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Adaptive Iterative Learning Control for Nonlinearly Parameterized Systems with Unknown Time-varying Delay and Unknown Control Direction

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Abstract: This paper proposes a new adaptive iterative learning control approach for a class of nonlinearly parameterized systems with unknown time-varying delay and unknown control direction. By employing the parameter separation technique and signal replacement mechanism, the approach can overcome unknown time-varying parameters and unknown time-varying delay of the nonlinear systems. By incorporating a Nussbaum-type function, the proposed approach can deal with the unknown control direction of the nonlinear systems. Based on a Lyapunov-Krasovskii-like composite energy function, the convergence of tracking error sequence is achieved in the iteration domain. Finally, two simulation examples are provided to illustrate the feasibility of the proposed control method.

Keywords: Nonlinearly time-varying parameterized systems, unknown time-varying delay, unknown control direction, composite energy function, adaptive iterative learning control.

1 Introduction

Iterative learning control (ILC) design is an important kind of control methodology dealing with repeated tracking control problems or periodic disturbance rejection problems[1−10]. The ILC strategies may be classified into two categories: non-adaptive and adaptive, according to whether the system parameters and/or learning gains are estimated or not. In the non-adaptive case, the control input is computed by adding the proportion term and/or the time derivative of system error to the previous control input[1−4]. In the adaptive case, the parameters of system or the learning gains are estimated, and then are used to generate the control input. Moreover, substantial research efforts have been made in this area[5−7,11−19]. Usually, the purpose of continuous-time adaptive ILC (AILC) design is to estimate the system parameters in time domain, and the discrete-time approach is to estimate the learning gains in iteration domain. In [7], the conventional adaptive control was combined with the ILC algorithm in such a way that the essential structure and properties of ILC algorithm were preserved. In [11], a novel AILC design has been proposed via Lyapunov technique, and some of common assumptions of nonlinear ILC are relaxed. Based on the universal adaptive scheme and high gain concepts, a full proof of convergence of the adaptive ILC scheme was given[12]. The closed-loop stability and uniform convergence of all error signals in the iteration domain are established. In [13], by adaptive robust iterative learning control with dead-zone scheme, the tracking system has converged to the error bound within finite iterations. For a class of unknown nonlinear systems, an adaptive fuzzy iterative learning control was designed for the varying tracking tasks problem[14]. A discrete-time adaptive ILC design was presented for systems with iteration-varying trajectory and random initial condition[15]. A uniform framework of AILC method was proposed for uncertain nonlinear systems[16]. In [17], a varied wavelet decomposition was used to design AILC algorithm, the terms of wavelet decomposition increased with the increase of iteration times, which leads to uniform convergence of the tracking error on the finite time interval. However, the unsatisfactory effects of the time delay on the tracking performance have not been considered in the above literatures.

The problem of designing an ILC algorithm for uncertain plants with time delays has not been fully investigated, and only a limited number of results are available. In [20], a new ILC algorithm for a class of linear dynamic systems with time delay was proposed. The algorithm used the holding mechanism, and the convergence of the proposed algorithm was given. In [21], a frequency-domain method to design an ILC method was proposed, which improved the performance of Smith predictor controller, and a necessary and sufficient condition was derived for convergence of iterative process for all admissible plant uncertainties. On the design of ILC for nonlinear time-delay systems, a high-order ILC algorithm for uncertain nonlinear time-delay systems was developed[22]. A learning control scheme was proposed to overcome the uncertainties in time-delay and/or in system parameters[23, 24]. They established robustness of the learning control in the presence of initial function errors. Recently, a novel AILC approach was proposed for first-order hybrid parametric nonlinear time-delay systems[18]. The approach consisted of a differential type updating law and a learning control law, and dealt with the non-uniform trajectory tracking problem, which avoids the restriction on the tracking trajectory in the traditional ILC design. Then, a sufficient condition of tracking error converging to zero in the sense of mean-square on the finite interval was obtained. Especially, based on composite energy function (CEF), a new AILC strategy has been proposed to achieve

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the perfect tracking under the repeatable control environment, which dealt with the time-varying nonlinearly parameterized systems with unknown time-varying delay[19].

It is well known that the unknown control direction problem is a challenging problem. As we know, in terms of task-space control, such as visual serving with an uncalibrated cameras, it could be difficult to determine the relationship between joint space actions and task-space measurements. Therefore, it is suggested to highlight the task-space control of a robot, although that is a MIMO problem obviously. The first result was proposed by Nussbaum[25]. Then, the idea was also extended to ILC design setting[26, 27]. But authors only concerned linearly parametric systems, the time-delay effects and nonlinear parametric problems have seldom been researched. The technique of Nussbaum-type "gains" was also applied to the output feedback adaptive control of nonlinear systems[28] and robust adaptive control of time-delay nonlinear systems[29].

Motivated by the previous works, in this paper, we will propose a new AILC method to deal with the tracking problem of a class of nonlinearly parameterized systems with unknown time-varying delay and unknown control direction. Then, based on a CEF, the convergence of the tracking error will be given in the mean-square sense on a finite time-interval. Two simulation examples can verify the feasibility of the control approach.

The contributions of this paper are as follows. 1) The new proposed control method can deal with the unknown direction of a class of nonlinearly parameterized systems with unknown time-varying delay; 2) the new AILC approach can ensure that the convergence of the tracking error sequence is achieved in the iteration domain.

The rest of this paper is organized as follows. Section 2 presents the new AILC scheme for the first-order time-delay system, and exhibits a rigorous analysis of learning convergence in $L^2_{[0,T]}$-norm using CEF. Section 3 presents a generalization for higher-order systems with mixed unknown parameters. Section 4 presents two simulation examples. Finally, conclusions are drawn in Section 5.

Notation. In this paper, $C\,[0,T]$ and $C^n\,[0,T]$ represent the space of continuous functions and n-th order continuous functions, respectively. $x_i(t)$ converges to zero in $L^2_{[0,T]}$-norm, i.e., $\lim_{i\to\infty}\int_0^T \|x_i\|^2 \mathrm{d}\sigma = 0,\ \ i\in \mathbf{Z}_+$, where i denotes the iteration number.

2 AILC for the first-order time-delay system

2.1 Problem description and preliminaries

Consider the following first-order nonlinear dynamic system

$$\left\{\begin{array}{l} \dot{x} = f\left(x(t-\tau(t)),\theta(t)\right)\xi(x) + bu \\ x = \varpi(t), \quad t\in[-\tau_{\max},0] \end{array}\right. \tag{1}$$

where $x\in\mathbf{R}$ and $u\in\mathbf{R}$ denote the measurable state and the control input of the system (1). $\tau(t)\in \mathrm{C}^1\,[0,T]$ represents the unknown time-varying delay of the system state x, and $\tau(t)$ satisfies $\tau(t)\leqslant\tau_{\max}$, $\forall t\in[0,T]$, with $\tau_{\max}$ being a known constant. $\theta(t)\in C^1\,[0,T]$ is an unknown continuous time-varying parameter. $b\neq 0$ is an unknown constant. The sign of b, which determines the control direction, is assumed unknown. $f:\mathbf{R}^2\to\mathbf{R}$ is an unknown continuous function, which is assumed to be locally Lipschitz continuous with respect to the first argument. $\xi:\mathbf{R}\to\mathbf{R}$ is a known continuous function, which is assumed to be local Lipschitz continuous. $x=\varpi(t),\ t\in[-\tau_{\max},0]$ is a known continuous function, and denotes the initial condition of system (1).

