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Limit equilibrium method (LEM) of slope stability and calculation of comprehensive factor of safety with double strength-reduction technique

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Abstract: When the slope is in critical limit equilibrium (LE) state, the strength parameters have different contribution to each other on maintaining slope stability. That is to say that the strength parameters are not simultaneously reduced. Hence, the LE stress method is established to analyze the slope stability by employing the double strength-reduction (DSR) technique in this work. For calculation model of slope stability under the DSR technique, the general nonlinear Mohr–Coulomb (M–C) criterion is used to describe the shear failure of slope. Meanwhile, the average and polar diameter methods via the DSR technique are both adopted to calculate the comprehensive factor of safety (FOS) of slope. To extend the application of the polar diameter method, the original method is improved in the proposed method. After comparison and analysis on some slope examples, the proposed method's feasibility is verified. Thereafter, the stability charts of slope suitable for engineering application are drawn. Moreover, the studies show that: (1) the average method yields similar results as that of the polar-diameter method; (2) compared with the traditional uniform strength-reduction (USR) technique, the slope stability obtained using the DSR technique tends to be more unsafe; and (3) for a slope in the critical LE state, the strength parameter φ, i.e., internal friction angle, has greater contribution on the slope stability than the strength parameters c, i.e., cohesion.

Keywords: Slope stability; Nonlinear Mohr–Coulomb (M-C) criterion; Double strength-reduction (DSR) technique; Slope Comprehensive Factor of Safety (FOS); Stability charts

Introduction

For stability analysis of a slope, the strength-reduction technique is usually used to obtain the critical limit equilibrium (LE) state of slope in the limit equilibrium method (LEM), limit analysis method (LAM), and numerical simulation method (NSM). Then, the reduction coefficient of strength parameters, i.e., the slope factor of safety (FOS), can be calculated to evaluate the slope stability (Dawson et al. 1999; Chen et al. 2007; Xiao et al. 2011; Nian et al. 2012; Tschuchnigg et al. 2015). Currently, the strength parameters are reduced using the uniform coefficient in the traditional

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strength-reduction technique (Cheng and Huang 2005; Zhang et al. 2013; Yang et al. 2014;Kelesoglu 2016;Tu et al. 2016; Sun et al. 2017), which is to say that the strength parameters have the same contribution on maintaining slope stability. However, when the slope reaches the critical LE state under some complex external factors, the reduction regularity of strength parameter is not consistent to each other. Considering the linear Mohr–Coulomb (M–C) criterion for the shear failure of a slope as an example, it has been confirmed that, when the slope transitions from stable state to unstable state, the strength parameters *c*, i.e., cohesion, and φ, i.e., internal friction angle, are not simultaneously reduced. Hence, different reduction coefficients should be adopted to reflect the contribution of each strength parameter on slope stability (Larochelle and Skempton 1965; Jiang et al. 2013). Accordingly, the actual failure process of a slope cannot be realistically simulated using the traditional uniform strength-reduction (USR) technique. Then, it is important to analyze the slope stability with the double strength-reduction (DSR) technique for further studying the destruction mechanism of slope.

In the term of slope stability analysis using the DSR technique, some tentative studies were conducted (Yuan et al. 2013; Bai et al. 2014; Zhao et al. 2015). However, most of these studies were carried out based on the LAM and NSM. For practical engineers, the LEM with simple theory is easy to be grasped. Thereby, some LEMs (Bishop 1955; Morgenstern and Price 1965; Spencer 1967) have been widely adopted in the slope design specifications. However, only a result, i.e., slope FOS, can be obtained from the traditional LEM. Moreover, it is also difficult to solve complex problems using the traditional LEM, such as the incorporation of nonlinear strength criterion (Jiang et al. 2003; Fu and Liao 2010; Eid 2010), which has hindered its development on slope stability. Compared to the LEM, both the LAM and NSM calculate the slope FOS by obtaining the stress distributions on a slip surface. Actually, after the stresses on a slip surface are known, it is convenient to solve the equation expressed in form of stress, such as the nonlinear M-C criterion. Accordingly, the LAM and NSM are both easy to perform stability analysis of a slope under complex conditions.

Based on the simple and practical characteristics of the LEM, the LE stress method for slope stability analysis using the DSR is established by incorporating the general nonlinear M–C criterion. Moreover, to assess the slope safety and construct the evaluation index consistent with the traditional LEM, the concept of slope comprehensive FOS via the DSR technique is adopted. For the slope comprehensive FOS, two methods, namely, average and polar diameter methods, are used, and the original polar diameter method is improved to extend its application in this study. By comparison and analysis on slope examples, the proposed method's feasibility is verified, and the rationality of the average method is studied. Meanwhile, to facilitate the engineering application, the stability charts suitable for guiding slope design under general conditions are also provided.

1 LEM for Slope Stability Analysis Using DSR Technique

1.1 Model of DSR in general nonlinear M–C criterion

In the stability analysis, the sliding of slope occurs because of the shear failure of slip surface, which is usually related to the M–C criterion. As shown in Figure 1, the general nonlinear M–C criterion is expressed as:

$$\tau_f = c_0 (1 + \frac{\sigma}{\sigma_t})^{\frac{1}{m}} \qquad (1)$$

Where σ is the normal stress of soil, τ_f is the shear strength of soil under the normal stress σ,

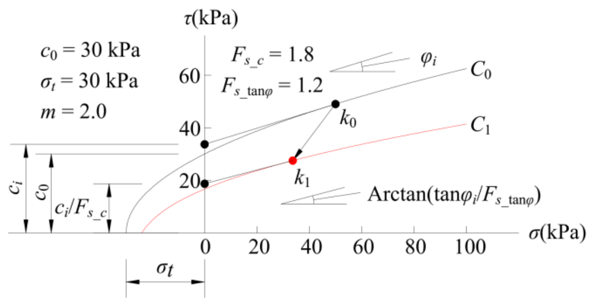


Figure 1 Model of DSR for general nonlinear M–C criterion. C_0 is the strength curve in general nonlinear M–C criterion; C_1 is the strength curve obtained using DSR technique.

and c_0, σ_t, and m ($m \geq 1$) are the strength parameters of soil.

In Equation (1) for $m = 1$, the linear M–C criterion is described. Then, Equation (1) can be written as $\tau_f = c + \sigma \times \tan\varphi$ when $c = c_0$ and $\tan\varphi = c_0 / \sigma_t$, where c is the cohesion of soil, and φ is the internal friction angle of soil.

Similar to the linear M–C criterion, the nonlinear M–C criterion can also have the same simple mathematical expression with the following two shear strength parameters, i.e., the instantaneous cohesion c_i and instantaneous internal friction angle φ_i. Then, the general nonlinear M–C criterion in Equation (1) is re-written as:

$$\tau_f = c_i + \sigma \tan\varphi_i \tag{2a}$$

$$c_i = c_0 (1 + \frac{\sigma}{\sigma_t})^{\frac{1-m}{m}} [1 + \frac{(m-1)\sigma}{m\sigma_t}] \tag{2b}$$

$$\tan\varphi_i = \frac{c_0}{m\sigma_t}(1 + \frac{\sigma}{\sigma_t})^{\frac{1-m}{m}} \tag{2c}$$

The definition of the traditional USR assumes that the two shear strength parameters, i.e., the cohesion and internal friction angle, are reduced simultaneously via the same reduction coefficient, which can be used as the slope FOS. However, the studies (Jiang et al. 2013; Isakov and Moryachkov 2014) show that the two shear strength parameters are not simultaneously reduced when the slope transitions from the stable state to the unstable state. Accordingly, the strength parameters have their different reduction coefficients. Thus, it is necessary to adopt the DSR technique in the slope stability analysis.

