Algebraic Stress Model with RNG ε-Equation for Simulating Confined Strongly Swirling Turbulent Flows

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Strongly swirl flow simulation are still under developing. In this paper, ε equation based on the Renormalization Group theory is used into algebraic stress model. Standard k-ε model, algebraic stress model by Jiang Zhang[5] and present model (RNG-ASM) are applied simultaneously to simulating the confined strongly swirling flow. The Simulating results by RNG-ASM model are compared to the results by other two model, it is shown that the predictions by this model display reasonable agreement with experimental data, and lead to greater improvement than Zhang's ASM turbulence model[5].

Key words: numerical simulating, Renormalization Group theory, swirling flow.

Introduction

Strongly swirling flows occur in a wide variety of engineering applications, for example, vortex reactors, cyclone separators, spray machines, swirling flows combustion engines and many fluid heaters. It is known that swirling flows are a strong swirl-turbulence interaction and have strongly nonisotropic effects. The use of the standard k-ε model and the modified k-ε model[1,2] often leads to a poor prediction of the central toroidal recirculation, axial and tangential velocities and the combined free and forced vortex motion. This is due to the isotropic hypothesis inherent in the eddy viscosity models of turbulence. The DSM can generally reproduce the major features of the strongly swirling flows, and the use of DSM usually have good improvement in the accuracy of the prediction, however, DSM introduces much extra computational effort. Six Reynolds stress transport equations have to be solved simultaneously, and there is obvious difficulty in specifying the inlet conditions of the six Reynolds stress[3]. Based on Rodi's approximation, the six Reynolds stress partial differential transport equations are simplified into algebraic expressions. The ASM leads to great improvement in the accuracy of the simulating results and requires less computational effort[4]. In 1992, Jian Zhang proposes a new version of ASM turbulence model[5]. The model retains the nongradient convection terms arising from the transformation from Cartesian to cylindrical coordinates, which were ignored in the original ASM. Zhang's ASM is better than Rodi's for simulating strongly swirling flows.

However, the transport equation for TKE k and its dissipation rate ε have the same form as in standard k-ε model in Zhang's ASM turbulence model. Numerous tests have revealed its limitations and shortcomings. The Renormalization Group Theory (RNG) of Yakhot and Orszag and Yakhot et al[6] offer a new theory for ε equations, the new feature of RNG ε equation has contributed more than other modifications to an apparent success of the model to predict the appropriate length of recirculating zones of several separating flows[7] (Orszag et al, 1993). RNG k-ε model was also applied to solve the turbulent separated flows in a 180 degree duct[8] (Shaoping Wang et al, 1996) and in pulverized coal pipe separator[9] (Jiangrong Xu et al, 1998), and the results by RNG k-ε model shows the greater improvement than that by standard k-ε model.

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Nomenclature			
$C_{\varepsilon 1}$ $C_{\varepsilon 2}$ C_{μ}	turbulence model constants	μ	dynamic viscosity
σ_{ε} σ_k	diffusion coefficient	v	kinematic viscosity
k	turbulent kinetic energy	ρ	density
ε	dissipation rate of turbulent kinetic energy	Φ	generalized dependent variable
p	pressure	R	additional term of RNG ε-equation
x	axial coordinate	η	the ratio of turbulence
r	radial coordinate		$\overline{u'v'}$ $\overline{v'w'}$ $\overline{u'w'}$ $\overline{u'u'}$ $\overline{v'v'}$ $\overline{w'w'}$ Reynolds stress components
S	source term		**Superscripts**
t	time	'	fluctuating quantity
u	axial velocity	-	Reynolds mean quantity
v	radialvelocity		**Subscripts**
w	tangential velocity	e	effective
β	turbulence model constants	t	turbulent
G	the TKE generation term	x y θ	axial, radial and tangential coordinate directions
λ	turbulence model constants		

In this paper, RNG ε-equation is used with Zhang's ASM. the algebraic stress model with RNG ε-equation (RNG-ASM) is applied to simulate the confined strongly swirling flow. The validation was conducted through successive comparisons of the calculated gas axial and tangential velocities with the test data.