For a given desired trajectory $x_r(t)\in C^1\,[0,T]$, we can define the tracking error $e_i = x_i - x_r$ at the i-th iteration.

The control objective is to find a sequence of control input $u_i\,(t)\,,\ i\in\mathbf{Z}_+$, such that the tracking error converges to zero in $L^2_{[0,T]}$-norm as the iteration number i approaches infinity.

To facilitate AILC approach design, we need the following preliminaries.

Assumption 1. The unknown time-varying delay $\tau\,(t)$ satisfies $\dot{\tau}\,(t)\leqslant\eta<1$, i.e., $-\dfrac{1-\dot{\tau}\,(t)}{1-\eta}\leqslant -1$.

Assumption 2. The unknown function $f(\cdot,\cdot)$ satisfies the following inequality

$$|f(x,\theta)-f(\bar{x},\theta)|\leqslant|x-\bar{x}|\,h\,(x,\bar{x})\,\lambda\,(\theta)$$

where $(x,\bar{x})\in\mathbf{R}^2,\forall\theta\in\mathbf{R}$, $h(\cdot,\cdot)$ is a known and nonnegative continuous function, and $\lambda\,(\cdot)$ is an unknown and nonnegative continuous function.

Assumption 3. The identical initial condition satisfies $e_i\,(t)=0,\ t\in[-\tau_{\max},0]\,,\ \forall i\in\mathbf{Z}_+$.

Definition 1[28]**.** An even smooth function $N\,(\varsigma):\ \mathbf{R}\to\mathbf{R}$, is called a Nussbaum-type function if it satisfies

$$\left\{\begin{array}{l} \lim\limits_{s\to\infty}\left(\sup\dfrac{1}{s}\displaystyle\int_0^s N\,(\varsigma)\,\mathrm{d}\varsigma\right)=+\infty \\ \lim\limits_{s\to\infty}\left(\inf\dfrac{1}{s}\displaystyle\int_0^s N\,(\varsigma)\,\mathrm{d}\varsigma\right)=-\infty. \end{array}\right.$$

For example, $\varsigma^2\cos\left(\frac{\pi\varsigma}{2}\right)$ and $\mathrm{e}^{\varsigma^2}\cos\left(\frac{\pi\varsigma}{2}\right)$ are Nussbaum-type functions. Throughout this paper, we choose $\mathrm{e}^{\varsigma^2}\cos\left(\frac{\pi\varsigma}{2}\right)$ as a Nussbaum-type function.

Lemma 1[28]**.** Let $V\,(t):[0,t_f)\to\mathbf{R}_+$ and $\varsigma:[0,t_f)\to\mathbf{R}$ be smooth functions with $V\,(t)\geqslant 0$. $N:[0,t_f)\to\mathbf{R}$ be a Nussbaum-type function. If the following inequality holds, i.e.,

$$V(t)\leqslant c_0+\int_0^t\left(gN\left(\varsigma\left(\tau\right)\right)+1\right)\mathrm{d}\varsigma\left(\tau\right),\ \forall t\in[0,t_f)$$

where $g\in\mathbf{R}$ is a nonzero constant, and $c_0\in\mathbf{R}$ represents a constant. Then, $\varsigma\,(t)$, $V\,(t)$ and $\int_0^t\left(gN\left(\varsigma\left(\tau\right)\right)+1\right)\mathrm{d}\varsigma\left(\tau\right)$ are bounded on $[0,t_f)$.

Remark 1. Assumption 1 is commonly made in solving the control problem of system with time-varying delays, which will be used to design AILC approach later. For example, the time-varying delays $\tau\,(t)=\arctan(t)$ and $\tau\,(t)=\mathrm{e}^{-t}$ all satisfy Assumption 1. Actually, this assumption was also used in [24].

Remark 2. In this paper, Assumption 2 is used to deal with the time-varying nonlinearly parameterized uncertain term $f\left(x\left(t-\tau\left(t\right)\right),\theta\left(t\right)\right)$ of the system (1). Actually, if $f(\cdot,\cdot)$ is assumed to be differentiable, we can obtain that $|f\left(x,\theta\right)-f\left(\bar{x},\theta\right)|\leqslant|x-\bar{x}|\,|f_1\left(x,\bar{x},\theta\right)|$ by the differential mean value theorem. Then, we can employ the parameter separation technique[24] to get $|f_1(x,\bar{x},\theta)| \leqslant h(x,\bar{x})\lambda(\theta)$. Therefore, the Assumption 2 holds, and can be seen in [24].

2.2 Design of control and adaptive law

In this section, a new AILC scheme is presented for the nonlinear system described by (1).

Note the tracking error $e_i(t) = x_i(t) - x_r(t)$ at the i-th iteration. By adding and subtracting $f(x_r(t-\tau(t)), \theta(t))$, the dynamics of tracking error $e_i(t)$ at the i-th iteration can be expressed as follows

$$\dot{e}_i(t) = \Theta(t)\xi(x_i) + bu_i - \dot{x}_r + \Lambda_i\xi(x_i) \tag{2}$$

where $\Theta(t) = f(x_r(t-\tau(t)), \theta(t))$ and $\Lambda_i = f(x_i(t-\tau(t)), \theta(t)) - f(x_r(t-\tau(t)), \theta(t))$.

Obviously, $\Theta(t)$ is related to the unknown time-varying parameter $\theta(t)$, which will be estimated later. Moreover, based on Assumption 2, the term Λ_i can be dealt with as follows

$$|\Lambda_i| \leqslant |e_i(t-\tau(t))| \times h(x_r(t-\tau(t)),\ x_i(t-\tau(t)))\lambda(\theta(t)). \tag{3}$$

Then, consider a Lyapunov-Krasovskii functional at the i-th iteration as

$$V_i(t) = \frac{1}{2}e_i^2(t) + \frac{1}{2(1-\eta)} \times \int_{t-\tau(t)}^{t} e_i^2(\sigma)h^2(x_r(\sigma), x_i(\sigma))\mathrm{d}\sigma. \tag{4}$$

It is easily seen from Assumption 3 that $V_i(0) = 0$. After taking the time derivative of (4) and using (2)–(3), we obtain the following expression

$$\begin{aligned}\dot{V}_i(t) =& e_i(t)[\Theta(t)\xi(x_i) + bu_i - \dot{x}_r + \Lambda_i\xi(x_i)] + \\ & \frac{1}{2}(1-\eta)e_i^2(t)h^2(x_r(t), x_i(t)) - \\ & \frac{1-\dot{\tau}(t)}{2(1-\eta)}e_i^2(t-\tau(t))h^2(x_r(t-\tau(t)), \\ & x_i(t-\tau(t))).\end{aligned} \tag{5}$$

The proposed learning control law at the i-th iteration is

$$u_i = N(\varsigma_i)G_i(t) \tag{6}$$

$$\dot{\varsigma}_i(t) = e_i(t)G_i(t),\ \varsigma_{-1}(t) = \text{const},\ \varsigma_i(0) = \varsigma_{i-1}(T) \tag{7}$$

where $G_i(t) = ke_i(t) + \frac{1}{2(1-\eta)}e_i(t)h^2(x_i(t), x_r(t)) + \hat{\beta}_i^{\mathrm{T}}(t)\phi_i - \dot{x}_r(t)$, $\phi_i = \left[\xi(x_i), \frac{1}{2}\xi^2(x_i)e_i\right]^{\mathrm{T}}$, and $\beta(t) = \left[\beta^{(1)}(t), \beta^{(2)}(t)\right]^{\mathrm{T}} = \left[\Theta(t), \lambda^2(\theta(t))\right]^{\mathrm{T}}$, and const represents a real number. $k > 0$ is the design parameter. $N(\varsigma_i)$ is a suitable Nussbaum-type function. $\hat{\beta}_i(t)$ denotes the estimation of $\beta(t)$ at the i-th iteration.

