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Robust Output Feedback Control for a Class of Nonlinear Systems with Input Unmodeled Dynamics

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Abstract: The robust global stabilization problem of a class of uncertain nonlinear systems with input unmodeled dynamics is considered using output feedback, where the uncertain nonlinear terms satisfy a far more relaxed condition than the existing triangular-type condition. Under the assumption that the input unmodeled dynamics is minimum-phase and of relative degree zero, a dynamic output compensator is explicitly constructed based on the nonseparation principle. An example illustrates the usefulness of the proposed method.

Keywords: Nonlinear systems, global stabilization, output feedback, input unmodeled dynamics, high gain.

1 Introduction

The problem of stabilization via output feedback is one of the most important problems in the field of nonlinear control. Unlike the case of linear systems, the so-called separation principle usually does not hold for nonlinear systems[1]. Due to this, the problem becomes exceptionally challenging and more interesting than full-state control problems. One of the valid methods to solve such a problem is to construct directly output feedback controllers for nonlinear systems based on the so-called nonseparation principle[2].

Over these years, a number of interesting results for the problem have been obtained under extra structural or growth conditions which are usually necessary. For example, it is assumed that the nonlinear terms satisfy some global Lipshitz-like condition[3], some triangular condition[2,4], or some relaxed triangular condition which was presented in [5] recently.

However, the results in [5] did not consider the input unmodeled dynamics which are likely to appear in practical nonlinear systems. The control issue of nonlinear systems with input unmodeled dynamics was firstly stated by Kristic et al.[6] It was shown that even in its simplest form, input unmodeled dynamics may result in dramatic shrinking of the region of attraction and finite escape time. Recently, the problem of input unmodeled dynamics has received a lot of attention[7−9], and all these results were obtained under the assumption that the input unmodeled dynamics was of relative degree zero and minimum phase. In addition, the case of nonminimum phase input unmodeled dynamics was considered in [10], the problem of input unmodeled dynamics with relative degree greater than zero was considered in [11], and more general cases of unmodeled input dynamics were addressed in [12]. In short, the problem of input unmodeled dynamics commonly exists in practice, and is challenging.

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In this paper, we consider the robust global stabilization for a class of uncertain nonlinear systems with input unmodeled dynamics using output feedback, where the nonlinear terms satisfy a far more relaxed triangular condition as shown in [5], and the input unmodeled dynamics is assumed to be relative degree zero and minimum phase. A high gain dynamic output compensator is explicitly constructed to guarantee the globally asymptotic stability of the closed-loop system.

This paper is organized as follows. In Section 2, the problem considered is described briefly. The main result, namely the design procedure of the dynamic output compensator is displayed in Section 3. Section 4 presents a simple numerical example. Conclusion is given in Section 5.

2 Problem statement

Consider the following system

$$\begin{cases} \dot{x}_1 = x_2 + f_1(t,x) \\ \quad\vdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(t,x) \\ \dot{x}_n = v + f_n(t,x) \\ y = x_1 \\ \dot{\varepsilon} = A_1(\varepsilon) + bu \\ v = c(\varepsilon) + u \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $u \in \mathbf{R}$, and $y \in \mathbf{R}$ are the state, the input, and the output of the system, respectively.

The ε-subsystem

$$\begin{cases} \dot{\varepsilon} = A_1(\varepsilon) + bu \\ v = c(\varepsilon) + u \end{cases} \tag{2}$$

is the input unmodeled dynamics of system (1), where $\varepsilon \in \mathbf{R}^p$, $v \in \mathbf{R}$, $A_1(\varepsilon)$ and $c(\varepsilon)$ are unknown continuous functions vanishing at zero, and $b \in \mathbf{R}^p$ is an unknown constant vector.

In this paper, it is supposed that system (1) satisfies the following two assumptions.

Assumption 1[4]**.** The admissible input unmodeled dynamics (2) is characterized by the following three assumptions:

1) There exists a constant $\bar{b}$ such that $\|b\| \leqslant \bar{b}$.

2) There exists a constant $\bar{c}$ such that $\|c(\varepsilon)\| \leqslant \bar{c}\|\varepsilon\|$.