RNG-ASM Model

The Renormalization Group theory offers a new theoretic support to the basic form of the ε-equation, and also perspectives to better account for the effects of extra strain rates. The first conclusion of RNG theory was that the theory yielded the same form of the dissipation equation for high Re number with an additional term, and produced the numerical values of the coefficients—considerably different from the standard values—without employing any experimental results. The values proposed by Oaszag et al is $C_{\varepsilon 1}$=1.4 and $C_{\varepsilon 2}$=1.68. Besides, a considerably smaller value of $C_{\varepsilon 2}$ from the standard value of about 1.9 is compensated for by a higher diffusion coefficient (by 80%, namely σ_k=σ_{ε}=0.7179) and the additional term[7].

$$R = \frac{\eta(1-\eta/\eta_0)}{1+\beta\eta^3} \tag{1}$$

where $\eta = SK/\varepsilon$ which is the ratio of turbulence to mean strain time scale and β=0.015 is the constant. Where

$$S = (2\overline{S_{ij}S_{ji}})^{1/2}$$

$$\overline{S_{ij}} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

η_0 is chosen as 4.38 and represents a typical value in homogeneous shear flows. This additional term deserves attention because it changes sign depending on whether the time scale ratio η is greater or smaller than the homogeneous value η_0, in such a way distinguishing the small from the large strain rates (the ratio of turbulence to mean strain). Some experience suggests that more research along this lines may result in a more general and universal form of the dissipation equation.

Zhang's ASM with RNG ε-equation can be cast into a generalized axisymmetric cylindrical coordinate form as follows:

$$\frac{\partial}{\partial x}(\rho u\Phi) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v\Phi) = \frac{\partial}{\partial x}(\Gamma_{\Phi x}\frac{\partial\Phi}{\partial x}) + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_{\Phi r}\frac{\partial\Phi}{\partial r}) + S(\Phi) \tag{2}$$

The meanings and detailed expressions of $\Gamma_{\phi x}$, $\Gamma_{\phi r}$ and $S(\phi)$ for each governing equation are listed in Table 1. The constants used in Zhang's ASM and RNG-ASM are compared in Table 2.

Six Reynolds stress components of Zhang's ASM in generalized axisymmetric cylindrical coordinate is given:

$$\overline{u'v'} = -\nu_{xr}\frac{\partial u}{\partial r} \tag{3}$$

$$\overline{v'w'} = -\nu_{r\theta}\, r\frac{\partial}{\partial r}(\frac{w}{r}) \tag{4}$$

$$\overline{u'w'} = -\lambda\frac{k}{\varepsilon}\overline{u'u'}\frac{\partial w}{\partial x} - \lambda\frac{k}{\varepsilon}\left[(\frac{\partial w}{\partial r} + \beta\frac{w}{r})\overline{u'v'} + \frac{\partial u}{\partial r}\overline{v'w'}\right] \tag{5}$$

$$\overline{u'u'} = \frac{2}{3}k + \frac{2}{3}\lambda\frac{k}{\varepsilon}\left[-2\frac{\partial u}{\partial r}\overline{u'v'} + r\frac{\partial}{\partial r}(\frac{w}{r})\overline{v'w'}\right] \tag{6}$$

$$\overline{v'v'} = \frac{2}{3}k + \frac{2}{3}\lambda\frac{k}{\varepsilon}\left[\frac{\partial u}{\partial r}\overline{u'v'} + [\frac{\partial w}{\partial r} + (2+3\beta)\frac{w}{r}]\overline{v'w'}\right] \tag{7}$$

$$\overline{w'w'} = \frac{2}{3}k + \frac{2}{3}\lambda\frac{k}{\varepsilon}\left[\frac{\partial u}{\partial r}\overline{u'v'} - [2\frac{\partial w}{\partial r} + (1+3\beta)\frac{w}{r}]\overline{v'w'}\right] \tag{8}$$

