International Journal of Automation and Computing 05(3), July 2008, 313-318
DOI: 10.1007/s11633-008-0313-7

An Approach to Polynomial NARX/NARMAX Systems Identification in a Closed-loop with Variable Structure Control

O. M. Mohamed Vall[1,*] R. M'hiri[2]

[1]Laboratory of Systems Analysis and Control, Department of Electrical Engineering, National School for Engineers of Tunis, le Belvédère 1002 Tunis, Tunisia

[2]Research Unit of Electrical Machines and Power Systems (RME), National Institute of Applied Sciences and Technology of Tunis, Tunisia

Abstract: Many physical processes have nonlinear behavior which can be well represented by a polynomial NARX or NARMAX model. The identification of such models has been widely explored in literature. The majority of these approaches are for the open-loop identification. However, for reasons such as safety and production restrictions, open-loop identification cannot always be done. In such cases, closed-loop identification is necessary. This paper presents a two-step approach to closed-loop identification of the polynomial NARX/NARMAX systems with variable structure control (VSC). First, a genetic algorithm (GA) is used to maximize the similarity of VSC signal to white noise by tuning the switching function parameters. Second, the system is simulated again and its parameters are estimated by an algorithm of the least square (LS) family. Finally, simulation examples are given to show the validity of the proposed approach.

Keywords: Identification, variable structure control (VSC), genetic algorithm (GA), NARX/NARMAX models.

1 Introduction

A wide class of nonlinear systems can be described by NARX/NARMAX models. These models were introduced by Leontaritis and Billings[1,2] and Chen and Billings[3] as a means of describing the input-output relationship of a nonlinear system[4].

The polynomial representation is the most common model, and it has been proven to work well in practical applications[5]. Compared to the Volterra or Wiener series (which are frequently used for the identification of nonlinear systems), these models could represent a nonlinear model with a smaller number of parameters[6].

Several approaches to NARX/NARMAX models identification have been developed (see e.g., [7–10]) and the majority of these approaches were proposed for open-loop identification. However, such identification operations cannot always be used or do not provide reliable models. The system to be identified might be unstable in open-loop; this system might contain inherent feedback mechanisms[11]; or for safety and/or economic reasons open, loop identification cannot be performed. Thus, we must carry out closed-loop identification so that we can identify the system.

On the other hand, the choice of the excitation signal is an important step in open- or closed-loop identification. The results and the performances obtained are closely related to the excitation sequence. In open-loop identification, we can freely choose an excitation signal having the necessary properties for identification. In closed-loop identification, the excitation signal is the controller output. However, it is well known that the identification under output feedback control and without persistent excitation gives biased results. In many conventional methods, to achieve closed-loop identification with unbiased results, one adds a persistently exciting test signal into the closed-loop. When no such external excitation can be available, it is then required that the controller order should be higher than that of the plant.

In this paper, we present an approach to identification of polynomial NARX/NARMAX systems in a closed-loop with variable structure control (VSC). The use of the VSC is motivated by its outstanding robust property in stabilizing highly nonlinear and uncertain processes[12−13]. Furthermore, the control signal in such a control technique is very rich in commutations and very interesting for identification. We will also show that an appropriate choice of the switching function parameters provides a control signal, whose spectral properties are very close to those of a white noise. Thus, we will not need to add an external test signal into the closed-loop for a better identification.

The problem of choosing the switching function parameters is resolved by a genetic algorithm (GA).

The paper is organized as follows. In Section 2, we present a formulation of the identification problem considered in this paper. The proposed genetic approach is developed in Section 3. Section 4 describes the NARX/NARMAX identification. The validity of the proposed approach is illustrated with simulation example in Section 5. Finally, conclusions is given in Section 6.

2 Problem formulation

This work deals with the identification of NARX/NARMAX closed-loop systems with variable structure control. Figs. 1 and 2 show polynomial NARX and polynomial NARMAX systems in the a closed-loop with VCS, respectively.

