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Determination of Effective Thermal Conductivity For Real Porous Media Using Fractal Theory

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In this paper, using fractal theory, the geometric structure of real soil was described with its section view and section particle area fractal dimension d of porous media was counted. The volumetric solid content and the relation between volumetric solid content and porous media particle arrangements as well as measure scale were obtainted. A heat conduction model was established and the effective thermal conductivity of real soil based on the volumetric solid content was calculated.

Keywords: effective thermal conductivity, porous media, fractal

INTRODUCTION

Porous media is a composite media that includes solid frame and fluid and existed widely in the earth biosphere. Heat and mass transfer in porous media is both a natural phenomenon in earth biosphere and a physical chemistry process in industries, agriculture and human life. Thus the study on heat and mass transfer in porous media has become an important task to scientists and engineers.

Heat and mass transfer in porous media is a very complex object. There are still many difficulties to describe the coupled heat and mass transfer phenomena. Among these difficulties, the most difficult one is determination of the physical properties of porous media, which is caused by the irregular geometric structures[2]. In traditional studies, the porous media is assumed to be a uniform, isotropic continuous media at macro scales and the geometric arrangement is described by the concepts of volume averaged. Usually thermal properties of porous media were determined based on this simplification, but in reality, homogeneous particle and void texture doesn't exist. So there is great difference between this ideal assumption and the reality. In order to determine the physical properties, a new mathematics is eagerly required to define the relationship between physical properties and porous structure.

Effective thermal conductivity for porous media has been long recognized as one of the most important properties in the study on heat and mass transfer in porous media. Various heat conduction models have been proposed for calculating the thermal conductivity for porous media. These models can be classified into the following groups[1]: (a) volume averaged model, (b) cell model, (c) statistical model, and (d) semiempirical model. However, the thermal conductivities that are calculated by these models depend on only one characteristic parameter of the porous media, e.g. porosity. Therefore, for real porous media that is disordered in its frame structure, a great error in prediction of thermal conductivity will be occured.

Liu et al[4] discussed pore characteristic of soils. The result showed that the soil structure was a fractal. Fractal theory opens a new way to determine the thermal parameters of composite media, it see ms that the study on heat and mass transfer in porous media can get more achievements. Pitchumany et al[7] and

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Adrian et al[1] used fractal techniques to study the effective thermal conductivity for porous media and got the expressions separately. But, their models for determining the local fractal dimension are still very hard and complex, there are some difficulties in practical use.

In our previous work, a new way for determining effective thermal conductivity using section particle area fractal dimension of porous media was suggested and a simplified model was established to determine the fractal dimension[2]. In this paper, the fractal characteristic of real porous media was studied further, section practical area fractal dimensions of porous media with different porosity were calculated, and the expression of the effective thermal conductivity for real soil was given by the fractal dimension and the model of heat transfer in soil.

FRACTAL STUDY ON REAL SOIL STRUCTURE

Fractal is a new approach of the geometry science to study the irregular objects. An important characteristic in fractal structure is its self-similarity. The greatest difference between fractal geometry and Euclidean geometry is in the range of dimension[5]. If an object is measured by scale δ, the value N will fit the following equation

$$N(\delta) \sim \delta^d \tag{1}$$

where d is dimension, for Euclidean geometry it is integer and for fractal geometry it can be noninteger. In the paper[2], the local fractal concept was developed and it was pointed out that an effective thermal parameter E could be expressed as a function of fractal dimension d of section area, each individual thermal parameter of different phases E_i, volume averaged porosity of porosity media ε, and scale l, that is

$$E = f(\sum E_i, \varepsilon, d, l) \tag{2}$$

By use of this basic expression we will determine the effective thermal conductivity of real soil in the following.

