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An estimation model for the fragmentation properties of brittle rock block due to the impacts against an obstruction

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Abstract: Mountain hazards with large masses of rock blocks in motion – such as rock falls, avalanches and landslides – threaten human lives and structures. Dynamic fragmentation is a common phenomenon during the movement process of rock blocks in rock avalanche, due to the high velocity and impacts against obstructions. In view of the energy consumption theory for brittle rock fragmentation proposed by Bond, which relates energy to size reduction, a theoretical model is proposed to estimate the average fragment size for a moving rock block when it impacts against an obstruction. Then, different forms of motion are studied, with various drop heights and slope angles for the moving rock block. The calculated results reveal that the average fragment size decreases as the drop height increases, whether for free-fall or for a sliding or rolling rock block, and the decline in size is rapid for low heights and slow for increasing heights in the corresponding curves. Moreover, the average fragment size also decreases as the slope angle increases for a sliding rock block. In addition, a rolling rock block has a higher degree of fragmentation than a sliding rock block, even for the same slope angle and block volume. Finally, to compare with others' results, the approximate number of fragments is estimated for each calculated example, and the results show that the proposed model is applicable to a relatively isotropic moving rock block.

Keywords: Rock block; Rock fragmentation; Rock movement process; Crushing work ratio; Average fragment size.

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Introduction

Among the dynamic processes of mountain hazards, such as rock avalanches and rock falls, the motion of rock blocks is a distinct phenomenon that is responsible for loss of life and damage to construction near the mountain (De Blasio 2011; Paluszny et al. 2016). Mechanical behaviors and responses of the brittle rock blocks in these disasters have been widely studied by many researchers, and the dynamic fragmentation of rock blocks has become a focus of research papers that determined the evolution of motion of the rock blocks (Bowman et al. 2012; Vocialta and Molinari 2015). As a precondition for dynamic fragmentation, the movement of rock blocks was studied first. Most of the studies aimed to determine the motion parameters of rockfalls, including the velocity and trajectory of free-falling blocks (Azzoni et al. 1992; Wyllie 2014), and other parameters relating to the movement of rock blocks, such as restitution coefficients or equivalent rolling friction coefficients (Chau et al. 2002; Giani et al. 2004). Through analysis of the motion process, the basic data for impact prediction can be determined, including velocity, direction and point of impact.

Rock fragmentation within an impacting load during motion of rock blocks is a complex process that involves several physical and mechanical mechanisms. The dynamic fragmentation of brittle materials including brittle rock is a consequence of the nucleation, growth and coalescence of microcracks (Zhang et al. 2004; De Blasio and Crosta 2014; Paluszny et al. 2016); initial or new cracks in a block propagate and finally coalesce to break the rock block into fragments. According to the complexity of rock fragmentation problem, some simplify assumptions are usually introduced, most of the studies consider the dynamic strength of rock materials and try to understand the rock fragmentation phenomenon through energy considerations (Grady and Kipp 1987; Zhang et al. 1999; Iqbal and Mohanty 2007). Only a few models have been proposed to determine the fragmentation of moving rock blocks (Giacomini et al. 2009). Crosta et al. (2006) implemented an energy approach to detection of fragmentation within their three-dimensional rockfall analysis code HY-STONE. Greco et al. (1981) quantified the energy required to break blocks with a stone hammer and considered that when the impacting kinetic energy reached the required energy, a rock block divided into several fragments, and the remaining kinetic energy was distributed among the fragments in proportion to the volume of the fragments. Moreover, a discrete element model was also effectively used to simulate impact-induced rock fragmentation in a rock-fall fly-rock analysis (Wang and Tonon 2009, 2010, 2011). Although much effort has been expended and many study results have been presented, a valid theoretical model for the evaluation and prediction of fragment size of rock blocks after impact needs additional work.

Among all of the studies of rock block fragmentation, the most notable and basic theories for the relation of fragment size and energy were conceived by Kick (1885) and Bond (1952); they explained the relation of work per volume and size reduction of rock blocks simply by using a proportional relationship. In this paper, these energy consumption theories are used to infer the average fragment size through considering the crushing work ratio in the theories as an energy determined by the impact energy. Then, according to the physical mechanics of rock block motion in rockfalls or avalanches, an appropriate theory is selected as the basis for the model of estimating average fragment size. Next, various forms of motion with different drop heights or slope angles for the rock blocks are analyzed using the proposed model, and the number of fragments is estimated based on the calculated average fragment size and finally compared with others' results to verify the applicability of the proposed model.

1 Theoretical Model

1.1 Impact of rock blocks

During a high-speed rock block motion process, a rock block may disintegrate into pieces due to the great force of impact. This fragmentation has a serious effect on the subsequent development of the disaster, which may evolve to a large-scale debris flow, and requires effective attention to prevent possible disasters. Therefore, clearly recognizing the fragmentation process and its results is important to the study of a rock fall disaster. In general, when the impact force on a rock block is greater than the maximum strength of the rock body, the block breaks. For a moving rock block, severe deformation resulting from compression may occur when it impacts a fixed obstruction on the slope at high speed. During this process, the kinetic energy of the rock block is constantly converted into strain energy and generates fracture energy, which can break the rock block and produce new fracture surfaces. However, not all of the strain energy converted from kinetic energy may be distributed to the rock block. Based on collision theory, when an impact occurs, the strain energy from kinetic energy will distribute to the two impactors in a certain proportion determined by the elastic moduli and Poisson's ratios of the impactors. The proportion can be determined as follows:

$$\frac{U_1}{U_2}=\frac{E_2}{E_1}\cdot\frac{1-\mu_1^2}{1-\mu_2^2} \tag{1}$$

where U_1 and U_2 are the distributed strain energies of the two impactors, and E_1 and E_2 and μ_1 and μ_2 are the elastic moduli and Poisson's ratios of the two impactors, respectively.

