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Game Modeling Research for Urbanization and Epidemic Control

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Abstract: To aid in the sustainable development of cities this paper examines methods for urbanization and epidemic control. Using, as a foundation, game theory from modern control theory, a set of strategies for modeling urbanization and epidemic control are examined by analyzing and studying the current condition of China including its population, economy, resources and city management methods. Urbanization and epidemic control solving strategies are probed and the solution to a simulated example is provided. The conclusion from this research is that the speed of Chinese urbanization should be slowed to match the condition of resources and level of city management available.

Keywords: Urbanization, epidemic, population, control, game.

1 Introduction

As China's high-speed economy has grown in recent years, a process of increased citification has been seen. Many cities are actively expanding in population and geographical area. They are also increasing in prosperity[1~3]. However, along with these changes, problems have appeared in such cities[3~9]. Such problems included the SARS outbreak in 2003, which made us wake up from our inebriation and calmed down our zealotry. Some people began to think and meditate. The fact that the number of victims was proportional to the population density in rapidly developing metropolises, like Beijing, Guangzhou, and Shanghai, made us look for a relation between population growth and epidemic, city expansion and epidemic, and at city development and its restraints, i.e. limited resources, management and waste processing etc. The harsh experience of problems such as SARS has forced us to search for a balance between development and environment, population growth and epidemic control, production and maintenance, exploitation and protection. Research probing a game between citification and epidemic control is therefore an urgent affair and is significant in contributing to the sustainable development of cities and, the quality and security of the lives of people living in such areas.

A problem on urbanization and population was firstly proposed in [5], in which an analyzing method was provided. In [3] an epidemic control countermeasure to the mobile population of cities was forwarded. Research in [9] probed problems in the control system model for a population with epidemic factors using an L2 state space. [10] discussed a control system model for a population containing epidemic factors and realized a mutual transformation between the model and a time variable big system population model. The varying trends of a healthy population subsystem and morbid population subsystem were also given. Despite this, there is little literature which probes the problem of urbanization and the control of a morbific population.

In this paper, a problem concerning the game between urbanization and epidemic control is put forward. Based on game theory, the game is used to attempt to provide a balance between urbanization and epidemic control. It attempts to model the current conditions of population, economy, resources, and management of cities in China, so that they can be analyzed to provide strategies to overcome the problems in this area. A simple simulated example is provided to illustrate the method by which models are developed.

2 Static models

An increase in a city's population can be a benefit, however, it can also place a burden on its resources (not only money, energy, water, land, social services, welfare, security, but also appropriation, waste disposal, and community management). Therefore, with a view to the benefit of citification, a population introducing decision should maximize population profit by counteracting the introduction of cost using a set of constraint conditions. Given set p as a population to be introduced, and v a population-citifying decision, an optimal model for the citifying process can be written as:

$$\begin{cases} \max\ [B(p,v) - C(r,w,m)] \\ \text{s.t. } G(p,v) - (r,w,m)^T = g \\ \qquad v \in V \\ \qquad (r,w,m)^T \geqslant 0 \end{cases} \tag{1}$$

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where $B(\cdot)$ is a benefit function matrix measured at some index for citifying, $r=r(p)$ is the quantity of resources consumed, $w = w(p)$ is the quantity of waste discharged, and $m = m(p)$ is management labor required. These variables are all functions with respect to the population p. $C(\cdot)$ is a cost function matrix measured at some index for citifying, which is mainly concerned with resource consumption, waste-processing expenditure and management expenditure. $G(\cdot)$ is a technical and economic constraint function matrix matched to a city's resource assignment, waste disposal and community management. g is a supply vector for the citifying process, and V is a set of feasible decision plans.