Manuscript received December 22, 2006; revised December 25, 2007
*Corresponding author. E-mail address: valmotaly@yahoo.fr

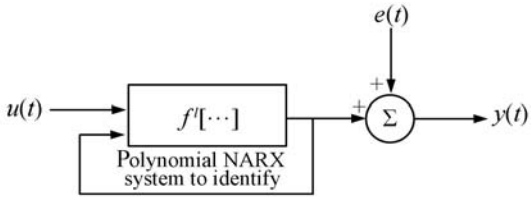

Fig. 1 Closed-loop polynomial NARX system to identify

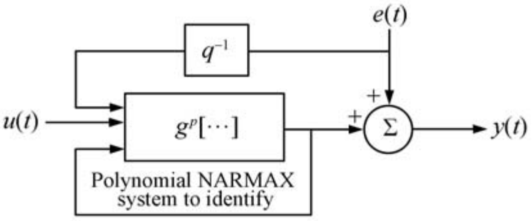

Fig. 2 Closed-loop polynomial NARMAX system to identify

These systems are described by the following equations. NARX system:

$$\begin{cases} y(t) = f^{\,l}[y(t-1), \cdots, y(t-n_y), u(t-1), \\ \qquad \cdots, u(t-n_u)] + e(t) \\ u(t) = F(S(t)) \end{cases} \tag{1}$$

NARMAX system:

$$\begin{cases} y(t) = g^p[y(t-1), \cdots, y(t-n_y), u(t-1), \cdots, \\ \qquad u(t-n_u), e(t-1), \cdots, e(t-n_e)] + e(t) \\ u(t) = F(S(t)) \end{cases} \tag{2}$$

where $y(t)$, $u(t)$, and $e(t)$ denote, respectively, the output, input, and noise signals, and n_y, n_u, and n_e are the maximum lags of past outputs, inputs, and noise entering the model. In this paper, f^l and g^p are assumed to be polynomial-type functions with nonlinearity degree $l, p \in \mathbf{R}^+$. F is a nonlinear function, which is often equal to the sign function defined as follows:

$$\text{sign}(S(t)) = \begin{cases} +K & \text{if} \quad S(t) \geqslant 0 \\ -K & \text{otherwise.} \end{cases} \tag{3}$$

$S(t)$ is called switching function and is defined by

$$S(t) = \varepsilon(t) + \lambda_1 \varepsilon^{(1)}(t) + \cdots + \lambda_{n_y-1} \varepsilon^{(n_y-1)}(t) \tag{4}$$

where $\varepsilon(t)$ is the error between the output signal $y(t)$ and the reference signal $r(t)$. $\lambda_i \, (i = 1, \cdots, n_y)$ are constants and $\varepsilon^{(i)}(t)$ is the i-th derivative of $\varepsilon(t)$.

The problem here is to estimate the models which describe, as well as possible, the considered systems behaviors, using the input and outputs measurements. Next, follows, we propose a method permitting a rich excitation signal.

As we have mentioned previously, the excitation signal has a significant importance in the identification procedure.

A pseudo-random binary sequence (PRBS) is usually used as the excitation signal. Such a signal provides a sufficient excitation in the case of linear systems identification[14]. However, the situation for nonlinear systems identification greatly differs from its linear counterpart. The PRBS is not sufficient as an excitation signal in this case, and can even lead to the loss of identifiability due to having only two possible levels[15,16]. Thus, a multilevel random signal is necessary in this case[16].

In our case, if F is the sign function, the VSC signal switches between two values. The shape of VSC signal remains in PRBS shape.

When F is the sign function, the VSC signal is a pseudo-random binary sequence (at least it has the form of this one). In a way similar to the PRBS, the VCS signal is not sufficient as an excitation signal for the nonlinear systems (due to having only two possible levels). Thus, we propose to use continuous VSC whose signal is multilevel. In this case, the function F is defined by

$$F(S(t)) = -K\left(\frac{S(t)}{|S(t)| + \eta}\right). \tag{5}$$

It is clear that if $\eta \to 0$, one tends towards the same discontinuous control law defined by (3).

We also propose an approach that we have already presented in a previous literature [17]. This approach, which is based on genetic programming, allows us to find the values of the switching function parameters which give a control signal whose properties approximate those of white noise.

