International Journal of Automation and Computing 3 (2006) 235-243

Modeling and Control of Hybrid Machine Systems — a Five-bar Mechanism Case

Hongnian Yu*

Faculty of Computing, Engineering and Technology, Staffordshire University, Stafford ST16 9DG, UK

Abstract: A hybrid machine (HM) as a typical mechatronic device, is a useful tool to generate smooth motion, and combines the motions of a large constant speed motor with a small servo motor by means of a mechnical linkage mechanism, in order to provide a powerful programmable drive system. To achieve design objectives, a control system is required. To design a better control system and analyze the performance of an HM, a dynamic model is necessary. This paper first develops a dynamic model of an HM with a five-bar mechanism using a Lagrangian formulation. Then, several important properties which are very useful in system analysis, and control system design, are presented. Based on the developed dynamic model, two control approaches, computed torque, and combined computed torque and slide mode control, are adopted to control the HM system. Simulation results demonstrate the control performance and limitations of each control approach.

Keywords: Hybrid machine (HM), Lagrangian systems, dynamics, computed torque control, sliding mode control.

1 Introduction

Robotics and intelligent machines have been identified as one of the five main important application areas for control technology, in a recent expert meeting of the control community[1]. The ideas and experiences from research in robotics and intelligent machines can be applied in the design, modelling, analysis, and control of hybrid machines (HMs). An HM is a useful tool to generate smooth motion, and is an adjustable mechanism with two motors, a constant velocity (CV) motor, and a servo-motor, which produces a programmable range of highly nonlinear output motions[2∼4]. HMs transmit power from the servo and CV motors, and power savings are obtained over the case where an output shaft is directly linked with a servomotor[5]. Meanwhile, the size of servomotor used in HMs can be minimized in comparison to the size of motor required to drive a load directly.

Output motion containing a dwell, is widely employed in common industrial applications, such as cutting, printing, and stamping. A cut-to-length machine is investigated in [4]. An HM for producing a reciprocating motion is studied in [2]. An HM with a five-bar mechanism is investigated in [3]. The optimal design for an HM with a four-bar mechanism for path generation, and motions with reduced harmonic content, is investigated in [6]. To produce more complicated paths, an HM with a five-bar mechanism was also optimally designed in [6]. An HM with a seven-bar mechanism was developed in [4]. Kinematics optimization for an HM with a seven-bar mechanism, using fuzzy logic and a neural network, was studied in [5], while basic control issues based on a simplified dynamic model were discussed in [7]. The benefits of using HMs, and a comparison of torques produced by a direct servo motor and an HM, were investigated in [7]. This work showed that an HM required less torque and less power than a direct drive motor when producing a required motion, as shown in Fig. 1.

A dynamic model plays a very important role in terms of system validation, analysis, and control system design. However, unlike robots with open-chain mechanisms, the derivation of dynamic equations of motion for a generic HM suitable for control system design, is still an active research topic; because of the complexity of the kinematics and dynamics analysis involved. The development of a dynamic model for HMs has received intensive research in recent years[6∼12]. An approximated dynamic model of an HM with a seven-bar mechanism was proposed in [7]. A full dynamic model was developed using a Lagrangian formulation in [8,9]. However, the developed model requires further validation. Modelling of an HM with a five-bar mechanism was investigated in [6,10] using a model reduction approach. In [6,12,13], several basic control approaches were proposed, based on the model in [6]. In this paper, we follow the procedures proposed in [8,9] to develop the kinematics and dynamic model for an HM with a five-bar mechanism. One of the main aims is to provide a validation means for the model developed in [8].

Based on the developed dynamic model, two well known control approaches are adopted to control a five-bar HM. The first approach is computed torque control[14], which gives excellent control performance when there is no uncertainty. However, it is sensitive

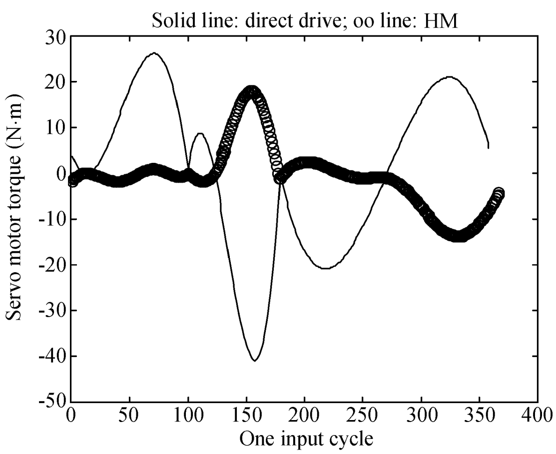
Manuscript received November 30, 2005; revised February 11, 2006.

The work was supported in part by the EPSRC research council (No. GR/M29108/01).

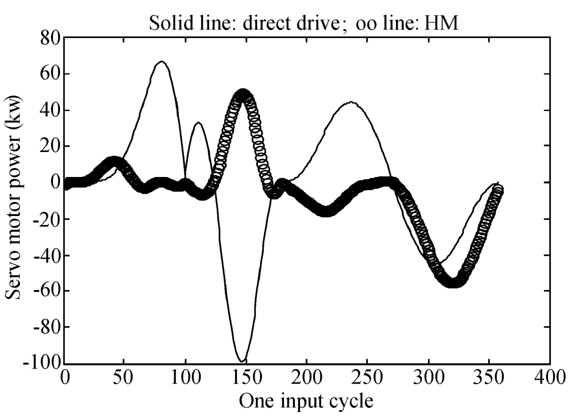
**E-mail address*: h.yu@staffs.ac.uk

to model uncertainty and other disturbance. The second approach is combined computed torque and sliding mode control[15], which provides good control performance when there is uncertainty.

This paper provides two contributions. First, a full dynamic model of an HM system with a five-bar mechanism is provided. The modelling method is based on a Lagrangian formulation. As a result, systematic and quantitative HM dynamics, i.e. a relation between the Lagrangian formulation (including the inertial matrix, the vector of centripetal and Coriolis torque, and the vector of gravitational torque) and linkages is presented. The properties of the dynamic model are summarised. The second, is that two control approaches are applied for controlling an HM system based on the developed dynamic model. Simulation results show the effectiveness of the investigated HM and control approaches.



(a) Servo motor torque



(b) Servo motor power

Fig. 1 Comparison of the torque and power produced by a direct servo motor and an HM

The paper is organized as follows: Section 2 develops the kinematics and dynamics for a five-bar mechanism. The dynamics development is based on a Lagrangian formulation. The computation procedures and properties of the developed dynamic model are also presented. Two control approaches are presented in Section 3. When the parameters of the HM are known, the computed torque control approach provides a good control policy and generates good tracking performance. However, when the HM parameters are unknown or changing with the environment, the computed torque control approach cannot provide an acceptable result. The sliding mode control approach is a good and simple robust control policy which can overcome system uncertainty well. To demonstrate the above argument, simulation results are presented in Section 4. Conclusions are given in Section 5.