On the other hand, the increase of a city's population in quantity is, in fact, a congregation of population. According to the relation formula "epidemic breaking out = number of sufferers × contagion potential × adjacence," commonly shown in literature[5,6], a congregation can result in a burden on epidemic control, such that sufferers increase, population crowding results, medical and sanitary resources are occupied, waste heaping arises and management is complicated. Thus, as regards the improvement of population quality and promotion of a city's stability and continuing development, an optimal integrity of epidemic control should be used to minimize the quantity of sufferers in a population, as well as the level of resources consumed (i.e. money, energy, land, water, city facilities etc.), waste discharged, and population density generated. A process used to describe the optimal integrity of epidemic control is

$$\begin{cases} \min\ [E(s,u) + C(r,w,m)] \\ \text{s.t. } H(s,u) + (r,w,m)^T = h \\ \quad u \in U \\ \quad (r,w,m)^T \geqslant 0 \end{cases} \tag{2}$$

where $E(\cdot)$ is a cost function matrix measured at some index for epidemic control, u is an epidemic control vector, $s = s(p)$ is the number of epidemic victims, $H(\cdot)$ is a technical and economic constraint function matrix matched to resource assignment, waste disposal, and community management for epidemic control; h is a demand vector for the epidemic control process and U is a set of feasible epidemic control plans.

Assuming that urbanisms and environmentalisms are cooperative, then simply combining (1) with (2), produces the following static optimal game model for population citifying and epidemic control:

$$\begin{cases} \min\limits_{u} \max\limits_{v}\ [B(p,v) + E(s,u)] \\ \text{s.t. } G(p,v) - (r,w,m)^T = g \\ \quad H(s,u) + T(r,w,m)^T = h \\ \quad E(s,r,w,p,m) \leqslant 0 \\ \quad u \in U,\ v \in V \\ \quad (r,w,m)^T \geqslant 0 \end{cases} \tag{3}$$

where $T(\cdot)$ is a cost transmission function matrix of resource consumption r, waste-processing fund w and management expenditure m. In accordance with different measuring indices, stressed content and focus of the game, detailed models can be given in a variety of forms.

Modeling example 1. Assuming that a citifying action puts stress only on the increased economic benefit from increasing population and epidemic control only on the decrease of epidemic sufferers and economic burden on population quality, then, resources can be taken as the focus for a two-part game. An objective function of citification therefore becomes

$$\max\{bp - C(r)\} \tag{4}$$

where b is the average contribution per person.

To promote the citifying part of the function to cooperate to improve population quality, an epidemic control-environment protecting technological parameter

$$e = e(u) \tag{5}$$

is provided to denote technologies adopted in part to improve population quality and also to protect resources such as garbage disposal, health care provision etc. The introduced population and occupied resources are therefore all functions of e. In addition the cost function is related to the citifying decision v and epidemic control u, and can be factorized as:

$$C(e,p,u,r) = c(p,u) + I(e) + C_f \tag{6}$$

where $I(e,v)$ is an increased input to improve population quality and C_f is a fixed cost without respect to e.

While the state of a country imposes a resource tax at a rate α from urbanism, and simultaneously grants a subsidy at a ratio β to its increased input in population quality improvement, the cost function (paying attention to (5), i.e. e as related to u) can be expressed as:

$$C(e,\alpha,\beta,r,u,v) = c(p,\alpha,u,v) + \\ (1-\beta)I(e,u,v) + \alpha r(e,\alpha,u,v) + C_f. \tag{7}$$

Therefore the citifying decision process is

$$\begin{cases} bp^* - C(e^*,\alpha,\beta,r^*,u^*,v^*) = \\ \qquad \max\limits_{e}\{bp - C(e,\alpha,\beta,r,u,v)\} \\ \text{s.t. } \beta I(e,u,v) \leqslant S \\ \quad Ae \leqslant q \end{cases} \tag{8}$$

where S is a set of subsidy limits and $Ae \leqslant q$ is a constraint for the input of technology. In order to minimize resource allocation conflicts, the state should determine an optimal subsidy ratio according to citifying action, allowing the decision process to be described as:

$$r(e^*, \alpha^*, \beta^*, u^*, v^*) = \min_{\beta} r(e^*, \alpha^*, \beta, u^*, v^*).$$

This process uses taxation and subsidy in the state as a lever, to focus resources. An optimal economic model for realizing citifying and epidemic control can therefore be written as:

$$\begin{cases} r(e^*, \alpha^*, \beta^*, u^*, v^*) = \min_{\beta} r(e^*, \alpha^*, \beta, u^*, v^*) \\ \text{s.t. } bp^* - C(e^*, \alpha, \beta, r^*, u^*, v^*) = \\ \qquad \max_{e}\{bp - C(e, \alpha, \beta, r, u, v)\} \\ \qquad \beta I(e, u, v) \leqslant S \\ \qquad Ae \leqslant q \end{cases} . \quad (9)$$

This produces a steady state Stackelberg game[13] for citifying and epidemic control.