The proposed adaptive learning law of $\beta(t)$ at the i-th iteration is

$$\hat{\beta}_i(t) = \hat{\beta}_{i-1}(t) + q\phi_i e_i(t),\quad \hat{\beta}_{-1}(t) = 0,\quad \forall t \in [0, T] \tag{8}$$

where $q > 0$ is the design parameter.

Substituting (6) into (5), we can obtain that

$$\begin{aligned}\dot{V}_i(t) =& e_i(t)[\Theta(t)\xi(x_i) + bN(\varsigma_i)G_i(t) - \dot{x}_r + \Lambda_i\xi(x_i)] + \\ & \frac{1}{2(1-\eta)}e_i^2(t)h^2(x_r(t), x_i(t)) - \frac{1-\dot{\tau}(t)}{2(1-\eta)} \times \\ & e_i^2(t-\tau(t))h^2(x_r(t-\tau(t)), x_i(t-\tau(t))).\end{aligned} \tag{9}$$

Using Young′s inequality[24], we have

$$\begin{aligned}& e_i(t)\Lambda_i\xi(x_i) \leqslant |e_i(t)||e_i(t-\tau(t))| \times \\ & \quad h(x_r(t-\tau(t)), x_i(t-\tau(t)))\lambda(\theta(t))|\xi(x_i)| \leqslant \\ & \quad \frac{1}{2}{e_i}^2(t)\lambda^2(\theta(t))\xi^2(x_i) + \\ & \quad \frac{1}{2}{e_i}^2(t-\tau(t))h^2(x_r(t-\tau(t)), x_i(t-\tau(t))).\end{aligned} \tag{10}$$

Using Assumption 1, (7), (9), and (10) yields

$$\begin{aligned}\dot{V}_i(t) \leqslant & e_i(t)[\Theta(t)\xi(x_i) - G_i(t) - \dot{x}_r(t)] + \\ & [bN(\varsigma_i) + 1]\dot{\varsigma}_i(t) + \frac{1}{2}{e_i}^2(t)\lambda^2(\theta(t))\xi^2(x_i) + \\ & \frac{1}{2}{e_i}^2(t-\tau(t))h^2(x_r(t-\tau(t)), x_i(t-\tau(t))) + \\ & \frac{1}{2(1-\eta)}e_i^2(t)h^2(x_r(t), x_i(t)) - \frac{1-\dot{\tau}(t)}{2(1-\eta)}e_i^2(t- \\ & \tau(t)) \times h^2(x_r(t-\tau(t)), x_i(t-\tau(t))) \leqslant \\ & e_i(t)\Bigg[\Theta(t)\xi(x_i) - G_i(t) + \frac{1}{2(1-\eta)}e_i(t) \times \\ & h^2(x_i(t), x_r(t)) + \frac{1}{2}e_i(t)\lambda^2(\theta(t))\xi^2(x_i) - \dot{x}_r(t)\Bigg] + \\ & [bN(\varsigma_i) + 1]\dot{\varsigma}_i(t) \leqslant \\ & -k{e_i}^2(t) + \tilde{\beta}_i^{\mathrm{T}}(t)\phi_i e_i(t) + [bN(\varsigma_i) + 1]\dot{\varsigma}_i(t)\end{aligned} \tag{11}$$

where $\tilde{\beta}_i(t) = \beta(t) - \hat{\beta}_i(t)$ denotes the estimation error of $\beta(t)$ at the i-th iteration.

2.3 Analysis of learning convergence

Theorem 1. Considering system (1) under the Assumptions 1–3, the control law (6) and the adaptive law (8) guarantee the convergence of the tracking error $e_i(t)$ and the boundedness of all signals $x_i(t)$, $\hat{\beta}_i(t)$, and $u_i(t)$ in the closed-loop system.

Proof. Define a Lyapunov-Krasovskii-like CEF at the i-th iteration as

$$E_i(t) = V_i(t) + \frac{1}{2q}\int_0^t \tilde{\beta}_i^{\mathrm{T}}\tilde{\beta}_i\mathrm{d}\sigma + \frac{1}{2q}\int_t^T \tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1}\mathrm{d}\sigma. \tag{12}$$

1) The difference of $E_i(t)$ at the i-th iteration is

$$\begin{aligned}\Delta E_i(t) =& E_i(t) - E_{i-1}(t) = \\ & V_i(t) + \frac{1}{2q}\int_0^t \left(\tilde{\beta}_i^{\mathrm{T}}\tilde{\beta}_i - \tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1}\right)\mathrm{d}\sigma + \\ & \frac{1}{2q}\int_t^{\mathrm{T}} \left(\tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1} - \tilde{\beta}_{i-2}^{\mathrm{T}}\tilde{\beta}_{i-2}\right)\mathrm{d}\sigma - V_{i-1}(t).\end{aligned} \tag{13}$$

Using Assumption 3 and (11), we can write the first term of (13) as follows

$$V_i(t)=\int_0^t \dot{V}_i\mathrm{d}\sigma+V_i(0)$$
$$-k\int_0^t {e_i}^2\mathrm{d}\sigma+\int_0^t \mu_i\mathrm{d}\sigma+\int_0^t [bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma \tag{14}$$

where $\mu_i=\tilde{\beta}_i^{\mathrm{T}}\phi_i e_i$.

Using the algebraic relation $(a-b)^{\mathrm{T}}(a-b)-(a-c)^{\mathrm{T}}\times(a-c)=(c-b)^{\mathrm{T}}[2(a-b)+(b-c)]$ (a,b,c are column vectors respectively), and the adaptive learning law (8), the second term of (13) can be expressed as follows

$$\frac{1}{2q}\int_0^t\left[\tilde{\beta}_i^{\mathrm{T}}\tilde{\beta}_i-\tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1}\right]\mathrm{d}\sigma=$$
$$\frac{1}{2q}\int_0^t\left[\left(\beta-\hat{\beta}_i\right)^{\mathrm{T}}\left(\beta-\hat{\beta}_i\right)-\left(\beta-\hat{\beta}_{i-1}\right)^{\mathrm{T}}\left(\beta-\hat{\beta}_{i-1}\right)\right]\mathrm{d}\sigma=$$
$$\frac{1}{2q}\int_0^t\left(\hat{\beta}_{i-1}-\hat{\beta}_i\right)^{\mathrm{T}}\left[2\left(\beta-\hat{\beta}_i\right)+\left(\hat{\beta}_i-\hat{\beta}_{i-1}\right)\right]\mathrm{d}\sigma=$$
$$-\int_0^t\mu_i\mathrm{d}\sigma-\frac{q}{2}\int_0^t\|\phi_i\|^2{e_i}^2\mathrm{d}\sigma. \tag{15}$$

