-2 is shown in Fig. 29 and corresponding XY curve is depicted in Fig. 30. The noise injected in links are shown in Fig. 31. Here also FOFP + FOFI + FOFD controller demonstrates much better noise suppression in comparison to its counterpart FP + FI + FD controller.

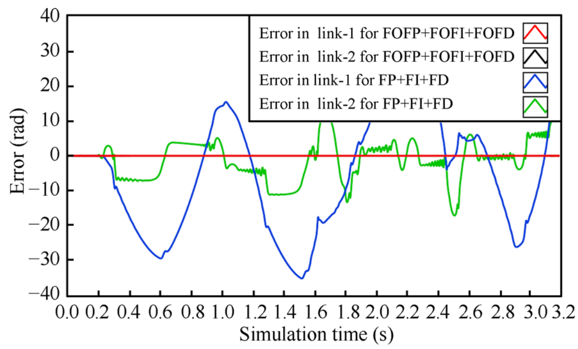

Fig. 26 Error signal for 25% decrease in length of both links

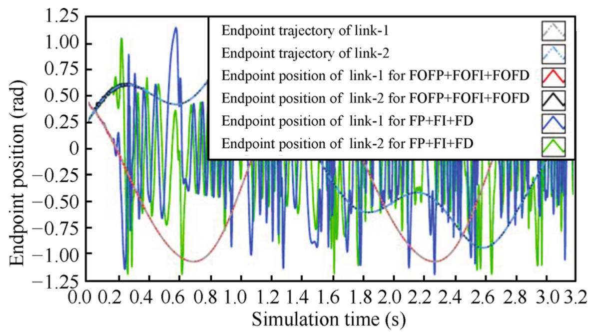

Fig. 27 Endpoint position versus time variation for 25% decrease in length of both links

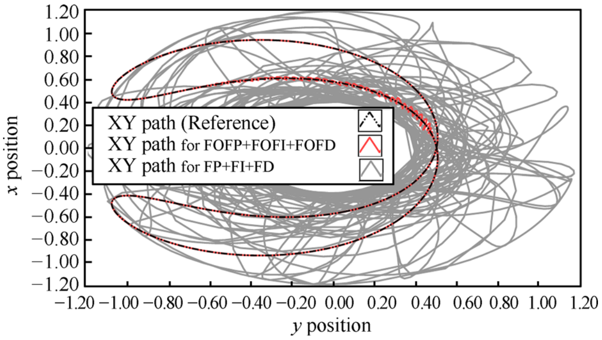

Fig. 28 XY curve for 25% decrease in length of both links

Table 8 Cost function comparison for variation in noise

Noise (in %)		Cost function	
Link-1	Link-2	FOFP + FOFI + FOFD	FP + FI + FD
0.0	—	0.003 259 66	0.008 318 61
0.1	—	0.012 879 4	0.030 514 20
0.2	—	0.016 451 9	34.854 900 0
0.3	—	0.025 449 4	63.087 100 0
0.4	—	0.029 431 0	127.134 000
0.5	—	0.033 317 1	43.478 900 0
0.6	—	0.040 773 3	96.153 000 0
0.7	—	0.046 592 8	11.739 300 0
0.8	—	0.042 077 7	30.676 900 0
0.9	—	0.068 988 9	133.210 000
1.0	—	0.060 377 3	18.576 600 0
2.0	—	0.112 330 0	22.170 200 0
3.0	—	0.169 198 0	157.565 000
—	0.01	0.006 454 09	0.016 772 6
—	0.02	0.008 971 38	0.026 228 9
—	0.03	0.013 855 00	0.028 898 3
—	0.04	0.015 176 70	1.062 600 0
—	0.05	0.014 436 50	80.049 400
—	0.06	0.018 019 10	233.408 00
—	0.07	0.017 078 80	184.099 00
—	0.08	0.017 081 70	144.253 00
—	0.09	0.020 090 70	112.422 00
—	0.10	0.017 405 50	206.945 00
—	0.20	0.027 146 60	20.070 580
—	0.30	0.031 712 40	72.789 800
0.1	0.01	0.010 518 50	0.029 722 8
0.2	0.02	0.018 260 60	36.941 400
0.3	0.03	0.026 191 90	84.946 400
0.4	0.04	0.034 642 40	20.346 800
0.5	0.05	0.040 292 40	104.602 00
0.6	0.06	0.046 916 80	71.977 100
0.7	0.07	0.053 713 70	158.234 00
0.8	0.08	0.073 732 00	101.872 00
0.9	0.09	0.070 970 40	48.240 300
1.0	0.10	0.087 889 40	72.623 000
2.0	0.20	0.191 557 00	31.653 200

