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Hysteretic Phenomenon of Shock Wave in a Supersonic Nozzle

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In recent years, hysteretic phenomena in fluid flow systems drew attention for their great variety of industrial and engineering applications. When the high-pressure gas is exhausted to atmosphere from the nozzle exit, the expanded supersonic jet with the Mach disk is formed at a specific condition. In two-dimensional expanded supersonic jet, the hysteresis phenomenon for the reflection type of shock wave is occurred under the quasi-steady flow and the transitional pressure ratio between the regular reflection and Mach reflection is affected by this phenomenon. However, so far, there are very few researches for the hysteretic phenomenon of shock wave in a supersonic internal flow and the phenomenon has not been investigated satisfactorily. The present study was concerned with the experimental and numerical investigations of hysteretic phenomena of shock wave in a supersonic nozzle, and discussed the relationship between hysteresis phenomenon and rate of the change of pressure ratio with time.

Keywords: compressible flow, hysteresis, internal flow, shock wave, supersonic nozzle

Introduction

Every process is irreversible. If a system has any reversed process, the system and surrounding would not return to their original conditions. Hysteretic phenomenon is itself an example of irreversible process. Hysteretic phenomena commonly occur in magnetic and ferromagnetic materials, as well as in the elastic, electric, and magnetic behavior of materials, in which a lag occurs between the application and the removal of a force or field and its subsequent effect. The term 'hysteresis' is sometimes also used in other fields, such as economics or biology; where it describes a memory, or lagging effect, in which the order of previous events can influence the order of subsequent events. Hysteresis was initially seen as problematic, but is now thought to be of great importance in technology.

In recent years, hysteretic phenomena in fluid flow systems draw attention of many researchers for their great variety of industrial and engineering applications. During the formation of flow in a certain condition, the flow will experience a transient state until the state return to the original state. In general, even a flow changes under quasi-steady, the flow characteristics is considered to accompany a hysteretic phenomenon (hysteresis loop). In case of a process of rapid change in the flow, this is distinguished from the phenomena for the delay in response time. Hysteretic phenomenon is well known for external

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Nomenclature

D	nozzle inlet diameter (mm)	***Greek letters***	
D_e	nozzle exit diameter (mm)	ϕ	pressure ratio (-)
D_t	throat diameter (mm)	**Subscripts**	
L	distance from nozzle throat to the first shock wave (mm)	0	stagnation
M	Mach number (-)	b	back
p	static pressure (Pa)	e	exit

flow such as reflection of shock waves in the jet. From some previous researches, it was clarified that when the high-pressure gas is exhausted to atmosphere from nozzle exit, the expanded supersonic jet with the Mach disk is formed at a specific condition. This type of jet is very important for some industrial devices [1, 2]. Teshima [3] has suggested the possibility of occurrence of the hysteresis in the supersonic jet formed by the rectangular orifice.

Recently, many researchers [4–13] have proven that the hysteresis phenomenon exists on the interference form of the shock wave in the flow. Most recently, the effect of non-equilibrium condensation on hysteresis phenomenon of underexpanded moist jets have been investigated [14]. However, so far, there are very few researches for the hysteretic phenomenon of a supersonic internal flow for shock waves and the phenomenon has not been clarified satisfactorily. From some previous experimental and numerical studies [15, 16], it was realized that in a supersonic nozzle flow the shock wave is exist in the supersonic part and in some cases normal shock also formed. On increasing the nozzle pressure ratio the shock position is moved towards the nozzle exit and vice versa of this phenomenon is also true. Therefore, it requires performing experimental investigations on the hysteretic phenomena of shock wave in a supersonic nozzle.

The purpose of this study is to clarify the hysteretic phenomena of shock wave in a supersonic nozzle experimentally and to discuss the relationship between hysteresis phenomenon and rate of the change of pressure ratio with time. Hysteretic phenomena for the location of shock wave in a supersonic nozzle were investigated experimentally. Moreover, in order to confirm the existence of hysteresis phenomena for shock wave in the supersonic nozzle numerical works have been conducted.

