Numerical Study for Hysteresis Phenomena of Shock Wave Reflection in Overexpanded Axisymmetric Supersonic Jet

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When the high-pressure gas is exhausted to the vacuum chamber from the supersonic nozzle, the overexpanded supersonic jet is formed at specific condition. In two-dimensional supersonic jet, furthermore, it is known that the hysteresis phenomena for the reflection type of shock wave in the flow field is occurred under the quasi-steady flow and for instance, the transitional pressure ratio between the regular reflection (RR) and Mach reflection (MR) is affected by this phenomenon. Many papers have described the hysteresis phenomena for underexpanded supersonic jet, but this phenomenon under the overexpanded axisymmetric jet has not been detailed in the past papers. The purpose of this study is to clear the hysteresis phenomena for the reflection type of shock wave at the overexpanded axisymmetric jet using the TVD method and to discuss the characteristic of hysteresis phenomena.

Keywords :compressible flow,axisymmetric flow, supersonic jet, regular reflection, Mach reflection
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Introduction

When the high-pressure gas is exhausted to the vacuum chamber from the supersonic nozzle, the overexpanded supersonic jet is formed at specific condition. This type of jet is very important for some industrial devices and has been investigated in past papers. The three-dimensional flow field is usually formed by jet and the jet structure is very complex involving some shock waves, as the oblique shock. In two-dimensional supersonic jet, furthermore, it is known that the hysteresis phenomena for the reflection type of shock wave is occurred under the quasi-steady flow in the recent study [1]-[5] and for instance, the transitional pressure ratio between the regular reflection (RR) and Mach reflection (MR) is affected by this phenomenon. Many papers have been described the hysteresis phenomena for underexpanded supersonic jet [1]-[3], but this phenomenon under the overexpanded axisymmetric jet has not been detailed in the past papers. The purpose of this study is to clear the hysteresis phenomena for the reflection type of shock wave at the overexpanded axisymmetric jet using the TVD method.

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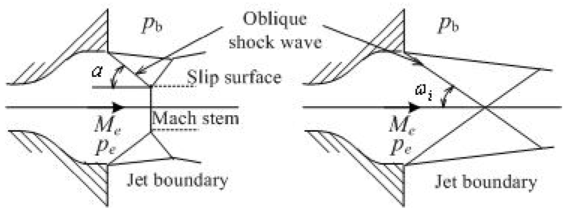
Computational Method

Typical flow field

When the pressure ratio of the reservoir and back pressure applies to overexpanded condition, the structure of overexpanded supersonic jet exhausted from the nozzle is schematically shown in Fig.1.



(a) (b)

Fig.1 Schematics of jet structure of overexpanded supersonic jet

In figure 1(a), the oblique shock formed at nozzle corner intersects with Mach stem. It is well known that this reflection type is a Mach reflection (MR). On the other hand, the oblique shock formed at nozzle corner intersects with each other at jet axis and this reflection type is a regular reflection (RR), as shown in Figure1 (b). As shown in these figures, the symbol α shows the incident angle of oblique shock, d_e is the nozzle exit diameter, M_e is the nozzle exit Mach number and p_0, p_e, p_b mean the reservoir, nozzle exit and vacuum pressures, respectively.

Governing equation and conditions

In the numerical calculation, the two-dimensional axisymmetric flow field is assumed and the *x-y* cylindrical coordinates system originated on the center of nozzle exit is considered, as shown in Fig. 2.

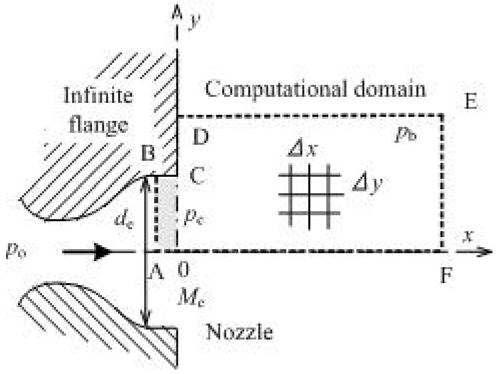


Fig.**2** Computational region and symbols used in this study

The governing equation used in this study is the compressible unsteady axisymmetric Euler's equation so that the inviscid gas is assumed. It can be written in Eq.(1) by the non-dimensional conservation forms.