In order to calculate the fractal dimension specifically, a typical section view of black soil that is from Heilongjiang province shown as Fig.1 is chosen. Consider that heat conduction in one point of porous media is only influenced by it's neighbourhood structure, we take section area D^2 of a smaller soil particle in the view to be the smallest scale, furthermore, take the area A^2 that can contain some particle group at the four directions to be the greatest scale. For different area scales X at the range $D^2 \to A^2$, if the value of averaged area of solid phase section S can satisfy the following expression

$$S(X) \sim X^d \tag{3}$$

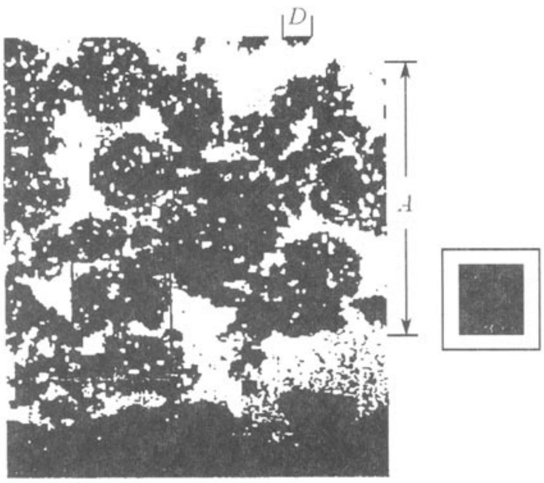

Fig.1 Section view of black soil

The structure of soil section is defined as a statistical self-similarity, d is the section area dimension of the solid phase. Thus the measurement of fractal dimension can be conducted as follows.

(i) Select area scale X in the range $D^2 \to A^2$ arbitrarily;

(ii) With area scale X locating in the ragion of the section view, select a point in a particle as a measuring center and measure the solid phase area using X;

(iii) Repeat step (ii) many times to ensure the area scale scanning every cell in whole section equiprobably, and take the average area of the solid phase as S;

(iv) Repeat step (ii), (iii) above for different values of X in the range $D^2 \to A^2$;

(v) Make a ln-ln plot of S against X, if X and S meet the following equation

$$\ln(S) = \ln c + d \ln(X) \tag{4}$$

then the structure is a fractal structure. In the equation, c is a proportional constant, the slope of the line which fit the points best is the fractal dimension d.

Fig.2 gives an evaluation of the section solid area fractal dimension for the given soil. From Fig.2, we can see that soil structure has obvious fractal feature.

Fig.3 is the section view of a rice soil from Taihu lake and the evaluation of the section solid area fractal dimension. Because Taihu lake soil is looser than black soil, its porosity is greater than black soil, so its section solid area fractal dimension is smaller.