Experimental Procedure

Experimental apparatus

Figure 1 and 2 shows the schematic diagram of the experimental apparatus and detailed nozzle configuration

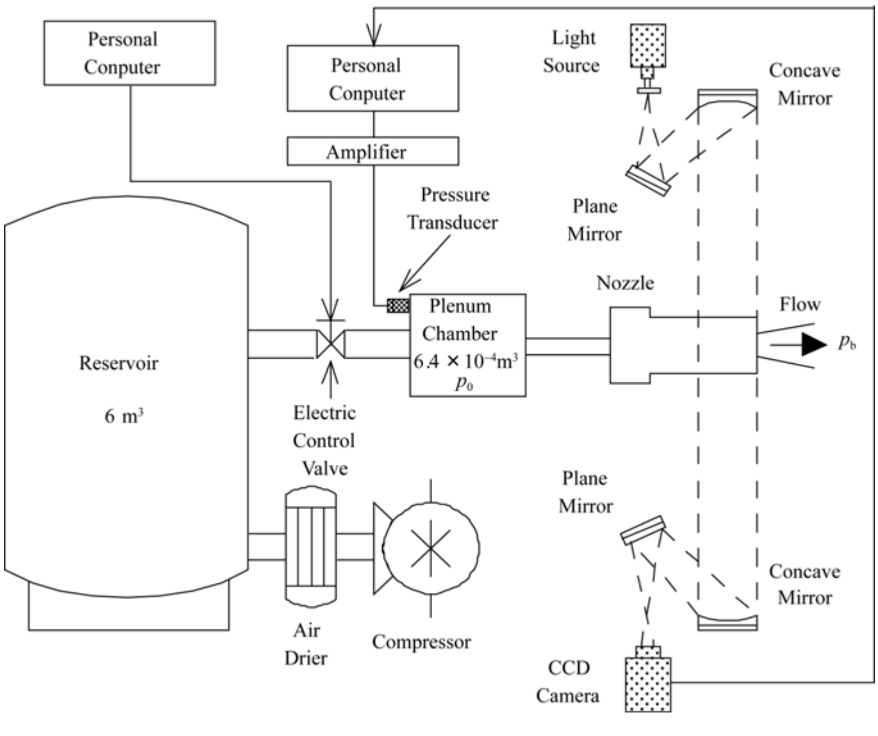

Fig. 1 Experimental apparatus

used in the present study, respectively. The apparatus is consisted of compressor, air drier, air reservoir, electronic control valve, plenum chamber and nozzle. Plenum chamber is placed upstream of the nozzle. The compressed air was used as a working gas. The test section is placed downstream of the nozzle throat and optical glass windows are installed on both side walls of the test section for flow visualization. In the present study, the nozzle was designed with the method of characteristics. The design Mach number of supersonic nozzle is M_e=1.86.

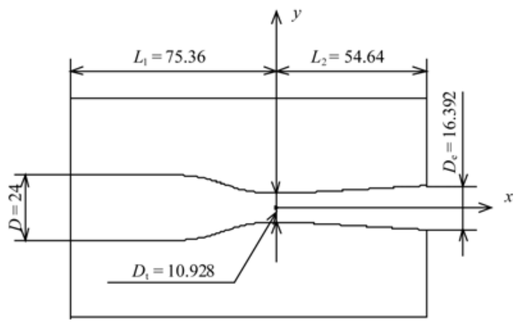

Fig.2 Supersonic nozzle geometry

Experimental procedure

In the present experiments, pressure ratio ($\phi=p_0/p_b$) was continuously changed with time using the electronic control valve. The symbol, p_0 and p_b represent the stagnation pressure of the plenum chamber and back pressure (atmospheric pressure), respectively. For shock wave in the nozzle the range of ϕ is from 1.63 to 3.14. The rate of the change of pressure ratio with time $\Delta\dot{\phi}$ is from 0.207 (1/s) to 0.459 (1/s).

Compressed dry air is discharged from nozzle exit. The flow field was investigated by a schlieren optical method. Visualization and measurement of pressure ratio were conducted simultaneously. The location of the first shock wave L was obtained from schlieren pictures. In the present experimental conditions, it is found that the time delay exists for response of change of flow to change of pressure ratio at $\Delta\dot{\phi}$ >0.342 (1/s). As described later, these results were determined from change of the shock wave position in the nozzle.