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \mathbf{W} = 0 \tag{1}$$

Where,

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (e+p) u \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (e+p) v \end{bmatrix},$$

$$\mathbf{W} = \frac{1}{y} \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 \\ (e+p) v \end{bmatrix}$$

Equation (1) is non-dimensionalized by the nozzle exit diameter d_e and the reservoir pressure p_o, density ρ_o and sound velocity a_o. The non-dimensionalized equation is solved by the TVD method [6] with the operator splitting technique [7]. The square grid such as $\Delta x = \Delta y$ is used, as shown in Fig.2 and the time step Δt is decided by the CFL condition.

In all the calculations, a symmetric boundary was imposed along the jet axis, namely, A-F in Fig.2. A fixed pressure boundary condition was applied to the upstream boundary A-B and the downstream boundaries D-E, E-F were taken as outlet condition, respectively.

As the initial conditions, the nozzle exit Mach number is M_e and the pressure ratio are varied in the range of $3 < M_e < 5$ and $20 < \phi < 95$, where, $\phi = p_0/p_b$ and p_0 is a reservoir pressure, p_b is a vacuum pressure. The ratio of the specific heats is $k = 1.4$.

The procedure of calculation is shown in Fig.3. In the first step, the steady jet under $\phi = \phi_{st}$ is solved and this solution is used as the initial condition for next step. In the second step, the pressure ratio is decreased just $\Delta\phi$ at the downstream boundary in computational domain and the calculation is repeated until this transient process becomes the steady state, shown by black circle in Fig.3. The result obtained by that process is regarded as quasi-steady solution and this solution is used as the initial condition for next step. That process is repeated until the pressure ratio ϕ becomes the lower limit ϕ_r and the calculation is started toward the upper limit ϕ_{st} by similarly calculation step.

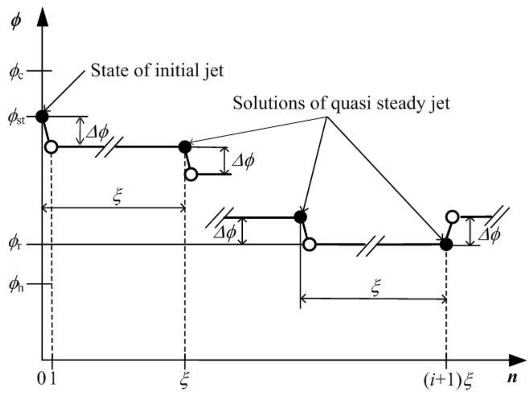
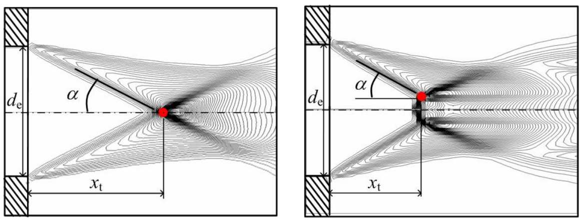


Fig.**3** Procedure of pressure change

Results and Discussions

Transition of reflection type

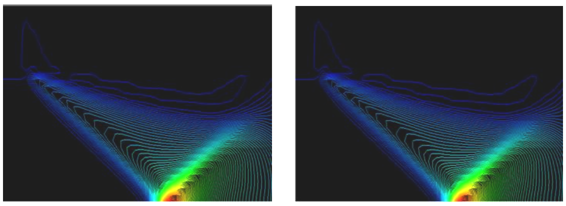
The typical isopycnics showing the jet structure for case of M_e=4 is shown in Fig.4. Figure (a) indicates the result of regular reflection (RR) and figure (b) represents the result of Mach reflection (MR). The MR type occurs at low pressure ratio domain and the RR type occurs at high pressure ratio domain. As shown in figure, the incident angle of oblique shock from a nozzle corner and the intersection position for oblique shocks or oblique shock and Mach stem are defined by symbol α and x_t, respectively.



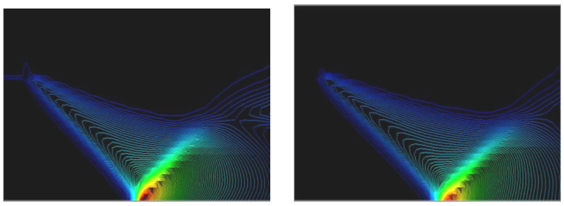
(a) ϕ=40 (b) ϕ=29

Fig.4 Typical isopycnics showing jet structure (M_e=4)

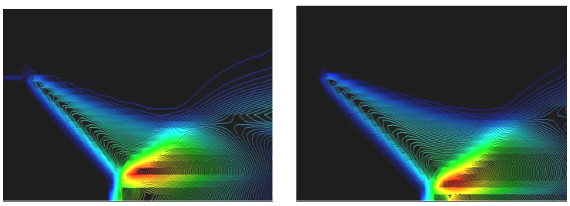
The typical isopycnics showing the hysteresis phenomena are shown in Fig.5 and Fig.6. All figures show the isopycnics of half area from jet axis when the pressure ratio ϕ is decreasing or increasing under the constant Mach number M_e .