2 Modelling an HM

This section concentrates on the development of the kinematics and dynamics for the five-bar HM shown in Fig. 2. Based on the developed model, several properties which will be used in the control system design in Section 3 are also provided.

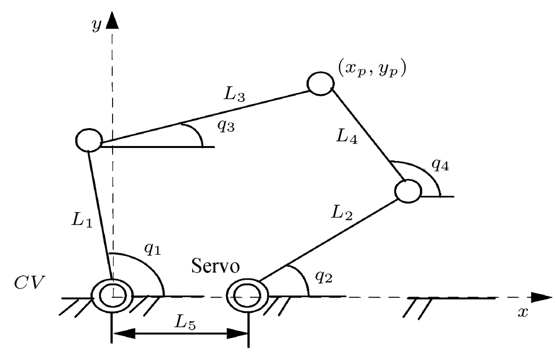


Fig. 2 An HM with a five-bar mechanism

2.1 Lagrangian equation

The general equation of motion for a mechanical linkage system, which is the basic structure of an HM, can be conveniently expressed through the application of Lagrangian equations[14]. Lagrangian equations for an HM are shown in Fig. 2, and can be written

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i \tag{1}$$

where $L = K - P$ is known as a Lagrangian function; K and P are respectively the total kinetic energy and potential energy of the linkage system, and τ_i is the generalised force or torque associated with the generalised co-ordinate q_i (angular displacement (rad)). The system has five links, $L1, \cdots, L_5$. Since link 5 is fixed, it can be neglected in the development of the dynamic equations. Let q_i denote the angle with respect to the reference position of the link parallel to, and having the same direction as the x co-ordinate shown in Fig. 2.

Then the kinetic energy of the HM system is:

$$K = \frac{1}{2}\sum_{i=1}^{4}[I_i\dot{q}_i^2 + m_i(\dot{x}_i^2 + \dot{y}_i^2)] = \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}D\dot{Q}_{12} \quad (2)$$

where I_i includes the inertia of the motor armature, the inertia of the load, and the inertia of the linkages, x_i is the x co-ordinate of mass centre of linkages L_i, y_i is the y co-ordinate of mass centre of linkages L_i, m_i is the mass of linkage L_i, $Q_{12} = [q_1 \quad q_2]^{\mathrm{T}}$ is a vector which represents two independent joints (inputs), i.e. the inputs from the server and CV motor respectively, and D is the generalised inertia matrix (which will be developed later). Potential energy is:

$$P = \sum_{i=1}^{4} m_i g y_i = gM^{\mathrm{T}}Y \quad (3)$$

where g is gravity acceleration, $M = [m_1\ m_2\ m_3\ m_4]^{\mathrm{T}}$ is the mass vector, and $Y = [y_1\ y_2\ y_3\ y_4]^{\mathrm{T}}$. By inserting (2) and (3) into (1) we have:

$$D\ddot{Q}_{12}+\dot{D}\dot{Q}_{12}-\frac{\partial K}{\partial Q_{12}}+\frac{\partial P}{\partial Q_{12}} = D\ddot{Q}_{12}+C\dot{Q}_{12}+G = \tau_{12} \quad (4)$$

where $C\dot{Q}_{12} = \dot{D}\dot{Q}_{12} - \dfrac{\partial K}{\partial Q_{12}}$ is a vector containing the effects of the Coriolis and centripetal torque, $G = \dfrac{\partial P}{\partial Q_{12}}$ is a vector of gravity torque, and $\tau_{12} = [\tau_1\ \tau_2]$ is a vector of the generalised torque of joints one and two, respectively. Our objective in dynamic development is to determine the details of the representation of D, C, and G in (4), for the HM shown in Fig. 2.

2.2 Kinematics of an HM

There is a loop $L_1L_3L_4L_2L_5$ in the HM shown in Fig. 2, which can be used to set up kinematic equations. Two equations from the loop are:

$$L1C1 + L3C3 - L2C2 - L4C4 - L5 = 0 \quad (5)$$
$$L_1S_1 + L_3S_3 - L_2S_2 - L_4S_4 = 0 \quad (6)$$

where $C_i = \cos(q_i)$ and $S_i = \sin(q_i)$ for simplicity reasons. Equations (5) and (6) can be used to determine the kinematics of the HM shown in Fig. 2.

2.2.1 Computation of q_3 and q_4

This HM has two degrees-of-freedom, and four joint variables q_1, q_2, q_3, and q_4. As discussed earlier, only q_1 and q_2 are independent, while the rest of the joints (q_3 and q_4) are functions of joints q_1 and q_2. The objective of this section is to find a representation for joints q_3 and q_4 in terms of q_1 and q_2. Using (5) and (6) and a simple algebra, we can find representations of q_3 and q_4 in terms of q_1 and q_2 as:

$$q_3 = \arctan\left(\frac{A}{C}\right) - \beta_1 \quad (7)$$

where $a = L_1C_1 - L_5 - L_2C_2$, $b = L_1S_1 - L_2S_2$, $A = L_4^2-(a^2+b^2+L_3^2)$, $C = 2L_3\sqrt{a^2+b^2}$, $\beta_1 = \arctan(a/b)$. Inserting (7) into (6), we have:

$$q_4 = \arctan\left(\frac{b + L_3S_3}{a + L_3C_3}\right). \quad (8)$$

2.2.2 Computation of $\dfrac{\partial q_i}{\partial q_1}$ and $\dfrac{\partial q_i}{\partial q_2}$ for $i = 3, 4$

Taking the partial differentiation of (5) and (6) with respect to q_1, and considering that $\dfrac{\partial q_2}{\partial q_1} = 0$; since q_1 and q_2 are independent of each other, we have:

$$L_1S_1 + L_3S_3\frac{\partial q_3}{\partial q_1} - L_4S_4\frac{\partial q_4}{\partial q_1} = 0$$
$$L_1C_1 + L_3C_3\frac{\partial q_3}{\partial q_1} - L_4C_4\frac{\partial q_4}{\partial q_1} = 0.$$

Solving these two equations gives:

$$\begin{bmatrix} \dfrac{\partial q_3}{\partial q_1} \\ \dfrac{\partial q_4}{\partial q_1} \end{bmatrix} = L_1\begin{bmatrix} -L_3S_3 & L_4S_4 \\ -L_3C_3 & L_4C_4 \end{bmatrix}^{-1}\begin{bmatrix} S_1 \\ C_1 \end{bmatrix} = L_1A_{34}^{-1}S_{c1} \quad (9)$$

where $A_{34} = \begin{bmatrix} -L_3S_3 & L_4S_4 \\ -L_3C_3 & L_4C_4 \end{bmatrix}$ and $S_{c1} = [S_1 \quad C_1]^{\mathrm{T}}$.