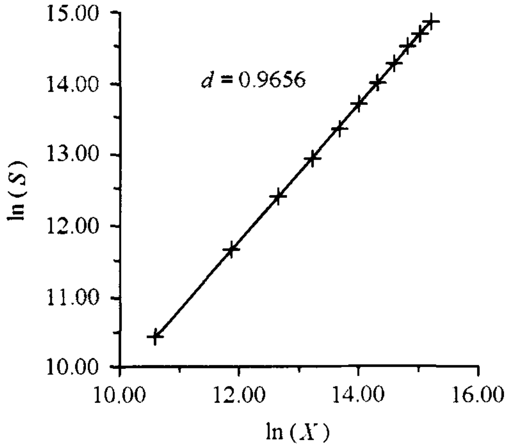

Fig.2 View of an evaluation of the section area fractal dimension for black soil

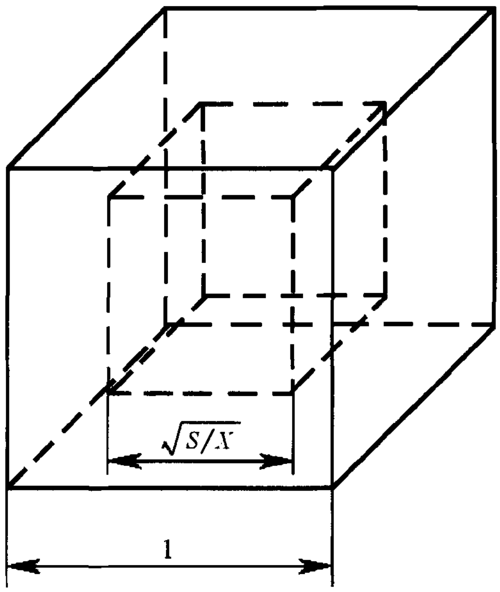

Fig.4 The volume cell for calculating the thermal conductivity

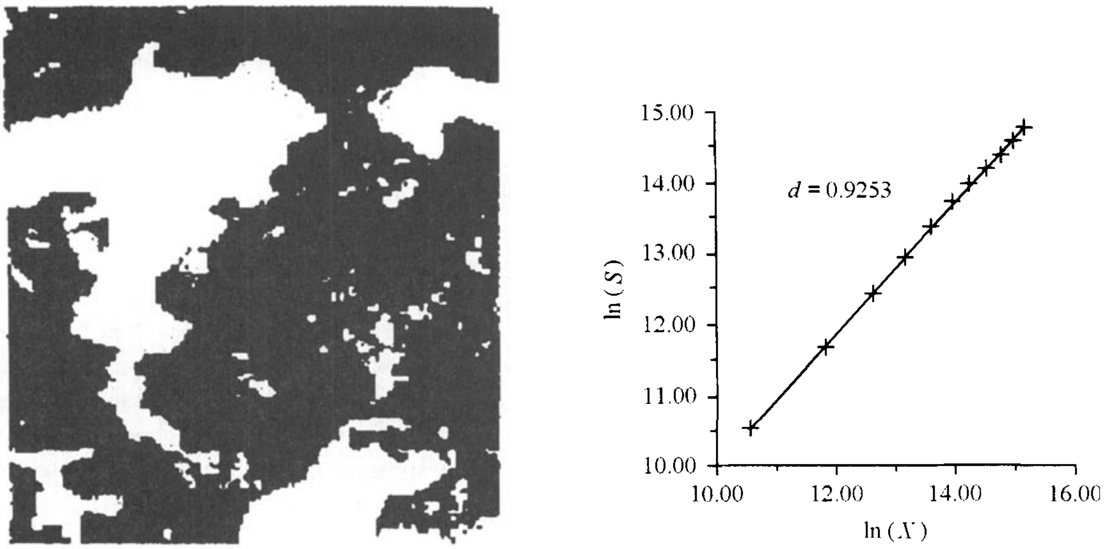

Fig.3 Section view of a rice soil from Taihu lake and the evaluation of the section area fractal dimension

FRACTAL MODEL OF THERMAL CONDUCTIVITY FOR SOIL

Geometric structure of real soil is complex and changeable, but, to this, based on the statistical concepts, the local fractal still exists. And fractal feature is proved to be existing really by the evaluation of the section area fractal dimension in soil. With this point of view, to calculate the effective thermal conductivity, a real complex structure can be simplified to be a rectangular structure that has the same area as the soil particle and the same fractal dimension as shown at right-lower side of Fig.1.

From equation (4), for a homogeneous soil structure, we can get the following equation

$$S = cX^d \tag{5}$$

Now, the network and the section area of soil particle are lessened to $1/X$ of the original area, the volume cell for calculating the thermal conductivity is shown in Fig.4, and its characteristics of heat conduction will be the same as in whole area.