Substituting (14)–(15) into (13), we arrive at

$$\Delta E_i(t)\leqslant-k\int_0^t{e_i}^2\mathrm{d}\sigma+\int_0^t[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma-$$
$$\frac{q}{2}\int_0^t\|\phi_i\|^2{e_i}^2\mathrm{d}\sigma-V_{i-1}(t)+$$
$$\frac{1}{2q}\int_t^T\left[\tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1}-\tilde{\beta}_{i-2}^{\mathrm{T}}\tilde{\beta}_{i-2}\right]\mathrm{d}\sigma\leqslant$$
$$-k\int_0^t{e_i}^2d\sigma+\int_0^t[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma+$$
$$\frac{1}{2q}\int_t^T\left[\tilde{\beta}_{i-1}^{\mathrm{T}}\tilde{\beta}_{i-1}-\tilde{\beta}_{i-2}^{\mathrm{T}}\tilde{\beta}_{i-2}\right]\mathrm{d}\sigma. \tag{16}$$

When $t=T$, the inequality (16) can be expressed as

$$\Delta E_i(T)\leqslant-k\int_0^T{e_i}^2\mathrm{d}\sigma+\int_0^T[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma. \tag{17}$$

2) Learning convergence property

Now we exhibit the learning convergence property, which is summarized as follows.

Applying (13) and (17) repeatedly, we have

$$E_j(T)=E_0(T)+\sum_{i=1}^j\Delta E_i(T)$$
$$E_0(T)-k\sum_{i=1}^j\int_0^T{e_i}^2\mathrm{d}\sigma+$$
$$\sum_{i=1}^j\int_0^T[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma. \tag{18}$$

We denote $\varsigma(t+(j-1)T)\triangleq\varsigma_j(t)$ and $\dot{\varsigma}(t+(j-1)T)\triangleq\dot{\varsigma}_j(t)$,$\forall j\geqslant 0$ and $j\in\mathbf{Z}$. Therefore, both $\varsigma(t)$ and $\dot{\varsigma}(t)$ are continuous functions for $\forall t\in[0,jT]$. Thus,

$$\sum_{i=1}^j\int_0^T[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma=$$
$$\int_0^T[bN(\varsigma_1)+1]\dot{\varsigma}_1\mathrm{d}\sigma+\int_0^T[bN(\varsigma_2)+1]\dot{\varsigma}_2\mathrm{d}\sigma+\cdots+$$
$$\int_0^T[bN(\varsigma_j)+1]\dot{\varsigma}_j\mathrm{d}\sigma=$$
$$\int_0^T[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma+\int_T^{2T}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma+\cdots+$$
$$\int_{(j-1)T}^{jT}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma=$$
$$\int_0^{jT}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma. \tag{19}$$

Denote $W(s+(j-1)T)\triangleq E_j(s)$, from (18)–(19), we have

$$W(jT)\leqslant$$
$$E_0(T)-k\sum_{i=1}^j\int_0^T{e_i}^2\mathrm{d}\sigma+\sum_{i=1}^j\int_0^T\times[bN(\varsigma_i)+1]\dot{\varsigma}_i\mathrm{d}\sigma=$$
$$E_0(T)-k\sum_{i=1}^j\int_0^T{e_i}^2\mathrm{d}\sigma+\int_0^{jT}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma. \tag{20}$$

Simultaneously, from (12), we also have

$$\dot{E}_i(t)=\dot{V}_i(t)+\frac{1}{2q}\left[\tilde{\beta}_i^{\mathrm{T}}(t)\tilde{\beta}_i(t)-\tilde{\beta}_{i-1}^{\mathrm{T}}(t)\tilde{\beta}_{i-1}(t)\right]. \tag{21}$$

Applying (11) and (15) into (21), we can obtain that

$$\dot{E}_i(t)\leqslant-k{e_i}^2+\tilde{\beta}_i^{\mathrm{T}}\phi_i e_i+[bN(\varsigma_i)+1]\dot{\varsigma}_i+$$
$$\left(-\mu_i-\frac{q}{2}\|\phi_i\|^2{e_i}^2\right). \tag{22}$$

Substituting $\mu_i=\tilde{\beta}_i^{\mathrm{T}}\phi_i e_i$ into (22), we have

$$\dot{E}_i(t)\leqslant[bN(\varsigma_i)+1]\dot{\varsigma}_i. \tag{23}$$

Thus, based on (22) and $W(s+(j-1)T)\triangleq E_j(s)$, we get

$$W(jT+t)=W(jT)+\int_0^t\dot{E}_{j+1}(\sigma)\mathrm{d}\sigma,\forall t\in[0,T]. \tag{24}$$

Using (20), (23) and (24), we can obtain that

$$W(jT+t)\leqslant E_0(T)-k\sum_{i=1}^j\int_0^T{e_i}^2\mathrm{d}\sigma+$$
$$\int_0^{jT}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma+\int_0^t[bN(\varsigma_{j+1})+1]\dot{\varsigma}_{j+1}\mathrm{d}\sigma$$
$$E_0(T)-k\sum_{i=1}^j\int_0^T{e_i}^2\mathrm{d}\sigma+\int_0^{jT}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma+$$
$$\int_{jT}^{(jT+t)}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma\leqslant$$
$$E_0(T)+\int_0^{(jT+t)}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma. \tag{25}$$

If $E_0(T)$ is a finite number, applying Lemma 1 to (25), we can obtain $W(jT+t) \leqslant M_1$ and $\int_0^{(jT+t)}[bN(\varsigma)+1]\dot{\varsigma}\mathrm{d}\sigma \leqslant M_2$, where M_1 and M_2 are nonnegative constants. Because $W(jT+t) \triangleq E_{j+1}(t) = V_{j+1}(t) + \frac{1}{2q}\int_0^t \tilde{\beta}_{j+1}^{\mathrm{T}}\tilde{\beta}_{j+1}\mathrm{d}\sigma + \frac{1}{2q}\int_t^T \tilde{\beta}_j^{\mathrm{T}}\tilde{\beta}_j\mathrm{d}\sigma$, all signals, which $E_{j+1}(t)$ contains, are bounded, and their upper bounds are independent of j. From (25), we further obtain $\sum_{i=1}^{j}\int_0^T {e_i}^2\mathrm{d}\sigma$, $\forall j \in \mathbf{Z}_+$ is bounded. Obviously, the boundedness of $\sum_{i=1}^{j}\int_0^T {e_i}^2\mathrm{d}\sigma$ implies the tracking error $e_i(t)$ converges to zero in $L^2_{[0,T]}$-norm as the iteration number i approaches infinity, i.e., $\lim\limits_{i\to\infty}\int_0^T {e_i}^2\mathrm{d}\sigma = 0$.

3) The finiteness of $E_0(T)$ is similar to the proof in [27].