Computational analysis

Numerical methods

Numerical investigations were performed under conditions that replicate the experimental investigations of hysteresis phenomena of shock waves in the supersonic nozzle. The flow under study was treated as compressible, viscous, unsteady and turbulent. The governing equations are mass averaged, time-dependent Navier-Stokes equations, with the k-ω SST (Shear-Stress Transport) turbulence model [17-19]. The resulting equations are expressed in an integral form:

$$\Gamma\frac{\partial}{\partial t}\int_v \mathbf{Q}dA + \oint[\mathbf{F}-\mathbf{G}]dA = 0 \quad (1)$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are the inviscid and viscous flux vectors in standard conservation form and $\boldsymbol{Q}$ is the dependent vector of primary variables.

$$\mathbf{F} = \left[\rho v, \rho v v_x + p\hat{i}, \rho v v_y + p\hat{j}, \rho v v_z + p\hat{k}, \rho v H\right]^T$$
$$\mathbf{G} = \left[0, \tau_{xi}, \tau_{yi}, \tau_{zi}, \tau_{ij}v_j + q\right]^T \quad (2)$$
$$\mathbf{Q} = \left[p, v_x, v_y, v_z, T\right]^T$$

In the above equations, H is total enthalpy per unit mass and is related to the total energy E by $H = E + p/\rho$, where E includes both internal and kinetic energies. The preconditioning matrix Γ is included in Eq. (1) to provide an efficient solution of the present axisymmetric compressible flow. This matrix is given by

$$\Gamma = \begin{bmatrix} \theta & 0 & 0 & 0 & \rho_T \\ \theta v_x & \rho & 0 & 0 & \rho_T v_x \\ \theta v_y & 0 & \rho & 0 & \rho_T v_y \\ \theta v_z & 0 & 0 & \rho & \rho_T v_x \\ \theta H & \rho v_x & \rho v_y & \rho v_z & \rho_T H + \rho C_p \end{bmatrix} \quad (3)$$

where ρ_T is the derivative of density with respect to temperature at constant pressure. The parameter θ is defined as

$$\theta = \frac{1}{U_r^2} - \left(\rho_T H + \rho C_p\right) \quad (4)$$

In Eq. (4), the reference velocity U_r is chosen such that the eigenvalues of the system remain well conditioned with respect to the convective and diffusive timescales and C_p is the specific heat at constant pressure.

The preconditioned governing equations are discretized spatially using a Finite volume scheme. For the time derivatives, an implicit multistage time stepping scheme, which is advanced from time t to time $t+\Delta t$ with a 2nd order Euler backward scheme for physical time and implicit pseudo-time marching scheme for inner iteration, is used.

The computational domain and boundary conditions are illustrated in Fig. 3. The boundary conditions used are the inlet total pressure and the outlet static pressure, respectively. The adiabatic no-slip conditions are applied to the solid walls. A structured clustered grid system was employed in the computations. A solution convergence was obtained when the residuals for each of the conserved variables were reduced below the order of magnitude 4. Another convergence criterion is to check the conserved quantities directly through the computational boundaries. The net mass flux was investigated when there was an applicable imbalance through the computational boundaries.

Computational procedure

Flows containing hysteresis phenomenon can be computed by numerical simulations [20], in which the flow boundary conditions are systematically changed to obtain each of the quasi-steady solutions. Figure 4 shows the computational procedure for the process of the startup transient, in which the pressure ratio is increased. The steady supersonic nozzle flow of $\phi=\phi_{st}$ is computed and the resulting solutions are used as the initial conditions for the first step of the process of the startup transient. In the second step, the pressure ratio is increased by $\Delta\phi$ and the computation is repeated until the transient process is completed, thus leading to a quasi-steady state, as indicated by the black circle. The computed quasi-steady solutions are used again as the initial conditions for the next step. Consequently the final quasi-steady solutions are obtained for the pressure ratio of ϕ_{fi}. On the contrary, in the process of the shutdown transients, the pressure ratio is decreased, the final quasi-steady solutions used again as the initial conditions. For reference, in the present study, ϕ_{st}=1.63, ϕ_{fi}=3.21, $\Delta\phi$=0.029 and ξ=2400. Through such a series of computations, the quasi-steady solutions obtained during the startup and shutdown transients are compared to investigate the hysteretic behaviors of shock waves generated in the supersonic nozzle.

