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Robust H_∞ Controller Design for Uncertain Neutral Systems via Dynamic Observer Based Output Feedback

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Abstract: In this paper, the dynamic observer-based controller design for a class of neutral systems with H_∞ control is considered. An observer-based output feedback is derived for systems with polytopic parameter uncertainties. This controller assures delay-dependent stabilization and H_∞ norm bound attenuation from the disturbance input to the controlled output. Numerical examples are provided for illustration and comparison of the proposed conditions.

Keywords: Neutral systems, delay-dependent conditions, H_∞ control design, observer, linear matrix inequality (LMI), time varying-delay systems, polytopic uncertainties.

1 Introduction

There are many practical systems containing time delays of neutral type, for example, population growth, distributed networks, and car chasing. During the last decades, stability and stabilization of neutral time-delay systems have attracted great attention[1−5] and references therein. The H_∞ control problem and a state observer reconstructing the states of a dynamic system have important applications in many aspects such as realization of feedback control, system supervision, and fault diagnosis. The states of a system are not always measurable in many physical control systems. Observer-based control is probably better suited than state feedback. These motivate us to consider observer-based control for neutral systems. Observer-based dynamic output feedback of dynamical time delay systems has received great interest in recent years (for example [6]). Depending on whether the stabilization criterion itself contains the sizes of delays, stabilization criteria of time-delay systems can be classified into two categories, namely delay-independent criteria[7−9] and delay-dependent criteria[10−13]. It is well-known that delay-independent criteria tends to be conservative, especially when the size of a delay is small. The main approaches rely on the use of either a Lyapunov-Krasovskii functional or a Lyapunov-Razumikhin function. Most of the criteria are expressed in terms of linear matrix inequalities (LMIs) and then easily solved using dedicated solvers[14, 15]. In [16], a descriptor model transformation and a corresponding Lyapunov-Krasovskii functional have been introduced for stability analysis of systems with time-varying delay. Zhang et al.[17] proposed another method called the integral inequality method that can be used to study the delay-dependent stabilization issue of the open problem for time-varying delays.

In [18], free matrices are introduced in the derivation of delay-dependent stabilization criteria for a system with a state delay. This idea is easily extended for delay-dependent robust observer-based H_∞ controller design for neutral systems with polytopic-type uncertainties that is based on the descriptor model transformation by taking few of the additional free matrix parameters to be zero, that provide more degrees of freedom. The latter method is obtained as a general case and lead to less conservative results than those obtained by [16, 17, 19].

The main aim of this paper is to develop a delay-dependent H_∞ control design method for neutral systems with time varying-delays and polytopic-uncertainties, via an observer-based output feedback. We design a robust H_∞ controller that stabilizes the system and reduces the effect of the disturbance input on the controlled output for all admissible uncertainties. Finally, some numerical examples are given to illustrate that the results are less conservative than previous works.

The following notations will be used throughout the paper. $\mathbf{R}^n$ denotes the n dimensional Euclidean space and $\mathbf{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. The notation $X \geqslant Y$ (respectively $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite).

2 Problem formulation and definitions

Consider the neutral systems with time-delay in state and control vectors described by the following state equation:

$$\begin{cases} \dot{x}(t) - F\dot{x}(t-\tau(t)) = A_0x(t) + A_1x(t-h(t)) + \\ \qquad\qquad B_0u(t) + B_1w(t) \\ z(t) = C_0x(t) + C_1u(t) + B_2w(t) \\ y(t) = C_2x(t) + C_3u(t) \\ x(\theta) = \phi(\theta), \quad \theta\in[-\bar{h}, 0], \quad t\geqslant 0 \end{cases} \tag{1}$$

where $x\in\mathbf{R}^n$, $u\in\mathbf{R}^m$, $w\in\mathbf{R}^l$, $y\in\mathbf{R}^p$, $z\in\mathbf{R}^d$ are respectively the state, the control input, the disturbance input (which is square integrable), the measured output, and controlled output. F, A_0, A_1, B_0, B_1, and C_i, $i = 0, 1, 2$ are known real constant matrices. The delays τ and h are assumed to be some unknown functions of time and are continuously differentiable, with their respective rates of change bounded as follows:

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$$0\leqslant h(t)\leqslant h_m,\ \ 0\leqslant\tau(t)<\infty,\ \ \dot h(t)\leqslant d_1<1,\ \ \dot\tau(t)\leqslant d_2<1 \tag{2}$$

where h_m, d_1, and d_2 are given positive constants. $\bar h=\max\{h(t),\tau(t)\}$, $\forall t\geqslant 0$ and $\phi(.)$ is a differentiable vector valued initial function. Note that in [13], the derivative of the delays is assumed to be positive, which does not happen in practice.

Let the difference operator $\Delta: C^1[-\bar h,0]\to\mathbf{R}^n$, given by $\Delta(x_t)=x(t)-Fx(t-\tau(t))$. Moreover, it is assumed that all the eigenvalues of matrix F are inside the unit circle. The latter guarantees that the difference operator $\Delta(x_t)=0$ is asymptotically stable for all τ.

Our aim in this paper, is to design the following observer-based dynamic output feedback control law for the system (1):

$$\begin{cases} u(t)=K\eta(t)\\ \dot\eta(t)-F\dot\eta(t-\tau(t))=A_0\eta(t)+A_1\eta(t-h(t))+\\ \qquad B_0u(t)+L(y(t)-C_2\eta(t)-C_3u(t))\end{cases} \tag{3}$$

where $\eta\in\mathbf{R}^n$, is the observer state vector and $K\in\mathbf{R}^{m\times n}$ is the controller gain matrix.