4.3.2 Disturbance rejection in presence of noise

Actually, noise is an integral part of any real-time control system and it is very difficult to totally remove it from the system. Therefore in this section a regulatory problem was considered in presence of noise. A random noise signal of 0.4% in link-1 and 0.04% in link-2 of the actual path as shown in Fig. 31 were added in the angular position of respective link value in feedback, as depicted in Fig. 13. Further, an additional step disturbance of magnitude of 0.2 was introduced in each link in the loop after 1s from the starting point of trajectory. The cost function was calculated for both the fuzzy controllers and corresponding disturbance rejection responses and XY curve are shown in Figs. 32 and 33. For FOFP + FOFI + FOFD controller cost function was 0.038 18 and for FP + FI + FD controller it was 111.223. It has been observed that FOFP + FOFI + FOFD controller performs excellently in noisy environment and shows very good disturbance rejection response as compared to its integer order counterpart.

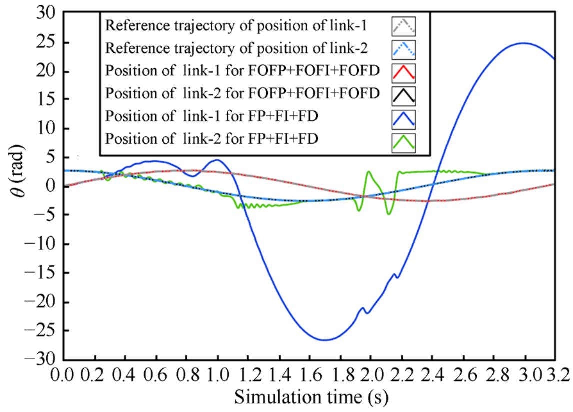

Fig. 29 Trajectory tracking response for 0.4% and 0.04% measurement noise in link-1 and link-2 respectively

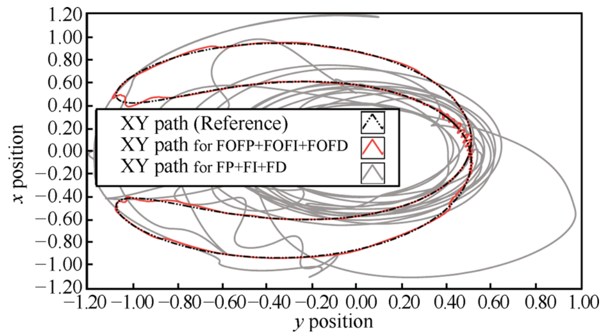

Fig. 30 XY curve for 0.4% and 0.04% measurement noise in link-1 and link-2 respectively

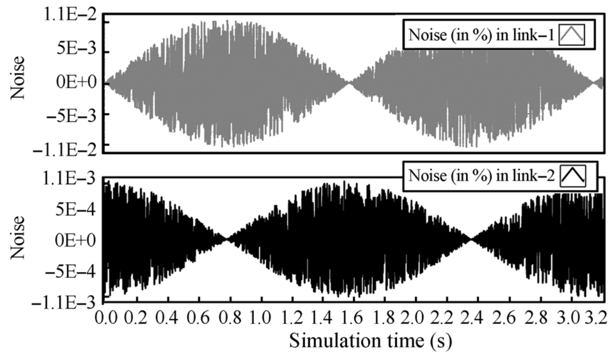

Fig. 31 Noise profile for 0.4% and 0.04% measurement noise in link-1 and link-2 respectively

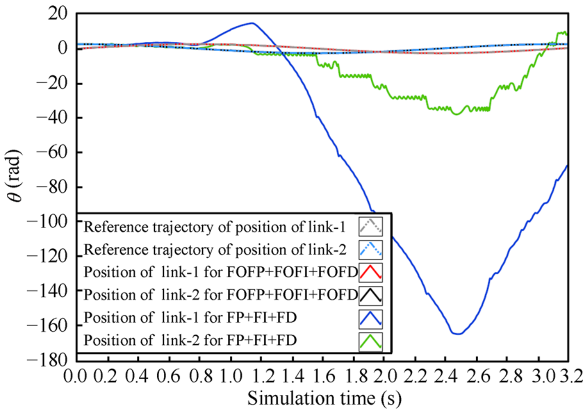

Fig. 32 Disturbance rejection response for step disturbance in both links in presence of measurement noise of 0.4% and 0.04% in link-1 and link-2 respectively