As shown in Figure 1, consistent with the physical definition of the USR, proposed by Bishop (1955), the two shear strength parameters of soil are reduced using the DSR technique, and then the potential sliding body of slope would transition into the critical LE state. Hence, by applying Equation (2), the shear stress on slip surface can be obtained using:

$$\tau = c_0 (1 + \frac{\sigma}{\sigma_t})^{\frac{1-m}{m}} \left\{ \frac{1}{F_{s\_c}} [1 + \frac{(m-1)\sigma}{m\sigma_t}] + \frac{1}{F_{s\_\tan\varphi}} \frac{\sigma}{m\sigma_t} \right\} \tag{3}$$

where τ is the shear stress on slip surface, F_{s_c} is the reduction coefficient of cohesion, and $F_{s\_\tan\varphi}$ is the reduction coefficient of internal friction angle.

For the relationship between the reduction coefficients F_{s_c} and $F_{s\_\tan\varphi}$ in the DSR technique, researchers (Jiang et al. 2013; Isakov and Moryachkov 2014) believe that a proportional relationship can be established. The equation is expressed as:

$$k = \frac{F_{s\_c}}{F_{s\_\tan\varphi}} \tag{4}$$

where k is the proportionality factor between reduction coefficients with the DSR technique.

In fact, there are many combinations for the relationship between the reduction coefficients F_{s_c} and $F_{s\_\tan\varphi}$. However, the proportional relationship between the reduction coefficients of two shear strength parameters, expressed in Equation (4), is feasible for all the combinations, and the only difference lies in the value of the proportionality factor k. Consequently, the calculated results using Equation (4) can reflect the general relationship between F_{s_c} and $F_{s\_\tan\varphi}$. Moreover, the above fact is verified by Isakov and Moryachkov (2014) using specific examples of slopes.

1.2 LE stress method for slope stability analysis

For a slope having a general shape, shown in Figure 2, when the x and y axes coordinate system is established with the slope toe as the origin, the equations of slope surface and slip surface can be written as $y = g(x)$ and $y = s(x)$, respectively. On the slip surface, A and B are the lower and upper sliding points, respectively. Taking a vertical micro-slice with width dx from the slope sliding body of slope to analyze the forces, the external forces include the gravity wdx, horizontal seismic force $k_H wdx$, vertical seismic force $k_V wdx$, horizontal

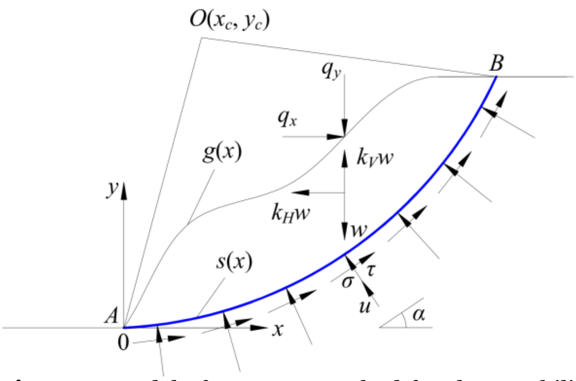


Figure 2 Model of LE stress method for slope stability analysis.

load $qxdx$ on slope surface, vertical load $qydx$ on slope surface, normal force $\sigma dx/cos\alpha$ on slip surface, the shear force $\tau dx/cos\alpha$ on slip surface, and the water force $udx/cos\alpha$ on slip surface. Here, k_H and k_V are the horizontal and vertical seismic force coefficients, respectively, σ, τ, and u are the normal stress, shear stress, and water pressure on slip surface, respectively, and α is the horizontal inclination angle of the tangent of slip surface in the vertical micro-slice.

The nonlinear failure characteristics of soil can be simulated by the nonlinear strength criterion. Moreover, for most of slopes, the shear strength of soil on slip surface show nonlinear behavior when the LE state is reached. Hence, compared with the linear strength criterion, the nonlinear strength criterion has wider applicability, and the linear strength criterion is only a special case of the nonlinear strength criterion. However, owing to the complex nonlinear relationship between the normal stress and the shear strength in the nonlinear strength criterion, it is difficult to integrate the nonlinear strength criterion into the theoretical solution of the traditional LEM. Thereby, the LE stress method (Deng et al. 2015, 2016) is adopted to analyze the slope stability by incorporating the nonlinear strength criterion.

In the LE stress method, the expression for the normal stress on slip surface is assumed as:

$$\sigma = \lambda_1 \sigma_0 \tag{5a}$$

$$\sigma_0 = \frac{[(1-k_V)w+q_y]+(-k_H w+q_x)s'}{1+(s')^2}-u \tag{5b}$$

where σ_0 is the initial normal stress on slip surface, which calculated from the stress analysis on the vertical micro-slice by neglecting the increment of inter-slice forces; λ_1, a dimensionless variable, is the correction coefficient of initial normal stress; and s' is the first derivative of slip surface equation $s(x)$ with respect to the x-axis, and $s' = \tan\alpha$.

Substituting Equation (4) into Equation (3), from Equation (3) the first derivative of τ with respect to σ can be obtained using:

$$\frac{d\tau}{d\sigma}=\frac{1}{F_{s\_c}}\frac{c_0}{\sigma_t}(1+\frac{\sigma}{\sigma_t})^{\frac{1-m}{m}}[\frac{(m+k-1)}{m^2}+\frac{(1-m)(1-k)}{m^2}(1+\frac{\sigma}{\sigma_t})^{-1}] \tag{6}$$

Then, expanding Equation (3) using the Taylor series with the initial normal stress σ_0 as the reference value, and substituting Equations (4) and (6) into the expanded equation, Equation (3) can be re-written as:

$$\begin{aligned}\tau=&\frac{1}{F_{s\_c}}c_0(1+\frac{\sigma_0}{\sigma_t})^{\frac{1-m}{m}}[1+\frac{(m+k-1)}{m}\frac{\sigma_0}{\sigma_t}]+\\&\frac{1}{F_{s\_c}}\frac{c_0}{\sigma_t}(1+\frac{\sigma_0}{\sigma_t})^{\frac{1-m}{m}}[\frac{(m+k-1)}{m^2}+\frac{(1-m)(1-k)}{m^2}(1+\frac{\sigma_0}{\sigma_t})^{-1}]\times\\&(\sigma-\sigma_0)+H(\sigma-\sigma_0)\end{aligned} \tag{7}$$

where $H(\sigma - \sigma_0)$ is the higher order of the error term.

In Equation (7), the higher order of error term $H(\sigma - \sigma_0)$ correlates with the first two terms in the right-hand side of the equation. Hence, the effect of $H(\sigma - \sigma_0)$ on the equation can be replaced by simply modifying the first two terms present in the right-hand side of the equation. Moreover, to simplify the calculation, another variable is used instead of $1/F_{s\_c}$. Then, the Equation (7) can be re-written as:

$$\tau = \lambda_2\tau_{01}+\lambda_3\tau_{02} \tag{8a}$$

$$\tau_{01}=c_0(1+\frac{\sigma_0}{\sigma_t})^{\frac{1-m}{m}}[1+\frac{(m+k-1)}{m}\frac{\sigma_0}{\sigma_t}] \tag{8b}$$

$$\tau_{02}=\frac{c_0}{\sigma_t}(1+\frac{\sigma_0}{\sigma_t})^{\frac{1-m}{m}}[\frac{(m+k-1)}{m^2}+\frac{(1-m)(1-k)}{m^2}(1+\frac{\sigma_0}{\sigma_t})^{-1}]\sigma_0 \tag{8c}$$

where λ_2 and λ_3 are the correction coefficients of shear stress on slip surface, and both the correction coefficients are dimensionless variables.