Table 1 Summary of the gas flow governing equation using ASM with ε-equation

	$\Gamma_{\phi x}$	$\Gamma_{\phi r}$	$S(\phi)$
u	μ_e	μ_{xr}	$\frac{\partial}{\partial x}(\mu_e \frac{\partial u}{\partial x}) + \frac{1}{r}\frac{\partial}{\partial r}(r\mu_{xr}\frac{\partial v}{\partial r}) - \frac{\partial p}{\partial x} - \frac{\partial}{\partial x}(\rho \overline{u'u'}) - 2\frac{\partial}{\partial x}(\mu_t \frac{\partial u}{\partial x})$
v	μ_{xr}	μ_e	$\frac{\partial}{\partial x}(\mu_{xr}\frac{\partial u}{\partial r}) + \frac{1}{r}\frac{\partial}{\partial r}(r\mu_e \frac{\partial v}{\partial r}) - \frac{\partial p}{\partial r} - \frac{\partial}{r\partial r}(r\rho\overline{v'v'}) - 2\frac{\partial}{r\partial r}(\mu_t r\frac{\partial v}{\partial r}) - 2\frac{\mu_e v}{r^2} + \frac{\rho w^2}{r} + \frac{\rho \overline{w'w'}}{r}$
w	$\mu_{\theta x}$	$\mu_{r\theta}$	$-\frac{\rho v w}{r} - \frac{w}{r^2}\frac{\partial}{\partial r}(r\mu_{r\theta}) - \frac{\partial}{\partial x}(\overline{u'w'} + \mu_{\theta x}\frac{\partial w}{\partial x})$
k	μ_e/σ_k	μ_e/σ_k	$G_k - \rho\varepsilon$
ε	μ_e/σ_ε	μ_e/σ_ε	$\frac{\varepsilon}{k}[(C_1 - R)G_k - C_2\rho\varepsilon]$

Table 2 Constants in gas phase governing equation

	λ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	β	C_μ
Zhang'sASM	0.135	0.9	1.22	1.44	1.92	0.8	0.09
RNG-ASM	0.135	0.7179	0.7179	1.42	1.68	0.8	0.085

where

$$v_{xr} = \frac{b_1 - a_1 b_2 r\frac{\partial}{\partial r}(\frac{w}{r})}{1 - a_1 a_2 \frac{\partial u}{\partial r} r \frac{\partial}{\partial r}(\frac{w}{r})} \tag{9}$$

$$v_{r\theta} = \frac{b_2 - a_2 b_1 \frac{\partial u}{\partial r}}{1 - a_1 a_2 \frac{\partial u}{\partial r} r \frac{\partial}{\partial r}(\frac{w}{r})} \tag{10}$$

where

$$a_1 = (\lambda\frac{k}{\varepsilon})^2\left[(\frac{7}{3} + 3\beta)\frac{w}{r} + \frac{2}{3}\frac{\partial w}{\partial r}\right]/A_1$$

$$a_2 = \frac{2}{3}(\lambda\frac{k}{\varepsilon})^2\frac{\partial u}{\partial r}/A_2$$

$$b_1 = \frac{2}{3}\lambda\frac{k^2}{\varepsilon}/A_1$$

$$b_2 = \frac{2}{3}\lambda\frac{k^2}{\varepsilon}/A_2$$

$$A_1 = 1 + (\lambda\frac{k}{\varepsilon})^2\left[\frac{2}{3}(\frac{\partial u}{\partial r})^2 + (1+\beta)\frac{w}{r}\frac{\partial w}{\partial r} + (1+\beta)\beta(\frac{w}{r})^2\right]$$