3 Approach proposed to determine the switching function parameters by genetic algorithm (GA)

The signal which is more adaptable to identification is the white noise. Indeed, this signal allows continuous excitation of the system to be identified.

The autocorrelation function of this signal is a Dirac delta function. It is given by

$$\delta(t) = \begin{cases} \infty & \text{if} \quad \tau = 0 \\ 0 & \text{if} \quad \tau \neq 0. \end{cases} \tag{6}$$

For a finished power signal, (6) is written

$$\delta(t) = \begin{cases} 1 & \text{if} \quad \tau = 0 \\ 0 & \text{if} \quad \tau \neq 0. \end{cases} \tag{7}$$

From (7), we have

$$\sum_{i=2}^{N} (\varphi_{uu}(i))^2 = 0 \tag{8}$$

where u and φ_{uu} are white noise and its autocorrelation function, respectively.

Since the spectral properties of the signal of VCS (in particular its autocorrelation function) highly depend on parameters of the switching function, we thus propose an approach based on the genetic programming allowing determination of the values of these parameters which give the nearest possible control signal to that of white noise. Based on the white noise propriety defined by (8), this means that the approach allows us to obtain the switching function parameters values which minimize the criterion:

$$J = \sum_{i=2}^{N} (\varphi_{uu}(i))^2 \tag{9}$$

where φ_{uu} is the autocorrelation of the VCS signal u.

Since the criterion J cannot be easily expressed analytically, we propose to solve this minimization problem via GA because these algorithms use only the values of the studied function and not its expression, its derivative or other auxiliary knowledge[18].

The application of the GA to determine the switching function parameters can be reformulated as follows:

Step 1. Starting with an initial population randomly generated (N vectors $(\lambda_1, \cdots, \lambda_{n-1})^{\mathrm{T}}$), where λ_i ($i = 1, \cdots, n-1$) is the switching function parameter;

Step 2. Calculation of the fitness function value for each individual (vector);

Step 3. Selection of the best individuals;

Step 4. Creation of a new population (from the old one) by the application of the operators, crossover and mutation;

Step 5. Insertion of the new population in the current generation;

Step 6. While the termination condition is not met, return to Step 2.

4 NARX/NARMAX systems identification

4.1 NARX system identification

The NARX system considered here is described by the NARX polynomial model. This model is linear in the parameters non-linear difference equations, and it is a function of past inputs and outputs.

$$\begin{aligned} y(t) = f^{\,l}[&y(t-1), \cdots, y(t-n_y), u(t-1), \cdots, \\ &u(t-n_u)] + e(t) \end{aligned} \tag{10}$$

where $y(t)$, $u(t)$, and $e(t)$ represent the output, input, and noise signals, respectively, n_y and n_u are their associated maximum lags, and f is some unknown nonlinear function. In this paper, f is assumed to be a polynomial function of degree $l \in \mathbf{R}^+$.

Equation (10) can be written as

$$y(t) = \psi^{\mathrm{T}}(t-1)\theta + e(t) \tag{11}$$

where $\psi^{\mathrm{T}}(t-1)$ includes all the output and input terms as well as all possible combinations up to degree l and up to time $(t-1)$, and θ is a vector which includes their corresponding coefficients.

Since the model described by (11) is linear in the linear in the parameters model, the recursive least square algorithm can be used to estimate the parameters vector θ.

The algorithm that we use here is described by

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\psi(k)\varepsilon(k) \\ P(k) = P(k-1) - \dfrac{P(k-1)\psi(k)\psi^{\mathrm{T}}(k)P(k-1)}{1+\psi^{\mathrm{T}}(k)P(k-1)\psi(k)} \\ \varepsilon(k) = y(k) - \hat{\theta}^{\mathrm{T}}(k-1)\psi(k). \end{cases} \tag{12}$$

4.2 NARMAX systems identification

Consider the NARMAX system described by[2]

$$\begin{aligned} y(t) = g^p[(&y(t-1), ..., y(t-n_y), u(t-1), \cdots, \\ &u(t-n_u), e(t-1), \cdots, \\ &e(t-n_e] + e(t) \end{aligned} \tag{13}$$

where n_y, n_u, and n_e are the maximum lags considered for the output, input, and noise terms, respectively. $u(t)$ and $y(t)$ are the input and output signals, respectively. The sequence $\{e(t)\}$ represents uncertainties, possible perturbation, un-modelled dynamics, etc., and g is some nonlinear function of $u(t)$, $y(t)$, and $e(t)$. In this paper, g is assumed to be a polynomial function with nonlinearity degree $p \in \mathbf{R}^+$.