The volumetric solid content based on the fractal concepts is

$$v = \left(\frac{S}{X}\right)^{3/2} / \left(\frac{X}{X}\right)^{3/2} = \left(\frac{S}{X}\right)^{3/2} \tag{6}$$

Combining with equations (5) and (6), we get

$$v = c^{3/2} X^{3(d-1)/2} \tag{7}$$

The volumetric solid content based on the large scale should be approximate to the volume averaged value from macro scale measurement, but there still exists a difference between them. On the one hand, it would be caused by large void, which may exist in the large volume domain, and on the other hand, it would be caused by the factor that we simplify the soil particle to be a cube. To compensate this difference, a volumetric factor m is introduced, and the effective volumetric solid content v_e become

$$v_e = mc^{3/2} X^{3(d-1)/2} \tag{8}$$

where

$$m = \frac{1-\varepsilon}{v(A^2)} \tag{9}$$

The simplified model of heat conduction by heat flux q through a unit structure is shown in Fig.5. Fig.6 is the schematic of thermal resistance of the system.

The total thermal resistance of the unit structure is

$$R = \frac{R_{g1} \cdot (R_{g2} + R_s)}{R_{g1} + R_{g2} + R_s} \tag{10}$$

where

$$R_{g1} = \frac{1}{\lambda_g \cdot (1 - v_e^{2/3})} \tag{11}$$

$$R_s + R_{g2} = \frac{\lambda_s \cdot (1 - v_e^{1/3}) + \lambda_g \cdot v_e^{1/3}}{\lambda_g \cdot \lambda_s \cdot v_e^{2/3}} \tag{12}$$

Substituting equation (11), (12) into equation (10), we get

$$R = \frac{\lambda_s \cdot (1 - v_e^{1/3}) + \lambda_g \cdot v_e^{1/3}}{\lambda_g \cdot \lambda_s \cdot (1 - v_e^{1/3} + v_e) + \lambda_g^2 \cdot (v_e^{1/3} - v_e)} \tag{13}$$

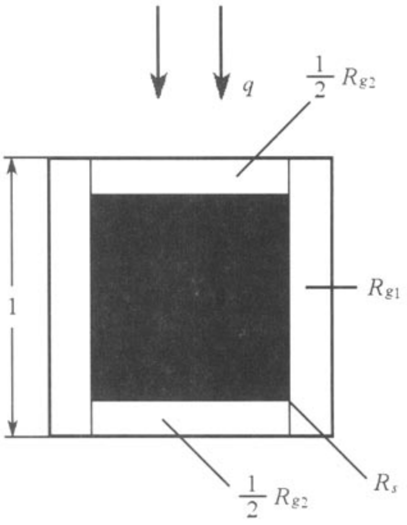

Fig.5 The simplified model of heat conduction

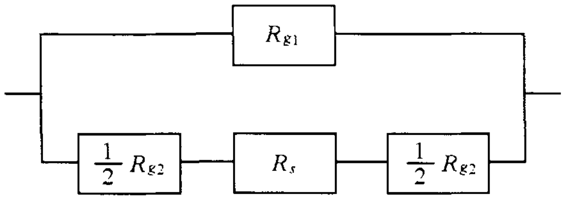

Fig.6 The schematic of thermal resistance of the system

Thus the effective thermal conductivity can be obtained as

$$\lambda_e = \frac{1}{R} = \frac{\lambda_g \cdot \lambda_g \cdot (1 - v_e^{1/3} + v_e) + \lambda_g^2 \cdot (v_e^{1/3} - v_e)}{\lambda_s \cdot (1 - v_e^{1/3}) + \lambda_g \cdot v_e^{1/3}} \tag{14}$$

Substituting equation (8) into equation (14), the final expression of the effective thermal conductivity for a porous structure based on fractal concept is

$$\lambda_e = \lambda_g \cdot \lambda_s \cdot (1 - m^{1/3} c^{1/2} X^{(d-1)/2} + mc^{3/2} X^{3(d-1)/2})$$
$$+ \lambda_g^2 \cdot (m^{1/3} c^{1/2} X^{(d-1)/2} - mc^{3/2} X^{3(d-1)/2})$$

$$/\lambda_s \cdot (1 - m^{1/3}c^{1/2}X^{(d-1)/2})$$
$$+\lambda_g \cdot (m^{1/3}c^{1/2}x^{(d-1)/2}) \quad (15)$$

where λ_g, λ_s are the thermal conductivity of gas-liquid mixture and solid phase respectively.