As shown in Figure 2, the force equilibrium conditions in the x and y directions and the moment equilibrium condition of all forces about the point (x_c, y_c) in the sliding body can be determined using:

$$\int[(-\sigma s'-us'+\tau)-(k_H w-q_x)]dx=0 \tag{9a}$$

$$\int[(\sigma+u+\tau s')-(1-k_V)w-q_y]dx=0 \tag{9b}$$

$$\begin{aligned}&\int[(-\sigma s'-us'+\tau)(y_c-s)+(\sigma+u+\tau s')(x-x_c)]dx-\\&\int\{[(1-k_V)w+q_y](x-x_c)+\\&k_H w[y_c-\frac{1}{2}(s+g)]-q_x(y_c-g)\}dx=0\end{aligned} \tag{9c}$$

By substituting Equation 9(a)-(b) into Equation 9(c) and further simplifying, the following equation can be obtained:

$$\int[(\sigma+u)(ss'+x)+\tau(xs'-s)]dx-\int\{[(1-k_V)w+q_y]x-\frac{1}{2}k_H w(s+g)+q_x g\}dx=0 \quad (10)$$

Then, by substituting Equations 5(a) and 8(a) into Equations 9(a)-(b) and (10), respectively, the following linear equations for the variables λ_1, λ_2, and λ_3 can be obtained:

$$\sum_{j=1}^{3} a_{ij}\lambda_j = b_i \ (i=1, 2, 3) \quad (11a)$$

$$a_{11} = -\int \sigma_0 s' dx \quad (11b)$$

$$a_{12} = \int \tau_{01} dx \quad (11c)$$

$$a_{13} = \int \tau_{02} dx \quad (11d)$$

$$b_1 = \int (k_H w - q_x + us')dx \quad (11e)$$

$$a_{21} = \int \sigma_0 dx \quad (11f)$$

$$a_{22} = \int \tau_{01} s' dx \quad (11g)$$

$$a_{23} = \int \tau_{02} s' dx \quad (11h)$$

$$b_2 = \int [(1-k_V)w + q_y - u]dx \quad (11i)$$

$$a_{31} = \int \sigma_0 (ss'+x)dx \quad (11j)$$

$$a_{32} = \int \tau_{01}(xs'-s)dx \quad (11k)$$

$$a_{33} = \int \tau_{02}(xs'-s)dx \quad (11l)$$

$$b_3 = \int \{[(1-k_V)w+q_y]x - \frac{1}{2}k_H w(s+g) + q_x g - u(ss'+x)\}dx \quad (11m)$$

By solving Equation (11), the variables λ_1, λ_2, and λ_3 are calculated. Thereafter, substituting these known variables into Equations (5) and (8), respectively, the normal stress σ and the shear stress τ acting on slip surface can be obtained.

According to the aforementioned analysis, after the two shear strength parameters of soil are reduced using the DSR technique, the potential sliding body of slope would transition into the critical LE state. Then, the anti-sliding force, which is obtained using the reduced strength parameters based on Equation (7), is equal to the sliding force on slip surface, solved using Equation (8). Thus, the strength reduction coefficients F_{s_c} and $F_{s\_\tan\varphi}$ can be respectively calculated using:

$$F_{s\_c} = \frac{\int\left\{c_0(1+\frac{\sigma}{\sigma_t})^{\frac{1-m}{m}}[1+\frac{(m+k-1)}{m}\frac{\sigma}{\sigma_t}]/\cos\alpha\right\}dx}{\int(\tau/\cos\alpha)dx} \quad (12a)$$

$$F_{s\_\tan\varphi} = \frac{F_{s\_c}}{k} \quad (12b)$$

where c_0, σ_t, m, k, and α have the same physical definitions as previously explained, and the normal stress σ and the shear stress τ can be calculated using Equations (5) and (8), respectively.

The above proposed method is still established on basic of the traditional LE theory. At present, numerical simulation software are often used to solve the slope stability under complex conditions. The reason is that the numerical simulation analysis has some advantages, such as diversification of output results and incorporation of nonlinear strength criterion of soil. However, numerical simulation results are greatly affected by the change of slope parameters. Compared with the numerical simulation method, the proposed method has the simple formulas and reliable calculation result. Moreover, the nonlinear M-C strength criterion is also adopted to analyze slope stability in this work.

2 Calculation of Slope Comprehensive FOS with DSR Technique

For the stability analysis of slope, a simple index, namely, FOS, is often used to evaluate the slope stability. The traditional USR technique can directly obtain the FOS, i.e., the strength-reduction coefficient. Comparing with the traditional USR technique, the contribution of the strength parameters of soil on maintaining slope stability can be truly reflected with the DSR technique when the slope has failed. However, the two coefficients, F_{s_c} and $F_{s\_\tan\varphi}$, are obtained to perform the slope stability analysis for the DSR. As the two coefficients have the opposite relationship, so it is difficult to solve the slope stability analysis using the DSR technique. To establish the same evaluation index with the USR technique, the concept of slope comprehensive FOS using the DSR technique is adopted. Moreover, the slope comprehensive FOS is the function of F_{s_c} and $F_{s\_\tan\varphi}$.

For solving the slope comprehensive FOS, two methods, i.e., average method and polar diameter method, are often used.

2.1 Average method

Jiang et al. (2013) believes that the slope comprehensive FOS can be calculated using the average of F_{s_c} and $F_{s\_\tan\varphi}$, named by the average method, and it is expressed as:

$$F_s = \frac{F_{s\_c} + F_{s\_\tan\varphi}}{2} \tag{13}$$

where F_s is the slope comprehensive FOS calculated using the DSR technique.

In Equation (13), when $F_{s\_c} = F_{s\_\tan\varphi}$, which implies that the traditional USR technique is adopted, the slope comprehensive FOS calculated using Equation (13) is consistent with the slope FOS calculated using the USR technique.

2.2 Polar diameter method

As shown in Figure 3a, a Cartesian coordinate system is established with $1/F_{s\_c}$ as x-axis and $1/F_{s\_\tan\varphi}$ as y-axis. By drawing the curve of strength-reduction coefficients in x and y axes coordinate system, Isakov and Moryachkov (2014) believe that the slope comprehensive FOS can be calculated by applying the strength reduction path, named by the polar diameter method, and it is expressed as:

$$F_s = \frac{1}{1 - L/\sqrt{2}} \quad (F_{s\_c} \geq 1 \text{ and } F_{s\_\tan\varphi} \geq 1) \tag{14}$$

where L is the length of the strength-reduction path, i.e., the length of the line M_0M_k, M_0 is the coordinate point for $1/F_{s\_c} = 1$ and $1/F_{s\_\tan\varphi} = 1$, M_k is the point on the curve of strength-reduction coefficients, and $L = \sqrt{(1-1/F_{s\_c})^2 + (1-1/F_{s\_\tan\varphi})^2}$.