3) The zero-dynamics of the ε-subsystem (2)

$$\dot{\varepsilon} = A_1(\varepsilon) - bc(\varepsilon) = A_0(\varepsilon) \tag{3}$$

satisfies the condition: when it is disturbed by (ω_1, ω_2), i.e.,

$$\dot{\varepsilon} = A_0(\varepsilon + \omega_1) + \omega_2 \tag{4}$$

there exist two positive scalars α_1 and α_2, and a continually differentiable, positive definite, and radially unbounded scalar function $V_3(\varepsilon)$ such that

$$\frac{\partial V_3}{\partial \varepsilon}[A_0(\varepsilon + \omega_1) + \omega_2] \leqslant \alpha_1(\|\omega_1\|^2 + \|\omega_2\|^2) - \alpha_2\|\varepsilon\|^2. \tag{5}$$

Clearly, the ε-subsystem (2) is minimum-phase and relative degree zero under Assumption 1.

Assumption 2[5]**.** The mappings $f_i(t,x) : \mathbf{R} \times \mathbf{R}^n \rightarrow \mathbf{R}, i = 1, 2, \cdots, n$ are continuous, and there exists a constant $\gamma > 0$, such that for any $s \in (0, 1)$, the inequality

$$\sum_{i=1}^{n} s^{i-1}|f_i(t,x)| \leqslant \gamma \sum_{i=1}^{n} s^{i-1}|x_i| \tag{6}$$

is satisfied.

It is easy to check that if $f_i(t,x), i = 1, 2, \cdots, n$ satisfies the triangular condition given in [2,4], i.e.,

$$|f_i(t,x)| \leqslant \lambda \sum_{j=1}^{i} |x_j| \tag{7}$$

where $\lambda > 0$, then Assumption 2 is always satisfied. In fact, if condition (7) is satisfied and if $\gamma = n\lambda$, then for any $0 < s < 1$, we have

$$\sum_{i=1}^{n} s^{i-1}|f_i(t,x)| \leqslant \sum_{i=1}^{n}\left(s^{i-1}\lambda \sum_{j=1}^{i}|x_j|\right) =$$

$$\lambda \sum_{j=1}^{n}\left(|x_j| \sum_{j=1}^{n} s^{i-1}\right) \leqslant \lambda \sum_{j=1}^{n} ns^{j-1}|x_j| = \gamma \sum_{j=1}^{n} s^{j-1}|x_j|.$$

This implies that Assumption 2 holds. But, the converse is not always true.

Remark 1. The relation between (6) and (7) has been revealed in [5]. Here, a different proof above is given not only for completeness but also for getting the explicit relation between the constants γ and λ, which is useful for comparing the result in [4] with the one presented in this paper.

The objective of this paper is to prove that there exists a dynamic output compensator with the form

$$\begin{cases} \dot{\chi} = g(\chi, y) \\ u = h(\chi, y) \end{cases} \tag{8}$$

where $g(0,0) = 0$ and $h(0,0) = 0$, such that the closed system described by (1) and (8) is globally asymptotically stable under Assumptions 1 and 2.

3 Main result

In this section, a dynamic output compensator is explicitly constructed for system (1). The designed dynamic output compensator consists of a linear high gain observer and a linear high gain controller. The main result is given in the following theorem.

Theorem 1. Under Assumptions 1 and 2, there exists a dynamic output compensator with the form (8), which can globally asymptotically stabilize the nonlinear system (1).

Proof. Letting $\eta = \varepsilon - bx_n$, system (1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 + f_1(t,x) \\ \quad \vdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(t,x) \\ \dot{x}_n = u + c(\eta + bx_n) + f_n(t,x) \\ y = x_1 \\ \dot{\eta} = A_0(\eta + bx_n) - bf_n(t,x). \end{cases} \tag{9}$$

We first introduce the following high gain state observer

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + La_1\kappa(y - \hat{x}_1) \\ \quad \vdots \\ \dot{\hat{x}}_{n-1} = \hat{x}_n + L^{n-1}a_{n-1}\kappa^{n-1}(y - \hat{x}_1) \\ \dot{\hat{x}}_n = u + L^n a_n \kappa^n (y - \hat{x}_1) \end{cases} \tag{10}$$

where $L > 1$ and $\kappa > 0$ are the design parameters which will be determined later; $a_i, i = 1, 2, \cdots, n$ are the coefficients of any Hurwitz polynomial $\rho^n + \sum_{i=1}^{n} a_i\rho^{n-i}$. Obviously, the polynomial $\rho^n + \sum_{i=1}^{n} a_i\kappa^i\rho^{n-i}$ is also a Hurwitz polynomial.