Taking the partial differentiation of (5) and (6) with respect to q_2, and considering $\dfrac{\partial q_1}{\partial q_2} = 0$, we have:

$$\begin{bmatrix} \dfrac{\partial q_3}{\partial q_2} \\ \dfrac{\partial q_4}{\partial q_2} \end{bmatrix} = -L_2\begin{bmatrix} -L_3S_3 & L_4S_4 \\ -L_3C_3 & L_4C_4 \end{bmatrix}^{-1}\begin{bmatrix} S_2 \\ C_2 \end{bmatrix} = -L_2A_{34}^{-1}S_{c2} \quad (10)$$

where $S_{c2} = [S_2\ C_2]^{\mathrm{T}}$.

2.2.3 Computation of $\dot{q}_3$ and $\dot{q}_4$

Taking the derivatives of (5) and (6) with the respect to time, gives:

$$L_1S_1\dot{q}_1 = L_2S_2\dot{q}_2 + L_3S_3\dot{q}_3 - L_4S_4\dot{q}_4 = 0$$
$$L_1C_1\dot{q}_1 - L_2C_2\dot{q}_2 + L_3C_3\dot{q}_3 - L_4C_4\dot{q}_4 = 0.$$

Writing the above equations in vector form gives:

$$\begin{bmatrix} \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} -L_3S_3 & L_4S_4 \\ -L_3C_3 & L_4C_4 \end{bmatrix}^{-1}\begin{bmatrix} L_1S_1 & -L_2S_2 \\ L_1C_1 & -L_2C_2 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}.$$

Let $Q_{34} = [q_3\ q_4]^{\mathrm{T}}$, and $A_{12} = \begin{bmatrix} L_1S_1 & -L_2S_2 \\ L_1C_1 & -L_2C_2 \end{bmatrix}$. Then the above equation can be written

$$\dot{Q}_{34} = A_{34}^{-1}A_{12}\dot{Q}_{12}. \quad (11)$$

In system design, it is necessary to avoid singular values of A_{34}, therefore its inverse is used in (9)∼(11). We can set $q_3 \neq q_4$ to avoid the singular value issue[5]. The singular value issue was not discussed in [12,13].

2.2.4 Computation of $\dot{x}_i$ and $\dot{y}_i$ where $i = 1, 2, 3, 4$

From Fig. 2, we can easily obtain:

$$\begin{array}{ll} x_1 = L_{c1}C_1, & y_1 = L_{c1}S_1 \\ x_2 = L_{c2}C_2, & y_2 = L_{c2}S_2 \\ x_3 = L_{c3}C_3 + L_1C_1, & y_3 = L_{c3}S_3 + L_1S_1 \\ x_4 = L_{c4}C_4 + L_2C_2, & y_4 = L_{c4}S_4 + L_2S_2 \end{array} \quad (12)$$

where $L_{ci}(i = 1, 2, 3, 4)$ are the distances from the joint to the centre of mass for the ith link. Taking the derivative with time for the above equations, we have:

$$\begin{array}{ll} \dot{x}_1 = -L_{c1}S_1\dot{q}_1, & \dot{y}_1 = L_{c1}C_1\dot{q}_1 \\ \dot{x}_2 = -L_{c2}S_2\dot{q}_2, & \dot{y}_2 = L_{c2}C_2\dot{q}_2 \\ \dot{x}_3 = -L_{c3}S_3\dot{q}_3 - L_1S_1\dot{q}_1, & \dot{y}_3 = L_{c3}C_3\dot{q}_3 + L_1C_1\dot{q}_1 \\ \dot{x}_4 = -L_{c3}S_4\dot{q}_4 - L_2S_2\dot{q}_2, & \dot{y}_4 = L_{c4}C_4\dot{q}_4 + L_2C_2\dot{q}_2. \end{array}$$

The above equations can be rewritten in vector form as:

$$\dot{X}_{12} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = -\begin{bmatrix} L_{c1}S_1 & 0 \\ 0 & L_{c2}S_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = -A_s\dot{Q}_{12} \quad (13)$$

$$\dot{Y}_{12} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} L_{c1}C_1 & 0 \\ 0 & L_{c2}C_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = A_c\dot{Q}_{12}. \quad (14)$$

Using (11), we have:

$$\dot{X}_{34} = \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = -(A_{s34}A_{34}^{-1}A_{12} + A_s)\dot{Q}_{12} \quad (15)$$

$$\dot{Y}_{34} = \begin{bmatrix} \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = (A_{c34}A_{34}^{-1}A_{12} + A_c)\dot{Q}_{12} \quad (16)$$

where

$$A_s = \begin{bmatrix} L_{c1}S_1 & 0 \\ 0 & L_{c2}S_2 \end{bmatrix}$$

$$A_c = \begin{bmatrix} L_{c1}C_1 & 0 \\ 0 & L_{c2}C_2 \end{bmatrix}$$

$$A_{s34} = \begin{bmatrix} L_{c3}S_3 & 0 \\ 0 & L_{c4}S_4 \end{bmatrix}$$

$$A_{c34} = \begin{bmatrix} L_{c3}C_3 & 0 \\ 0 & L_{c4}C_4 \end{bmatrix}.$$

2.2.5 Computation of $\frac{\partial y_i}{\partial q_1}$ and $\frac{\partial y_i}{\partial q_2}$

From (12), and considering that q_1 and q_2 are independent of each other, we have the following:

$$\frac{\partial y_1}{\partial q_1} = L_{c1}C_1, \quad \frac{\partial y_1}{\partial q_2} = 0$$

$$\frac{\partial y_2}{\partial q_1} = 0, \quad \frac{\partial y_2}{\partial q_2} = L_{c2}C_2$$

$$\frac{\partial y_3}{\partial q_1} = L_{c3}C_3\frac{\partial q_3}{\partial q_1} + L_1C_1, \quad \frac{\partial y_3}{\partial q_2} = L_{c3}C_3\frac{\partial q_3}{\partial q_2}$$

$$\frac{\partial y_4}{\partial q_1} = L_{c4}C_4\frac{\partial q_4}{\partial q_1}, \quad \frac{\partial y_4}{\partial q_2} = L_{c4}C_4\frac{\partial q_4}{\partial q_2} + L_2C_2.$$

Rewriting the above equations in vector form gives:

$$\frac{\partial Y_{12}}{\partial q_1} = \begin{bmatrix} L_{c1}C_1 \\ 0 \end{bmatrix} = L_{c1}C_1B_1 \quad (17)$$

$$\frac{\partial Y_{12}}{\partial q_2} = \begin{bmatrix} 0 \\ L_{c2}C_2 \end{bmatrix} = L_{c2}C_2B_2 \quad (18)$$

$$\frac{\partial Y_{34}}{\partial q_1} = \begin{bmatrix} L_{c3}C_3 & 0 \\ 0 & L_{c4}C_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial q_3}{\partial q_1} \\ \frac{\partial q_4}{\partial q_1} \end{bmatrix} + \begin{bmatrix} L_1C_1 \\ 0 \end{bmatrix} = L_1A_{c34}A_{34}^{-1}S_{c1} + L_1C_1B_1 \quad (19)$$

$$\frac{\partial Y_{34}}{\partial q_2} = \begin{bmatrix} L_{c3}C_3 & 0 \\ 0 & L_{c4}C_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial q_3}{\partial q_2} \\ \frac{\partial q_4}{\partial q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ L_2C_2 \end{bmatrix} = -L_2A_{c34}A_{34}^{-1}S_{c2} + L_2C_2B_2 \quad (20)$$

2.3 Dynamic model of an HM

In this section, we will develop the dynamics of an HM using the kinematics described in Section 2.2.