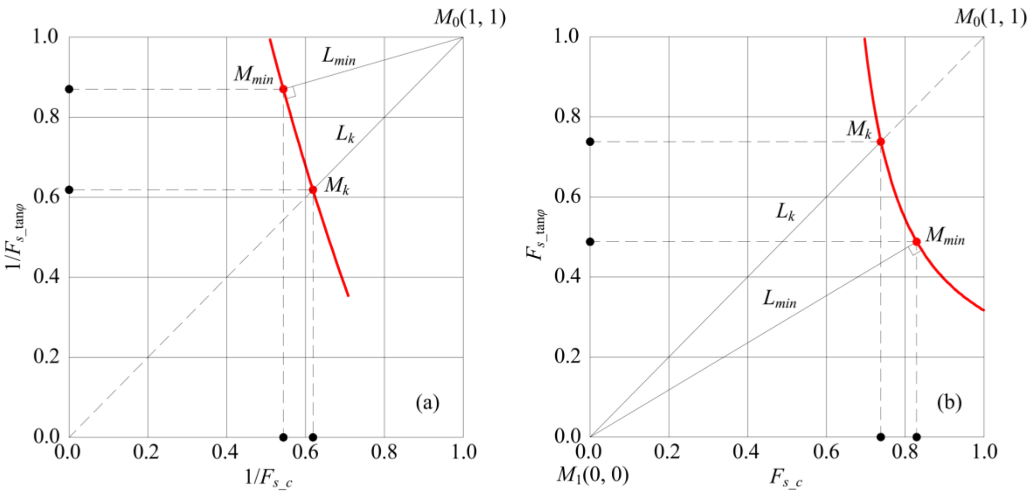
From the viewpoint of the special Cartesian coordinate system established by Isakov and Moryachkov (2014), Equation (14) is only used for the cases when $F_{s\_c} \geq 1$ and $F_{s\_\tan\varphi} \geq 1$. In fact, when the strength reduction coefficients are less than 1, the strength parameters should be enhanced to make the slope be in critical LE state. However, the original polar diameter method could not solve the slope comprehensive FOS for the condition of $F_{s\_c} < 1$ and $F_{s\_\tan\varphi} < 1$. To make up for the shortcoming of the original polar diameter method, shown in Figure 3b, another Cartesian coordinate system is established with F_{s_c} as x-axis and $F_{s\_\tan\varphi}$ as y-axis. Similarly, by drawing the curve of the strength-reduction coefficients in the x and y axes coordinate system, the slope comprehensive FOS is also calculated by applying the same principle, i.e., the strength-reduction path. The equation is given by:

$$F_s = L/\sqrt{2} \quad (F_{s\_c} < 1 \text{ and } F_{s\_\tan\varphi} < 1) \tag{15}$$

where L is the length of strength-reduction path, i.e., the length of line M_1M_k, M_1 is the coordinate point for $F_{s\_c} = 0$ and $F_{s\_\tan\varphi} = 0$, M_k is the point on the curve of strength-reduction coefficients, and $L = \sqrt{F_{s\_c}^2 + F_{s\_\tan\varphi}^2}$.

2.3 Application of two calculation methods

After the equation for calculating the slope comprehensive FOS is selected, the most important



(a) $F_{s\_c} \geq 1$ and $F_{s\_\tan\varphi} \geq 1$ (b) $F_{s\_c} < 1$ and $F_{s\_\tan\varphi} < 1$

Figure 3 Calculation model for slope comprehensive FOS using polar diameter method via DSR technique.

aspect of solving the slope stability is to determine the minimum slope comprehensive FOS and its corresponding critical slip surface. For the average method, different proportionality factor k from the linear relationship between F_{s_c} and $F_{s\_\tan\varphi}$ are adopted, and the slope comprehensive FOS is obtained using Equation (13) by searching the critical slip surface for every k. Then, among these slope comprehensive FOSs, the minimum value is the final result, and its corresponding slip surface is the final critical slip surface. For the polar diameter method, as shown in Figure 3, similarly, different proportionality factor k from the linear relationship between F_{s_c} and $F_{s\_\tan\varphi}$ are adopted, and the slope comprehensive FOS is obtained using Equations (14) or (15) by searching the critical slip surface for every k. Then, the curves for the strength-reduction coefficients are drawn. Thereafter, based on the shortest length L_{min} of strength-reduction path, the minimum slope comprehensive FOS can be obtained.

The above analysis shows that the average method has a simple calculation equation and it is suitable to calculate the slope comprehensive FOS in all case. However, the physical definition of the average method on slope comprehensive FOS is unclear.

Compared with the average method, the polar diameter method calculates the minimum slope comprehensive FOS by applying the shortest length of strength-reduction path, and it has a clear physical definition. However, the original equation of polar diameter method, proposed by Isakov and Moryachkov (2014), is only suitable when $F_{s\_c} \geq 1$ and $F_{s\_\tan\varphi} \geq 1$. Accordingly, in this work, the application of the original polar diameter method is extended, and the new established equation is suitable for the condition of $F_{s\_c} < 1$ and $F_{s\_\tan\varphi} < 1$, shown in Figure 3b. Thus, for the polar diameter method, it has different formulas of slope comprehensive FOS for the above two cases so as to make its calculation process more complicated.

3 Comparison and Analysis on Slope Examples

Slope example 1: A homogeneous slope has a slope height H = 15 m and slope angle β = 60°. In the slope, the soil natural unit weight is γ = 17.8 kN/m³, and the shear failure of soil is subject to the linear M–C criterion for slope sliding, i.e., m = 1 in Equation (1). Then, based on the different combinations of strength parameters of soil, i.e., cohesion c and internal friction angle φ, 16 cases are listed to solve the slope stability using the DSR technique. Table 1 gives the obtained results using the circular slip surface. Moreover, to verify the feasibility of the LE stress method proposed in this work, the traditional LE Morgenstern–Price (M–P) method is adopted, and both the results are compared and analyzed. Additionally, for case 13, the curve is drawn to show the relationship between the reduction coefficients and slope comprehensive FOS in Figure 4 (for stress method)

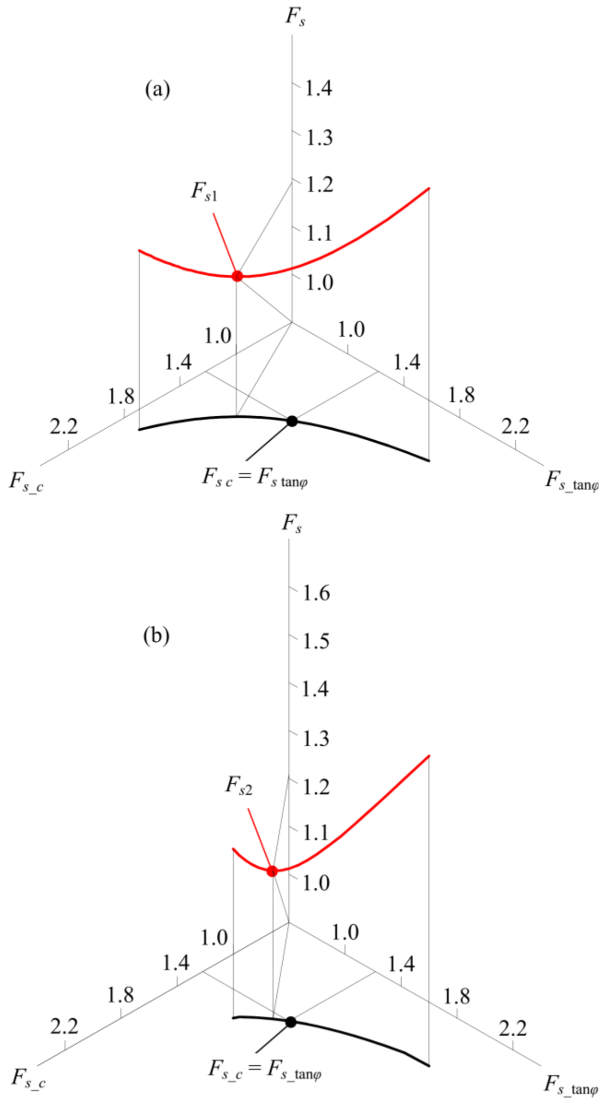


Figure 4 Results of limit equilibrium (LE) stress method in example 1 for case 13(a) Average method; (b) Polar diameter method.