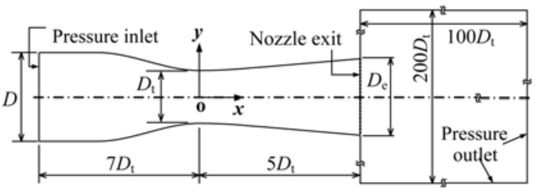

Fig. 3 Schematic view of computational domain and boundary conditions

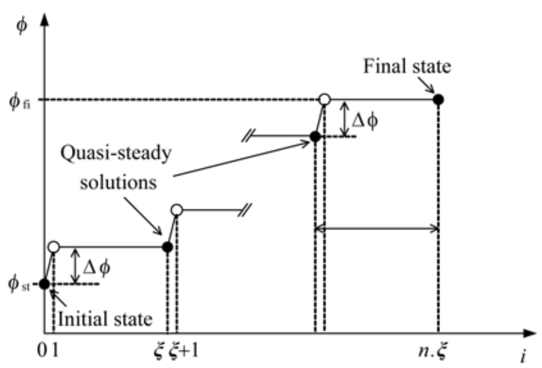

Fig. 4 Computational procedure to simulate the processes of startup and shutdown transients

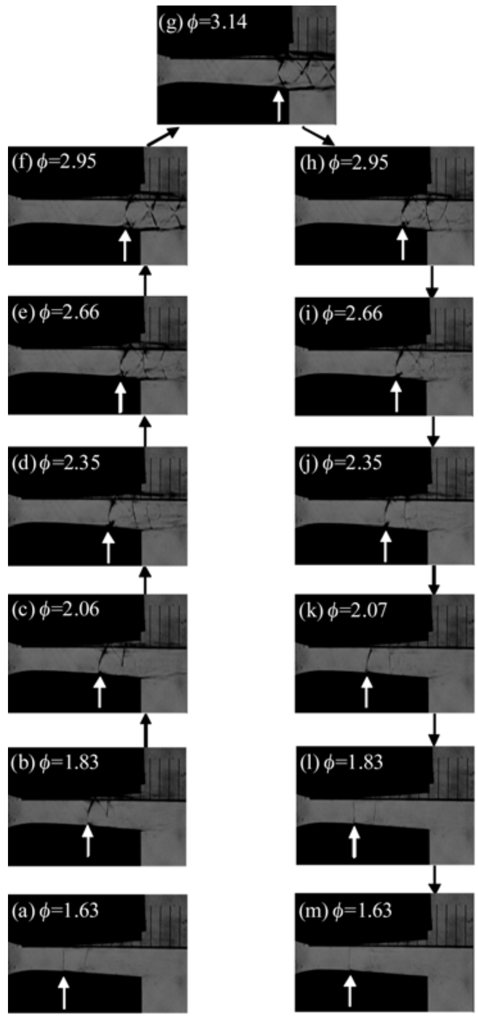

Fig.5 Schlieren photographs ($\Delta\dot{\phi}$ =0.286 (1/s))

Results and discussions

Figure 5 shows schlieren pictures of flow field of supersonic nozzle. The value of rate of change of the pressure ratio with time $\Delta\dot{\phi}$ is 0.286 (1/s). In this figure, the left side sequence represents the increasing process of pressure ratio ϕ and the right side for the decreasing process of pressure ratio. As seen from these picture sequences, there are differences between the locations of the first shock wave even in the same pressure ratio.

In the present study, investigating the hysteresis phenomena of shock wave in the supersonic part of the nozzle, the experiment is conducted in a range of pressure ratios, and the rate of change of pressure ratio with time ranges from $\Delta\dot{\phi}$=0.207 (1/s) to 0.459 (1/s). Figure 6 shows the effect of $\Delta\dot{\phi}$ on the location of the first shock wave L/D_t (D_t : diameter of nozzle throat), measured from the nozzle throat, in the range from 0.207 (1/s) to

0.459 (1/s). As is evident from this figure, hysteresis loops exist at the course between A and B. It is noticed that processes of variation between A and B follow the same course below $\Delta\dot{\phi}$=0.342 (1/s).