By introducing the observer error

$$e(t)=x(t)-\eta(t) \tag{4}$$

we get the following augmented system

$$\begin{cases} \dot\xi(t)-\widetilde F\dot\xi(t-\tau(t))=\widetilde A_0\xi(t)+\widetilde A_1\xi(t-h(t))+\\ \qquad \widetilde B_0u(t)+\widetilde B_1w(t)\\ z(t)=\widetilde C_0\xi(t)+C_1u(t)+B_2w(t)\end{cases} \tag{5}$$

where

$$\xi(t)=\begin{pmatrix} x^{\mathrm T}(t) & -e^{\mathrm T}(t)\end{pmatrix}^{\mathrm T} \tag{6}$$

and the corresponding augmented matrices:

$$\widetilde A_0=\begin{pmatrix} A_0 & 0\\ 0 & A_0-LC_2\end{pmatrix},\quad \widetilde A_1=\begin{pmatrix} A_1 & 0\\ 0 & A_1\end{pmatrix},$$
$$\widetilde F=\begin{pmatrix} F & 0\\ 0 & F\end{pmatrix}$$

$$\widetilde B_0=\begin{pmatrix} B_0\\ -LC_3\end{pmatrix},\quad \widetilde B_1=\begin{pmatrix} B_1\\ -B_1\end{pmatrix},$$
$$\widetilde C_0=\begin{pmatrix} C_0 & 0\end{pmatrix},\quad \widetilde K=\begin{pmatrix} K & K\end{pmatrix} \tag{7}$$

with

$$u(t)=\widetilde K\xi(t) \tag{8}$$

Definition 1.[13] The neutral system (1)–(2) (with $w(t)=0$) is said to be asymptotically stabilizable via dynamic output feedback control law (3), if there exist positive definite matrices P, Q, R, W, and a positive constant π, such that the derivative of the Lyapunov functional

$$V(\xi,t)=\xi^{\mathrm T}(t)P_1\xi(t)+\int_{t-h(t)}^{t}\xi^{\mathrm T}(s)Q\xi(s)\mathrm ds+$$
$$\int_{-h_m}^{0}\int_{t+\theta}^{t}\dot\xi^{\mathrm T}(s)R\dot\xi(s)\mathrm ds\mathrm d\theta+\int_{t-\tau(t)}^{t}\dot\xi^{\mathrm T}(s)W\dot\xi(s)\mathrm ds \tag{9}$$

with respect to time t satisfies

$$\dot V(\xi,t)\leqslant-\pi\|\xi\|^2 \tag{10}$$

for all pairs $(\xi(t),t)\in\mathbf{R}^{2n}\times\mathbf{R}$. Then, the control law (3) is called a dynamic output feedback stabilizing controller and the closed loop system (5) (with $w=0$) is said to be asymptotically stable.

Definition 2.[13] For a given $\gamma>0$, the neutral system (1) and (2) is said to be asymptotically stabilizable via dynamic output feedback control law (3) with H_∞ norm bound γ if the following conditions hold:

1) The resulting closed-loop system (5) with $w=0$ is asymptotically stable.

2) Subject to the zero initial condition, the controlled output z satisfies

$$\|z\|_2\leqslant\gamma\|w\|_2 \tag{11}$$

for square integrable disturbance input w.

3 Main results

This section presents delay-dependent stabilization conditions. The closed-loop system constructed by (5) and (8) is given by

$$\begin{cases} \dot\xi(t)-\widetilde F\dot\xi(t-\tau(t))=\widetilde A_{0c}\xi(t)+\widetilde A_1\xi(t-h(t))+\widetilde B_1w(t)\\ z(t)=\widetilde C_{0c}\xi(t)+B_2w(t)\end{cases} \tag{12}$$

with

$$\begin{cases} \widetilde A_{0c}=\widetilde A_0+\widetilde B_0\widetilde K\\ \widetilde C_{0c}=\widetilde C_0+C_1\widetilde K.\end{cases} \tag{13}$$

The following Lemma is obtained for system (12).

Lemma 1. For a given $\gamma>0$, the system (12) is asymptotically stable with H_∞ norm bound γ if there exist positive definite symmetric matrices $P_1=P_1^{\mathrm T}\in\mathbf{R}^{2n\times 2n}, Q=Q^{\mathrm T}\in\mathbf{R}^{2n\times 2n}, R=R^{\mathrm T}\in\mathbf{R}^{2n\times 2n}$ and $W=W^{\mathrm T}\in\mathbf{R}^{2n\times 2n}$, and matrices $P_i\in\mathbf{R}^{2n\times 2n}, i=2,\cdots,6$ such that the LMI (14) holds.