and Figure 5 (for M–P method). In Table 1, for the stress method, the minimum slope comprehensive FOSs calculated using the average and polar diameter methods are named by F_{s1} and F_{s2}, respectively, and for the M–P method, the minimum slope comprehensive FOSs calculated using the average and polar diameter methods are named F_{s3} and F_{s4}, respectively.

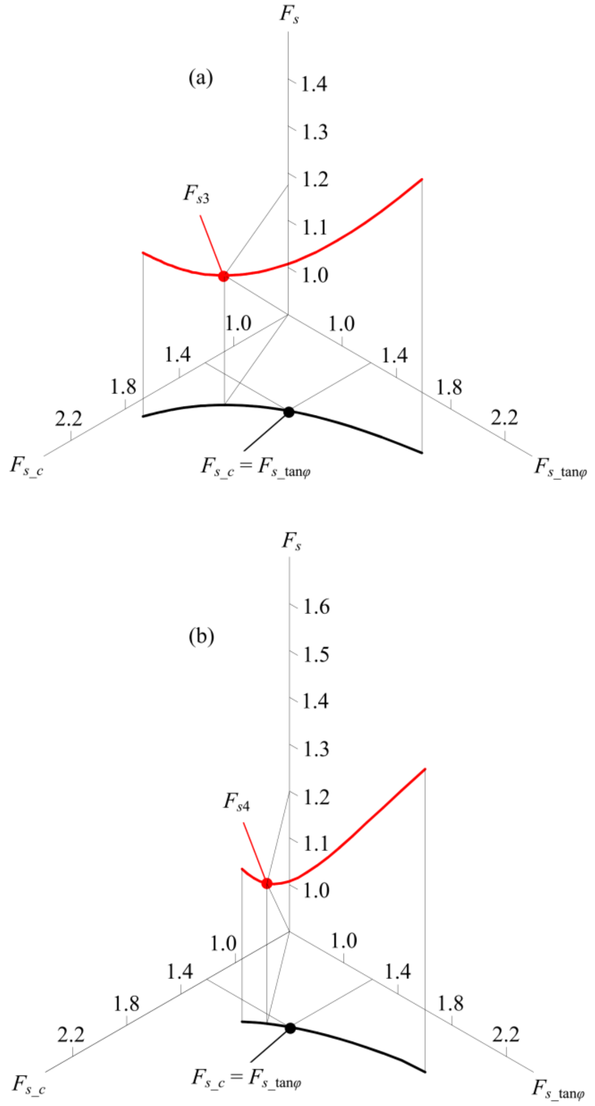


Figure 5 Results of limit equilibrium (LE) M–P method in example 1 for case 13(a) Average method; (b) Polar diameter method.

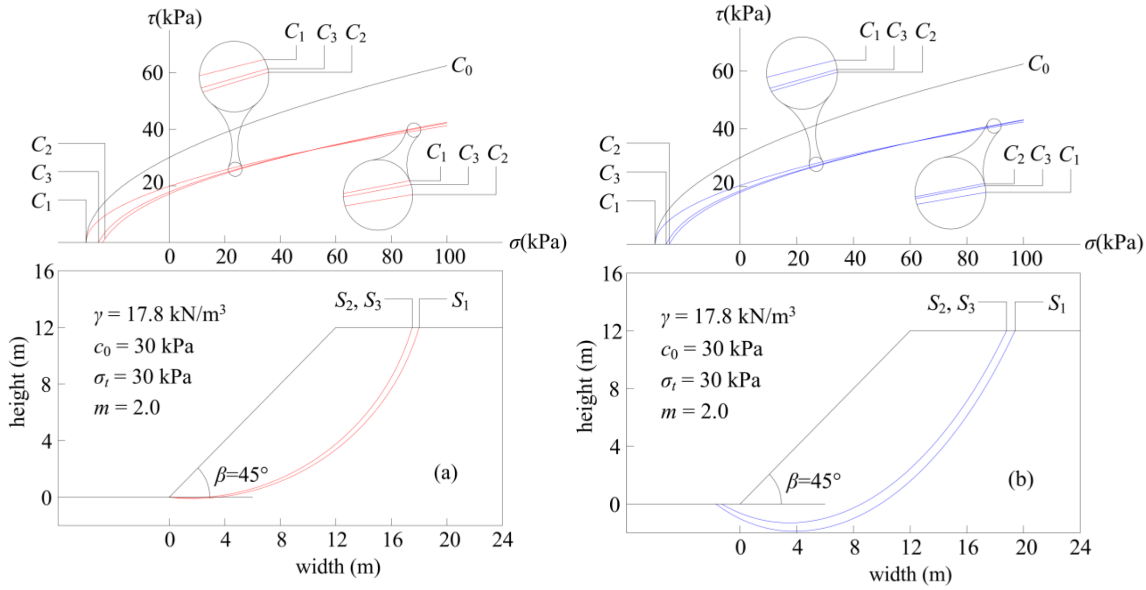
Table 1, Figure 4, and Figure5 show that: (1) the results obtained using the traditional LE M–P method and the LE stress method are in good agreement, verifying the feasibility of the LE stress method; (2) the results of the average and polar diameter methods are the same to each other, verifying the reasonability of the average method; (3) compared with the traditional USR technique, the minimum slope comprehensive FOS is smaller from the DSR technique; and (4) despite a linear proportionality is assumed between the reduction coefficients, the results exhibit their nonlinear relationship with slope comprehensive FOS.

Slope example 2: A homogeneous slope has the slope height H = 12 m and slope angle β = 45°. In the slope, the soil natural unit weight is γ = 17.8 kN/m3, and the shear failure of soil is subject to the nonlinear M–C criterion for slope sliding, i.e., m>1 in Equation (1). Then, based on the different combinations of strength parameters of soil, i.e., c_0, σ_t, and m, 32 cases are listed to solve the slope stability using the DSR technique via the LE stress method. Table 2 gives the obtained results. Additionally, for case 26, the strength curve and critical slip surface are drawn in Figure 6. In Table 2, the two types of slip surfaces, i.e., the circular slip surface and arbitrary curved slip surface, are adopted. Using the circular slip surface, the minimum slope comprehensive FOS calculated from the average and polar diameter methods are named by F_{s1_CSS} and F_{s2_CSS}, respectively. Using the arbitrary curved slip surface, the minimum slope comprehensive FOS calculated from the average and polar diameter methods are named by F_{s1_ACSS} and F_{s2_ACSS}, respectively.