Now we introduce

$$\zeta = [\zeta_1, \zeta_2, \cdots, \zeta_n]^{\mathrm{T}} \in \mathbf{R}^n$$

with

$$\zeta_i = L^{-(i-1)}\hat{x}_i, \ i = 1, 2, \cdots, n. \tag{11}$$

Then, it is easily derived that

$$\begin{cases} \dot{\zeta}_1 = L\zeta_2 + La_1\kappa(y - \hat{x}_1) \\ \quad \vdots \\ \dot{\zeta}_{n-1} = L\zeta_n + La_{n-1}\kappa^{n-1}(y - \hat{x}_1) \\ \dot{\zeta}_n = L^{-(n-1)}u + La_n\kappa^n(y - \hat{x}_1) \end{cases} \tag{12}$$

Next, we consider the estimation error

$$e_i = x_i - \hat{x}_i, \ i = 1, 2, \cdots, n. \tag{13}$$

Further, we introduce the scaled estimation error

$$\xi = [\xi_1, \xi_2, \cdots, \xi_n]^{\mathrm{T}} \in \mathbf{R}^n$$

with

$$\xi_i = L^{-(i-1)}e_i, \ i = 1, 2, \cdots, n. \tag{14}$$

Then, we have

$$\dot{\eta} = A_0(\eta + bL^{n-1}(\xi_n + \zeta_n)) - bf_n(t, \zeta, \xi) \tag{15}$$

and

$$\begin{cases} \dot{\xi}_1 = L\xi_2 - La_1\kappa\xi_1 + f_1(t,\xi,\zeta) \\ \quad\vdots \\ \dot{\xi}_{n-1} = L\xi_n - La_{n-1}\kappa^{n-1}\xi_1 + L^{-(n-2)}f_{n-1}(t,\xi,\zeta) \\ \dot{\xi}_n = -La_n\kappa^n\xi_1 + L^{-(n-1)}c(\eta + bL^{n-1}(\xi_n+\zeta_n)) + \\ \qquad L^{-(n-1)}f_n(t,\xi,\zeta) \end{cases} \tag{16}$$

where

$$f_i(t,\xi,\zeta) = f_i(t,\xi_1+\zeta_1,\cdots,L^{n-1}(\xi_n+\zeta_n)),\ i=1,2,\cdots,n.$$

With the above relations, we can get the following system

$$\begin{cases} \dot{\xi}_1 = L\xi_2 - La_1\kappa\xi_1 + f_1(t,\xi,\zeta) \\ \quad\vdots \\ \dot{\xi}_{n-1} = L\xi_n - La_{n-1}\kappa^{n-1}\xi_1 + L^{-(n-2)}f_{n-1}(t,\xi,\zeta) \\ \dot{\xi}_n = -La_n\kappa^n\xi_1 + L^{-(n-1)}c(\eta + bL^{n-1}(\xi_n+\zeta_n)) + \\ \qquad L^{-(n-1)}f_n(t,\xi,\zeta) \\ \dot{\zeta}_1 = L\zeta_2 + La_1\kappa\xi_1 \\ \quad\vdots \\ \dot{\zeta}_{n-1} = L\zeta_n + La_{n-1}\kappa^{n-1}\xi_1 \\ \dot{\zeta}_n = L^{-(n-1)}u + La_n\kappa^n\xi_1 \\ \dot{\eta} = A_0(\eta + bL^{n-1}(\xi_n+\zeta_n)) - bf_n(t,\xi,\zeta) \end{cases} \tag{17}$$

or equivalently,

$$\begin{cases} \dot{\xi} = LA(\kappa)\xi + L^{-(n-1)}Ec(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) + F \\ \dot{\zeta} = LB\zeta + L^{-(n-1)}Eu + LC(\kappa)\xi \\ \dot{\eta} = A_0(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) - L^{n-1}bE^{\mathrm{T}}F \end{cases} \tag{18}$$

where

$$A(\kappa) = \begin{bmatrix} -a_1\kappa & 1 & 0 & \cdots & 0 \\ -a_2\kappa^2 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1}\kappa^{n-1} & 0 & \cdots & 0 & 1 \\ -a_n\kappa^n & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$C(\kappa) = \begin{bmatrix} -a_1\kappa & 0 & 0 & \cdots & 0 \\ -a_2\kappa^2 & 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1}\kappa^{n-1} & 0 & \cdots & 0 & 0 \\ -a_n\kappa^n & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$F = \begin{bmatrix} f_1(t,\xi_1+\zeta_1,\cdots,L^{n-1}(\xi_n+\zeta_n)) \\ \vdots \\ L^{-(n-2)}f_{n-1}(t,\xi_1+\zeta_1,\cdots,L^{n-1}(\xi_n+\zeta_n)) \\ L^{-(n-1)}f_n(t,\xi_1+\zeta_1,\cdots,L^{n-1}(\xi_n+\zeta_n)) \end{bmatrix}.$$