The effective thermal conductivity for black soil was calculated by using the equation (15) and the results are given out at Fig.7. The average porosity of black soil at macro volume is 0.5, volumetric moisture content is 10%, the average diameter of particle is 0.05 mm. During the calculation the scale at the range 4×10^4 to 4×10^6 (as 2.5×10^{-4} mm^2 to 2.5×10^{-2} mm^2), λ_s is 1.8 W/m·°C, λ_g is 0.145 W/m·°C ($\lambda_g = 0.1/0.5\lambda_w + 0.4/0.5\lambda_a, \lambda_w$ is the thermal conductivity of water and λ_a is of air). From Fig.7, we found that the values of λ_e are at the range 0.22~ 0.28 W/m·°C. The local effective thermal conductivity of the loam whose moisture content was 10% was measured by probe and the value is 0.242 W/m·°C (Ministry of Agriculture, 1996). This value is almost equal to the calculated value corresponding to the scale 6×10^5 (as 3.75×10^{-3} mm^2). The averaged area of a soil particle and the neighbour void is $(m(0.05)^2/(1-\varepsilon))^{2/3} = 3.72 \times 10^{-3}$ mm^2, and the scale above is approximately equal to it. This means that if we choose a given scale to calculate, we can get a calculated effective thermal conductivity very close to the ture value of effective thermal conductivity for real soil. Therefore we define this scale which is just fit the real physical process as the best cell scale. In general, we take the averaged area of a soil particle and the neighbour void as the best cell scale, the predicted value of a physical property by the best cell scale would be the closest one to the ture value of local porous media, and because we assumed that the volumetric solid content based on the large scale is equal to the average volumetric solid content at macro scale, the calculated value corresponding to large scale is much closer to the apparent thermal conductivity for large volume soil. On the contrary, the calculated value corresponding to the scale that is less than the best cell scale are the local thermal conductivity for a part of a soil particle and a small neighbour–hood. This local value is quite greater and it isn't fit the true physical process, and might be no practical significance. It is clear that we can get a variable effective thermal conductivities range by selecting different fractal scale and there exists one value at the best cell scale which can fit the measured local thermal conductivity of real porous material.

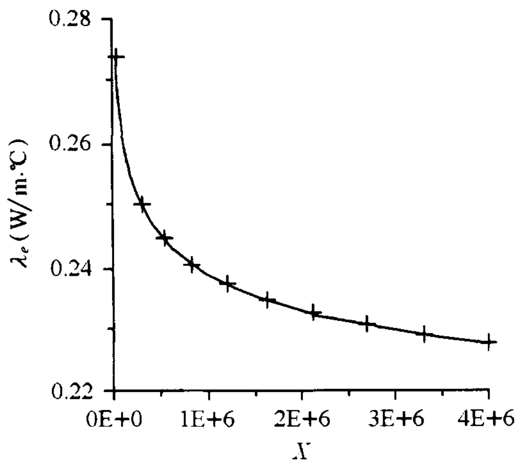

Fig.7 The results of effective thermal conductivity for black soil

CONCLUSIONS

Based on the section view of real soil, the section area fractal dimension is defined and calculated. A heat conduction model was developed to calculate the effective thermal conductivities of real soil by using the fractal dimension. The predicted value of thermal conductivity corresponding to the best cell scale is the closest to the true value of effective thermal conductivity for local soil; the value corresponding to large scale (A^2) is much close to the apparent thermal conductivity for large volume soil. So when we need to determine the effective thermal conductivity for local soil using fractal technique, the best cell scale should be used; and if we want to get the apparent thermal conductivity for large volume soil, we should select the large area scale. The results obtained in this paper indicate that the method to determine the effective thermal conductivity by using section area fractal dimension is effective, and it gives new way to forecast the thermal conductivity of different kinds of porous media.

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