Table 2 and Figure 6 show that: (1) the results obtained using the arbitrary curved slip surface are slightly close but less than the results obtained using the circular slip surface; (2) the failure area of the critical arbitrary curved slip surface is larger than that of the critical circular slip surface; (3) for the shear failure of a slope subject to the nonlinear M–C criterion, both the average and polar diameter methods also have the same results; and (4) when the normal stress σ on slip surface is in the low stress region, i.e., $\sigma \le$ 20kPa, the strength curve drawn using the DSR technique has the lower position than that drawn using the USR technique, which is the reason of the differences on their calculation results.

4 Stability Charts of Slope

The analysis from the above examples shows that the average method can obtain the same slope



(a) Strength curve and critical circular slip surface (b) Strength curve and critical arbitrary curved slip surface

Figure 6 Contrast of strength curve and critical slip surface in example 2 for case 26. C_0 is the strength curve in nonlinear M–C criterion; C_1 is the strength curve obtained using uniform strength-reduction (USR) technique; C_2 is the strength curve obtained using double strength-reduction (DSR) technique via average method; C_3 is the strength curve obtained using DSR technique via polar diameter method; S_1 is the slope critical slip surface obtained using USR technique; S_2 is the slope critical slip surface obtained using DSR technique via average method; S_3 is the slope critical slip surface obtained using DSR technique via polar diameter method.

Table 1 Contrast of the results of slope stability analysis in example 1

Case	Slope parameters		Soil parameters			Minimum slope comprehensive FOS and their differences					
						Stress method			M-P method		
	H (m)	β (°)	γ (kN/m³)	c (kPa)	φ (°)	F_{s1}	F_{s2}	$[F_{s1} - F_{s2}] / F_{s2}$ × 100%	F_{s3}	F_{s4}	$[F_{s3} - F_{s4}] / F_{s4}$ × 100%
1	15	60	17.8	20	20	0.874	0.875	-0.11%	0.854	0.855	-0.12%
2	15	60	17.8	20	25	0.985	0.985	0.00%	0.963	0.963	0.00%
3	15	60	17.8	20	30	1.097	1.097	0.00%	1.074	1.075	-0.09%
4	15	60	17.8	20	35	1.213	1.215	-0.16%	1.189	1.192	-0.25%
5	15	60	17.8	25	20	0.986	0.994	-0.80%	0.965	0.975	-1.03%
6	15	60	17.8	25	25	1.105	1.108	-0.27%	1.080	1.084	-0.37%
7	15	60	17.8	25	30	1.225	1.226	-0.08%	1.198	1.198	0.00%
8	15	60	17.8	25	35	1.350	1.350	0.00%	1.322	1.322	0.00%
9	15	60	17.8	30	20	1.091	1.105	-1.27%	1.072	1.093	-1.92%
10	15	60	17.8	30	25	1.219	1.223	-0.33%	1.192	1.201	-0.75%
11	15	60	17.8	30	30	1.346	1.347	-0.07%	1.316	1.317	-0.08%
12	15	60	17.8	30	35	1.478	1.478	0.00%	1.446	1.446	0.00%
13	15	60	17.8	35	20	1.192	1.209	-1.41%	1.175	1.199	-2.00%
14	15	60	17.8	35	25	1.327	1.332	-0.38%	1.301	1.312	-0.84%
15	15	60	17.8	35	30	1.461	1.462	-0.07%	1.429	1.431	-0.14%
16	15	60	17.8	35	35	1.600	1.599	0.06%	1.564	1.563	0.06%

comprehensive FOS with the polar diameter method. Meanwhile, compared with the polar diameter method, the average method is relatively simple without application restriction on slope stability. Thus, the average method is adopted to draw the stability charts of slope. Moreover, the linear M–C criterion for the shear failure of soil has less strength parameters, which can be obtained by simple laboratory experiments. Hence, in practical scenarios, it is simple and practical for the linear

Table 2 Contrast of the results of slope stability analysis in example 2

Case	Slope parameters		Soil parameters				Minimum slope comprehensive FOS and their differences					
	H (m)	β (°)	γ (kN/m³)	c_0 (kPa)	σ_t (kPa)	m	Circular slip surface		Arbitrary curved slip surface (Deng et al. 2011)		Difference	
							F_{s1_CSS}	F_{s2_CSS}	F_{s1_ACSS}	F_{s2_ACSS}	DIFF_1	DIFF_2
1	12	45	17.8	15	30	1.5	0.904	0.906	0.892	0.894	1.35%	1.34%
2	12	45	17.8	15	30	2.0	0.731	0.740	0.717	0.724	1.95%	2.21%
3	12	45	17.8	15	25	1.5	0.980	0.983	0.969	0.972	1.14%	1.13%
4	12	45	17.8	15	25	2.0	0.781	0.789	0.767	0.773	1.83%	2.07%
5	12	45	17.8	15	20	1.5	1.086	1.088	1.076	1.077	0.93%	1.02%
6	12	45	17.8	15	20	2.0	0.849	0.856	0.835	0.840	1.68%	1.90%
7	12	45	17.8	15	15	1.5	1.248	1.248	1.238	1.238	0.81%	0.81%
8	12	45	17.8	15	15	2.0	0.950	0.957	0.936	0.941	1.50%	1.70%
9	12	45	17.8	20	30	1.5	1.205	1.211	1.190	1.194	1.26%	1.42%
10	12	45	17.8	20	30	2.0	0.974	1.009	0.955	0.983	1.99%	2.64%
11	12	45	17.8	20	25	1.5	1.307	1.309	1.292	1.294	1.16%	1.16%
12	12	45	17.8	20	25	2.0	1.041	1.068	1.023	1.047	1.76%	2.01%
13	12	45	17.8	20	20	1.5	1.448	1.449	1.434	1.435	0.98%	0.98%
14	12	45	17.8	20	20	2.0	1.132	1.151	1.113	1.130	1.71%	1.86%
15	12	45	17.8	20	15	1.5	1.664	1.664	1.651	1.651	0.79%	0.79%
16	12	45	17.8	20	15	2.0	1.267	1.276	1.247	1.256	1.60%	1.59%
17	12	45	17.8	25	30	1.5	1.506	1.506	1.487	1.487	1.28%	1.28%
18	12	45	17.8	25	30	2.0	1.218	1.242	1.194	1.216	2.01%	2.14%
19	12	45	17.8	25	25	1.5	1.633	1.631	1.615	1.614	1.11%	1.05%
20	12	45	17.8	25	25	2.0	1.302	1.317	1.278	1.292	1.88%	1.93%
21	12	45	17.8	25	20	1.5	1.810	1.808	1.793	1.792	0.95%	0.89%
22	12	45	17.8	25	20	2.0	1.415	1.422	1.391	1.398	1.73%	1.72%
23	12	45	17.8	25	15	1.5	2.080	2.079	2.063	2.064	0.82%	0.73%
24	12	45	17.8	25	15	2.0	1.583	1.580	1.559	1.557	1.54%	1.48%
25	12	45	17.8	30	30	1.5	1.807	1.798	1.785	1.778	1.23%	1.12%
26	12	45	17.8	30	30	2.0	1.462	1.469	1.433	1.440	2.02%	2.01%
27	12	45	17.8	30	25	1.5	1.960	1.952	1.938	1.933	1.14%	0.98%
28	12	45	17.8	30	25	2.0	1.562	1.599	1.534	1.533	1.83%	4.31%
29	12	45	17.8	30	20	1.5	2.173	2.167	2.152	2.148	0.98%	0.88%
30	12	45	17.8	30	20	2.0	1.698	1.686	1.669	1.660	1.74%	1.57%
31	12	45	17.8	30	15	1.5	2.496	2.495	2.476	2.476	0.81%	0.77%
32	12	45	17.8	30	15	2.0	1.900	1.878	1.871	1.854	1.55%	1.29%

Notes: DIFF_1 = [F_{s1_CSS} – F_{s1_ACSS}] / F_{s1_ACSS} × 100%; DIFF_2 = [F_{s2_CSS} – F_{s2_ACSS}] / F_{s2_ACSS} × 100%.