4) Boundedness property of all signals in the closed-loop system

From the above sections, we demonstrate the boundedness of $E_0(t)$ and $E_i(t)$ on the interval $[0,T]$. Therefore, $x_i(t)$, $\hat{\beta}_i(t)$, $e_i(t)$, $N(\varsigma(t))$, and $\varsigma(t)$ are bounded obviously. Moreover, $h(\cdot,\cdot)$ and $\xi(\cdot)$ represent two continuous functions such that their boundedness is obvious on the interval $[0,T]$. From (6) and (7), we can also obtain that $u_i(t)$ is bounded. □

3 AILC for higher-order systems with mixed unknown parameters

Consider the following higher-order nonlinear dynamic systems

$$\begin{cases} \dot{X} = AX + B_c\left[bu + \varphi^{\mathrm{T}}(X)\vartheta + \right. \\ \quad \left. f^{\mathrm{T}}(X(t-\tau(t)),\theta(t))\xi(X)\right] \\ X = \varpi(t), \quad t \in [-\tau_{\max}, 0] \end{cases} \tag{26}$$

where $X = [x_1, x_2, \dots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$ is the measurable state, and $u \in \mathbf{R}$ is the control input of the system (26). $A = \begin{bmatrix} 0 & I_{n-1} \\ \vdots & \\ 0 & 0 \end{bmatrix}$. $B_c = [0,0,\cdots,1]^{\mathrm{T}}$. $b \neq 0$ is an unknown constant. The sign of b, which determines the control direction, is also assumed unknown. Both $\xi: \mathbf{R}^n \to \mathbf{R}^m$ and $\varphi: \mathbf{R}^n \to \mathbf{R}^q$ are known continuous functions, which are assumed to be local Lipchitz continuous. $\vartheta \in \mathbf{R}^q$ is an unknown time-invariant parameter vector. $\theta(t) \in \mathbf{R}^q$ is an unknown time-varying parameter vector. $f: \mathbf{R}^{n+q} \to \mathbf{R}^m$ is an unknown continuous function, which is assumed to be local Lipchitz continuous with respect to the first argument. $X = \varpi(t), t \in [-\tau_{\max}, 0]$ is a known continuous function, and denotes the initial condition of system (26). The expression of $\tau(t)$ and $\tau_{\max}$ is the same as in Section 2.

For a given desired trajectory $X_r(t) = \left[y_r(t), \dot{y}_r(t), \cdots, y_r^{(n-1)}(t)\right]^{\mathrm{T}}$, $t \in [0,T]$, we can define the tracking error $e_i = X_i - X_r$ at the i-th iteration. The control objective is similar to Section 2.

The following assumption will be used in AILC design.

Assumption 4. The unknown function $f(\cdot,\cdot)$ satisfies the following inequality

$$\left|f(X,\theta) - f(\bar{X},\theta)\right| \leqslant \left|X - \bar{X}\right| h\left(X,\bar{X}\right)\lambda(\theta),$$

$\forall X$, $\bar{X} \in \mathbf{R}^n$, $\theta \in \mathbf{R}^q$, where $h(\cdot,\cdot)$ is a known and nonnegative continuous function, $\lambda(\cdot)$ is an unknown and nonnegative continuous function.

To facilitate AILC algorithm design, we reconstruct the dynamics of tracking error e_i at the i-th iteration as

$$\begin{aligned} \dot{e}_i =& A_ce_i + B_c\left[\alpha^{\mathrm{T}}e_i - {y_r}^{(n)} + bu_i + \varphi^{\mathrm{T}}(X_i)\vartheta + \right. \\ & \left. f^{\mathrm{T}}(X_i(t-\tau(t)),\theta(t))\xi(X_i)\right] = A_ce_i + B_c\times \\ & \left[bu_i + \beta_1^{\mathrm{T}}\psi_i + \Xi^{\mathrm{T}}(t)\xi(X_i) + \Lambda_i\xi(X_i)\right] \end{aligned} \tag{27}$$

where $A_c = \begin{bmatrix} 0 & I_{(n-1)\times(n-1)} \\ \vdots & \\ -a_1 \ \cdots & -a_n \end{bmatrix}$, $\alpha = [a_1, \cdots, a_n]^{\mathrm{T}}$, $\beta_1 = \left[\beta_1^{(1)}, \beta_1^{(2)}\right]^{\mathrm{T}} = [1, \vartheta]^{\mathrm{T}}$, $\psi_i = \left[a^{\mathrm{T}}e_i - y_r^{(n)}, \varphi^{\mathrm{T}}(X_i)\right]^{\mathrm{T}}$, $\Xi(t) = f(X_r(t-\tau(t)),\theta(t))$, and $\Lambda_i = f^{\mathrm{T}}(X_i(t-\tau(t)),\theta(t)) - f^{\mathrm{T}}(X_r(t-\tau(t)),\theta(t))$.

A suitable vector α enables that $s^n + a_1s^{n-1} + a_2s^{n-2} + \cdots + a_{n-1}s^1 + a_n$ is a Hurwitz polynomial. Therefore, for a given constant $l > 0$, there exists a matrix $P = P^{\mathrm{T}} > 0$ such that

$$A_c^{\mathrm{T}}P + PA_c = -lI \tag{28}$$

holds.

Consider a Lyapunov-Krasovskii functional at the i-th iteration as follows

$$\begin{aligned} V_i =& \frac{1}{2}e_i^{\mathrm{T}}Pe_i + \frac{1}{2q_1}\tilde{\beta}_{1,i}^{\mathrm{T}}\tilde{\beta}_{1,i} + \\ & \frac{\varepsilon}{2(1-\eta)}\int_{t-\tau(t)}^{t} e_i^{\mathrm{T}}(\sigma)e_i(\sigma)h^2\left(X_r(\sigma), X_i(\sigma)\right)\mathrm{d}\sigma \end{aligned} \tag{29}$$

where $\varepsilon > 0$ and $q_1 > 0$ are the design parameters. $\tilde{\beta}_{1,i} = \beta_1 - \hat{\beta}_{1,i}$ denotes the estimation error of β_1.

Using (27) and taking the time derivative of (28), we obtain the following expression

$$\begin{aligned} \dot{V}_i =& -\frac{l}{2}e_i^{\mathrm{T}}e_i + e_i^{\mathrm{T}}PB_c\left[bu_i + \beta_1^{\mathrm{T}}\psi_i + \Xi^{\mathrm{T}}(t)\xi(X_i) + \right. \\ & \left. \Lambda_i\xi(X_i)\right] + \frac{1}{q_1}\dot{\tilde{\beta}}_{1,i}^{\mathrm{T}}\tilde{\beta}_{1,i} + \frac{\varepsilon h^2\left(X_r(\sigma), X_i(\sigma)\right)}{2(1-\eta)}e_i^{\mathrm{T}}e_i - \\ & \varepsilon\frac{1-\dot{\tau}(t)}{2(1-\eta)}e_i^{\mathrm{T}}(t-\tau)e_i(t-\tau)\times \\ & h^2\left(X_r(t-\tau), X_i(t-\tau)\right). \end{aligned} \tag{30}$$

From Young's inequality[24] and Assumption 4, we have

$$\begin{aligned} & \left|e_i^{\mathrm{T}}PB_c\Lambda_i\xi(X_i)\right| \leqslant (e_i^{\mathrm{T}}PB_c)^2\xi^{\mathrm{T}}(X_i)\xi(X_i)\frac{1}{2\varepsilon}\lambda^2(\theta(t)) + \\ & \varepsilon\frac{1}{2}e_i^{\mathrm{T}}(t-\tau)e_i(t-\tau)\times h^2(X_r(t-\tau), X_i(t-\tau)). \end{aligned} \tag{31}$$

Since $h(X_i, X_r)$ is a continuous function on the interval $[0,T]$. There exists a constant $H > 0$ so that $\left|h^2(X_i, X_r)\right| \leqslant H < \infty$ holds.