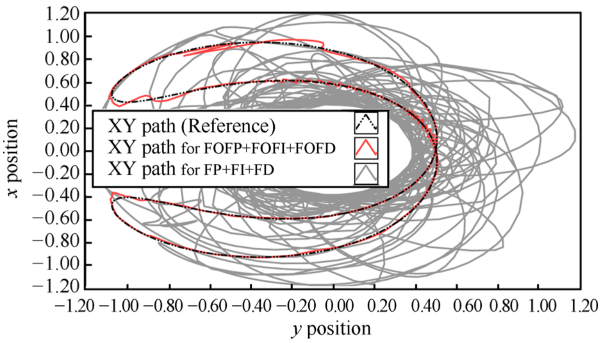

Fig. 33 XY curve for step disturbance in both links in presence of measurement noise of 0.4% and 0.04% in link-1 and link-2 respectively

4.3.3 Uncertainty in length of links in presence of noise

Further, to critically check the robustness of controllers under study, model uncertainty was also studied in a noisy situation. The length of the both links are decreased by 25%, i.e., 0.6 m and 0.3 m and the mass of the links were also reduced accordingly, i.e., 0.075 kg for each link. A sensor noise signal of 0.4% in link-1 and 0.04% in link-2 of the actual reference trajectory, as shown in Fig. 31 was added in the angular position of link-1 and link-2. The servo problem was studied in this noisy environment. The trajectory tracking performances and XY curve of controllers are shown in Figs. 34 and 35. The cost function for FOFP + FOFI + FOFD controller was 0.040 028 1 and for FP + FI +FD controller it was 34.106 7. Again, FOFP + FOFI + FOFD controller demonstrated its supremacy over FP + FI + FD controller and perfectly tracked the trajectory in noisy environment and did not depart. Whereas FP + FI + FD controller is not able to sustain in the noisy condition and fails to track the reference path and becomes unstable.

Therefore, it can be concluded from the noise suppression study that FOFP + FOFI + FOFD controller shows very good noise attenuation characteristic as compared to its integer order counterpart. The proposed control system can trace the reference trajectory even in the presence of noise. Also, FOFP + FOFI + FOFD controller successfully demonstrates the disturbance rejection performance in noisy atmosphere. Further, it shows very strong robust behaviour by tracing the reference path under 25% variation in length of links and having noisy environment.

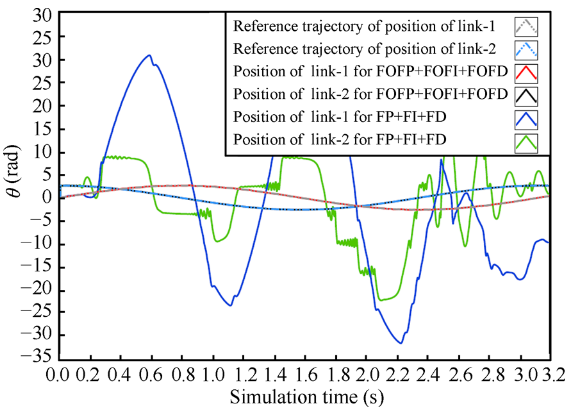

Fig. 34 Trajectory tracking performance for 25% decrease in length of both links in presence of measurement noise of 0.4% and 0.04% in link-1 and link-2 respectively

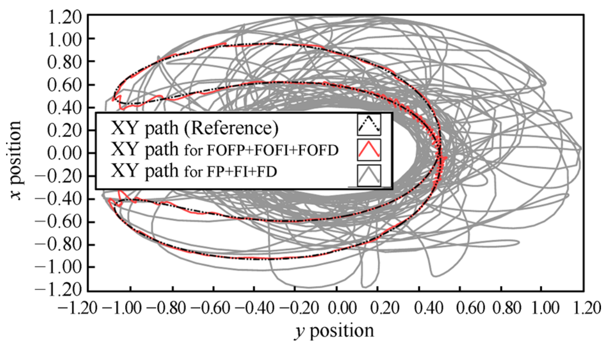

Fig. 35 XY curve for 25% decrease in length of both links in presence of measurement noise of 0.4% and 0.04% in link-1 and link-2 respectively

5 Stability analysis

BIBO stability analysis of nonlinear FOFP + FOFI + FOFD / FP + FI + FD control is performed using the well-known Small Gain Theorem[7−12, 14, 17, 24, 25]. The two fuzzy controllers differ by the use of integrating or differentiating operators. The FOFP + FOFI + FOFD controller use non-integer operators while FP + FI + FD controller use integer operators. In case of FOFP + FOFI + FOFD controller input is changed due to non-integer operators and its internal structure remains the same as FP + FI + FD controller. Therefore, the stability conditions will remain same as in case of FP + FI + FD controller[24]. In the present work, different adjustable parameters, such as L_p, L_i and L_d are considered for fuzzy P/I/D components respectively in contrast to single adjustable parameter L used in [22, 24]. So, there is a slight change in the BIBO stability conditions as in [24] and are presented in subsequent section.