M–C criterion to be widely adopted by engineers to analyze the slope stability.

Taking a homogeneous slope with a slope angle β and slope height of H = 10 m as an example, the shear failure of soil for slope sliding is assumed to be subjected to the linear M–C criterion. The soil has the following parameters: the natural unit weight γ, cohesion c, and internal friction angle φ, where γ = 17.8 kN/m³. To facilitate the engineering application, a Cartesian coordinate system is established with the dimensionless variable $c/(\gamma H)$ as the horizontal coordinate and the minimum slope comprehensive FOS as the vertical coordinate. Using circular slip surface, the slope stability is analyzed with the LE stress method for slope angles β = 30°, 40°, 50°, 60°, 70°, 80°, and 90°, respectively. Then, the slope stability charts are drawn in Figure 7. In Figure 7, the stability charts have the applicable scope of 5° ~ 35° for the internal friction angle φ. Additionally, in Figure 7, corresponding to the minimum slope comprehensive FOS, the curve of proportionality factor k is also drawn. Although the stability charts in Figure 7 are obtained for the slope with slope height of H = 10 m, it is suitable for the slope with different slope height to be quickly analyzed its stability.

From Figure 7, it shows that: (1) the proportionality factor k corresponding to the minimum slope comprehensive FOS is usually more than 1, implying that when the slope is in the

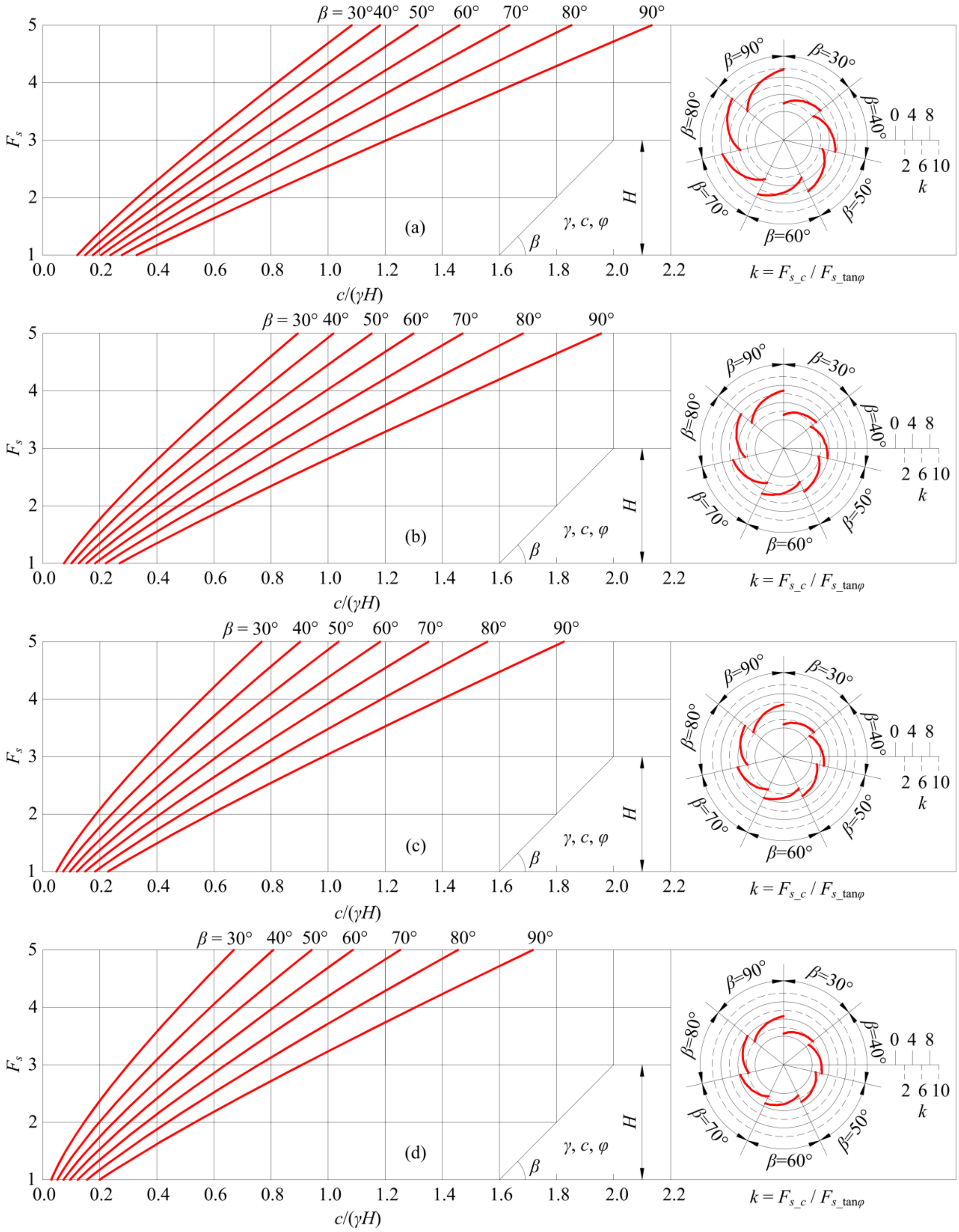


Figure 7 Stability charts of slope using double strength-reduction (DSR) technique(a) φ = 5°; (b) φ = 10°; (c) φ = 15°; (d) φ = 20°; (e) φ = 25°; (f) φ = 30°; (g) φ = 35°; (h) φ = 40°

(-To be continued-)

(-Continued-)