2.3.1 Dynamic model

Kinetic energy (2) can be rewritten

$$K = \frac{1}{2}\sum_{i=1}^{4}[I_i\dot{q}_i^2 + m_i(\dot{x}_i^2 + \dot{y}_i^2)] = \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}I_{12}\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{34}^{\mathrm{T}}I_{34}\dot{Q}_{34} + \frac{1}{2}\dot{X}_{12}^{\mathrm{T}}M_{12}\dot{X}_{12} + \frac{1}{2}\dot{X}_{34}^{\mathrm{T}}M_{34}\dot{X}_{34} + \frac{1}{2}\dot{Y}_{12}^{\mathrm{T}}M_{12}\dot{Y}_{12} + \frac{1}{2}\dot{Y}_{34}^{\mathrm{T}}M_{34}\dot{Y}_{34}$$

where $I_{12} = diag[I_1 \quad I_2]$, $I_{34} = diag[I_3 \quad I_4]$, $M_{12} = diag[M_1 \quad M_2]$, $M_{34} = diag[M_3 \quad M_4]$. Using the results obtained in Section 2.2, kinetic energy can be written:

$$K = \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}I_{12}\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}A_{12}^{\mathrm{T}}A_{34}^{-\mathrm{T}}I_{34}A_{34}^{-1}A_{12}\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}A_s^{\mathrm{T}}M_{12}A_s\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}(A_{s34}A_{34}^{-1}A_{12}+A_s)^{\mathrm{T}}M_{34}(A_{s34}A_{34}^{-1}A_{12}+A_s)\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}A_c^{\mathrm{T}}M_{12}A_c\dot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}(A_{c34}A_{34}^{-1}A_{12}+A_c)^{\mathrm{T}}M_{34}(A_{c34}A_{34}^{-1}A_{12}+A_c)\dot{Q}_{12} = \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}D\dot{Q}_{12} \quad (21)$$

K can also be represented as:

$$K = \frac{1}{2}[(I_1 + m_1L_{c1}^2 + m_3L_1^2)\dot{q}_1^2 + (I_2 + m_3L_{c2}^2 + m_4L_2^2)\dot{q}_2^2 + 2m_3L_2L_{c4}C_{1-3}\dot{q}_1\dot{q}_3 + (I_3 + m_3L_{c3}^2)\dot{q}_3^2 + 2m_4L_2L_{c4}C_{2-4}\dot{q}_2\dot{q}_4 + (I_4 + m_4L_{c4}^2)\dot{q}_4^2]. \qquad (22)$$

Using (22), we have:

$$\frac{\partial K}{Q_{12}} = \begin{bmatrix} \dfrac{\partial K}{\partial q_1} \\ \dfrac{\partial K}{\partial q_2} \end{bmatrix} =$$

$$\begin{bmatrix} -2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & -2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \dot{Q}_{34} +$$

$$\begin{bmatrix} [\frac{\partial q_3}{\partial q_1} \;\; \frac{\partial q_4}{\partial q_1}] \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \\ [\frac{\partial q_3}{\partial q_2} \;\; \frac{\partial q_4}{\partial q_2}] \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \end{bmatrix} \dot{Q}_{34}$$

$$= C_{s34}\dot{Q}_{34} + C_{34}\dot{Q}_{34} =$$

$$(C_{s34} + C_{34})A_{34}^{-1}A_{12}\dot{Q}_{12} = C_{KQ}\dot{Q}_{12} \qquad (23)$$

where

$$C_{s34} = \begin{bmatrix} -2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & -2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix}$$

$$C_{34} = \begin{bmatrix} [\frac{\partial q_3}{\partial q_1} \;\; \frac{\partial q_4}{\partial q_1}] \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \\ [\frac{\partial q_3}{\partial q_2} \;\; \frac{\partial q_4}{\partial q_2}] \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \end{bmatrix} =$$

$$\begin{bmatrix} (L_1A_{34}^{-1}S_{c1})^{\mathrm{T}} \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \\ (L_1A_{34}^{-1}S_{c2})^{\mathrm{T}} \begin{bmatrix} 2m_3L_1L_{c3}S_{1-3}\dot{q}_1 & 0 \\ 0 & 2m_4L_2L_{c4}S_{2-4}\dot{q}_2 \end{bmatrix} \end{bmatrix}$$

$$C_{KQ} = (C_{s34} + C_{34})A_{34}^{-1}A_{12}.$$

From (21), we can easily obtain:

$$D = I_{12} + (A_{34}^{-1}A_{12})^{\mathrm{T}}I_{34}A_{34}^{-1}A_{12} + A_s^{\mathrm{T}}M_{12}A_s + (A_{s34}A_{34}^{-1}A_{12} + A_s)^{\mathrm{T}}M_{34}(A_{s34}A_{34}^{-1}A_{12} + A_s) + A_c^{\mathrm{T}}M_{12}A_c + (A_{c34}A_{34}^{-1}A_{12} + A_c)^{\mathrm{T}}M_{34}(A_{c34}A_{34}^{-1}A_{12} + A_c). \qquad (24)$$