In order to estimate the parameters of the NARMAX model, (13) should be expressed as

$$y(t) = \psi^{\mathrm{T}}(t-1)\theta + e(t) \tag{14}$$

where $\psi^{\mathrm{T}}(t-1)$ includes all the output, input, and noise terms as well as all possible combinations up to degree p and up to time $(t-1)$, and θ is a vector which includes their corresponding coefficients.

The model (14) is linear in its parameters, so the estimation of these parameters can be done by an algorithm of the least square (LS) family[9]. If the noise $\{e(t)\}$ is known, an ordinary recursive least-squares (RLS) algorithm can be used for estimation. However, in general, the noise is not measurable, and sequence $\{e(t)\}$ is estimated iteratively. The problem can be resolved by a recursive extended least square (RELS) algorithm. Notice that there is a different version of this algorithm. In this paper, we use the one based on a posterior estimation of the noise sequence.

From the expression of the model NARMAX (14), we have

$$e(t) = y(t) - \psi^{\mathrm{T}}(t-1)\theta. \tag{15}$$

Equation (15) shows that the sequence of noise can be replaced by residues $\{\varepsilon^0(t)\}$. In other words, the sequence $\{\varepsilon^0(t)\}$ is a rebuilding of the noise sequence $\{e(t)\}$, by using experimental measurements (vector of observation) and of the vector of estimated parameters.

The residue $\varepsilon^0(t)$, which corresponds to the a posterior prediction error, can be calculated as follows:

$$\varepsilon^0(t) = y(t) - \hat{\psi}^{0^{\mathrm{T}}}(t-1)\hat{\theta} \tag{16}$$

where the vector $\hat{\psi}^{0^{\mathrm{T}}}(t-1)$ represents an approximation of the observation vector $\psi^{\mathrm{T}}(t-1)$. The vector $\hat{\psi}^{0^{\mathrm{T}}}(t-1)$ includes all output, input, and estimated noise terms.

The RELS algorithm, which allows estimating vector parameters in the model (14) using a posterior estimation, is described by

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\hat{\psi}^0(k)\varepsilon(k) \\ P(k) = P(k-1) - \dfrac{P(k-1)\hat{\psi}^0(k)\hat{\psi}^{0^{\rm T}}(k)P(k-1)}{1+\hat{\psi}^{0{\rm T}}(k)P(k-1)\hat{\psi}^0(k)} \\ \varepsilon(k) = y(k) - \hat{\theta}(k-1)\hat{\psi}^0(k). \end{cases} \tag{17}$$

5 Simulation examples

The proposed method was implemented and tested to identify several polynomials NARX/NARMAX systems. All simulations carried out showed the effectiveness of this method.

In this paper, two simulation examples, where polynomial NARX/NARMAX models are considered, are used to evaluate the proposed approach validity.

5.1 The NARX system

In this first simulation example, the following NARX model is used to generate simulation data which will be used for identification. This example is taken from [7].

$$y(t) = 0.5y(t-1) + 0.8u(t-2) + u(t-1)^2 - 0.05y(t-1)^2 + 0.5 + \frac{0.11}{1-0.5q^{-1}}e(t) \tag{18}$$

where $y(t)$ and $u(t)$ are the output and input, respectively. $e(t)$ is a zero mean Gaussian noise with variance 0.05.

Consider that the system is in closed-loop with variable structure controller, where

$$\begin{cases} u(t) = 0.1\dfrac{S(t)}{|S(t)|+0.025} \\ S(t) = \varepsilon(t) + \lambda\varepsilon^{(1)}(t) \\ \varepsilon(t) = r(t) - y(t) \end{cases} \tag{19}$$

where $\varepsilon(t)$ is the tracking error between the reference signal, chosen to be equal to 0.9, and the process output. $\varepsilon^{(1)}(t)$ is the derivative of $\varepsilon(t)$.