$$A_2 = 1 + \frac{2}{3}(\lambda\frac{k}{\varepsilon})^2\left[(\frac{\partial w}{\partial r})^2 + (4+6\beta)\frac{w}{r}\frac{\partial w}{\partial r} + (1+6\beta+6\beta^2)(\frac{w}{r})^2\right]$$

where $\mu_e = \rho C_\mu \frac{k^2}{\varepsilon}$ $\quad \mu_t = \rho C_\mu \frac{k^{1.5}}{\varepsilon}$

$$\mu_{xr} = \rho v_{xr} \qquad \mu_{r\theta} = \rho v_{r\theta}$$

$$\mu_{\theta x} = \rho\lambda\frac{k}{\varepsilon}\overline{u'u'}$$

$$G_k = -\rho\frac{\partial u}{\partial r}\overline{u'v'} - \rho r\frac{\partial}{\partial r}(\frac{w}{r})\overline{v'w'}$$

Simulating Results and Discussion

The experiment results of the confined strongly swirling flow were described in Ref. [1]. The present work consists of two parts: numerical simulation of the confined strongly swirling flow using standard k-ε model and Zhang's ASM is used to compare the experimental data to testify the procedure, after that, RNG-ASM is used to calculate the confined strongly swirling flow, the predication is also compared with the results of Zhang's ASM and experimental data.

The procedure adopts the staggered grid system for pressure and velocity, the SIMPLE algorithm and TDMA, under relaxation, line by line, sweeping technique. The boundary conditions are described in Ref. [1], Wall function is employed in u, w, k and ε equations. The turbulent viscosity coefficients of u, w equations at near-wall points use the recommended expressions[10]:

$$y^+ = \frac{\Delta d C_\mu^{1/4} k_p^{1/2}}{v}, \quad u^+ = \frac{1}{\kappa}\ln(Ey^+),$$

$$\mu_t = \frac{y^+ \mu}{u^+} \tag{11}$$

where κ=0.4, E=9.0, values of ε at near-wall points are determined by:

$$\varepsilon = \frac{C_{\mu}^{3/4} k^{3/2}}{\kappa \Delta d} \tag{12}$$

Fig.1 shows a comparision of the calculated gas axial and tangential velocities using standard k-ε model, Zhang's ASM with experimental results. It is seen that the prediction of standard k-ε model leads to poor agreement with the experimental data. Zhang's ASM leads to a good improvement in predicating the central toroidal recirculation zone, axial and tangential velocity. However, tangential velocity is less than the experimental result. Fig.2 shows the stream lines of the confined strongly swirling flow. It is obvious that the central toroidal recirculation zone calculated by Zhang's ASM is longer than that by standard k-ε model, and is in good agreement with the experimental findings.

Fig.3 shows simulating gas axial and tangential velocities using RNG-ASM, which are compared with simulating results of Zhang's ASM and experimental results. It is shown that axial velocity in the central toroidal recirculation zone is in better agreement with the experimental data than the results by Zhang's ASM. Tangential velocity about the axisymmetric line is larger than the results by Zhang's model and in good agreement with experimental results.

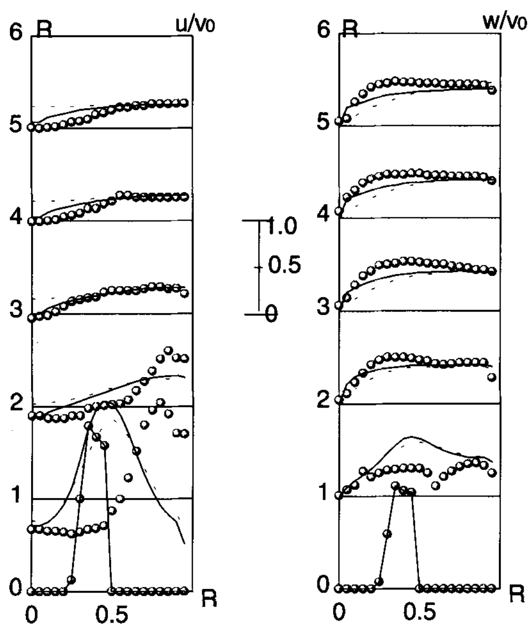

Fig.1 Comparision of calculating results by standard k-ε model and Zhang's ASM with experimental results
(··· standard k-ε model, — Zhang's ASM, • exp. data)