$$\Gamma=\begin{pmatrix} \Gamma_{11} & \Gamma_{21}^{\mathrm T} & \Gamma_{31}^{\mathrm T} & -P_4^{\mathrm T} & P_2^{\mathrm T}\widetilde F & P_2^{\mathrm T}\widetilde B_1+\widetilde C_{0c}^{\mathrm T}B_2\\ \Gamma_{21} & \Gamma_{22} & \Gamma_{32}^{\mathrm T} & -P_5^{\mathrm T} & P_3^{\mathrm T}\widetilde F & P_3^{\mathrm T}\widetilde B_1\\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & -P_6^{\mathrm T} & 0 & 0\\ -P_4 & -P_5 & -P_6 & -\dfrac{1}{h_m}R & 0 & 0\\ \widetilde F^{\mathrm T}P_2 & \widetilde F^{\mathrm T}P_3 & 0 & 0 & -(1-d_2)W & 0\\ \widetilde B_1^{\mathrm T}P_2+B_2^{\mathrm T}\widetilde C_{0c} & \widetilde B_1^{\mathrm T}P_3 & 0 & 0 & 0 & -\gamma^2I+B_2^{\mathrm T}B_2\end{pmatrix}<0 \tag{14}$$

where

$$\begin{cases} \Gamma_{11} = P_2^{\mathrm{T}}\widetilde{A}_{0c} + \widetilde{A}_{0c}^{\mathrm{T}}P_2 + P_4 + P_4^{\mathrm{T}} + \widetilde{C}_{0c}^{\mathrm{T}}\widetilde{C}_{0c} + Q \\ \Gamma_{21} = P_1 + P_3^{\mathrm{T}}\widetilde{A}_{0c} + P_5^{\mathrm{T}} - P_2 \\ \Gamma_{22} = h_m R + W - P_3 - P_3^{\mathrm{T}} \\ \Gamma_{31} = \widetilde{A}_1^{\mathrm{T}}P_2 - P_4 + P_6^{\mathrm{T}} \\ \Gamma_{32} = \widetilde{A}_1^{\mathrm{T}}P_3 - P_5 \\ \Gamma_{33} = -P_6 - P_6^{\mathrm{T}} - (1-d_1)Q. \end{cases} \tag{15}$$

Proof. To prove our Lemma 1, first we calculate the time derivative of $V(\xi,t)$ along the trajectory of (12)

$$\dot{V}(\xi,t) = 2\xi^{\mathrm{T}}(t)P_1\dot{\xi}(t) + \xi^{\mathrm{T}}(t)Q\xi(t) - (1-\dot{h}(t))\xi^{\mathrm{T}}(t-h(t))\cdot Q\xi(t-h(t)) + h_m\dot{\xi}^{\mathrm{T}}(t)R\dot{\xi}(t) - \int_{t-h_m}^{t}\dot{\xi}^{\mathrm{T}}(s)R\dot{\xi}(s)\mathrm{d}s + \dot{\xi}^{\mathrm{T}}(t)W\dot{\xi}(t) - (1-\dot{\tau}(t))\dot{\xi}^{\mathrm{T}}(t-\tau(t))W\dot{\xi}(t-\tau(t)). \tag{16}$$

From (2), it is clear that the following is true:

$$-\int_{t-h_m}^{t}\dot{\xi}^{\mathrm{T}}(s)R\dot{\xi}(s)\mathrm{d}s \leqslant -\int_{t-h(t)}^{t}\dot{\xi}^{\mathrm{T}}(s)R\dot{\xi}(s)\mathrm{d}s. \tag{17}$$

Let $P = \begin{pmatrix} P_1 & 0 & 0 \\ P_2 & P_3 & 0 \\ P_4 & P_5 & P_6 \end{pmatrix}$, with P_1 is a symmetric and positive-definite matrix and $P_2 = \mathrm{diag}(P_{20}, P_{20})$. From the Leibniz-Newton formula,

$$0 = \xi(t) - \xi(t-h(t)) - \int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s \tag{18}$$

and using (12), we can write the first term of (16) as follows:

$$2\xi^{\mathrm{T}}(t)P_1\dot{\xi}(t) = 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} \dot{\xi}(t) \\ 0 \\ 0 \end{pmatrix} = 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\cdot \begin{pmatrix} \dot{\xi}(t) \\ -\dot{\xi}(t)+\widetilde{F}\dot{\xi}(t-\tau(t))+\widetilde{A}_{0c}\xi(t)+\widetilde{A}_1\xi(t-h(t))+\widetilde{B}_1 w(t) \\ \xi(t)-\xi(t-h(t))-\int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s \end{pmatrix} \tag{19}$$

where $\widetilde{\xi}(t) = \begin{pmatrix} \xi^{\mathrm{T}}(t) & \dot{\xi}^{\mathrm{T}}(t) & \xi^{\mathrm{T}}(t-h(t)) \end{pmatrix}^{\mathrm{T}}$.

Substituting (19) into (16), we obtain

$$\dot{V}(\xi,t) = \widetilde{\xi}^{\mathrm{T}}(t)\Xi\widetilde{\xi}(t) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ 0 \\ -I \end{pmatrix}\int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s - (1-\dot{h}(t))\xi^{\mathrm{T}}(t-h(t))Q\xi(t-h(t)) - (1-\dot{\tau}(t))\dot{\xi}^{\mathrm{T}}(t-\tau(t))\cdot W\dot{\xi}(t-\tau(t)) + \xi^{\mathrm{T}}(t)Q\xi(t) + h_m\dot{\xi}^{\mathrm{T}}(t)R\dot{\xi}(t) + \dot{\xi}^{\mathrm{T}}(t)W\dot{\xi}(t) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ \widetilde{F} \\ 0 \end{pmatrix}\dot{\xi}(t-\tau(t)) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ \widetilde{B}_1 \\ 0 \end{pmatrix}w(t) - \int_{t-h_m}^{t}\dot{\xi}^{\mathrm{T}}(s)R\dot{\xi}(s)\mathrm{d}s \tag{20}$$

where

$$\Xi = P^{\mathrm{T}}\begin{pmatrix} 0 & I & 0 \\ \widetilde{A}_{0c} & -I & \widetilde{A}_1 \\ I & 0 & -I \end{pmatrix} + \begin{pmatrix} 0 & \widetilde{A}_{0c}^{\mathrm{T}} & I \\ I & -I & 0 \\ 0 & \widetilde{A}_1^{\mathrm{T}} & -I \end{pmatrix}P.$$