Initially, a GA is used in order to determine the value of λ which minimizes (9). The parameters for the GA are set as follows:

1) Initial population size: 15;
2) Maximum number of generation: 100;
3) Crossover: uniform crossover with probability 0.9;
4) Mutation probability: 0.01;
5) Fitness function used to evaluate individuals is: $F = 1/(1+J)$. It is obvious that this fitness function increases as J decreases. J is the criterion defined by (9).

Fig. 3 shows λ and fitness function evolution during the optimization operation.

The optimum value of the switching function parameter obtained by GA is $\lambda_{\rm opt} = 0.2320$. Using the new switching function, the data to be used identification are generated.

Fig. 4 shows the input and output of the system, which can be measured for identification.

Fig. 5 shows the actual system and estimated model outputs.

Fig. 6 shows the error between the real system and estimated model outputs. It can be noted that the agreement between the estimated model and the actual system is good.

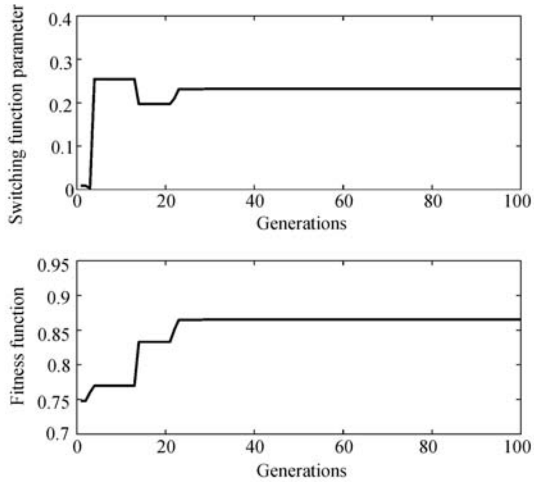

Fig. 3 Switching function parameter and fitness function evolution during the optimisation operation

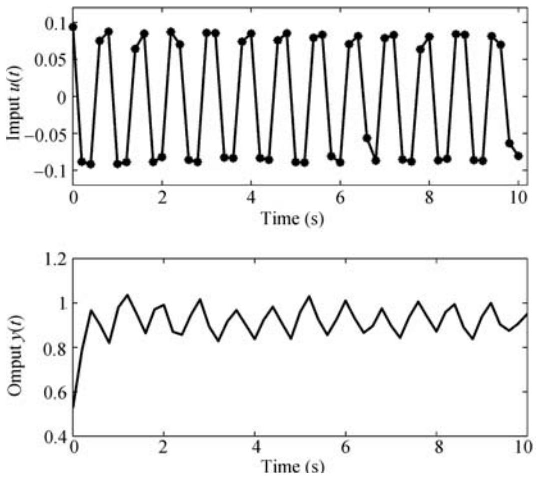

Fig. 4 Input and output signals of the system to identify

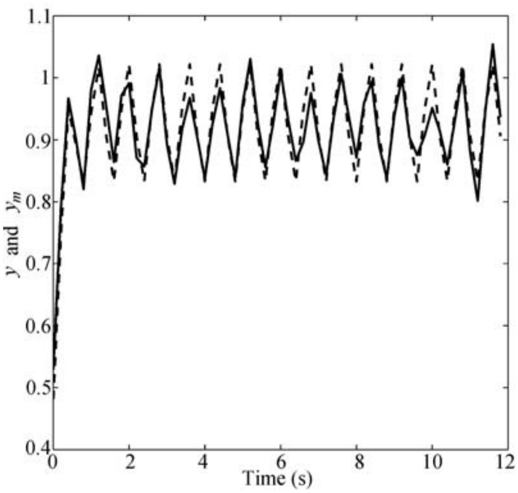

Fig. 5 Actual system output y (solid) and estimated model output y_m (dashed)