Taking the time derivative of D, we have:

$$\dot{D} = 2(A_{34}^{-1}A_{12})^{\mathrm{T}}I_{34}A_{34}^{-1}(\dot{A}_{12} - \dot{A}_{34}A_{34}^{-1}A_{12}) + 2A_s^{\mathrm{T}}M_{12}\dot{A}_s + 2(A_{s34}A_{34}^{-1}A_{12} + A_s)^{\mathrm{T}}M_{34}((\dot{A}_{s34} - A_{s34}A_{34}^{-1}\dot{A}_{34})A_{34}^{-1}A_{12} + A_{s34}A_{34}^{-1}\dot{A}_{12} + \dot{A}_s) + 2A_c^{\mathrm{T}}M_{12}\dot{A}_c + 2(A_{c34}A_{34}^{-1}A_{12} + \dot{A}_s)M_{34}((\dot{A}_{c34} - A_{c34}A_{34}^{-1}\dot{A}_{34})A_{34}^{-1}A_{12} + A_{c34}A_{34}^{-1}\dot{A}_{12} + \dot{A}_c)$$

where

$$\dot{A}_{12} = \begin{bmatrix} L_1C_1\dot{q}_1 & -L_2C_2\dot{q}_2 \\ -L_1S_1\dot{q}_2 & L_2S_2\dot{q}_2 \end{bmatrix}$$

$$\dot{A}_s = \begin{bmatrix} -L_{c1}C_1\dot{q}_1 & 0 \\ 0 & -L_{c2}C_2\dot{q}_2 \end{bmatrix}$$

$$\dot{A}_c = \begin{bmatrix} -L_{c1}S_1\dot{q}_1 & 0 \\ 0 & -L_{c2}S_2\dot{q}_2 \end{bmatrix}$$

$$\dot{A}_{s34} = \begin{bmatrix} L_{c3}C_3\dot{q}_3 & 0 \\ 0 & L_{c4}C_4\dot{q}_4 \end{bmatrix}$$

$$\dot{A}_{c34} = -\begin{bmatrix} L_{c3}S_3\dot{q}_3 & 0 \\ 0 & L_{c4}S_4\dot{q}_4 \end{bmatrix}$$

$$\dot{A}_{34} = \begin{bmatrix} -L_3C_3\dot{q}_3 & L_4C_4\dot{q}_4 \\ L_3S_3\dot{q}_3 & -L_4S_4\dot{q}_4 \end{bmatrix} = \begin{bmatrix} \dot{Q}_{34}^{\mathrm{T}}A_{-34} \\ \dot{Q}_{34}^{\mathrm{T}}A_{3-4} \end{bmatrix}.$$

Let $D = \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix}$, and we have:

$$\dot{D} = \begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{11}}{\partial q_1} \\ \dfrac{\partial d_{11}}{\partial q_2} \end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} \end{bmatrix} \\ \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} \end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{22}}{\partial q_1} \\ \dfrac{\partial d_{22}}{\partial q_2} \end{bmatrix} \end{bmatrix} \qquad (25)$$

$$\dot{D}\dot{Q}_{12} = \begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{11}}{\partial q_1} \\ \dfrac{\partial d_{11}}{\partial q_2} \end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} \end{bmatrix} \\ \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} \end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{22}}{\partial q_1} \\ \dfrac{\partial d_{22}}{\partial q_2} \end{bmatrix} \end{bmatrix} \dot{Q}_{12} =$$

$$\begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{11}}{\partial q_1} & \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{11}}{\partial q_2} & \dfrac{\partial d_{12}}{\partial q_2} \end{bmatrix} \dot{Q}_{12} \\ \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{12}}{\partial q_1} & \dfrac{\partial d_{22}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} & \dfrac{\partial d_{22}}{\partial q_2} \end{bmatrix} \dot{Q}_{12} \end{bmatrix}$$

$$\frac{\partial K}{\partial Q_{12}} = \frac{1}{2} \begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{11}}{\partial q_1} & \dfrac{\partial d_{12}}{\partial q_1} \\ \dfrac{\partial d_{12}}{\partial q_2} & \dfrac{\partial d_{22}}{\partial q_2} \end{bmatrix} \dot{Q}_{12} \\ \dot{Q}_{12}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial d_{11}}{\partial q_2} & \dfrac{\partial d_{12}}{\partial q_2} \\ \dfrac{\partial d_{12}}{\partial q_2} & \dfrac{\partial d_{22}}{\partial q_2} \end{bmatrix} \dot{Q}_{12} \end{bmatrix} =$$

$$\frac{1}{2}\begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{11}}{\partial q_1}\\ \frac{\partial d_{12}}{\partial q_1}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{12}}{\partial q_1}\\ \frac{\partial d_{22}}{\partial q_1}\end{bmatrix} \\ \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{11}}{\partial q_2}\\ \frac{\partial d_{12}}{\partial q_2}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{12}}{\partial q_2}\\ \frac{\partial d_{22}}{\partial q_2}\end{bmatrix}\end{bmatrix}\dot{Q}_{12}.$$

Using the above two equations, we have:

$$\dot{D}\dot{Q}_{12} - \frac{\partial K}{\partial Q_{12}} =$$
$$\frac{1}{2}\begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{11}}{\partial q_1}\\ 2\frac{\partial d_{11}}{\partial q_2}-\frac{\partial d_{12}}{\partial q_1}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{12}}{\partial q_1}\\ 2\frac{\partial d_{12}}{\partial q_2}-\frac{\partial d_{22}}{\partial q_1}\end{bmatrix} \\ \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}2\frac{\partial d_{12}}{\partial q_1}-\frac{\partial d_{11}}{\partial q_2}\\ \frac{\partial d_{12}}{\partial q_2}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}2\frac{\partial d_{22}}{\partial q_1}-\frac{\partial d_{12}}{\partial q_2}\\ \frac{\partial d_{22}}{\partial q_2}\end{bmatrix}\end{bmatrix}\dot{Q}_{12}.$$

Therefore, C in (4) is:

$$C =$$
$$\frac{1}{2}\begin{bmatrix} \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{11}}{\partial q_1}\\ 2\frac{\partial d_{11}}{\partial q_2}-\frac{\partial d_{12}}{\partial q_1}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}\frac{\partial d_{12}}{\partial q_1}\\ 2\frac{\partial d_{12}}{\partial q_2}-\frac{\partial d_{22}}{\partial q_1}\end{bmatrix} \\ \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}2\frac{\partial d_{12}}{\partial q_1}-\frac{\partial d_{11}}{\partial q_2}\\ \frac{\partial d_{12}}{\partial q_2}\end{bmatrix} & \dot{Q}_{12}^{\mathrm{T}}\begin{bmatrix}2\frac{\partial d_{22}}{\partial q_1}-\frac{\partial d_{12}}{\partial q_2}\\ \frac{\partial d_{22}}{\partial q_2}\end{bmatrix}\end{bmatrix} =$$
$$\dot{D} - C_{KQ}. \tag{26}$$

Using the results in Section 2.2.5, the terms of gravity torque are:

$$G = \begin{bmatrix}\frac{\partial P}{\partial q_1}\\ \frac{\partial P}{\partial q_2}\end{bmatrix} = \begin{bmatrix} g(M_{g12}\frac{\partial Y_{12}}{\partial q_1} + M_{g34}\frac{\partial Y_{34}}{\partial q_1})\\ g(M_{g12}\frac{\partial Y_{12}}{\partial q_2} + M_{g34}\frac{\partial Y_{34}}{\partial q_2})\end{bmatrix} =$$
$$\begin{bmatrix} gL_{c1}C_1M_{g12}B_1 + gL_1M_{g34}(A_{c34}A_{34}^{-1}S_{c1} + C_1B_1)\\ gL_{c2}C_2M_{g12}B_2 + gL_2C_2M_{g34}B - gL_2M_{g34}A_{c34}A\end{bmatrix} \tag{27}$$

where $M_{g12} = [m_1 \quad m_2]$, and $M_{g34} = [m_3 \quad m_4]$.