The sufficient conditions for nonlinear FOFP + FOFI + FOFD / FP + FI + FD control systems to be stable are

1) the nonlinear process under control (denoted by $\mathbf{R}$) has a bounded norm (gain) i.e., $\|\mathbf{R}\| < \infty$; and

2) the parameters of FOFP + FOFI + FOFD / FP + FI + FD controllers satisfy

$$\alpha_1 \|\mathbf{R}\| < \infty \tag{10}$$

where α_1 is given in Table 9. More details about the BIBO stability of FP + FI + FD controller are presented in [24].

6 Conclusions

In the present work, a robust Fractional Order Fuzzy P + Fuzzy I + Fuzzy D controller is proposed. It is a nonlinear fuzzy controller having variable gains and demonstrating self-tuning feature. A comparative study has also been performed with its integer order counterpart i.e., Fuzzy P + Fuzzy I + Fuzzy D controller to evaluate the relative performance of the controllers. The applications of fractional order operators offered additional degrees of freedom to Fractional Order Fuzzy P + Fuzzy I + Fuzzy D controller thereby yielding a superior performance. A 2-link planar rigid manipulator was considered as a plant for this study. Both the controllers were tuned using GA for a weighted cost function comprising of IAE of both the loops. Intensive simulations were performed and controllers were evaluated for servo and regulatory problem with and without the presence of model uncertainty and noise. Simulation results clearly revealed that Fractional Order Fuzzy P + Fuzzy I + Fuzzy D controller outperformed Fuzzy P + Fuzzy I + Fuzzy D controller in every aspect of study and demonstrated very strong robust behaviour in case of model uncertainties and noise attenuation. Also, the sufficient BIBO stability conditions are established for the fuzzy controllers using Small Gain Theorem.

Additional number of tunable parameters offered additional advantage to the Fractional Order Fuzzy P + Fuzzy I + Fuzzy D controller over Fuzzy P + Fuzzy I + Fuzzy D controller. Further, as a future scope of this study, complex system such as single or multiple link flexible manipulator and flexible joint manipulator may also be explored.

Acknowledgements

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Table 9 Gains α_1 of subsystem (FOFP + FOFI + FOFD / FP + FI + FD) in the IC regions

IC#	Value of α_1
IC I & IC III	$\left\lvert \frac{L_pK_{UP}(TK_p^2 - K_p^1)}{2T(2L_p - K_p^1M_e)} + \frac{L_iK_{UI}(TK_i^1 + K_i^2)}{2T(2L_i - K_i^1M_e)} + \frac{L_dK_{UD}(TK_d^2 - K_d^1)}{2T(2L_d - K_d^2M_e)} \right\rvert$
IC II & IC IV	$\left\lvert \frac{L_pK_{UP}(TK_p^2 - K_p^1)}{2T(2L_p - K_p^2M_r)} + \frac{L_iK_{UI}(TK_i^1 + K_i^2)}{2T(2L_i - K_i^2M_r)} + \frac{L_dK_{UD}(TK_d^2 - K_d^1)}{2T(2L_d - K_d^1M_r)} \right\rvert$
IC V	$\left\lvert \frac{K_{UP}K_p^2}{2T} + \frac{K_{UI}K_i^2}{2T} - \frac{K_{UD}K_d^1}{2T} \right\rvert$
IC VI	0
IC VII	$\left\lvert \frac{K_{UI}K_i^1}{2} + \frac{K_{UD}K_d^2}{2} - \frac{K_{UP}K_p^1}{2} \right\rvert$
IC VIII	0
IC IX	$\left\lvert \frac{K_{UP}K_p^2}{2T} + \frac{K_{UI}K_i^2}{2T} - \frac{K_{UD}K_d^1}{2T} \right\rvert$
IC X	0
IC XI	$\left\lvert \frac{K_{UI}K_i^1}{2} + \frac{K_{UD}K_d^2}{2} - \frac{K_{UP}K_p^1}{2} \right\rvert$
IC XII	0

where $M_r = \sup\limits_{n \geq 1} |r(nT)| = \sup\limits_{n \geq 1} \dfrac{1}{T}|e(nT) - e(nT-T)| \leq \dfrac{2}{T}M_e$.

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