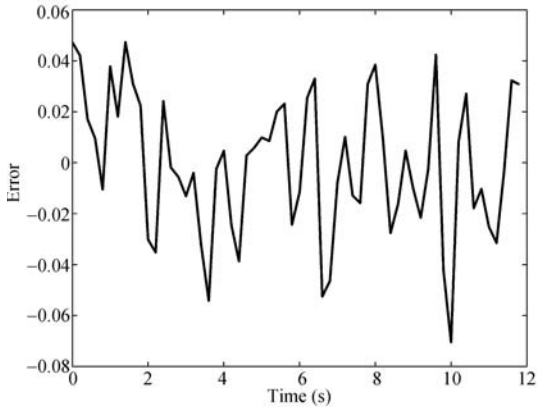

Fig. 6 Error between the actual system and estimated model outputs

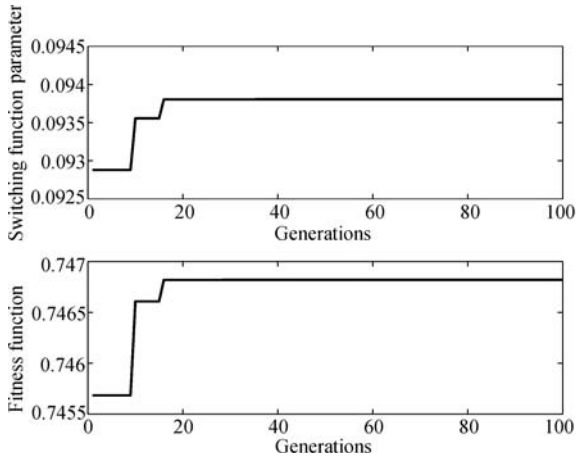

Fig. 7 Switching function parameter and fitness function evolution during the optimisation operation

5.2 The NARMAX system

In the second simulation example, we consider the following NARMAX model. The structure of this model is taken from [8].

$$\begin{aligned} y(t) = {} & 1.25y(t-1) + 0.8u(t-2)+ \\ & 0.012u(t-1)y(t-2) - 0.05y(t-2)^2+ \\ & 0.007y(t-2)e(t-2) + e(t) \end{aligned} \tag{20}$$

where $y(t)$ and $u(t)$ are the output and input system, respectively; $e(t)$ is a zero mean Gaussian noise with variance 0.001.

This NARMAX system is simulated in closed-loop with variable structure controller whose parameters are set as follows:

$$\begin{cases} u(t) = 0.5\dfrac{S(t)}{|S(t)| + 0.125} \\ S(t) = \varepsilon(t) + \lambda\varepsilon^{(1)}(t) \\ \varepsilon(t) = r(t) - y(t) \end{cases} \tag{21}$$

where $\varepsilon(t)$ is the tracking error between the reference signal $r(t)$ (taken here $r(t) = 5$) and the system output. $\varepsilon^{(1)}(t)$ is the derivative of $\varepsilon(t)$.

Firstly, a GA is used to determine the value of the switching function parameter which maximizes the similarity of the VCS signal to the white noise. The parameters of this AG are the same as those considered in the first example.

The switching function parameter and fitness function evolutions during the GA running are shown in Fig. 7.

The value of the switching parameter obtained by the GA is $\lambda_{opt} = 0.0938$. Fig. 8 shows the system input and output.

The parameters of the model are estimated using an ERLS algorithm (17). Figs. 9 and 10 show the estimation results. In Fig. 9, we compare the actual system and estimated model outputs. We can note that the output model reproduces very well the actual system output.