Remark. It is not yet deeply understood what chemical, biological, population, environmental or ecological physical phenomena result in epidemic situations. Therefore there are great uncertainties when describing the problems of population and epidemic control. At the same time, the problems, to a great extent, depend on the value tropism of deciders. These complicated factors make the solving of population and epidemic control problems so difficult that systematic tactics have to be applied in synthetic management by means of the newest results of control theory and related techniques. The problems in urban, population and epidemic control, however, refer to a lot of parameters and variables related to time and space. It's therefore not sufficient for steady state models to describe the population-citifying and epidemic control problem. The problem must instead be investigated with a dynamic long-term view.

3 Dynamic models

The developing trends of population urbanization and epidemic control are impacted by many factors such as the lasted, periodic or stochastic variety, etc. The dynamic equation for the game of urbanization and epidemic control can be generally described as:

$$\dot{x} = f(x, u, v) \quad (10)$$

where x is a state vector. The components of this equation may be variables with some determined meanings such as the economic effect of a city, its population and epidemic sufferers, etc. There may also be some inner states without practical significance in the system. u is an epidemic control vector, v is a vector of population introducing decision and $f(\cdot)$ is a dynamic function vector of the system and is nonlinear and therefore may be coupled in a general way. Equation (10) expresses the dynamics of the citifying process under common effects provided by population introduction and epidemic control processes.

In a citifying action, a deciding objective should be to maximize some benefits or population size. This can be expressed as:

$$\max_{v} J_v[v(\cdot)] = \max_{v} \int_0^{t_f} g(x, u, v)dt \quad (11)$$

where $g(\cdot) > 0$ is an index function of benefits or population. In an epidemic control action, a deciding objective should be to minimize the number of sufferers or some related cost, that is

$$\min_{u} J_u[u(\cdot)] = \min_{u} \int_0^{t_f} h(x, u, v)dt \quad (12)$$

where $h(\cdot) > 0$ is an index function of epidemic sufferers or costs. It has to be noticed that full realization of objective (11) results in a worst case scenario in epidemic control.

The optimal result of the game should be that while v maximizes its benefits or other related indices of urbanization, u minimizes the quantity of epidemic sufferers or some related costs. A combination of (11) and (12) can be expressed as:

$$\begin{aligned} &\min_{n} \max_{v} J[u(\cdot), v(\cdot)] = \min_{n} \max_{v} (J_v + J_u) \\ &= \min_{n} \max_{v} \int_0^{t_f} [g(x, u, v) + h(x, u, v)]dt. \quad (13) \end{aligned}$$

The dynamic equation (10) and objective function (13) comprise a parametric min-max problem[14] or "two-player zero-sum game". The problem can be written in a form in accordance with definite appreciating indices and gaming focuses, etc. Furthermore, it can be solved by transforming it into a single control design.

Modeling example 2. By allowing the variable x_1 to express the current population size, in the Logistics equation

$$\dot{x}_1 = \frac{r}{K} x_1 (K - x_1) \quad (14)$$

the natural growing dynamics of a city's population can be described. The parameter r is called the intrinsic growth rate, and K is the population carrying capacity of a city. The quantities r, K, and x_1 are all positive. The solution to this differential equation where x_1 is initially small gives an "S" shaped growth curve over time, with exponential growth from a small population and then leveling off as the population nears K.

If v is used to represent a deciding variable for population introducing, then the product vx_1 can express the speed of population growth in urbanization. By applying it to the right-hand side of (14), we get

$$\dot{x}_1 = \frac{r}{K}x_1(K - x_1) + vx_1 \tag{15}$$

which is used to describe the general growth dynamics of the population during a citifying process.