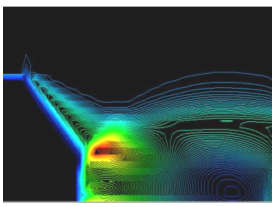
(a) ϕ=40 (RR) (g) ϕ=40 (RR)



(b) ϕ=29.5 (RR) (f) ϕ=35 (RR)



(c) ϕ=29 (MR) (e) ϕ=34.5 (MR)

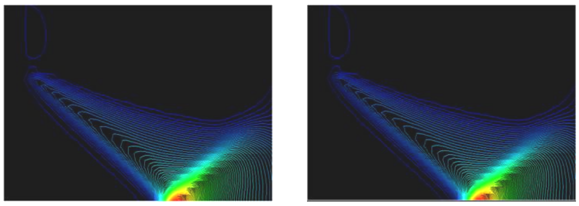


(d) ϕ=20 (MR)

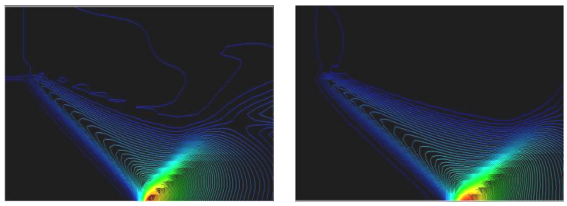
Fig.5 Typical isopycnics showing hysteresis phenomena (M_e=4)

Figure5 shows the results of M_e=4 and the figure (a), (b), (c) denote the results when the pressure ratio ϕ decreases and the figure (d), (e), (f) represent the results when the pressure ratio ϕ increases, respectively. In figure (a) and (b), the regular reflection (RR) is observed. In figure (c), the Mach reflection (MR) occurs at this condition. The reflection type transforms from RR to MR in region 29.5>ϕ>29 at this case. But in figure (d), (e), (f), namely, when ϕ increases, it is remarkable that the reflection type transforms from MR to RR in region 34.5<ϕ<35 and the transitional pressure ratio of reflection type differs compared with figure (c), (d), namely, when ϕ decreases. The condition occurred the hysteresis phenomena, therefore, are affected by the pressure ratio ϕ of flow field.

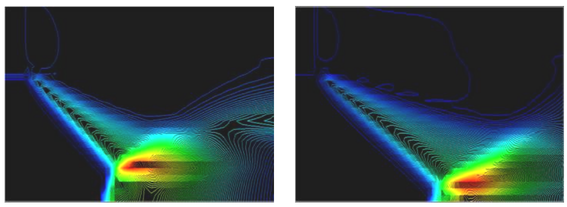
Figure 6 shows the result of M_e=5 and the reflection type transforms from RR to MR or MR to RR because of



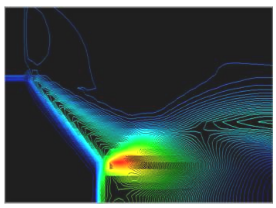
(a) ϕ=95 (RR) (g) ϕ=95 (RR)



(b) ϕ=63.5 (RR) (f) ϕ=93 (RR)



(c) ϕ=63 (MR) (e) ϕ=92.5 (MR)



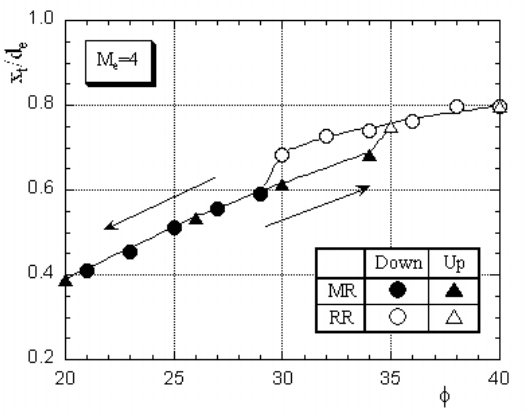
(d) ϕ=55 (MR)

Fig.6 Typical isopycnics showing hysteresis phenomena (M_e=5) change of pressure ratio ϕ. But, the region of transitional pressure ratio differs compared by the result of Fig.5 so that the hysteresis phenomena are affected by nozzle exit Mach number M_e.