We can summarise the procedures of computing the dynamics of HM (4) as follows:

1) Input the parameters of the mechanical linkage system, L_i, L_{ci}, m_i, I_i, for $i = 1, 2, 3, 4$.

2) Loop: Perform the following computation, for $t = 1, 2, \cdots, N$

a) Input the two independent joint variables q_1 and q_2, velocity $\dot{q}_1$ and $\dot{q}_2$, acceleration $\ddot{q}_1$ and $\ddot{q}_2$.

b) Compute the other joint variables q_3 and q_4 using (7) and (8).

c) Compute D using (24).

d) Compute C using (26).

e) Compute G using (27).

f) Compute $\tau_1 2$ using (4).

The above procedures are very useful for system analysis, simulation study, and controller design.

2.3.2 Properties

It can be proved that (4) has the following properties:

a) Inertia matrix D is symmetric, positive definite, and bounded above and below, i.e. $D^{\mathrm{T}} = D, X^{\mathrm{T}}DX > 0$ (X is an arbitrary vector), and there exist constants d_m and d_M, such that $d_m I \leqslant D \leqslant d_M I$ (I is a 2×2 identity matrix).

b) Matrices D and C satisfy:

$$\dot{Q}_{12}^{\mathrm{T}}(\dot{D} - 2C)\dot{Q}_{12} = 0.$$

c) The term $C\dot{Q}_{12}$ has:

$$\|C\dot{Q}_{12}\| \leqslant d\|\dot{Q}_{12}\|^2$$

where d is a scalar constant.

d) The gravity term G is bounded above, i.e.:

$$\|G\| \leqslant d_q$$

where d_g is a scalar constant.

e) Dynamics (4) defines a passive mapping from the input τ_{12} to the generalised velocity $\dot{Q}_{12}$, i.e.:

$$\int_0^t \dot{Q}_{12}^{\mathrm{T}}(u)\tau_{12}(u)du \geqslant 0$$

f) Dynamics (4) can be linearised in terms of unknown (or uncertain) parameters as follows:

$$\tau_{12} = D\ddot{Q}_{12} + C\dot{Q}_{12} + G = \Theta W + W_0.$$

The above properties are very useful in control system design, and to validate the design of an HM. Their proofs are given in Appendix.

3 Control of HM

The highly non-linear nature of HM dynamics (4), means that a linear control approach will only provide local and approximate results. In order to obtain global results, more advanced and quite different techniques from non-linear control theory are required. Many nonlinear based approaches have been proposed to control robot manipulators, e.g. geometric control, computed torque control, and sliding mode control, *etc*. In these approaches, a non-linear control law can be derived either using a feedforward (computed torque), or a feedback (feedback linearization, inverse dynamics) method, from which the intent of linearization and decoupling of non-linear robot dynamics can be established. Computed torque (also called inverse dynamics) is commonly referred to as a special case of feedback linearization in robot control.

In this section, we introduce two control approaches to control an HM. One is the popular computed torque control method, while the other is a combination of computed torque and sliding mode control.

3.1 Computed torque control of an HM

To control the HM dynamic model (4), a selected computed torque control law is:

$$\tau_{12} = DQ_r + C\dot{Q}_{12} + G \tag{28}$$

where $Q_r = \ddot{Q}_{d12} - K_v\dot{e}_{12} - K_p e_{12}$, $e_{12} = Q_{12} - Q_{d12}$, Q_{d12} is the desired (or reference) trajectory vector of Q_{12}, and K_p and K_d are linear control parameters. Inserting control law (28) into dynamic (4), gives:

$$D(\ddot{Q}_{12} - \ddot{Q}_{d12} + K_v\dot{e}_{12} + K_p e_{12}) = 0.$$

Using property (a) in section 2.3.2, we have the following error equation:

$$\ddot{e}_{12} + K_v\dot{e}_{12} + K_p e_{12} = 0. \tag{29}$$

Therefore, standard control methods from linear control theory can be used to design gains K_v and K_p, and to achieve desired tracking performance. If the gain matrices K_v and K_p are chosen as diagonal matrices with positive elements, then the closed-loop system is linear, decoupled, and exponentially stable. This is a very appealing control approach. However, the chief drawback of the computed torque control approach arises from the fact that it relies on an exact cancellation of non-linear terms in order to obtain linear input-output behaviour. Therefore, we have to combine computed torque control with robust control - sliding mode control, in order to improve the robustness of the closed loop system.

3.2 Combining computed torque and sliding mode control for an HM

Since the dynamic parameters of an HM in a real system are difficult to obtain, we can only use estimated values. Therefore, the following combined control law is adopted:

$$\tau_{12} = \hat{\tau}_{12} + \tau_s = \hat{D}\dot{v} + \hat{C}v + \hat{G} - K_s\text{sgn}(s) \tag{30}$$

where $\hat{D}$, $\hat{C}$, $\hat{G}$ and are estimations of D, C, and G, $v = \dot{Q}_{d12} - P_{12}e_{12}$ is reference desired joint signal, $s = \dot{e}_{12} + P_{12}e_{12}$ is reference error, P_{12} is a positive matrix selected by a designer, K_s is a positive sliding gain matrix, and $\text{sgn}(\cdot)$ is a signal function. $s = \dot{e}_{12} + P_{12}e_{12} = 0$ is also called the sliding surface in [6].

Theorem 1. For HM (4), if combined control law (29) is applied and the following condition satisfied:

$$k_i \geqslant |\tilde{D}(Q_{12})\dot{v} + \tilde{C}(Q_{12}, \dot{Q}_{12})\dot{Q}_{12} + \tilde{G}(Q_{12})|_i + \eta_i \tag{31}$$

where $\tilde{D} = \hat{D} - D$, $\tilde{C} = \hat{C} - C$, $\tilde{G} = \hat{G} - G$, and η_i, are small positive values, then for a reasonably small positive constant, all signals in the system are bounded, and tracking error e_{12} tends to zero as time tends to infinity.