Substituting (30) into (29) and using Assumption 1, we get

$$\dot{V}_i \leqslant -\left[\frac{l}{2}-\frac{\varepsilon H}{2(1-\eta)}\right] e_i^{\mathrm{T}} e_i + e_i^{\mathrm{T}} P B_c \left[b u_i + \beta_1^{\mathrm{T}} \psi_i \ \beta_2^{\mathrm{T}} \phi_i\right] + \frac{1}{q_1}\dot{\tilde{\beta}}_{1,i}^{\mathrm{T}} \tilde{\beta}_{1,i} \tag{32}$$

where $\beta_2(t)=\left[\beta_2^{(1)}(t),\beta_2^{(2)}(t)\right]^{\mathrm{T}}=\left[\Xi^{\mathrm{T}}(t),\frac{1}{2\varepsilon}\lambda^2(\theta(t))\right]^{\mathrm{T}}$, $\phi_i=\left[\xi(X_i),e_i^{\mathrm{T}}PB_c\xi^{\mathrm{T}}(X_i)\xi(X_i)\right]^{\mathrm{T}}$.

The proposed learning control law at the i-th iteration is

$$u_i = N(\varsigma_i)\, G_i(t) \tag{33}$$

$$\dot{\varsigma}_i(t) = e_i^{\mathrm{T}} P B_c G_i(t), \varsigma_{-1}(t) = \text{const}, \varsigma_i(0) = \varsigma_{i-1}(T) \tag{34}$$

where $G_i = \hat{\beta}_{1,i}^{\mathrm{T}}\psi_i + \hat{\beta}_{2,i}^{\mathrm{T}}\phi_i$, and const represents a real number. $N(\varsigma_i)$ is a suitable Nussbaum-type function.

Thus, we can obtain the following expression

$$\begin{aligned}\dot{V}_i \leqslant& -\left[\frac{l}{2}-\frac{\varepsilon H}{2(1-\eta)}\right] e_i^{\mathrm{T}} e_i + e_i^{\mathrm{T}} P B_c \left[b N(\varsigma_i) G_i(t) + \right.\\ &\left.\beta_1^{\mathrm{T}}\psi_i + \beta_2^{\mathrm{T}}\phi_i\right] + \frac{1}{q_1}\dot{\tilde{\beta}}_{1,i}^{\mathrm{T}}\tilde{\beta}_{1,i} = -\left[\frac{l}{2}-\frac{\varepsilon H}{2(1-\eta)}\right]\times\\ &e_i^{\mathrm{T}} e_i + \left[b N(\varsigma_i)+1\right]\dot{\varsigma}_i + \frac{1}{q_1}\dot{\tilde{\beta}}_{1,i}^{\mathrm{T}}\tilde{\beta}_{1,i} - \\ &e_i^{\mathrm{T}} P B_c \left[G_i(t) - (\beta_1^{\mathrm{T}}\psi_i + \beta_2^{\mathrm{T}}\phi_i)\right].\end{aligned} \tag{35}$$

The proposed adaptive law for time-invariant parameter at the i-th iteration is

$$\begin{aligned}&\dot{\hat{\beta}}_{1,i} = q_1 {e_i}^{\mathrm{T}} P B_c \psi_i\\ &\hat{\beta}_{1,i}(0) = \hat{\beta}_{1,i-1}(T),\ \ \hat{\beta}_{1,-1}(0) = 0,\ \ \forall t\in[0,T].\end{aligned} \tag{36}$$

The proposed adaptive law for time-varying parameter at the i-th iteration is

$$\hat{\beta}_{2,i} = \hat{\beta}_{2,i-1} + q_2 {e_i}^{\mathrm{T}} P B_c \phi_i,\ \hat{\beta}_{2,-1}(t) = 0, \forall t\in[0,T] \tag{37}$$

where $q_2 > 0$ is the design parameter. There exist two design parameters l, ε such that $c=\frac{l}{2}-\frac{\varepsilon H}{2(1-\eta)}>0$ holds. Then, substituting c into (33), we have

$$\dot{V}_i \leqslant -c e_i^{\mathrm{T}} e_i + \left[b N(\varsigma_i) G_i(t) + 1\right]\dot{\varsigma}_i + {e_i}^{\mathrm{T}} P B_c \tilde{\beta}_{2,i}^{\mathrm{T}}\phi_i \tag{38}$$

where $\tilde{\beta}_{2,i} = \beta_2 - \hat{\beta}_{2,i}$ denotes the estimation error of β_2 at the i-th iteration.

Therefore, we can get the following result.

Theorem 2. Considering system (26) under the Assumptions 1 and 3–4, the control law (32) and the adaptive law (34)–(35) enable the convergence of the tracking error $e_i(t)$ and the boundedness of all signals $X_i, \hat{\beta}_{1,i}, \hat{\beta}_{2,i}$, and $u_i(t)$ in the closed-loop system.

Proof. Being similar to the proof of Theorem 1, this proof is omitted. □

4 Simulation examples

Example 1. Consider the first-order nonlinear system

$$\begin{cases}\dot{x} = 2x^2 f\left(x(t-\tau(t)),\theta(t)\right) + bu\\ x = \varpi(t), \quad t\in[-\tau_{\max},0].\end{cases} \tag{39}$$

In this simulation, we suppose $b = 1.5$. The unknown time-varying parameter is $\theta(t) = |\cos(2t)\sin(t)|$. $\tau(t) = 1 - 0.5\sin(t)$, $\tau_{\max} = 1$, $\dot{\tau}(t) \leqslant 0.5$ (Assumption 1 is satisfied). The time-varying nonlinearly parameterized uncertain term $f(x(t-\tau(t)),\theta(t)) = \mathrm{e}^{-\theta(t)x^2(t-\tau(t))}$ satisfies Assumption 2, i.e., $\left|\mathrm{e}^{-\theta(t)x^2(t-\tau(t))} - \mathrm{e}^{-\theta(t){x_r}^2(t-\tau(t))}\right| \leqslant |x(t-\tau(t)) - x_r(t-\tau(t))|\sqrt{2|\theta|}\mathrm{e}^{-0.5}$. Moreover, the desired trajectory is $x_r(t) = \sin(t)$, $t\in[0,10]$ (Assumption 3 is satisfied).

Let $\varsigma_{-1}(t) = -0.135$, $k = 0.015$, $q = 0.0081$, and $\eta = 0.5$. Applying the learning control law (6) and adaptive law (8) into the system (37), Fig. 1 shows that a perfect tracking performance will be achieved as the iteration number increases. Fig. 2 verifies the boudedness of u_i when the iteration number is 150, and implies the boundedness of u_i at each iteration. Fig. 3 depicts that the estimation of unknown time-varying parameter is bounded in iteration domain. Fig. 4 implies that two curves indicate the boundedness of $N(\varsigma_i(\cdot))$ and $\varsigma_i(\cdot)$ at each iteration. Finally, Fig. 5 shows the evolution of the Nussbaum-type gain function in time domain when the iteration number is 150.