From Assumption 1 and the fact that $L > 1$, we have

$$\begin{aligned} \|F\| &= \sum_{i=1}^{n} L^{-(i-1)}|f_i(t,\xi_1+\zeta_1,\cdots,L^{n-1}(\xi_n+\zeta_n)| \leqslant \\ &\gamma\sum_{i=1}^{n} L^{-(i-1)}|L^{i-1}(\xi_i+\zeta_i)| \leqslant \gamma\sum_{i=1}^{n}|\xi_i+\zeta_i|. \end{aligned} \tag{19}$$

Then, we design a linear high gain controller with the form

$$u = -L^n(b_n\zeta_1 + b_{n-1}\zeta_2 + \cdots + b_1\zeta_n) \tag{20}$$

where $b_i, i = 1,2,\cdots,n$ are the coefficients of any Hurwitz polynomial $\rho^n + \sum_{i=1}^{n} b_i\rho^{n-i}$.

It is easy to verify that the compensator (10) and (20) globally asymptotically stabilizes system (1) if the closed-loop system (18) and (20) is globally asymptotically stable.

In the end, we consider the stability of the closed-loop system (18) and (20), which can be equivalently written as

$$\begin{cases} \dot{\xi} = LA(\kappa)\xi + L^{-(n-1)}Ec(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) + F \\ \dot{\zeta} = LB_c\zeta + LC(\kappa)\xi \\ \dot{\eta} = A_0(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) - L^{n-1}bE^{\mathrm{T}}F \end{cases} \tag{21}$$

where

$$B_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ -b_n & -b_{n-1} & \cdots & -b_2 & -b_1 \end{bmatrix}.$$

Since both polynomials $\rho^n + \sum_{i=1}^{n} a_i\kappa^i\rho^{n-i}$ and $\rho^n + \sum_{i=1}^{n} b_i\rho^{n-i}$ are Hurwitz polynomials, there exist two positive definite symmetric matrices $P(\kappa)$ and Q, such that

$$A^{\mathrm{T}}(\kappa)P(\kappa) + P(\kappa)A(\kappa) = -I \tag{22}$$

and

$$B_c^{\mathrm{T}}Q + QB_c = -I. \tag{23}$$

Now, we consider the following two continuously differentiable, positive definite and radially unbounded scalar functions

$$V_1(\xi) = \xi^{\mathrm{T}}P(\kappa)\xi$$

and

$$V_2(\zeta) = \zeta^{\mathrm{T}}Q\zeta.$$

By a simple computation, we can get the time derivatives of $V_1(\xi)$ and $V_2(\zeta)$ along the solution of (21) as follows:

$$\dot{V}_1(\xi) = L\xi^{\mathrm{T}}[A^{\mathrm{T}}(\kappa)P(\kappa) + P(\kappa)A(\kappa)]\xi +$$
$$2\xi^{\mathrm{T}}P(\kappa)[L^{-(n-1)}Ec(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) + F] \leqslant$$
$$2\|\xi\|\|P(\kappa)\|\|L^{-(n-1)}Ec(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta)) + F\|$$
$$-L\|\xi\|^2 \leqslant -L\|\xi\|^2 + 2\|\xi\|\|P(\kappa)\|\|F\| +$$
$$2L^{-(n-1)}\|\xi\|\|P(\kappa)\|\|c(\eta + bL^{n-1}E^{\mathrm{T}}(\xi+\zeta))\| \leqslant$$
$$-L\|\xi\|^2 + 2\|\xi\|\|P(\kappa)\|(\gamma\sum_{i=1}^{n}|\xi_i+\zeta_i|) +$$
$$2\bar{c}L^{-(n-1)}\|\xi\|\|P(\kappa)\|\|\eta\| + 2\bar{b}\bar{c}\|\xi\|\|P(\kappa)\|\|\xi+\zeta\| \leqslant$$
$$-L\|\xi\|^2 + 2\sqrt{n}\gamma\|\xi\|\|P(\kappa)(\|\zeta\| + \|\xi\|) + 2\bar{c}L^{-(n-1)}\cdot$$
$$\|\xi\|\|P(\kappa)\|\|\eta\| + 2\bar{b}\bar{c}\|\xi\|\|P(\kappa)\|(\|\xi\| + \|\zeta\|) \leqslant$$
$$2\sqrt{n}\gamma\|P(\kappa)\|\xi\|^2 + \sqrt{n}\gamma\|P(\kappa)\|(\|\zeta\|^2 + \|\xi\|^2) +$$
$$\bar{c}\|P(\kappa)\|(\|\xi\|^2 + L^{-2(n-1)}\|\eta\|^2) + 2\bar{b}\bar{c}\|P(\kappa)\|\|\xi\|^2 +$$
$$\bar{b}\bar{c}\|P(\kappa)\|(\|\xi\|^2 + \|\zeta\|^2) - L\|\xi\|^2 =$$
$$[-L + (3\sqrt{n}\gamma + \bar{c} + 3\bar{b}\bar{c})\|P(\kappa)\|]\|\xi\|^2 +$$
$$(\sqrt{n}\gamma + \bar{b}\bar{c})\|P(\kappa)\|\|\zeta\|^2 + L^{-2(n-1)}\bar{c}\|P(\kappa)\|\eta\|^2$$

and

$$\dot{V}_2(\zeta) = L\zeta^{\mathrm{T}}[B_c^{\mathrm{T}}Q + QB_c]\zeta + L\zeta^{\mathrm{T}}[C^{\mathrm{T}}(\kappa)Q +$$
$$QC(\kappa)]\xi \leqslant -L\|\zeta\|^2 + L\|\zeta\|\|C^{\mathrm{T}}(\kappa)Q + QC(\kappa)\|\|\xi\| \leqslant$$
$$-L\|\zeta\|^2 + \frac{1}{2}L\|C^{\mathrm{T}}(\kappa)Q + QC(\kappa)\|(\|\zeta\|^2 + \|\xi\|^2) \leqslant$$
$$-L\|\zeta\|^2 + \frac{1}{2}L(\|C^{\mathrm{T}}(\kappa)Q\| + \|QC(\kappa)\|)(\|\zeta\|^2 + \|\xi\|^2) =$$
$$-L\|\zeta\|^2 + L\|QC(\kappa)\|(\|\zeta\|^2 + \|\xi\|^2) =$$
$$-L(1 - \|QC(\kappa)\|)\|\zeta\|^2 + L\|QC(\kappa)\|\|\xi\|^2$$

According to the third item of Assumption 2, there exists a continuously differentiable, positive definite and radially unbounded scalar function $V_3(\eta)$, the time derivative of which along the solution of (21) satisfies

$$\dot{V}_3(\eta) = \alpha_1(\|L^{n-1}bE^{\mathrm{T}}(\xi+\zeta)\|^2 + \|L^{n-1}bE^{\mathrm{T}}F\|^2)$$
$$-\alpha_2\|\eta\|^2 \leqslant 2\alpha_1\bar{b}^2L^{2(n-1)}(\|\xi\|^2 + \|\zeta\|^2) + \alpha_1\bar{b}^2L^{2(n-1)}$$
$$\|F\|^2 - \alpha_2\|\eta\|^2 \leqslant 2\alpha_1\bar{b}^2L^{2(n-1)}(\|\xi\|^2 + \|\zeta\|^2)$$
$$-\alpha_2\|\eta\|^2 + \alpha_1\bar{b}^2L^{2(n-1)}\left(\gamma\sum_{i=1}^{n}|\xi_i+\zeta_i|\right)^2 \leqslant$$
$$2\alpha_1\bar{b}^2L^{2(n-1)}(\|\xi\|^2 + \|\zeta\|^2) - \alpha_2\|\eta\|^2 +$$
$$\alpha_1\bar{b}^2L^{2(n-1)}[\sqrt{n}\gamma(\|\xi\| + \|\zeta\|)]^2 \leqslant$$
$$2\alpha_1\bar{b}^2L^{2(n-1)}(\|\xi\|^2 + \|\zeta\|^2) - \alpha_2\|\eta\|^2 +$$
$$2n\alpha_1\bar{b}^2L^{2(n-1)}\gamma^2(\|\xi\|^2 + \|\zeta\|^2) =$$
$$2(n\gamma^2+1)\alpha_1\bar{b}^2L^{2(n-1)}(\|\xi\|^2 + \|\zeta\|^2) - \alpha_2\|\eta\|^2$$