Using the Jensen's inequality[20], the last term can be bounded as follows:

$$-\int_{t-h(t)}^{t}\dot{\xi}^{\mathrm{T}}(s)R\dot{\xi}(s)\mathrm{d}s \leqslant -\int_{t-h(t)}^{t}\dot{\xi}^{\mathrm{T}}(s)\mathrm{d}s\frac{R}{h_m}\int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s. \tag{21}$$

We apply the inequality (21) to the term of (20). Therefore we get

$$\dot{V}(\xi,t) \leqslant \widetilde{\xi}^{\mathrm{T}}(t)\Xi\widetilde{\xi}(t) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ 0 \\ -I \end{pmatrix}\int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s - (1-d_1)\xi^{\mathrm{T}}(t-h(t))Q\xi(t-h(t)) - (1-d_2)\dot{\xi}^{\mathrm{T}}(t-\tau(t))\cdot W\dot{\xi}(t-\tau(t)) + \xi^{\mathrm{T}}(t)Q\xi(t) + h_m\dot{\xi}^{\mathrm{T}}(t)R\dot{\xi}(t) + \dot{\xi}^{\mathrm{T}}(t)W\dot{\xi}(t) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ \widetilde{F} \\ 0 \end{pmatrix}\dot{\xi}(t-\tau(t)) + 2\widetilde{\xi}^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{pmatrix} 0 \\ \widetilde{B}_1 \\ 0 \end{pmatrix}w(t) - \frac{1}{h_m}\int_{t-h(t)}^{t}\dot{\xi}^{\mathrm{T}}(s)\mathrm{d}sR\int_{t-h(t)}^{t}\dot{\xi}(s)\mathrm{d}s. \tag{22}$$

Now, we consider the asymptotic stability of system (12) with $w(t) = 0$, then (22) becomes

$$\dot{V}(\xi,t) \leqslant \widetilde{\Omega}^{\mathrm{T}}(t,s)\Upsilon\widetilde{\Omega}(t,s) \tag{23}$$

where

$$\Upsilon = \begin{pmatrix} \Upsilon_{11} & \Upsilon_{21}^{\mathrm{T}} & \Upsilon_{31}^{\mathrm{T}} & -P_4^{\mathrm{T}} & P_2^{\mathrm{T}}\widetilde{F} \\ \Upsilon_{21} & \Upsilon_{22} & \Upsilon_{32}^{\mathrm{T}} & -P_5^{\mathrm{T}} & P_3^{\mathrm{T}}\widetilde{F} \\ \Upsilon_{31} & \Upsilon_{32} & \Upsilon_{33} & -P_6^{\mathrm{T}} & 0 \\ -P_4 & -P_5 & -P_6 & -\dfrac{1}{h_m}R & 0 \\ \widetilde{F}^{\mathrm{T}}P_2 & \widetilde{F}^{\mathrm{T}}P_3 & 0 & 0 & -(1-d_2)W \end{pmatrix} \tag{24}$$

$$\begin{cases} \Upsilon_{11} = P_2^{\mathrm{T}}\widetilde{A}_{0c} + \widetilde{A}_{0c}^{\mathrm{T}}P_2 + P_4 + P_4^{\mathrm{T}} + Q \\ \Upsilon_{21} = P_1 + P_3^{\mathrm{T}}\widetilde{A}_{0c} + P_5^{\mathrm{T}} - P_2 \\ \Upsilon_{22} = h_m R + W - P_3 - P_3^{\mathrm{T}} \\ \Upsilon_{31} = \widetilde{A}_1^{\mathrm{T}}P_2 - P_4 + P_6^{\mathrm{T}} \\ \Upsilon_{32} = \widetilde{A}_1^{\mathrm{T}}P_3 - P_5 \\ \Upsilon_{33} = -P_6 - P_6^{\mathrm{T}} - (1-d_1)Q \end{cases}$$

and

$$\widetilde{\Omega}(t,s) = \begin{pmatrix} \widetilde{\xi}^{\mathrm{T}}(t) & \int_{t-h(t)}^{t}\dot{\xi}^{\mathrm{T}}(s)\mathrm{d}s & \dot{\xi}^{\mathrm{T}}(t-\tau(t)) \end{pmatrix}^{\mathrm{T}}.$$

It is clear that if Γ is negative definite, then Υ is negative definite, so the conditions 1) of Definition 2 is verified.

To establish the upper bound $\gamma\|w\|_2$ for the $L_2[0,\infty)$-norm of z, assume that $x(t) = \phi(t) = 0$ for $t \in [-\bar{h}, 0]$ (the assumption of the zero initial condition). Let

$$J = \int_0^{\infty}\Big(z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)\Big)\mathrm{d}t \tag{25}$$

then, we have

$$J = \int_0^{\infty}\Big(z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(\xi,t)\Big)\mathrm{d}t - \Pi \tag{26}$$

where

$$\Pi=\lim_{t\to\infty}\left\{\int_{t-h(t)}^{t}\xi^{\mathrm T}(s)Q\xi(s)\mathrm ds+\int_{-h_m}^{0}\int_{t+\theta}^{t}\dot\xi^{\mathrm T}(s)R\dot\xi(s)\mathrm ds\mathrm d\theta+\int_{t-\tau(t)}^{t}\dot\xi^{\mathrm T}(s)W\dot\xi(s)\mathrm ds\right\}+\xi^{\mathrm T}(\infty)P_1\xi(\infty).$$