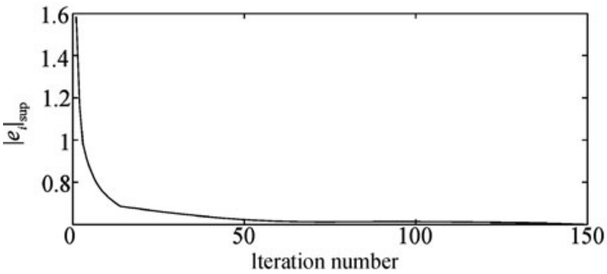

Fig. 1 Learning convergence of AILC

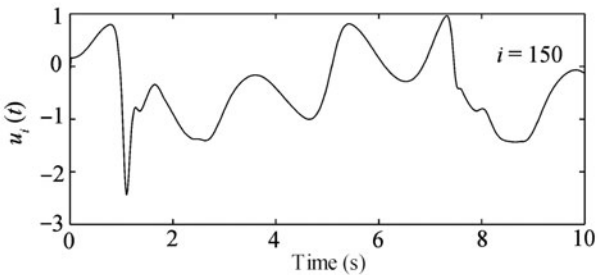

Fig. 2 The control input

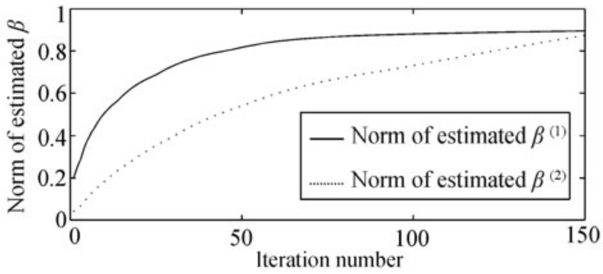

Fig. 3 The norm of $\hat{\beta}(t)$

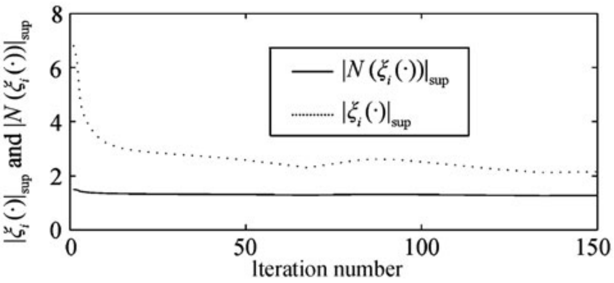

Fig. 4 Evolution of the Nussbaum-type function

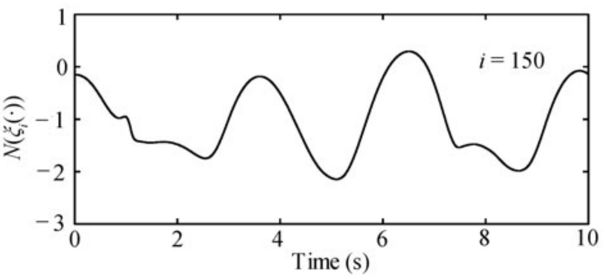

Fig. 5 The Nussbaum-type function

Example 2. Consider the higher-order nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = bu + \vartheta \dfrac{x_1 x_2}{2 + \cos(x_1 x_2)} + \\ \qquad \sin(x_1) x_2 f\left(X(t - \tau(t)), \theta(t)\right) \\ x_1(t) = 0, \ t \in [-\tau_{\max}, 0] \\ x_2(t) = 1, \ t \in [-\tau_{\max}, 0]. \end{cases} \tag{40}$$

In the simulation, suppose $b = 4$, $\vartheta = -1.4$, $\theta(t) = |\cos(2t)|$, $\tau(t) = 1 - 0.5\sin(t)$, $\tau_{\max} = 1$, $\dot{\tau}(t) \leqslant 0.5$, where Assumption 1 is satisfied. And $f(X, \theta) = \mathrm{e}^{-\theta(t)[{x_1}^2(t-\tau(t)) - {x_2}^2(t-\tau(t))]}$ satisfies Assumption 4. The desired trajectories are $X_r(t) = [y_r(t), \dot{y}_r(t)]^{\mathrm{T}} = [\sin(t), \cos(t)]^{\mathrm{T}}$, $t \in [0, 10]$ (Assumption 3 is satisfied).

Let $q_1 = 4.5$, $q_2 = 0.045$, and $\varsigma_{-1}(t) = -0.012$. When $a = [1, 2]^{\mathrm{T}}$ and $l = 12$, we can obtain $P = \begin{bmatrix} 18 & 6 \\ 6 & 6 \end{bmatrix}$ by the help of (28). Applying the learning control law (32) and adaptive law (34)–(35) into the system (38), from Fig. 6 we can infer the convergence of tracking error in iteration domain. Fig. 7 gives evolution of the control input signal u_i when the iteration number equals 50, and implies the boundedness of u_i in iteration domain. Fig. 8 depicts that the estimation of the unknown time-invariant and time-varying parameters is bounded at each iteration. Fig. 9 gives that two curves denote the boundedness of $N(\varsigma_i(\cdot))$ and $\varsigma_i(\cdot)$ in iteration domain. Finally, Fig. 10 shows the evolution of the Nussbaum-type gain function when the iteration number is 50.