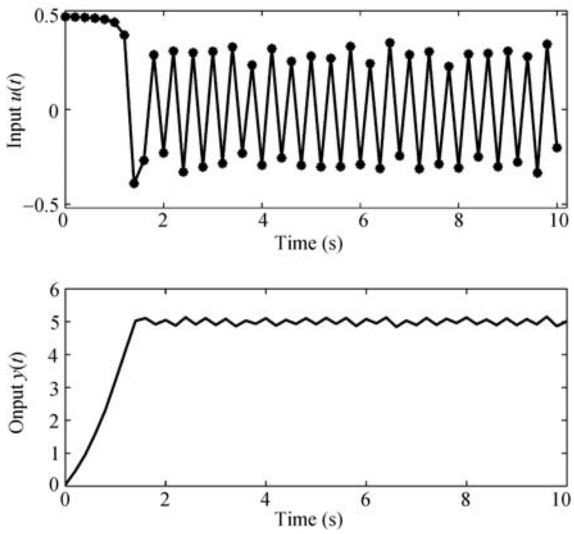

Fig. 8 Input and output signals of the system to identify

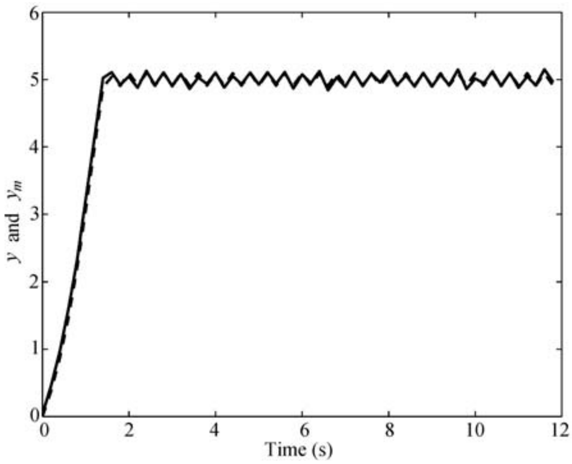

Fig. 9 Actual system output y (solid) and estimated model output y_m (dashed)

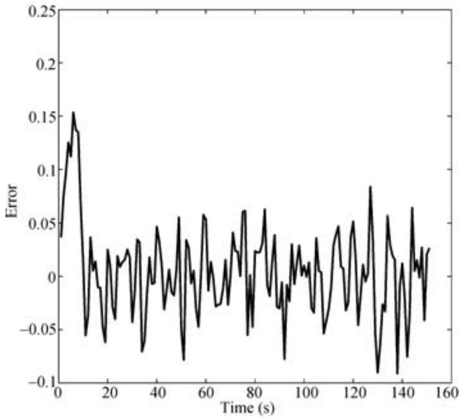

Fig. 10 Error between the actual system and estimated model outputs

6 Conclusions

In this paper, we present an approach to the closed-loop identification of the polynomial NARX/NARMAX systems. The systems to identify are assumed to be the closed-loop with variable structure control (VCS). We have chosen to use such control because the VCS has an outstanding robust property in stabilizing highly nonlinear and uncertain processes.

Furthermore, the control signal is very rich in commutations and is very interesting for identification. The approach that we have proposed here includes two stages. In the first stage, a GA algorithm is used to determine the values of the switching function parameters which give a control signal whose properties are closest to those of white noise. In the second stage, the model parameters are estimated. Finally, the provided simulation examples show the validity of the proposed approach.

References

[1] I. J. Leontaritis, S. A. Billings. Input-output Parametric Models for Non-linear Systems, Part I: Deterministic Non-linear Systems. *International Jouranl of Control*, vol. 41, no. 2, pp. 303–328, 1985.

[2] I. J. Leontaritis, S. A. Billings. Input-output Parametric Models for Non-linear Systems, Part II: Stochastic Non-linear Systems. *International Jouranl of Control*, vol. 41, no. 2, pp. 329–344, 1985.

[3] S. Chen , S. A. Billings. Representations of Non-linear Systems: The NARMAX Model. *International Jouranl of Control*, vol. 49, no. 3, pp. 1013–1032, 1989.

[4] N. Chiras, C. Evans, D. Rees, M. Solomou. Nonlinear System Modelling: How to Estimate the Highest Significant Order. In *Proceedings of IEEE Instrumentation and Measurement Technology Conference*, IEEE Press, Anchorage, USA, pp. 353–358, 2002.

[5] A. E. Ruano, P. J. Fleming, C. Teixeira, K. Odrguez-Vzquez, C. M. Fonseca. *Nonlinear Identification of Aircraft Gas-turbine Dynamics*. *Neurocomputing*, vol. 55, no. 3-4, pp. 551–579, 2003.