Now, we consider the function

$$V(\xi,\zeta,\eta) = V_1(\xi) + V_2(\zeta) + \alpha_2^{-1}L^{-2(n-1)}(\bar{c}\|P(\kappa)\|+1)V_3(\eta)$$

which is a continuously differentiable, positive definite and radially unbounded scalar function, and satisfies

$$\dot{V}(\xi,\zeta,\eta) = \dot{V}_1(\xi) + \dot{V}_2(\zeta) + \alpha_2^{-1}L^{-2(n-1)}\cdot$$
$$(\bar{c}\|P(\kappa)\|+1)\dot{V}_3(\eta) \leqslant [-L + (3\sqrt{n}\gamma + \bar{c} + 3\bar{b}\bar{c})\|P(\kappa)\|]\cdot$$
$$\|\xi\|^2 + (\sqrt{n}\gamma + \bar{b}\bar{c})\|P(\kappa)\|\|\zeta\|^2 + L^{-2(n-1)}\bar{c}\|P(\kappa)\|\cdot$$
$$\|\eta\|^2 - L(1 - \|QC(\kappa)\|)\|\zeta\|^2 + L\|QC(\kappa)\|\|\xi\|^2 +$$
$$\alpha_2^{-1}L^{-2(n-1)}(\bar{c}\|P(\kappa)\|+1)[2(n\gamma^2+1)\alpha_1\bar{b}^2L^{2(n-1)}\cdot$$
$$(\|\xi\|^2 + \|\zeta\|^2) - \alpha_2\|\eta\|^2] \leqslant [-(1 - \|QC(\kappa)\|)L +$$
$$(3\sqrt{n}\gamma + \bar{c} + 3\bar{b}\bar{c})\|P(\kappa)\| + 2(n\gamma^2+1)\alpha_1\alpha_2^{-1}\bar{b}^2\cdot$$
$$(\bar{c}\|P(\kappa)\|+1)]\|\xi\|^2 + [-(1 - \|QC(\kappa)\|)L +$$
$$(\sqrt{n}\gamma + \bar{b}\bar{c})\|P(\kappa)\| + 2(n\gamma^2+1)\alpha_1\alpha_2^{-1}\bar{b}^2\cdot$$
$$(\bar{c}\|P(\kappa)\|+1)]\|\zeta\|^2 - L^{-2(n-1)}\|\eta\|^2.$$

Let

$$\mu = (3\sqrt{n}\gamma + \bar{c} + 3\bar{b}\bar{c})\|P(\kappa)\| + 2(n\gamma^2+1)\alpha_1\alpha_2^{-1}\bar{b}^2(\bar{c}\|P(\kappa)\|+1)$$

and

$$L_0 = (1 - \|QC(\kappa)\|)^{-1}(\mu+1). \tag{24}$$

Then, it is obvious that the following inequality

$$\dot{V}(\xi,\zeta,\eta) \leqslant -\|\xi\| - \|\zeta\|^2 - L^{-2(n-1)}\|\eta\|$$

holds when $L \geqslant L_0$. This implies that the closed-loop system (18) and (20) is globally asymptotically stable, hence the compensator (10) and (20) globally asymptotically stabilizes system (1).

According to the above design and analysis, we can give the dynamic output compensator as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + La_1\kappa(y - \hat{x}_1) \\ \quad\vdots \\ \dot{\hat{x}}_{n-1} = \hat{x}_n + L^{n-1}a_{n-1}\kappa^{n-1}(y - \hat{x}_1) \\ \dot{\hat{x}}_n = u + L^n a_n\kappa^n(y - \hat{x}_1) \\ u = -(b_nL^n\hat{x}_1 + b_{n-1}L^{-(n-1)}\hat{x}_2 + \cdots + b_1L\hat{x}_n) \end{cases} \tag{25}$$

which takes the form (8). □

Remark 2. Clearly, the result in this paper is the extension of the two ones in [4, 5]. In fact, on one hand, if there is no input unmodeled dynamics in system (1), then the problem in this paper reduces to the case in [5], and L_0 is computed by $L_0 = (1 - \|QC(\kappa)\|)^{-1}(3\sqrt{n}\gamma\|P(\kappa)\| + 1)$. On the other hand, if the nonlinear terms satisfy the triangular condition (7) as shown in [4], they also satisfy the relaxed triangular condition (6) when $\gamma = n\lambda$, that is to say that the problem in [4] can be solved by the result in this paper. However, the converse is not always true, which will be further illuminated by the example in Section 4.