It follows from (22) and (26) that

$$J=\int_0^\infty\left\{\widetilde\xi^{\mathrm T}(t)\Xi\widetilde\xi(t)+2\widetilde\xi^{\mathrm T}(t)P^{\mathrm T}\begin{pmatrix}0\\0\\-I\end{pmatrix}\int_{t-h(t)}^{t}\dot\xi(s)\mathrm ds-(1-d_1)\xi^{\mathrm T}(t-h(t))Q\xi(t-h(t))-(1-d_2)\dot\xi^{\mathrm T}(t-\tau(t))\cdot W\dot\xi(t-\tau(t))+\xi^{\mathrm T}(t)Q\xi(t)+h_m\dot\xi^{\mathrm T}(t)R\dot\xi(t)-\gamma^2w^{\mathrm T}(t)w(t)+z^{\mathrm T}(t)z(t)+\dot\xi^{\mathrm T}(t)W\dot\xi(t)+2\widetilde\xi^{\mathrm T}(t)P^{\mathrm T}\begin{pmatrix}0\\\widetilde F\\0\end{pmatrix}\dot\xi(t-\tau(t))+2\widetilde\xi^{\mathrm T}(t)P^{\mathrm T}\begin{pmatrix}0\\\widetilde B_1\\0\end{pmatrix}w(t)-\frac{1}{h_m}\int_{t-h(t)}^{t}\dot\xi^{\mathrm T}(s)\mathrm ds\cdot R\int_{t-h(t)}^{t}\dot\xi(s)\mathrm ds\right\}\mathrm dt-\Pi. \tag{27}$$

We have

$$J\leqslant\int_0^t\Omega^{\mathrm T}(t,s)\Gamma\Omega(t,s)\mathrm dt-\Pi \tag{28}$$

with

$$\Omega(t,s)=\begin{pmatrix}\widetilde\xi^{\mathrm T}(t) & \int_{t-h(t)}^{t}\dot\xi^{\mathrm T}(s)\mathrm ds & \dot\xi^{\mathrm T}(t-\tau(t)) & w^{\mathrm T}(t)\end{pmatrix}^{\mathrm T}.$$

Since (14) holds, it follows that $J<0$, i.e., $\|z\|_2\leqslant\gamma\|w\|_2$ for any non-zeros $w\in L_2[0,\infty)$, so the conditions 2) of Definition 2 is verified. □

Theorem 1. For some given scalars $\gamma>0$, system (1)–(2) is asymptotically stabilizable via dynamic output feedback control law (3) and satisfies (11) for all non-zeros $w\in L_2[0,\infty)$, any $h(t)$ and $\tau(t)$ satisfying (2), if there exist positive definite symmetric matrices $X_1=X_1^{\mathrm T}\in\mathbf R^{2n\times2n}$, $\overline Q=\overline Q^{\mathrm T}\in\mathbf R^{2n\times2n}$, $\overline R=\overline R^{\mathrm T}\in\mathbf R^{2n\times2n}$, and $\overline W=\overline W^{\mathrm T}\in\mathbf R^{2n\times2n}$, some matrices $X_i\in\mathbf R^{2n\times2n}$, $i=3,4,5$, $X_{20}\in\mathbf R^{n\times n}$, $U\in\mathbf R^{m\times n}$, and a scalar α such that the condition (29) is verified[1].

$$\Psi=\begin{pmatrix}\Psi_{11} & * & * & * & * & * & *\\ \Psi_{21} & \Psi_{22} & * & * & * & * & *\\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * & * & * & *\\ -X_3 & -X_4 & -X_5 & -\frac{1}{h_m}\overline R & * & * & *\\ X_2^{\mathrm T}\widetilde F^{\mathrm T} & \alpha X_2^{\mathrm T}\widetilde F^{\mathrm T} & 0 & 0 & -(1-d_2)\overline W & * & *\\ \widetilde B_1^{\mathrm T}+B_2^{\mathrm T}\widetilde C_0X_2+B_2^{\mathrm T}C_1\widetilde U & \alpha\widetilde B_1^{\mathrm T} & 0 & 0 & 0 & -\gamma^2I+B_2^{\mathrm T}B_2 & *\\ \widetilde C_0X_2+C_1\widetilde U & 0 & 0 & 0 & 0 & 0 & -I\end{pmatrix}<0 \tag{29}$$

where

$$\begin{cases}\Psi_{11}=\widetilde A_0X_2+X_2^{\mathrm T}\widetilde A_0^{\mathrm T}+\widetilde B_0\widetilde U+\widetilde U^{\mathrm T}\widetilde B_0^{\mathrm T}+X_3+X_3^{\mathrm T}+\overline Q\\ \Psi_{21}=\alpha\widetilde A_0X_2+\alpha\widetilde B_0\widetilde U+X_4^{\mathrm T}+X_1-X_2^{\mathrm T}\\ \Psi_{22}=-\alpha(X_2+X_2^{\mathrm T})+h_m\overline R+\overline W\\ \Psi_{31}=X_5^{\mathrm T}+X_2^{\mathrm T}\widetilde A_1^{\mathrm T}-X_3\\ \Psi_{32}=\alpha X_2^{\mathrm T}\widetilde A_1^{\mathrm T}-X_4\\ \Psi_{33}=-X_5-X_5^{\mathrm T}-(1-d_1)\overline Q\end{cases} \tag{30}$$

$$X_2=\mathrm{diag}(X_{20},X_{20}),\ \widetilde U=\begin{pmatrix}U & U\end{pmatrix}.$$