Figure 7 shows the relationship between L/D_t and pressure ratio ϕ in case of occurrence of hysteresis phenomena in supersonic nozzle ($\Delta\dot{\phi}$=0.286 (1/s)). In this figure, symbols from (a) to (m) correspond to those in Fig.5. As is evident from the figure, the existence of two values of L/D_t is confirmed in the ranges of ϕ=1.63 - 3.14.

In order to confirm the existence of hysteresis phenomena for shock wave in the supersonic nozzle, a computational study is conducted in a range of pressure ratio from ϕ=1.63 to 3.21. Figure 8 shows the computed density contours in the supersonic part of the nozzle. Here, the computed results have confirmed the formation of shock wave in the nozzle, and some normal shocks are also formed due to the sudden rise in temperature across the shock as explained in the previous research [15, 16].

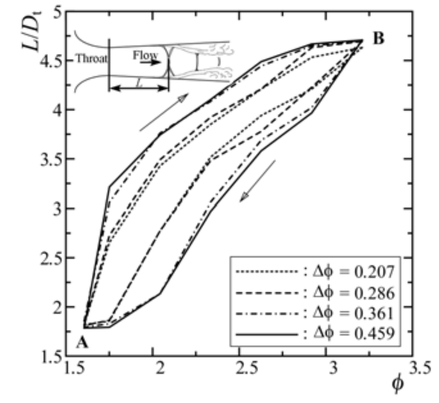

Fig. 6 Effect of $\Delta\dot{\phi}$ on L/D_t

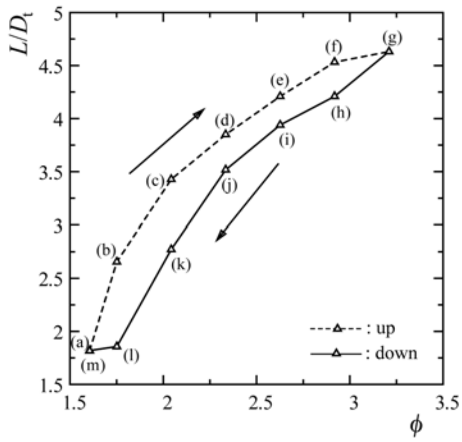

Fig. 7 Hysteresis diagram for location of Mach disk ($\Delta\dot{\phi}$= 0.286 (1/sec))

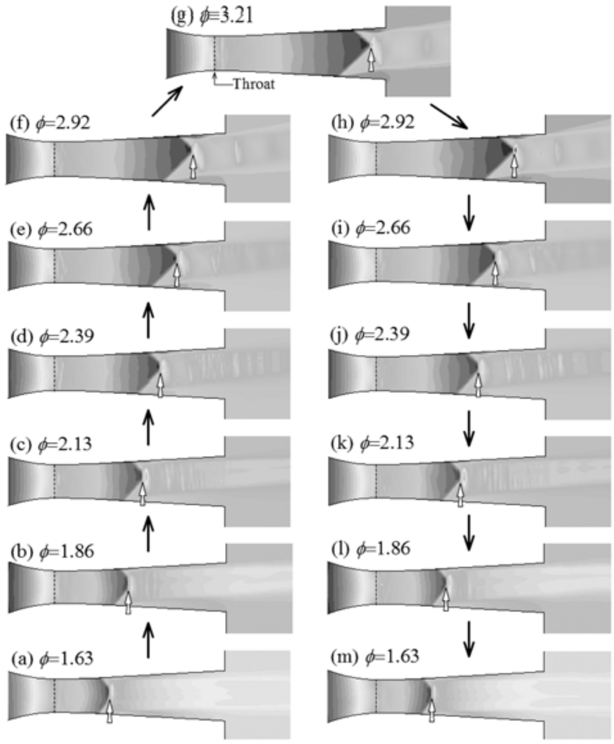

Fig. 8 Density contours showing the hysteresis phenomena in the supersonic nozzle