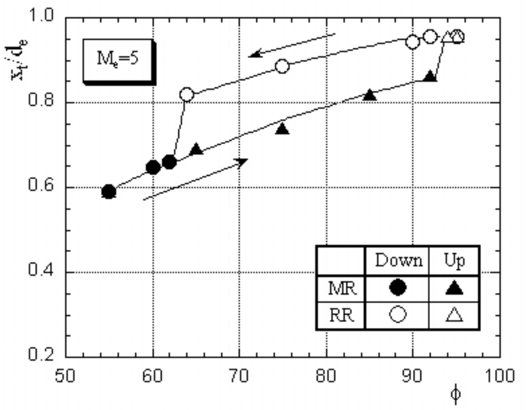
From the results of Fig.5 and Fig.6, it is remarkable that the hysteresis phenomena occur at the transitional pressure ratio between RR and MR.

Effect of hysteresis phenomena

The relation between the non-dimensional intersection position x_t/d_e and pressure ratio ϕ is shown in Fig.7. Figure (a) indicates the result for M_e=4 and figure (b) represents the result for M_e=5, respectively. The black symbol shows the MR type and white symbol indicates the RR type. The difference of symbol form denotes the increment or decrement of pressure ratio ϕ.



(a) M_e=4



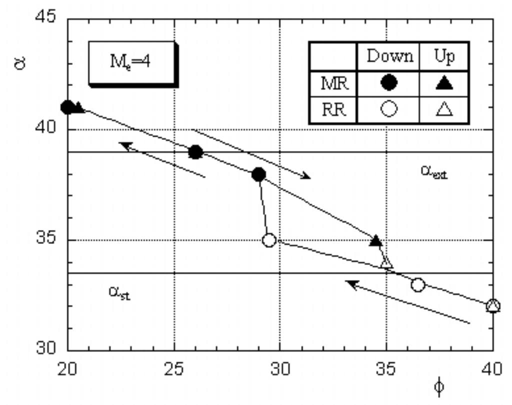
(b) M_e=5

Fig.7 Relation between non-dimensional intersection position x_t/d_e and pressure ratio ϕ

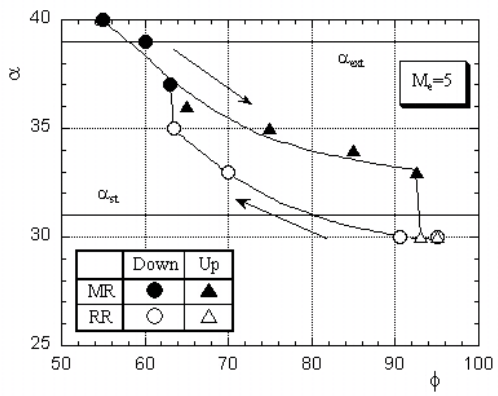
In figure (a), the value of x_t/d_e increases with increasing of ϕ and this fact means that the jet size extends with increasing of ϕ. The value of x_t/d_e, furthermore, differs at region 30<ϕ<34 because of increment or decrement of pressure ratio ϕ. The value of x_t/d_e, obtained at RR is larger than that obtained at MR, because the intersection position at RR transfers toward downstream. The reflection type changes in this region compared with the isopycnics and the effect of hysteresis phenomena occur at intersection position x_t/d_e.

On the other hand, in figure (b), the transition of reflection type occurs caused by change of pressure ratio, as same as figure (a), but the disagreement of x_t/d_e observes at region 64<ϕ<92 because of hysteresis phenomena. This region, in which the disagreement of x_t/d_e is observed, becomes high pressure ratio and this area extends compared with result of M_e=4. The condition occurred the hysteresis phenomena, therefore, are affected by the nozzle exit Mach number M_e and pressure ratio of flow field ϕ, as shown in Fig.4. These characteristics for overexpanded supersonic jet are in good agreement with the result for underexpanded supersonic jet [3].