Proof. Define a Lypunov-like positive function:

$$V = \frac{1}{2}s^{\mathrm{T}}Ds.$$

Taking the derivative of the above equation with respect to time, and using the properties in 2.3.2, control law (30) and (31) have:

$$\begin{aligned}\dot{V} =& s^{\mathrm{T}}D\dot{s} + \frac{1}{2}s^{\mathrm{T}}\dot{D}s = \\ & s^{\mathrm{T}}(\tau_{12} - C\dot{Q}_{12} - G - D\ddot{Q}_{d12} + DP_{12}\dot{e}_{12} + \frac{1}{2}\dot{D}s) = \\ & s^{\mathrm{T}}(\hat{D}\dot{v} + \hat{C}v + \hat{G} - K_s\text{sgn}(s) - Cv - Cs - G - \\ & D\dot{v} + \frac{1}{2}\dot{D}s) = \\ & s^{\mathrm{T}}(\tilde{D}\dot{v} + \tilde{C}v + \tilde{G}) - \sum_{i=1}^{2}K_{si}|s_i| \geqslant -\sum_{i=1}^{2}\eta_i|s_i|.\end{aligned}$$

Hence $s_i \to 0$ as $t \to \infty$. This leads to $e_{12} \to 0$ as $t \to \infty$. □

4 Simulation study

The purpose of the simulations in this paper is 1) to test and validate the adopted control approaches, and 2) to compare the combined computed torque control and sliding model control approach with the computed torque control approach.

Simulation studies were conducted using the five-bar HM shown in Fig. 2. Numerical values for link parameters used in the simulations were as shown in Table 1. The estimated values of the link parameters used in the control law were shown in Table 2. During simulation, the servo and CV motor were instructed to complete the following motion:

$$q_{d1} = \pi/8 + 1.5t$$
$$q_{d2} = 0.6\sin(0.5\pi t) + 0.3\sin(1.5\pi t) + 0.1\sin(3\pi t)$$

Table 1 Parameters for a five-bar HM

Link i	L_i(m)	L_{ci}(m)	m_i(kg)	I_i(kg·m^2)
1	0.08	0.006	0.91	0.000 847
2	0.1	0.028	0.28	0.000 63
3	0.25	0.125	0.38	0.004 002
4	0.25	0.125	0.38	0.004 002
5	0.25	-	-	-

Table 2 Estimated parameters for a five-bar HM

Link i	L_i(m)	L_{ci}(m)	m_i(kg)	I_i(kg·m^2)
1	0.08	0.006	0.90	0.000 8
2	0.11	0.025	0.29	0.000 6
3	0.24	0.12	0.39	0.004
4	0.26	0.13	0.37	0.003 5
5	0.25	-	-	-

The trajectories q_{d1}, q_{d2}, q_{d3}, and q_{d4}, are shown in Fig. 3.

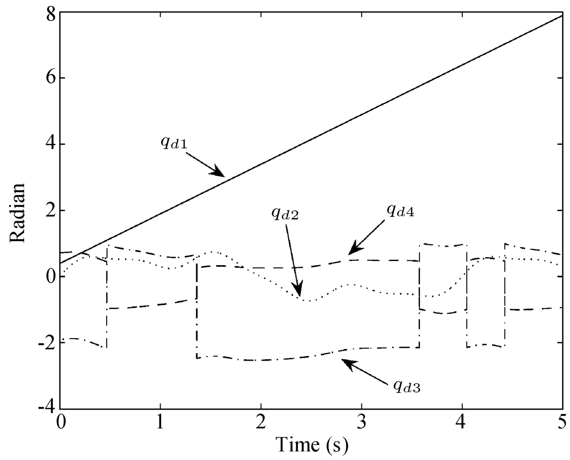


Fig. 3 Desired joints for an HM

4.1 Computed torque control

The parameters of the computed torque control law were: $K_v = \begin{bmatrix} 10 & 0 \\ 0 & 40 \end{bmatrix}$ and $K_p = \begin{bmatrix} 50 & 0 \\ 0 & 100 \end{bmatrix}$. When there was no parameter uncertainty, i.e. true link parameters were used, tracking performance was good. When estimated link parameters were used, simulation results were as shown in Fig. 4. It can be seen that there are significant errors.

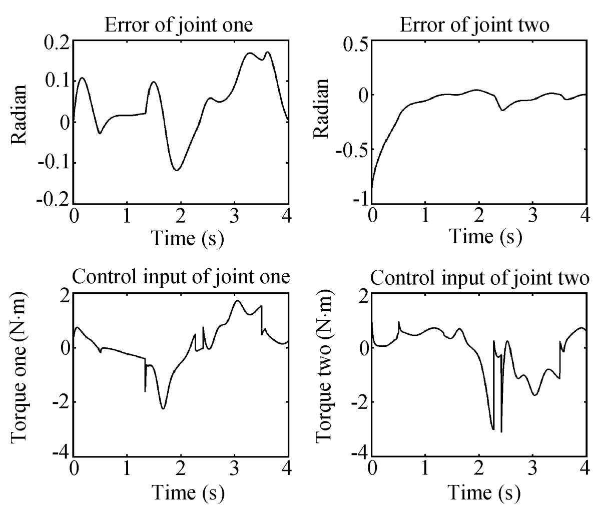


Fig. 4 Tracking performance of a computed torque controller

4.2 Sliding mode control

The parameters of the sliding mode control law were: $P_{12} = \begin{bmatrix} 100 & 0 \\ 0 & 200 \end{bmatrix}$, $K_s = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ and $\eta_1 = \eta_2 = 0.4$. When the estimated link parameters were used, the simulation results were shown in Fig. 5. It can be seen that the tracking performance shown in Fig. 5 is much better than that shown in Fig. 4.

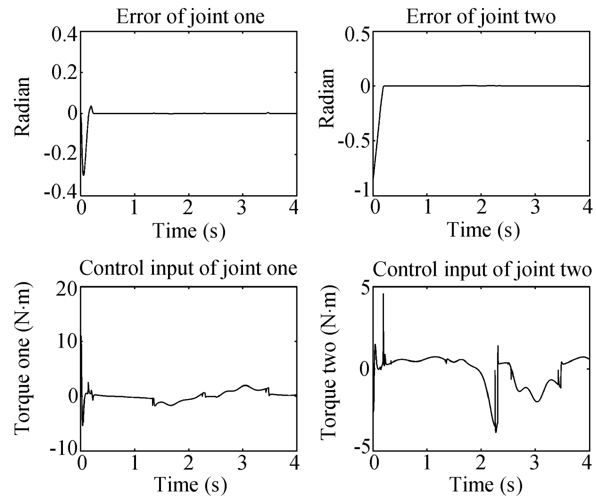


Fig. 5 Tracking performance of a sliding mode controller

5 Conclusions

In this paper, a general dynamic model for an HM with a five-bar mechanism has been developed using a Lagrangian formulation. The dynamic model has a closed form, and can be used to validate mechanical systems, to design a controller, and to perform simulation studies. Several important properties have been obtained from the dynamic model. These properties, which are similar to those of robot manipulators, can play an important role in advanced control system design. Furthermore, two popular control approaches, computed torque control, and sliding mode control, have been adopted based on the developed dynamic model. Simulation results have demonstrated good control performance.

Appendix

Proofs of the properties presented in Section 2.3.2 are given below. It is easy to prove that D is symmetric, positive definite, and bounded above and below. The symmetric and positive definite properties of D are straightforward, since matrices I_{12}, I_{34}, M_{12}, and M_{34}, are diagonal and positive definite. The boundedness of D is due to the fact that all elements of the matrices in (23) are sine or cosine functions of joint variables. We have already proved property (a).