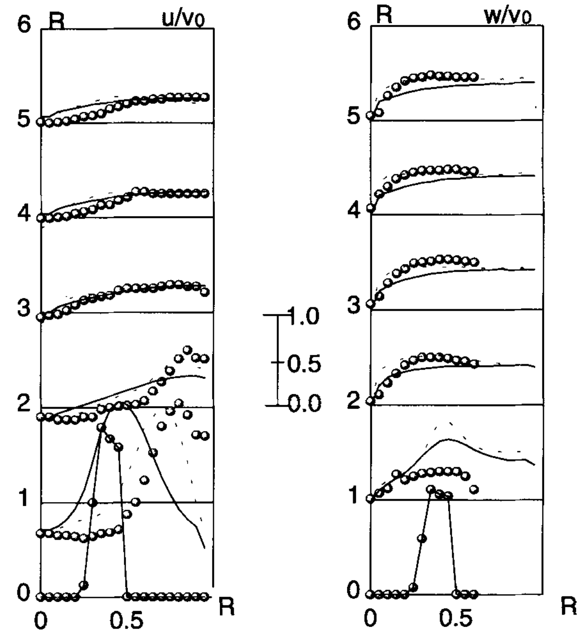

Fig.3 Comparision of calculating results by Zhang's ASM and RNG-ASM with experimental results
(··· RNG-ASM, — Zhang's ASM, • exp. data)

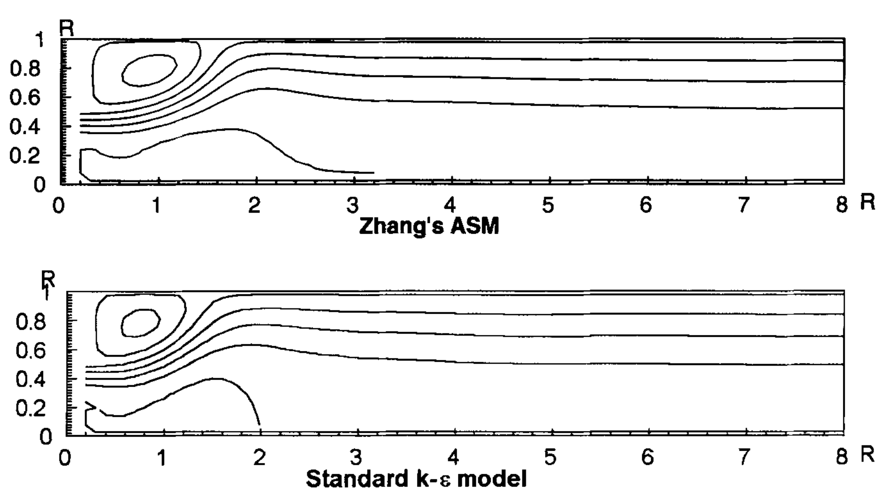

Fig.2 Comparision of calculating stream line by standard k-ε model with ASM

Fig.4 shows the stream lines simulating by RNG-ASM and Zhang's ASM. The central toroidal recirculation zone calculated by RNG-ASM is a bit longer than that by Zhang's model. The nonisotropic effects in confined strongly swirling flow is displayed in Fig.5. It is shown that six Reynolds stress components in the line of X/R=1 are strongly nonisotropic. After X/R=2, the nonisotropic effects decrease.