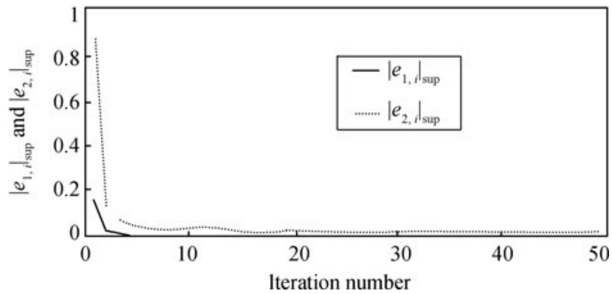

Fig. 6 Learning convergence of AILC

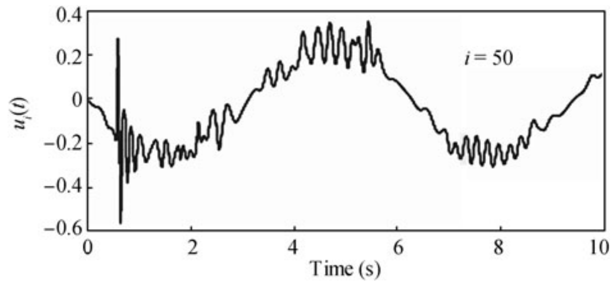

Fig. 7 The control input

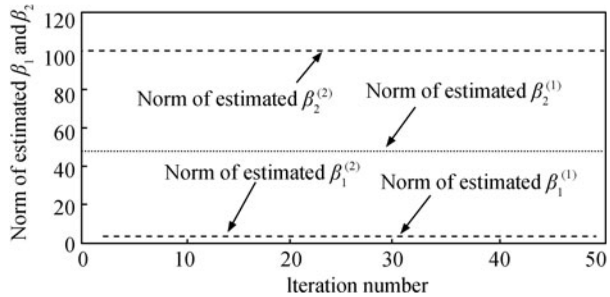

Fig. 8 The norm of $\hat{\beta}_1$ and $\hat{\beta}_2(t)$

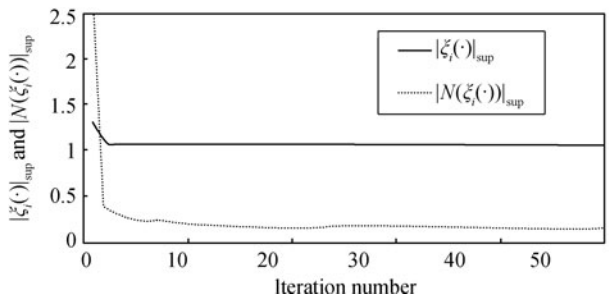

Fig. 9 Evolution of the Nussbaum-type function

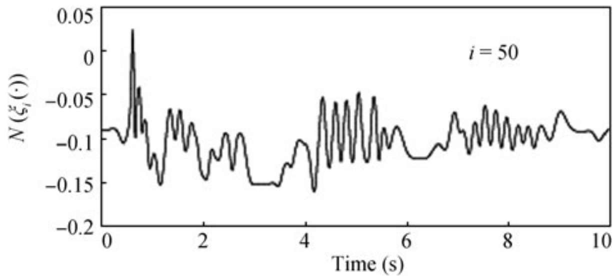

Fig. 10 The Nussbaum-type function

5 Conclusions

In this paper, we have extended AILC algorithms to a class of nonlinearly parameterized systems with unknown time-varying delay and unknown control direction. Based on Nussbaum-type function, a new AILC scheme is proposed by incorporating the parameter separation principle and reparametrization method. Then, the convergence of closed-loop system is proved by constructing a Lyapunov-Krasovskii-like composite energy function. The feasibility of control method proposed is demonstrated through two simulation examples. Moreover, the robustness to noise for Nussbaum gain is also an important problem, therefore the robustness of the Nussbaum gain to the sensor noise will be investigated.

References

[1] S. Arimoto, S. Kawamura, F. Miyazaki. Bettering operation of robots by learning. *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] K. L. Moore. *Iterative Learning Control for Deterministic Systems*, London, UK: Springer-Verlag, pp. 152, 1993.

[3] J. X. Xu, Z. H. Qu. Robust iterative learning control for a class of nonlinear systems. *Automatica*, vol. 34, no. 8, pp. 983–988, 1998.

[4] H. S. Ahn, Y. Q. Chen, K. L. Moore. Iterative learning control: Brief survey and categorization. *IEEE Transactions on Systems, Man, and Cybernetics – Part C, Applications and Reviews*, vol. 37, no. 6, pp. 1099–1121, 2007.

[5] W. S. Chen, L. Zhang. Adaptive iterative learning control for nonlinearly parameterized systems with unknown time-varying delays. *International Journal of Control, Automation and Systems*, vol. 8, no. 2, pp. 177–186, 2010.

[6] W. S. Chen, R. H. Li, J. Li. Observer-based adaptive iterative learning control for nonlinear systems with time-varying delays. *International Journal of Automation and Computing*, vol. 7, no. 4, pp. 438–446, 2010.

[7] J. Y. Choi, J. S. Lee. Adaptive iterative learning control of uncertain robotic systems. *IEE Proceedings – Control Theory and Applications*, vol. 147, no. 2, pp. 217–223, 2000.

[8] Y. Q. Wang, F. R. Gao, F. J. Doyle III. Survey on iterative learning control, repetitive control, and run-to-run control. *Journal of Process Control*, vol. 19, no. 10, pp. 1589–1600, 2009.

[9] P. Jiang, H. D. Chen, L. C. A. Bamforth. A universal iterative learning stabilizer for a class of MIMO systems. *Automatica*, vol. 42, no. 6, pp. 973–981, 2006.

[10] C. L. Zhang, J. M. Li. Adaptive iterative learning control for nonlinear time-delay systems with periodic disturbances using FSE-neural network. *International Journal of Automation and Computing*, vol. 8 no. 4, pp. 403–410, 2011.

[11] M. French, E. Rogers. Non-linear iterative learning by an adaptive Lyapunov technique. *International Journal of Control*, vol. 73, no. 10, pp. 840–850, 2000.

[12] D. H. Owens. Universal iterative learning control using adaptive high-gain feedback. *International Journal of Adaptive Control and Signal Processing*, vol. 7, no. 5, pp. 383–388, 1993.

[13] J. X. Xu, B. Viswanathan. Adaptive robust iterative learning control with dead zone scheme. *Automatica*, vol. 36, no. 1, pp. 91–99, 2000.

[14] C. J. Chien. A combined adaptive law for fuzzy iterative learning control of nonlinear systems with varying control tasks. *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 1, pp. 40–51, 2008.

[15] R. H. Chi, Hou Z S, J. X. Xu. Adaptive ILC for a class of discrete-time systems with iteration-varying trajectory and random initial condition. *Automatica*, vol. 44, no. 8, pp. 2207–2213, 2008.

[16] A. Tayebi, C. J. Chien. A unified adaptive iterative learning control framework for uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, vol. 52, no. 10, pp. 1907–1913, 2007.

[17] J. X. Xu, R. Yan. Adaptive learning control for finite interval tracking based on constructive function approximation and wavelet. *IEEE Transactions on Neural Networks*, vol. 22, no. 6, pp. 893–905, 2011.

[18] J. M. Li, Y. P. Sun, Y. Liu. Hybrid adaptive iterative learning control of non-uniform trajectory tracking. *Control Theory and Applications*, vol. 25, no. 1, pp. 100–104, 2008. (in Chinese)

[19] J. M. Li, Y. L. Wang, X. M. Li. Adaptive iterative learning control for nonlinearly parameterized-systems with unknown time-varying delays. *Control Theory and Application*, vol. 28, no. 6, pp. 861–868, 2011. (in Chinese)

[20] K. H. Park, Z. Bien, D. H. Hwang. Design of an iterative learning controller for a class of linear dynamic systems with time delay. *IEE Proceedings – Control Theory and Applications*, vol. 145, no. 6, pp. 507–512, 1998.

[21] Q. P. Hu, J. X. Xu, T. H. Lee. Iterative learning control design for Smith predictor. *Systems and Control Letters*, vol. 44, no. 3, pp. 201–210, 2001.

[22] Y. Q. Chen, Z. M. Gong, C. Y. Wen. Analysis of a high-order iterative learning control algorithm for uncertain nonlinear systems with state delays. *Automatica*, vol. 34, no. 3, pp. 345–353, 1998.

[23] M. X. Sun, D. W. Wang. Iterative learning control design for uncertain dynamic systems with delayed states. *Dynamics and Control*, vol. 10, no. 4 , pp. 341–357, 2001.

[24] M. X. Sun, D. W. Wang. Initial condition issues on iterative learning control for non-linear systems with time delay. *International Journal of Systems Science*, vol. 32, no. 11, pp. 1365–1375, 2001.

[25] R. D. Nussbaum. Some remarks on a conjecture in parameter adaptive control. *Systems & Control Letters*, vol. 3, no. 5, pp. 243–246, 1983.

[26] H. D. Chen, P. Jiang. Adaptive iterative feedback control for nonlinear system with unknown high-frequency gain. In *Proceedings of the 4th World Congress on Intelligent Control and Automation*, IEEE, Shanghai, China, vol. 2, pp. 847–851, 2002.

[27] J. X. Xu, R. Yan. Iterative learning control design without a prior knowledge of the control direction. *Automatica*, vol. 40, no. 10, pp. 1803–1809, 2004.

[28] X. D. Ye, J. P. Jiang. Adaptive nonlinear design without a priori knowledge of control directions. *IEEE Transactions on Automatic Control*, vol. 43, no. 11, pp. 1617–1621, 1998.

[29] C. C. Hua, Q. G. Wang, X. P. Guan. Robust adaptive controller design for nonlinear time-delay systems via T-S fuzzy approach. *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 4, pp. 901–910, 2009.

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