During the startup transient of supersonic nozzle flow, at pressure ratio ϕ=1.63, the shock wave (first shock) is located downstream of the nozzle throat. As ϕ increases to 3.21, the first shock wave moves downstream with stronger magnitude. At ϕ=3.21 which corresponds to the final steady state in the startup transient, the computed flow field is nearly the same to that employed as the initial conditions in the shutdown transient. In the shutdown transient, as ϕ decreases again to 1.63, the shock wave moves upstream and the strength seems to be stronger than those found in the startup transient. From a series of computations, it is found that the location and strength of the first shock wave is significantly different in both the processes of the startup and shutdown transients. This clearly revealed that there exist a hysteretic behavior in the formation of shock wave in the supersonic nozzle for the range of pressure ratio ϕ=1.63 to 3.21.

More quantitative data for hysteresis phenomena is presented in Fig. 9, where the location of first shock wave L/D_t, measured from the nozzle throat, in the nozzle during the processes in the shutdown and startup transients are plotted against pressure ratio ϕ. In the process of the startup transient, the position of the first shock moves downstream with an increase in ϕ. On the contrary, in the process of the shutdown transient, the shock position moves upstream with a decrease in ϕ. Hysteresis loop in the location of shock wave is found in both of the transient processes.

Figure 10 shows the hysteretic behavior in the shock strength which is corresponds to the static pressures p/p_b behind the shock. In the process of the startup transient, the strength of the first shock increase with an increase in ϕ. On the contrary, in the process of the shutdown transient, the shock strength decreases with a decrease in ϕ. Therefore, the hysteretic behavior is produced during both the processes of the startup and shutdown transients.

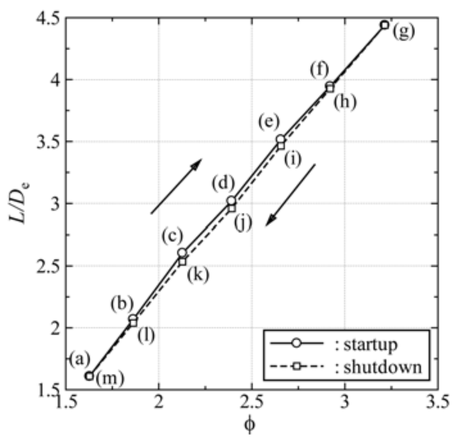

Fig. 9 Hysteretic behavior in the location of shock wave during startup and shutdown transients of supersonic flow in the nozzle

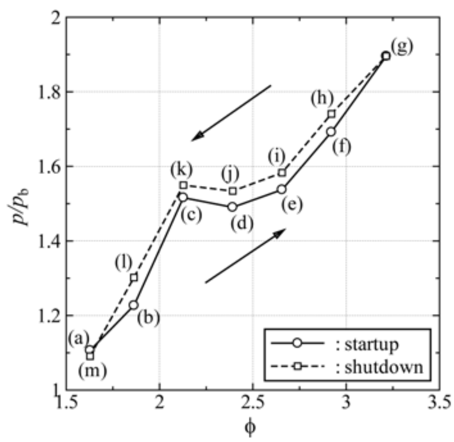

Fig. 10 Hysteretic behavior in the shock strength during startup and shutdown transients of supersonic flow in the nozzle

Conclusions

The present study deals with the experimental and computational works to investigate the hysteretic behavior in the formation of shock waves in supersonic nozzle. The mass averaged, time-dependent, compressible Navier-Stokes equations have been solved numerically to simulate the flow field concerning with the hysteretic behavior in both the processes of shutdown and startup transients of supersonic nozzle flow. Hysteresis phenomena for the location of shock wave in a supersonic nozzle were investigated experimentally. As the results, hysteresis phenomenon for shock wave in the supersonic nozzle was confirmed at a certain specific condition. The relationship between hysteresis phenomenon and the range of the rate of change of pressure ratio with time was shown experimentally. The existence of hysteretic behavior in the formation, both the location and strength, of shock wave in the supersonic part of the nozzle with a range of pressure ratio has also been confirmed numerically.

Acknowledgement

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