Relation between the incident angle of oblique shock α and the pressure ratio ϕ is shown in Fig.8. Figure (a)



(a) M_e=4



(b) M_e=5

Fig.8 Relation between incident angle of oblique shock and pressure ratio indicates the result of M_e=4 and figure (b) represents the result of M_e=5, respectively. The black symbol shows the result of MR type and the white symbol indicates the result of RR type. The difference of symbol form denotes the increment or decrement pressure ratio. The two solid lines, furthermore, show the result obtained by theoretical criteria deduced from the analysis of shock wave angle [8], namely, two or three shock theory for two-dimensional flow. The upper line indicates the result of two shock theory and the symbol α_{ext} denotes the extreme angle. The lower line indicates the result of the three shock theory and the symbol α_{st} denotes the stationary angle. The region surrounded by two solid lines, therefore, means the dual solution domain.

In figure (a), the value of α decreases with increasing of x_t/d_e, because the intersection position increases, and is larger than that of RR, because the intersection position x_t/d_e transfers toward radius direction. Furthermore, the value of α differs at region 30<ϕ<34, caused by the increment or decrement of pressure ratio ϕ. As the same as Fig.7(a), the reflection type differs in this region from the isopycnics so that the effect of hysteresis phenomena occurs at the incident angle α of oblique shock.

In figure (b), the transition of reflection type occurs caused by change of pressure ratio, as the same as figure (a), but the disagreement of α is observed at the region 64<ϕ<92, caused by the hysteresis phenomena. This region becomes high pressure ratio and this area extends compared with the result of M_e=4.

From these figures, it is important that the hysteresis phenomena occur at the dual solution domain obtained for two-dimensional flow.

Figure 9 shows the relation between the incident angle of oblique shock α and the nozzle exit Mach number M_e. The solid line indicates the result by the two or three shock theory for regular reflection or Mach reflection of oblique shock in two-dimensional flow and the symbol <RR or MR> shows the dual solution domain. The symbol α_{ext} denotes the extreme angle and the symbol α_{st} denotes the stationary angle, respectively. In addition to the result of M_e=4 and M_e=5, the result of M_e=3 is shown in this figure. It is clear that the hysteresis phenomena obtained by this study for overexpanded axisymmetric jet are almost observed in the dual solution domain and this phenomenon occurred in the region of incident angle extends with increasing of nozzle exit Mach number M_e. This characteristics agree well with the result of underexpanded supersonic axisymmetric jet [3].

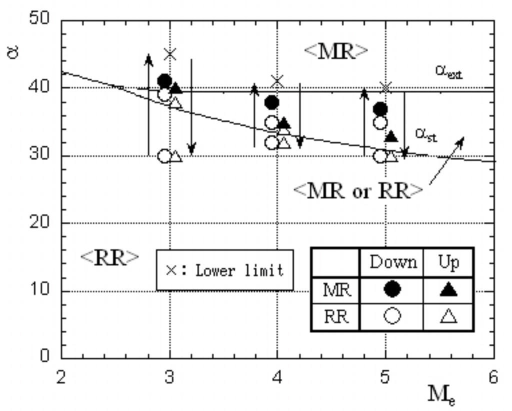


Fig.9 Relation between incident angle of oblique shock α and nozzle exit Mach number M_e

From all figures, it is concluded that the hysteresis phenomena for overexpanded axisymmetric jet is generally occurred in the dual solution domain and the incident angle of oblique shock could be predicted by the two or three shock theory for regular or Mach reflection of oblique shock and this characteristic agrees with the result of underexpanded supersonic axisymmetric jet.

Conclusions

The paper aims to clear the hysteresis phenomena for the reflection type of shock wave at the overexpanded axisymmetric jet using the TVD method. The results are as follows.

(1) The hysteresis phenomena occur in overexpanded supersonic axisymmetric jet at the specific condition. The effect by this phenomenon is observed at the intersection position x_t/d_e and the incident angle of oblique shock α.

(2) The non-dimensional intersection position x_t/d_e increases with increasing of pressure ratio ϕ and the value of x_t/d_e differs at specific condition caused by hysteresis phenomena. The disagreement region of x_t/d_e becomes high pressure ratio domain and this area extends with increasing of nozzle exit Mach number M_e.

(3) The incident angle of oblique shock α decreases with increasing of ϕ and the value of α differs at specific condition caused by hysteresis phenomena. The disagreement region of α becomes high pressure ratio domain and this area extends with increasing of nozzle exit Mach number M_e.

(4) The hysteresis phenomena for overexpanded axisymmetric jet is generally occurred in the dual solution domain and the incident angle of oblique shock could be predicted by the two or three shock theory for regular or Mach reflection of oblique shock. This characteristic agrees well with the result of underexpanded supersonic axisymmetric jet.

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