[6] C. J. Li, Y. Jeon. Genetic Algorithm in Identifying Nonlinear Autoregressive with Exogenous Input Models for Nonlinear Systems. In *Proceedings of American Control Conference*, IEEE Press, San Francisco, CA, USA, pp. 2305–2309, 1993.

[7] L. Piroddi, W. Spinelli. An Identification Algorithm for Polynomial NARX Models Based on Simulation Error Minimization. *International Journal of Control*, vol. 76, no. 17, pp. 1767–1781, 2003.

[8] W. Spinelli. An identification Algorithm for Polynomial NARX Models, M. Sc. dissertation, Politecnico di Milano, Italy, 2002.

[9] L. Piroddi, W. Spinelli. A Pruning Method for the Identification of Polynomial NARMAX Models. In *Proceedings of the 13rd IFAC Symposium on System Identification*, Rotterdam, the Netherlands, pp. 1108–1113, 2003.

[10] D. H. Zhou, P. M. Frank. A Real-time Estimation Approach to Time-varying Time Delay and Parameters of NARX Processes. *Computers & Chemical Engineering*, vol. 23, no. 11-12, pp. 1763–1772, 2000.

[11] U. Forssell, L. Ljung. Closed-loop Identification Revisited. *Automatica*, vol. 35, no. 7, pp. 1215–1241, 1999.

[12] J. Y. Hung, W. B. Gao, J. C. Hung. Variable Structure Control: A survey. *IEEE Transactions on Industrials Electronics*, vol. 40, no. 1, pp. 2–22, 1993.

[13] V. I. Utkin. Sliding Mode Control Design Principles and Applications to Electric Drives. *IEEE Transactions on Industrials Electronics*, vol. 40, no. 1, pp. 23–36, 1993.

[14] O. Toker, H. E. Emara-Shabaik. Pseudo-random Multilevel Sequences: Spectral Properties and Identification of Hammerstein Systems. *IMA Journal of Mathematical Control and Information*, vol. 21, no. 2, pp. 183–205, 2004.

[15] S. A. Billings, S. Y. Fakhouri. Identification of Nonlinear Systems Using Correlation Analysis and Pseudorandom Sequences. *International Journal of Systems Science*, vol. 11, no. 3, pp. 261–279, 1980.

[16] R. D. Nowak, B. D. Van Veen. Random and Pseudorandom Inputs for Volterra Filter Identification. *IEEE Transactions on Signal Processing*, vol. 42, no. 8, pp. 2124–2135, 1994.

[17] O. M. Mohamed Vall, M. Radhi. Using Genetic Algorithms in Closed Loop Identification of the Systems with Variable Structure Controller. *Enformatika*, vol. 7, no. 1, pp. 415–418, 2005.

[18] F. Herrera, M. Lozano, J. L. Verdegay. Tackling Real-coded Genetic Algorithms: Operators and Tools for the Behaviour Analysis. *Artificial Intelligence Review*, vol. 12, no. 4, pp. 265–319, 1998.

O. M. Mohamed Vall received his B. Sc. degree in electronic from the Faculty of Sciences of Tunis, Tunisia, in 1998 and the M. Sc. degree in automatic control and signal processing and the Ph. D. degree in electric engineering from National School of Engineers of Tunis (ENIT), in 2001 and 2007, respectively. He is teaching in the Computer Sciences Department at the Faculty of Sciences of Bizerte, Tunisia.

His research interests include robust automatic control, sliding mode, and systems identification.

R. M′hiri received his M. Sc. and Ph. D. degrees in automatic control from the Tunis University, Higher School of Sciences and Technology of Tunis, (ENSET), Tunisia, and his Habilitation in electrical engineering from the Tunis El Manar University (ENIT), Tunisia, in 2000. Currently, he is a professor in the Department of Physics at Tunis El Manar University, Tunisia. He is collaborator member of the Center for Research in Higher Education (CRHE) at University of Sherbrooke, Canada. He is the head of a research group (AIA) on automatic control at his university, he has published about 100 refereed journal and conference papers.

His research interests include feedback control systems, control theory, hybride system, and e-learning.