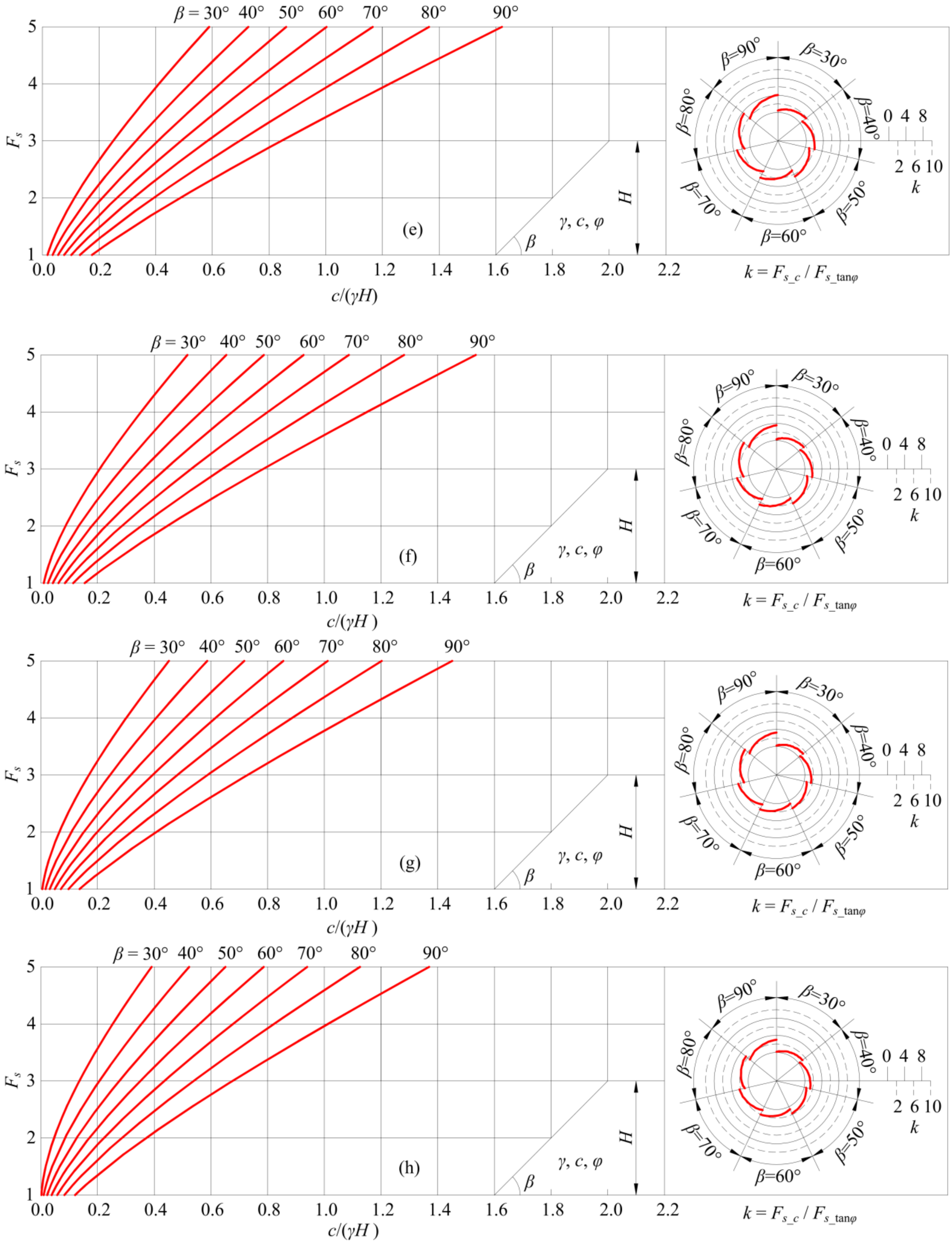


Figure 7 Stability charts of slope using double strength-reduction (DSR) technique(a) $\varphi = 5°$; (b) $\varphi = 10°$; (c) $\varphi = 15°$; (d) $\varphi = 20°$; (e) $\varphi = 25°$; (f) $\varphi = 30°$; (g) $\varphi = 35°$; (h) $\varphi = 40°$

critical LE state, the strength parameter φ has greater contribution on the slope stability than the strength parameter c; and (2) the obtained proportionality factor k corresponding to the minimum slope comprehensive FOS is large for a slope with the large value of c and small value of φ.

Table 3 Contrast of the minimum slope FOS from different methods

Example	Slope parameters		Soil parameters			Minimum slope factor of safety					
	H (m)	β (°)	γ (kN / m³)	c (kPa)	φ (°)	F_s	F_{s_a}	F_{s_b}	F_{s_c}	F_{s_d}	F_{s_e}
1	10	45	17.8	35	15	1.648	1.645	1.732	1.688	1.714	1.701
2	10	45	17.8	35	20	1.860	1.855	1.910	1.854	1.891	1.878
3	10	45	17.8	35	25	2.066	2.059	2.091	2.023	2.072	2.058
4	15	60	17.8	50	15	1.317	1.317	1.411	1.390	1.364	1.420
5	15	60	17.8	50	20	1.472	1.476	1.542	1.515	1.491	1.544
6	15	60	17.8	50	25	1.622	1.628	1.674	1.641	1.622	1.668
7	10	45	17.8	40	15	1.794	1.790	1.906	1.860	1.885	1.872
8	10	45	17.8	40	20	2.017	2.010	2.088	2.030	2.066	2.052
9	10	45	17.8	40	25	2.231	2.223	2.273	2.203	2.251	2.236
10	15	60	17.8	60	15	1.480	1.484	1.616	1.595	1.564	1.631
11	15	60	17.8	60	20	1.648	1.654	1.752	1.724	1.694	1.760
12	15	60	17.8	60	25	1.814	1.817	1.889	1.855	1.827	1.889

The following 12 slope examples are listed to verify the feasibility of charts in Figure 7. Meanwhile, the LE Swedish method, Simplified Bishop method, and M-P method used in the current calculation software are also adopted to analyze the slope stability, and the results are given in Table 3. In Table 3, F_s is the minimum slope FOS obtained from the charts in Figure 7, F_{s_a} is the minimum slope FOS obtained using the LE stress method with the DSR technique, and F_{s_b}, F_{s_c}, F_{s_d}, and F_{s_e} are the minimum slope FOS obtained using the LE stress method, Swedish method, Simplified Bishop method, and M-P method, respectively, with the USR technique. It should be noted that for application of charts in Figure 7 on slope stability, the linear interpolation method can be used to calculate the slope FOS, and an example is that the slope with slope angle β = 45° has the FOS equal to the average value of the FOS of slopes with β = 40° and β = 50° from the charts.

Table 3 shows that the obtained slope FOS from the charts in Figure 7 has the close values with that of the traditional LEM, thus verifying the applicability of these charts. Meanwhile, as the same with the current calculation software, for the given the slope height H, slope angle β, cohesion c, and slope angle φ in a slope, the slope stability can be quickly obtained using the stability charts, and the corresponding proportionality factor k between strength-reduction coefficients is also got. In addition, when the three parameters among the slope height H, slope angle β, cohesion c, and slope angle φ for a slope are given, the unknown parameter can be solved using the stability charts for the slopes with safety requirement. In other words, the charts can be used to design the slope parameters. It is helpful for the above feature to optimize the relationship between slope height and slope angle in the actual project.

5 Conclusions

When the slope is in the critical LE state, the strength parameters of soil have actually different contributions on slope stability. Hence, compared with the traditional USR technique, the application of the DSR technique to analyze the slope stability is more in line with the actual situation. In this study, based on the simple and practical characteristics of the LEM, the LE stress method for slope stability analysis using the DSR technique is established by combining with the general nonlinear M–C criterion. In addition, to achieve the goal of assessing the slope safety and constructing the evaluation index consistent with the traditional LEM, the concept of the slope comprehensive FOS is employed. For the slope comprehensive FOS, two methods, namely, average and polar diameter methods, are adopted. Moreover, this study improves the original polar diameter method and makes it suitable for the cases where the strength-reduction coefficients are less than unity. By comparison and analysis on slope examples, the proposed method's feasibility was verified. The following conclusions can be obtained:

(1) The results obtained using the average and polar diameter methods are the same to each other, indicating the reasonability of the average method.

(2) Compared with the USR technique, the strength curve obtained using the DSR technique

has the lower position when the normal stress acting on slip surface is in the low stress region, which results in the differences on their calculation results.

(3) Using the stability charts of slope, the minimum slope comprehensive FOS can be quickly obtained, and an unknown parameter, including slope parameter or soil material parameter, can be also determined for the slope with safety requirement.

(4) When the slope is in the critical LE state, the proportionality factor k is usually more than 1, indicating that the strength parameter φ has greater contribution on the slope stability than the strength parameter c.

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