Using (25) and (26), we can prove property (b). Using (24) and (26), and the boundedness of the sine and cosine functions, we can show property (c).

Property (d) can be shown using (27).

Property (e) may be shown using Hamilton's equation of motion[14]. Let Hamilton's H be:

$$H = K + P = \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}} D\dot{Q}_{12} + P.$$

Taking the derivative of the above equation with respect to time, gives:

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t} =& \dot{Q}_{12}^{\mathrm{T}} D\ddot{Q}_{12} + \frac{1}{2}\dot{Q}_{12}^{\mathrm{T}}\dot{D}\dot{Q}_{12} + \dot{Q}_{12}^{\mathrm{T}}\frac{\partial P}{\partial Q_{12}} = \\ & \dot{Q}_{12}^{\mathrm{T}}(D\ddot{Q}_{12} + \frac{1}{2}\dot{D}\dot{Q}_{12} + \frac{\partial P}{\partial Q_{12}}).\end{aligned} \quad \text{(A1)}$$

Using (4), we have:

$$\dot{Q}_{12}^{\mathrm{T}}\tau_{12} = \dot{Q}_{12}^{\mathrm{T}}(D\ddot{Q}_{12} + C\dot{Q}_{12} + \frac{\partial P}{\partial Q_{12}}). \quad \text{(A2)}$$

Comparing (A1) and (A2), and using property (b), we have

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \dot{Q}_{12}^{\mathrm{T}}\tau_{12}.$$

Integrating the above equation gives:

$$\int_0^t \dot{Q}_{12}^{\mathrm{T}}(u)\tau_{12}(u)du = H(t) - H(0) \geqslant -H(0).$$

This proves the passivity of the mapping between τ_{12} and $\dot{Q}_{12}$[14].

References

[1] R. M. Murray, K. J. Astrom, S. P. Boyd, R. W. Brockett, G. Stein. Future Directions in Control in an Information-Rich World. *IEEE Control Systems Magazine*, vol. 23, no. 2, pp. 20−33, April 2003.

[2] L. Tokuz. Hybrid Machine Modelling and Control. Ph.D. dissertation, Liverpool John Moores University, Liverpool, UK, 1992.

[3] A. Kirecci. Motion Design for High Speed Machines. Ph.D. dissertation, Liverpool John Moores University, Liverpool, UK, 1993.

[4] W. K. Bradshaw. Control of Hybrid Machines. Ph.D. dissertation, Liverpool John Moores University, Liverpool, UK, 1997.

[5] Z. Yuan. Design and Control of Hybrid Machines. Ph.D. dissertation, Liverpool John Moores University, Liverpool, UK, 2000.

[6] F. H. Ghorbel, O. Chetelat, R. Gunawardam, R. Longchamp. Modeling and Set Point Control of Closed-Chain Mechanisms: Theory and Experiment. *IEEE Transactions On Control Systems Technology*, vol. 8, no. 5, pp. 801−815, 2000.

[7] H. Yu, Z. Yuan, M. J. Gilmartin, S. Lloyd. Modelling and Control of a Slider Hybrid Seven-bar System. *International Association of Science and Technology for Development (IASTED) International Conference on Robotics and Applications*, Santa Barbara, CA, USA, pp. 319−323, October 28−30, 1999.

[8] H. Yu, M. Deng, M. J. Gilmartin, T. C. Yang. Full Dynamic Model of a Hybrid Seven-bar System. In *Proceedings of Third World Manufacturing Congress*, CD ROM, Rochester, NY, USA, September 24−27, 2001.

[9] M. Deng, H. Yu, M. J. Gilmartin, T. C. Yang. Lagrangian Dynamics and Analysis of a Hybrid Linkage System. *International Journal of Computers, Systems and Signals*, vol. 2, no. 1, pp. 54−70, 2001.

[10] P. R. Ouyang, Q. Li, W. J. Zhang, L. S. Guo. Design, Modeling and Control of a Hybrid Machine System. *Mechatronics*, vol. 14, no. 10, pp. 1197−1217, 2004.

[11] Z. Yuan, M. J. Gilmartin, S. S. Douglas. Optimal Mechanism Design for Path Generation and Motions with Reduced Harmonic Content. *Journal of Mechanical Design*, vol. 126, no. 1, pp. 191−196, 2004.

[12] Q. Li, F. X. Wu. Control Performance Improvement of a Parallel Robot via the Design for Control Approach. *Mechatronics*, vol. 14, no. 8, pp. 947−964, 2004.

[13] F. X. Wu, W. J. Zhang, Q. Li, P. R. Ouyang, Z. X. Zhou. Control of Hybrid Machines with 2-DOF for Trajectory Tracking Problems. *IEEE Transactions on Control Systems Technology*, vol. 13, no. 2, pp. 338−342, 2005.

[14] R. Ortega, A. Loria, P. J. Nicklasson, H. Sira-Ramirez. *Passivity-based Control of Euler-Lagrange Systems*. Springer-Verlag, London, 1998.

[15] H. Yu, S. Lloyd, Variable Structure Adaptive Control of Robot Manipulators. *IEE Proceeding – Control Theory Applications*, vol. 144, no. 2, pp. 167−176, 1997.

Hongnian Yu is a Professor of Computer Science at Staffordshire University. He was a lecturer in Control and Systems Engineering at Yanshan University, China, from 1985-1990, he conducted his Ph.D. in Robotics at King's College London (1990-1994), and was a research fellow in Manufacturing Systems at Sussex University (1994-1996), a lecturer in Artificial Intelligence at Liverpool John Moore's University (1996-1999), a lecturer in Control and Systems Engineering at the University of Exeter (1999-2002), and a Senior Lecturer in Computing at the University of Bradford (2002-2004). He now leads the Mobile Computing and Distributed Systems Research Group at Staffordshire University. He was a founding member of the Modelling Optimisation Scheduling and Intelligent Control research group at the University of Bradford.

He has published over 100 research papers focusing on the following: neural networks, computer networks, adaptive and robust control of robot manipulators, analysis and control of hybrid machines, control of time delayed systems, predictive control, manufacturing system modelling and scheduling, planning, and supply chains. He has extensive research experience in neural networks, mobile computing, modelling, control of robot manipulators; and modelling, scheduling, planning, and simulation of large discrete event dynamic systems with applications in manufacturing systems, supply chains, transportation networks, and computer networks.

Professor Yu is an EPSRC college member, a member of the IEEE, and a committee member for several conferences and journal editorial boards. He has held several research grants from the EPSRC, the Royal Society, and the EU, as well as from industry. He was awarded the F.C. William Premium for his paper on adaptive and robust control of robot manipulators by the IEE Council in 1997.