Moreover, the gain of the controller is given by

$$\widetilde K=\widetilde UX_2^{-1}. \tag{31}$$

Proof. After substituting (13) into (14), taking $P_3=\alpha P_2$, where α is a tuning scalar parameter, pre and post-multiply both sides of (14) with $\Delta=\mathrm{diag}\{P_2^{-1},P_2^{-1},P_2^{-1},P_2^{-1},P_2^{-1},I\}$ and its transpose, respectively. Introduce some change of variables such that

$$X_2=P_2^{-1},\ X_1=X_2^{\mathrm T}P_1X_2,\ X_3=X_2^{\mathrm T}P_4X_2,\\ X_4=X_2^{\mathrm T}P_5X_2,\ X_5=X_2^{\mathrm T}P_6X_2,\ \overline Q=X_2^{\mathrm T}QX_2,\\ \overline R=X_2^{\mathrm T}RX_2,\ \overline W=X_2^{\mathrm T}WX_2,\ \widetilde U=\widetilde KX_2. \tag{32}$$

Form Schur complement, we find that the condition (29) holds. Therefore, the resulting closed-loop system (5) is asymptotically stable, and the desired controller is defined by (8) with $\widetilde K=\widetilde UX_2^{-1}$. □

Remark 1. Theorem 1 may be easily applied to the special case of stabilizing delayed systems

$$\dot x(t)=A_0x(t)+A_1x(t-h(t))+B_0u(t) \tag{33}$$

with state feedback control low

$$u(t)=Kx(t). \tag{34}$$

In this case, the dimensions of the closed-loop system is reduced and (19) is replaced by

$$2x^{\mathrm T}(t)P_1\dot x(t)=2\widetilde x^{\mathrm T}(t)P^{\mathrm T}\begin{pmatrix}\dot x(t)\\-\dot x(t)+(A_0+B_0K)x(t)+A_1x(t-h(t))\\x(t)-x(t-h(t))-\int_{t-h(t)}^{t}\dot x(s)\mathrm ds\end{pmatrix} \tag{35}$$

where $\widetilde x(t)=\begin{pmatrix}x^{\mathrm T}(t) & \dot x^{\mathrm T}(t) & x^{\mathrm T}(t-h(t))\end{pmatrix}^{\mathrm T}$ and the following corollary is derived.

[1]The symbol * stands for symmetric block in matrix inequalities.

Corollary 1. The control law of (34) asymptotically stabilizes (33) for all the delay $h(t)$ satisfying (2), if there exist positive definite symmetric matrices $X_1 = X_1^{\rm T} \in \mathbf{R}^{n\times n}$, $\overline{Q} = \overline{Q}^{\rm T} \in \mathbf{R}^{n\times n}$, and $\overline{R} = \overline{R}^{\rm T} \in \mathbf{R}^{n\times n}$, some matrices $X_i \in \mathbf{R}^{n\times n}$, $i = 2, 3, 4, 5$, $U \in \mathbf{R}^{m\times n}$, and a scalar α such that the following condition is verified:

$$\Omega = \begin{pmatrix} \Omega_{11} & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & * \\ -X_3 & -X_4 & -X_5 & -\dfrac{1}{h_m}\overline{R} \end{pmatrix} < 0 \qquad (36)$$

where

$$\begin{cases} \Omega_{11} = A_0X_2 + X_2^{\rm T}A_0^{\rm T} + B_0U + U^{\rm T}B_0^{\rm T} + X_3 + X_3^{\rm T} + \overline{Q} \\ \Omega_{21} = \alpha A_0X_2 + \alpha B_0U + X_4^{\rm T} + X_1 - X_2^{\rm T} \\ \Omega_{22} = -\alpha(X_2 + X_2^{\rm T}) + h_m\overline{R} \\ \Omega_{31} = X_5^{\rm T} + X_2^{\rm T}A_1^{\rm T} - X_3 \\ \Omega_{32} = \alpha X_2^{\rm T}A_1^{\rm T} - X_4 \\ \Omega_{33} = -X_5 - X_5^{\rm T} - (1-d_1)\overline{Q}. \end{cases} \qquad (37)$$

The state-feedback gain is then given by

$$K = UX_2^{-1}. \qquad (38)$$

Remark 2. When deriving our results, we have used both Leibniz-Newton formula and Jonsen′s inequality, and we have taken a matrix P that contains free matrices P_2, P_3, P_4, P_5, and P_6. The descriptor method of [16, 21] uses a matrix P of the form $P = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix}$. Thus, our result is more general since it posses more degrees of freedom. In [18], some null terms containing free matrices have been added in the derivative of the Lyapunov-Krasovskii. It has been shown that the introduction of free matrices may lead to less restrictive results. However, this is not true in general[22]. To avoid products of Lyapunov matrices and system matrices, we have adopted a method like descriptor model transform[1, 16]. Then, to avoid the difficulty in handling the Lyapunov functional and the cross product between $x(t)$, $\dot{x}(t)$, and $x(t-h(t))$, we have retained these terms and $\int_{t-h(t)}^{t}\dot{x}(s){\rm d}s$. The method we have adopted in manipulating these techniques may have the potential to give less conservative results. Note that the technique applied in [12], with the augmented Lyapunov functional and free matrices has not given the best results in our case.