On the other hand, if x_2 is used to represent the quantity of epidemic sufferers, making use of the mechanism that "epidemic breaking out = the number of sufferers × potential contagion × adjacence," then equation

$$\dot{x}_2 = qx_2d \tag{16}$$

can be used to describe the dynamics of an epidemic situation breaking out in a city. Where q is the velocity of epidemic intensity and d is population density. This means that an epidemic can be expressed by:

$$d = (p - u_1p)/K \tag{17}$$

where p is the gross current population size of a city, u_1 is the percentage of immunized population and K is the population carrying capacity of a city. The solution to differential equation (16) with x_2 initially small gives an exponential growth curve over time.

While $u = (u_1, u_2)$ is used to express the vector of epidemic control, u_2x_2 can express the velocity of the percentage of healed population. Therefore equation (16) can be rewritten as:

$$\dot{x}_2 = qx_2d - u_2x_2. \tag{18}$$

Combining (15) and (18), we notice that population-citifying and epidemic control proceed simultaneously ($p = x_1$). Formula (17) can therefore be written as:

$$\begin{cases} \dot{x}_1 = \frac{r}{K}x_1(K - x_1 - kx_2) + vx_1 \\ \dot{x}_2 = \frac{q}{K}(x_1 - u_1x_1 - kx_2)x_2 - u_2x_2 \end{cases} \tag{19}$$

where kx_2 is the number of dead epidemic sufferers and k is the mortality of them. Equations (19) are a set of state equations for the dynamic model for the game of population-citifying and epidemic control.

An objective function of the game can be expressed as:

$$\min_u \max_v J = \min_u \max_v \int_0^T (x_1^2 + x_2^2 + \rho u^T u - \sigma v^2)dt \tag{20}$$

where T is time anticipation, σ is a weighted function for the cost of population urbanizing decision and ρ is a weighted function for the cost of epidemic control. All of these variables depend on the value tropism deciding criterion of each part.

Putting equations (19) and function (20) together, a nonlinear dynamic model for the differential game of population urbanization and epidemic control is composed. This model directly appears as a game for both sides in population quantity. An optimal solution $[u^*(x_1, x_2), v^*(x_1, x_2)]$ when obtained will provide a zero-sum optimal value on the saddle point (x_1^*, x_2^*, u^*, v^*). This is a game of cooperation formed impersonally in urbanization and epidemic control, and quantity and quality of population.

Modeling example 3. If it is assumed that resources are taken as a focus for a game, and that there is a given quantity of resources C, this can be allocated between a population urbanization part and epidemic controlling environmentalisms. If it is assumed that the urbanization part has a supply of resources x_1 which is aimed at increasing the quantity of a city's population transferred at a rate m_1; and environmentalism has a supply of resources x_2, transferred at a rate m_2, aimed at improving the quality of the city's citizens, then v can be the fraction of resources urbanization part voluntarily used to control epidemics in a city, and u the fraction used to control epidemics by environmentalism. A subsidy from v for the epidemic control of a city would increase x_2; whereas, in economic benefit law, an input from u to epidemic control would activate resources to flow to urbanization. Thus, in resources' flowing law, the dynamics can be given with

$$\begin{cases} \dot{x}_1 = m_1 - c_2(1 - u)x_1x_2 \\ \dot{x}_2 = m_2 - c_1(1 - v)x_1x_2 \end{cases} \tag{21}$$

where c_1 and c_2 are the resources' effectiveness for the urbanization part and environmentalism respectively.

As a game, the urbanization part seeks to minimize v, while the environmentalisms seek to maximize u. The payoff to environmentalisms is the excess resources environmentalism put into epidemic-control

$$J = \int_0^T (ux_2 - vx_1)dt \tag{22}$$

subject to the constraints

$$\begin{cases} x_1 + x_2 = C \\ 0 \leqslant u \leqslant 1 \\ 0 \leqslant v \leqslant 1 \end{cases}. \tag{23}$$