Conclusions

In the present study, ε equation based on the Renormalization Group theory is used into algebraic stress model. The model (RNG-ASM) is applied to simulate the confined strongly swirling flow. The predictions by this model display reasonable agreement with experimental data, and lead to greater improvement than Zhang's ASM turbulence model. Then, this study also suggests that besides the nonisotropic model like algebraic stress model, a more reasonable ε equation is also very important for simulating strongly swirling flows, because strongly swirling flows are strongly swirl-turbulence interaction, which have different dissipation rate. The use of a single time-and-length scale is one of the basic approximations pertinent to all single-scale models. the Renormalization Group theory offers a new theory to refine the turbulence model.

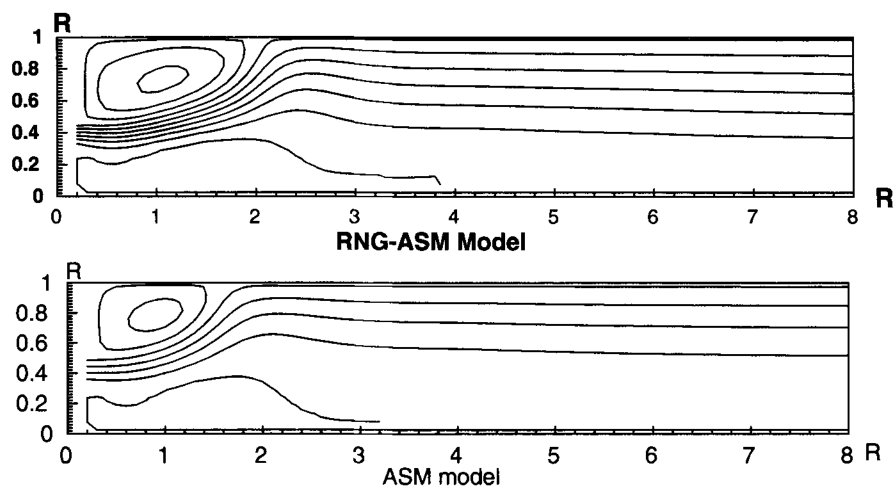

Fig.4 Comparision of calculating stream line by RNG-ASM with ASM

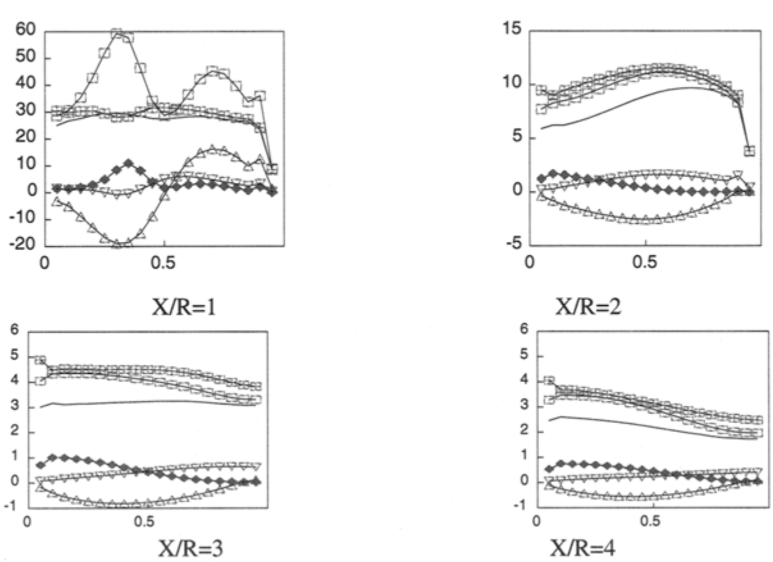

Fig.5 Six Reynolds stress components calculated by RNG-ASM

($\overline{u'u'}$, ⊞ $\overline{v'v'}$, ● $\overline{w'w'}$, Δ $\overline{u'v'}$, ∇ $\overline{v'w'}$, ◊ $\overline{u'w'}$)

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