Remark 3. When the scalar parameter α is fixed, the conditions (29) and (36) become LMIs. However, choosing an arbitrary α does not lead to a best result. As same as that in [17], we will propose a tuning procedure for the parameter α in order to obtain good performance. Choose a cost function to be $f(\alpha) = t_{\min}$, for which $\Psi \leqslant t_{\min}I$ where Ψ is defined in (29). Therefore, the tuning parameter is defined for the following condition:

$$-\alpha < 0. \qquad (39)$$

The parameter $t_{\min}$ is obtained by solving the feasibility problem with the solver *feasp* in the LMI Toolbox[23]. It is positive when there exists no feasible solution to the set of LMIs under consideration. Finally, applying a numerical optimization algorithm, such as program *fmincon* in the Optimization Toolbox[24], to $f(\alpha)$ under the constraint (39), a locally convergent solution to the problem is obtained. If the resulting minimum value of the cost function is negative, then the tuning parameter that solves the problem is found. This procedure can be derived as follows:

Algorithm (finding α that maximizes h_m).

Step 1. Set a step length, h_{mstep}, for h_m. Choose an upper bound, ub, and a lower bound, lb, on α satisfying (39). Fix γ, h_{m0}, and α_0, where h_{m0} is small enough to have a feasible solution for (29) and (36). From simulation results, it is found that $\alpha_0 = 1$ works in a large number of cases.

Step 2. Solve the following problem:

$$\min_{\alpha} f(\alpha) \text{ subject to } (39) \qquad (40)$$

using the function *fmincon* with α_0,h_{m0}, ub, and lb, and obtain a new value for the tuning parameter α.

Step 3. If $f(\alpha) < 0$, let $\alpha_0 = \alpha$ and go to Step 4, otherwise go to Step 5.

Step 4. If γ is less than some pre-specified performance value, set $h_{m0} = h_{m0} + h_{mstep}$ and go to Step 2, otherwise set $\gamma = \gamma - \varepsilon$ with ε sufficiently small positive scalar, and go to Step 2.

Step 5. Stop. The maximal time delay bound $h_m = h_{m0} + h_{mstep}$, α, and $\gamma = \gamma + \varepsilon$ are obtained.

Assume that the system matrices are not precisely known but belong to a given convex set of finitely many vertices. The matrices A_0, A_1, B_0, B_1, B_2, C_0, C_1, $C2$, and C_3 are subject to uncertainties and satisfy the real convex polytopic model

$$\begin{pmatrix} A_0(\lambda) & A_1(\lambda) & B_0(\lambda) & B_1(\lambda) & B_2(\lambda) \\ C_0(\lambda) & C_1(\lambda) & C_2(\lambda) & C_3(\lambda) & \end{pmatrix} = \sum_{j=1}^{N}\lambda_j \begin{pmatrix} A_0^j & A_1^j & B_0^j & B_1^j & B_2^j \\ C_0^j & C_1^j & C_2^j & C_3^j & \end{pmatrix} \qquad (41)$$

where $\lambda_j \geqslant 0$ are positive and their sum is one: $\sum_{j=1}^{N}\lambda_j = 1$. Based on the result of Theorem 1, proving robust stability for the resulting uncertain system can be achieved by finding parameter-dependent matrices $P(\lambda)$, $Q(\lambda)$, $R(\lambda)$, and $W(\lambda)$ such that (29) holds for all admissible values of λ, where $P_1(\lambda) = \sum_{j=1}^{N}\lambda_jP_1^j$, $Q(\lambda) = \sum_{j=1}^{N}\lambda_jQ^j$, $R(\lambda) = \sum_{j=1}^{N}\lambda_jR^j$, and $W(\lambda) = \sum_{j=1}^{N}\lambda_jW^j$ with positive definite vertices P_1^j, Q^j, R^j, and W^j.

Theorem 2. For some given scalars $\gamma > 0$, system (1)–(2) with polytopic-type uncertainties (41) is robustly stabilizable via dynamic output feedback control law (3) and satisfies (11) for all non-zeros $w \in L_2[0,\infty)$, any $h(t)$ and $\tau(t)$ satisfying (2), if there exist $X_1^j > 0$, $\overline{Q}^j > 0$, $\overline{R}^j > 0$, $\overline{W}^j > 0$, X_{20}, X_3^j, X_4^j, X_5^j, U, and a scalar α, such that the condition (29) holds, where the matrices $X_1 > 0$, $\overline{Q} > 0$, $\overline{R} > 0$, $\overline{W} > 0$, X_3, X_4, X_5, $\widetilde{A}_0$, $\widetilde{A}_1$, $\widetilde{C}_0$, C_1, $\widetilde{B}_0$, and $\widetilde{B}_1$ are taken with the superscript j. The feedback gain controller is given by (31).

The proof is omitted because it is now classical in the robust analysis context.

4 Numerical examples

Example 1. Consider system (33) where

$$A_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & -1 \\ 0 & -0.9 \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{42}$$

We address the problem of finding a state-feedback controller for guaranteeing stability of the above system. The upper bound on the time-delay was found to be 1.510 in [16], 3.200 in [19], and 6.000 in [17]. Their results are listed in Table 1 along with the results obtained by Corollary 1 for $\alpha = 13$. Clearly, our method produces much less conservative results.