If the equations of motion could yield an equilibrium $(\hat{x}_1, \hat{x}_2)$, then the urbanization part would aim to maximize $\hat{x}_1$ and minimize $\hat{x}_2$ under the first constraint in (23); while environmentalism would like to maximize $\hat{x}_2$ and minimize $\hat{x}_1$ under the same constraint. At the same time, the population citifying part would like to minimize J, while environmentalism would like to maximize J. An objective function therefore becomes

$$\max_u \min_v J = \max_u \min_v \int_0^T (ux_2 - vx_1)dt \tag{24}$$

By combining (24), (21) and (23), a resources optimal model for the differential game of urbanization and epidemic control can be produced. This model appears as a game of two sides for resource assignment. The optimal value J^* if obtained appears as the gross resources used in epidemic control during an anticipation. It is a game of cooperation formed subjectively in urbanization and epidemic control, and quantity and quality of population.

4 Simulating example

In the dynamic model combining (15), (18) and (20), we have

$$\begin{cases} \dot{x}_1 = \frac{r}{K}x_1(K - x_1 - kx_2) + vx_1 \\ \dot{x}_2 = \frac{q}{K}(x_1 - u_1x_1 - kx_2)x_2 - u_2x_2 \\ \min\limits_u \max\limits_v J = \min\limits_u \max\limits_v \int_0^T (x_1^2 + \\ \qquad x_2^2 + \rho_1 u_1^2 + \rho_2 u_2^2 - \sigma v^2)dt \end{cases}. \tag{25}$$

If $x = [x_1 \quad x_2]^T$ is the state vector of the system, and set

$$h_1(x) = \begin{bmatrix} x \\ 0 \end{bmatrix}, h_2(x) = \begin{bmatrix} 0 \\ \rho^{\frac{1}{2}}(x) \end{bmatrix}, \rho(x) = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} \frac{r}{K}x_1(K - x_1 - kx_2) \\ \frac{q}{K}(x_1 - kx_2)x_2 \end{bmatrix}, g_1(x) = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$

$$g_2(x) = \begin{bmatrix} 0 & 0 \\ -\frac{q}{K}x_1x_2 & -x_2 \end{bmatrix}$$

the parametric min-max problem (25) (i.e. the control u minimizes the number of sufferers of epidemic or some related cost, as the policy v is maximizing the benefit of citifying or other related indices) can be condensed to an H_∞ state feedback control problem in a nonlinear system as follows[20].

In a closed system:

$$\begin{cases} \dot{x} = f(x) + g_1(x)v + g_2(x)u_g \\ z = h_1(x) + h_2(x)u_g \\ u = u(x) \end{cases}$$

seek the state feedback controller u, such that: 1) the L_2 gain is smaller or equal to $\sigma^{\frac{1}{2}}$ (i.e. $\| z \|_T \leqslant \sigma^{\frac{1}{2}} \| v \|_T$); 2) the corresponding autonomous system ($v = 0$) is asymptotically stable.

This is the state feedback control problem that guarantees that some performance index is optimal in the worst conditions for the epidemic control part (i.e. $\sup_{\|v\|=1} \| z \|_T$ is minimized). Obviously, all conditions for the problem to be solvable hold in practical circumstances. By applying Hamilton-Jacobi inequality:

$$\frac{\partial V}{\partial x}f(x) + \frac{1}{2}\frac{\partial V}{\partial x}[\sigma^{-1}g_1(x)g_1^T(x) \\ - g_2(x)g_2^T(x)]\frac{\partial V^T}{\partial x} + \frac{1}{2}h_1^T(x)h_1(x) \leqslant 0$$

and assuming $x_1, x_2 \geqslant 1$ (the assumption sufficiently hold in practical circumstances), the solution

$$\frac{\partial V}{\partial x} = [0 \quad \eta(x_1, x_2)]$$

therefore exists, where the component

$$\eta(x_1, x_2) = [(\frac{q}{K}x_1x_2)^2 + x_2^2]^{-1}\{qx_2^2 - \frac{q}{K}x_1x_2 \\ \pm [(\frac{q}{K}x_1x_2 - qx_2^2)^2 + [(\frac{q}{K}x_1x_2)^2 + x_2^2](x_1^2 + x_2^2)]^{\frac{1}{2}}\}.$$