4 Design example

To check the effect of our result, we consider the following simple system

$$\begin{cases} \dot{x}_1 = x_2 + \theta\sqrt{|x_1|} \\ \dot{x}_2 = v + \sqrt{|x_2|} \\ y = x_1 \\ \dot{\varepsilon} = -\sigma\varepsilon + u \\ v = \varepsilon + u \end{cases} \tag{26}$$

where $\theta \in [0.1, 1]$ and $\sigma \in [2, 3]$ are unknown constants.

The above system fails to satisfy the triangular condition given in [4]. Consequently, the global stabilization problem of system (26) via output feedback can not be solved by the result of [4]. Fortunately, the system (26) satisfies Assumptions 1 and 2 if $\gamma = \bar{c} = \bar{b} = 1$, $\alpha_1 = 4$, and $\alpha_2 = 2$. Hence, by Theorem 1, there exists a dynamic output compensator globally asymptotically stabilizing system (26). By the proof of Theorem 1, to construct such a compensator, we choose $a_1 = a_2 = 1$, $b_1 = 2$, $b_2 = 1$, $\kappa = 0.25$, and $L = 1400$. Then, we can get the compensator as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + 350\,(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = u + 122500\,(y - \hat{x}_1) \\ u = -1960000\hat{x}_1 - 2800\hat{x}_2. \end{cases} \tag{27}$$

For the numerical simulation, we choose $\theta = 1$, $\sigma = 2$, and the initial states to be

$$(x_1(0), x_2(0), \varepsilon(0), \hat{x}_1(0), \hat{x}_2(0)) = (1, 5, 1, 3, 5).$$

The simulation results are shown in Figs. 1 and 2. They show the effect of the output dynamic compensator (27).

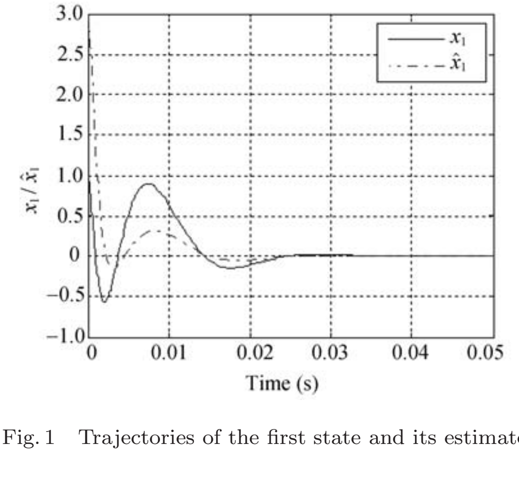

Fig. 1 Trajectories of the first state and its estimate

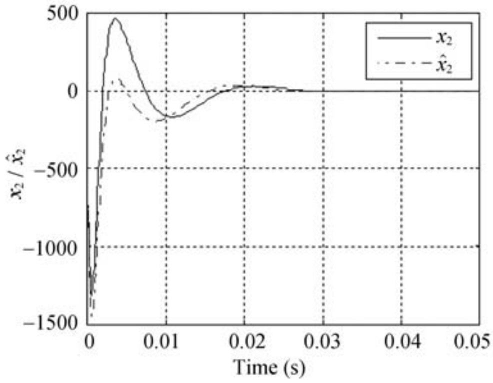

Fig. 2 Trajectories of the second state and its estimate

Remark 3. From the above design example and the example in [5], we can see that the scheme of stabilizing nonlinear systems by high gain dynamic output compensators will be, on some occasions, of limited practical use due to the very high gains required, which cannot be physically implemented in practical applications. Hence, the problem of how to effectively use this scheme in practice must be further considered.

5 Conclusions

In this paper, we studied the problem of robust global stabilization using output feedback for a class of uncertain nonlinear systems with input unmodeled dynamics under a far more relaxed condition than the existing triangular-type condition. Based on the nonseparation principle, a robust dynamic output compensator consisting of a linear high gain observer and a linear high gain controller was designed to guarantee the globally asymptotic stability of the closed-loop system. The effect and usefulness of the proposed method was illustrated by a simple example.

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