Table 1 Stability bound of h_m and control gain K

	h_m	Control gain
Fridman and Shaked[16]	$h_m \leqslant 1.51$	$K = -(58.31 \quad 294.935)$
Gao and Wang[19]	$h_m \leqslant 3.200$	$K = -(7.964 \quad 14.77)$
Zhang et al.[17]	$h_m \leqslant 6.000$	$K = -(70.18 \quad 77.67)$
Corollary 1 in this paper	$h_m \leqslant 10$	$K = -(176.1863 \quad 189.4912)$

To compare the magnitude of the controller gain, let us take $h_m = 6$, the feedback gain matrix $K = -\begin{pmatrix} 44.9103 & 52.0388 \end{pmatrix}$ guarantees the stability with $\alpha = 3.1801$. In [17], the control gain K is given by $K = -\begin{pmatrix} 70.18 & 77.67 \end{pmatrix}$ whose magnitude is larger than that obtained.

Now, trying to obtain the largest bound of time delay for which the system can be stabilized by our approach, we apply the above algorithm. We obtain the feasibility of the condition (36) with the following upper bound $h_m \leqslant 554.396$ and $\alpha = 13$.

Now, suppose that the considered system is subject to uncertainties such as

$$A_0 = \begin{pmatrix} \rho & 0 \\ 0 & 1+\rho \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1+\rho & -1 \\ 0 & -0.9+\rho \end{pmatrix},$$
$$B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad C_0 = \begin{pmatrix} 0 & 1 \end{pmatrix},$$
$$C_1 = C_2 = F = 0 \tag{43}$$

where we assume that $|\rho| \leqslant 0.2$.

From Theorem 2 in this paper, a desired robust H_∞ state feedback controller is obtained as $K = -\begin{pmatrix} 0.0000 & 9.5407 \end{pmatrix} \times 10^3$, for $h_m = 1.2909$, $\gamma = 0.0062$, and $\alpha = 1.4276$.

Example 2. Consider system (1) with the following parameters

$$A_0 = \begin{pmatrix} 2 & 0.5 & 1 \\ -0.5 & -2 & 1 \\ 0 & 0 & 0.1 \end{pmatrix}, A_1 = \begin{pmatrix} -1 & 0.1 & 0.2 \\ 0 & -1 & -0.1 \\ 0 & -0.1 & -2 \end{pmatrix},$$

$$B_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} -0.1 & 0.1 & -0.2 \\ -0.1 & -0.2 & 0.1 \\ 0 & 0 & -0.1 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \quad C_0 = C_1 = B_1 = 0. \tag{44}$$

By Theorem 1 of [25], it was found that the observer-based controller stabilizes system (44) for $h_m \leqslant 0.4$. Applying Theorem 1 in this paper, we found that the system (1) will be stabilized by the observer-based control (3) for the upper bound on the time delay $h_m = 0.614$ with $\alpha = 1.1625$ and

$$L = \begin{pmatrix} 5.3943 \\ 1.6841 \\ 2.17 \end{pmatrix}.$$

It can be seen that this comparison shows that the observer-based control in Theorem 1 for time delay systems in this paper is less conservative than those in [25].

Fig. 1 shows the state trajectories of system (1) with (44) for $h_m = 0.614$, the observer output feedback controller gain K given by

$$K = \begin{pmatrix} -2.3305 & -2.4390 & -1.5827 \\ -1.7686 & -2.1587 & -1.4682 \end{pmatrix} \tag{45}$$

and the conditions

$$x(\theta) = \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix}, \eta(\theta) = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, \forall \theta \in [-\bar{h}, 0].$$

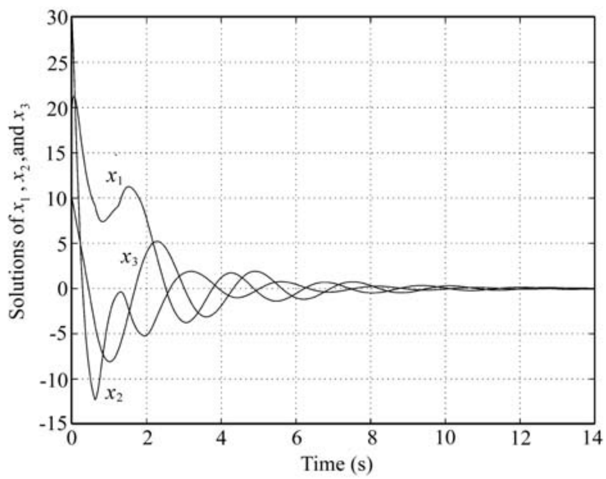

Fig. 1 Trajectories for observer-based control with $h_m = 0.614$

Assume that the system matrices A_0 and A_1 are subject to perturbation as

$$A_0 = \begin{pmatrix} 2 & 0.5 & 1 \\ -0.5 & -2+\sigma_1 & 1 \\ \sigma_1 & 0 & 0.1 \end{pmatrix}$$
$$A_1 = \begin{pmatrix} -1+\sigma_2 & 0.1 & 0.2 \\ 0 & -1 & -0.1 \\ 0 & -0.1 & -2+\sigma_2 \end{pmatrix}$$

where $\sigma_1 \in [0.08, 0.2]$ and $\sigma_2 \in [0.004, 0.03]$. Constructing the four vertices of the system and applying Theorem 2, the stability is guaranteed for $h_m = 0.598$ with $\alpha = 0.845$, the same L and

$$K = \begin{pmatrix} -2.7684 & -2.8059 & -1.7139 \\ -2.3012 & -2.8763 & -1.8368 \end{pmatrix}.$$

5 Conclusions

The observer-based H_∞ controller design for neutral systems with polytopic type uncertainties has been addressed. The derived conditions depend both on the size and rate of delays. The observer-based output feedback is obtained by solving conditions that depend on a tuning parameter α. An iterative procedure based on numerical optimization is proposed in order to determine the value of α that gives the better performances.

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