This allows us to construct a feedback controller as:

$$u(x) = -g_2^T(x)\frac{\partial V^T}{\partial x} = -\begin{bmatrix} 0 & -\frac{q}{K}x_1x_2 \\ 0 & -x_2 \end{bmatrix}\begin{bmatrix} 0 \\ \eta(x_1, x_2) \end{bmatrix} \\ = \begin{bmatrix} \frac{q}{K}x_1x_2 \\ x_2 \end{bmatrix}\eta(x_1, x_2).$$

Assuming K, q, r and k are given as 100 000, 10%/month, 1%/month and 5% respectively, and the weights ρ_1, ρ_2 and σ are all 1, we can obtain a relationship between the vector composed with the population variable x_1, the epidemic population variable x_2, and the population-introducing decision variable v. We can simulate the result of the varying states of the population variable x_1 and epidemic population variable x_2, under the given population introducing decision (say $v = 3\%$ as shown in Fig.1, and 10% as shown in Fig.2).

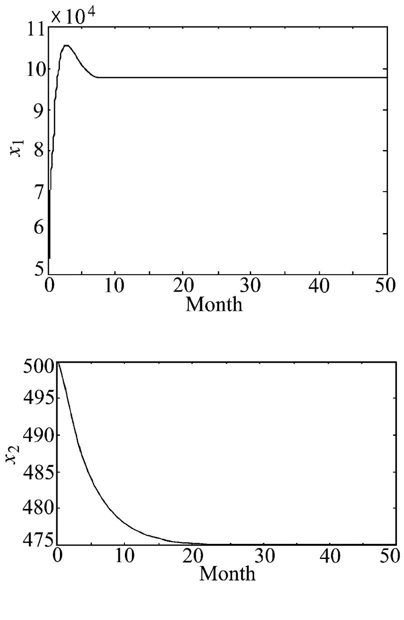

Fig.1 The sumulation for $v = 3\%$

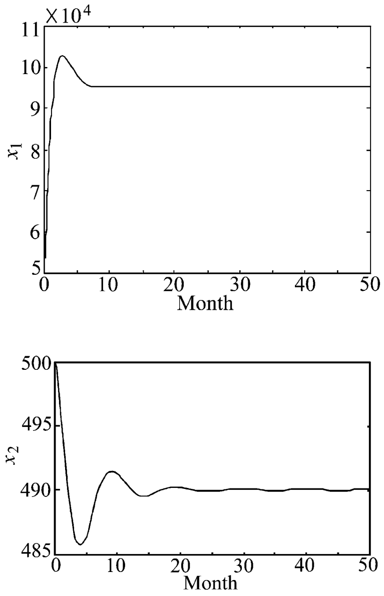

Fig.2 The sumulation for $v = 10\%$

5 Conclusions

The problem of urbanization and epidemic control forms a complicated game, which contains many certain and uncertain factors. However, the optimal modeling, allowing key problems to be solved, should highlight contradictions and give prominence to emphases.

In the course of modeling the game of urbanization and epidemic control, it was realized and perceived that urbanization can be described as the protection and utility of resources, the control of a population's epidemic, the process of waste management and community management, and so on. Such control requires a long term process which must conform the development of a cities' utilizable resources and population-carrying capacity, etc. This highlights that the problem of urbanization can't be overcome in a single step or action. It must be researched, probed by means of system tactics and the newest results from control theory, and resolved gradually.

Urbanization highlights the contradiction between high-speed consumption and high-speed growth. This contradiction forms an intrinsic suppression in the future economic development of China. If this is not resolved, the economy of China may plunge into a trap of long term future decline. Urbanization should be accompanied by economic growth. It is not only a population's and laborers' concentration in space, but a course of epidemic factors' accumulation and centralization. The duplicate effect must be clearly realized. Although some cities possess abundant elements for urbanization, lots of problems need to be solved, such as the duplication of industrial frameworks, outstanding contradictions between humans and land, the demolition of traditions and environmental pollution, etc. Therefore a population transferring model with Chinese characteristics must be sought as the chief task to allow the sustainable modern economic development of the Chinese economy and its cities.

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