For the moving rock block, the process of impact includes two stages as shown in Figure 1. First, the impact force increases constantly, starting with the contact of the rock block and the obstruction, and the kinetic energy begins to transform into strain energy (Figure 1b). When the deformation of the rock block reaches its maximum, the impact force is also at the maximum value (Figure 1c), and all of the kinetic energy of the rock is converted into strain energy because the obstruction is fixed on the slope. Second, the deformation of the rock block recovers to some extent, and a resilient velocity v is imparted to the rock block (Figure 1d). Based on this principle, the strain energy that is distributed to the rock block can be written as:

$$U_{1\max}=\frac{1}{2}mv^2\cdot\frac{E_2\left(1-\mu_1^2\right)}{E_1\left(1-\mu_2^2\right)+E_2\left(1-\mu_1^2\right)} \tag{2}$$

where m is the mass of the moving rock block and v is the velocity of the moving rock block before fragmentation.

Next, the volume strain energy strength theory can be used to estimate whether fragmentation occurs during the impact, as in Liu (2003). The controlling equation is as follows:

$$v\geq\frac{\sigma_c}{\sqrt{\rho}}\sqrt{\frac{1-\mu_2^2}{E_2\left(1-\mu_1^2\right)}+\frac{1}{E_1}} \tag{3}$$

where ρ is the density of the rock block and σ_c is its compressive strength.

When the impact velocity is higher than the above calculated critical velocity, the distributed strain energy is greater than the maximum energy that the rock block can withstand.

1.2 Estimation model

1.2.1 Energy consumption theory

Fragmentation of a rock block is an extremely complicated process that involves many mechanical mechanisms due to the block's various internal structures, such as the distribution of cracks. It is very difficult to analyze the process and results of rock fragmentation under impact loading through analysis of only the force and stress evolution. To overcome this difficulty, energy analysis is widely applied to study the relationship between impact loading and fragment size.

The crushing work ratio is an important parameter that denotes the work used to break a volume of rock in a fragmentation event. This ratio can reveal the relationship between energy and size reduction of a rock block and thus has interested many researchers for more than a century. Notable theories were conceived by Rittinger, Kick and Bond (Balaz 2008; Jankovic et al. 2010; Morrell 2004). Rittinger (1867) proposed that the work done in crushing is proportional to the new surface area, which means that the crushing work ratio is proportional to the value of (1/d-1/D), if the size of

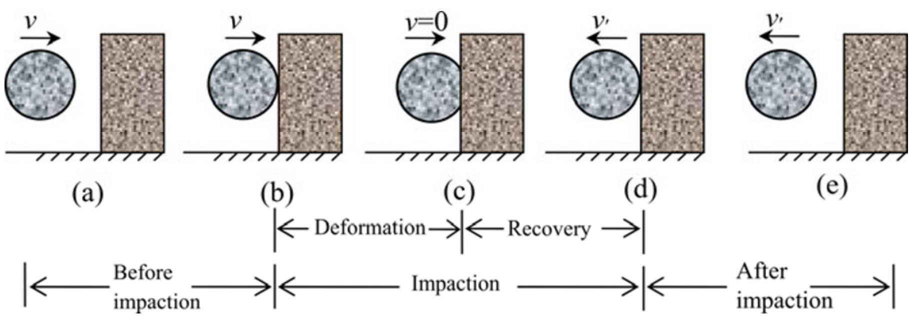

Figure 1 Impacting process of brittle rock.

the rock is reduced to d from D. Therefore, the crushing work ratio can be written as:

$$W_R = K_R\left(\frac{1}{d} - \frac{1}{D}\right) = \frac{K_R}{D}(I-1) \tag{4}$$

where K_R is the Rittinger's constant, which is related to the properties of the rock block, and I is the reduction ratio, which is determined by D/d.

Kick (1885) proposed that for a homogeneous rock, the work input is proportional to the reduction in particle size; his theory rests on the proportionality of the required energy to the lost volume, and the crushing work ratio by this theory is changed as follows:

$$W_K = K_K\left(\lg\frac{1}{d} - \lg\frac{1}{D}\right) = K_K \lg I \tag{5}$$

where K_K is the Kick's constant, which is related to the properties of the rock block.

According to Bond, the formation of defects drives the consumption of energy, so he proposed that the crushing work ratio is inversely proportional to the value of $d^{0.5}$ because of cracks hidden in the rock (Bond 1952, 1960). According to this theory, the crushing work ratio is determined as:

$$W_B = K_B\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{D}}\right) \tag{6}$$

where K_B is the Bond's constant, which is related to the properties of the rock block, and it can be estimated by K_B=10E_B, where E_B is the Bond's crushing work index. The specific energy consumption data were fitted to these three models.

Hukki (1962) evaluated these energy-size relationships stating that each of Rittinger, Kick and Bond theories might be applicable for different narrow size ranges. Ghorbani et al. (2010) had found that the specific energy consumption data were fitted to these three models and estimated their parameters based on some correlative experiments using hammer mill. A large amount of more researches on rock fragmentation using these three theories reveals that Rittinger's theory may be used for finer grinding and applies to the condition of intensely smashed rock fragments with fragment size less than 0.5 mm. Kick's theory is applicable for crushing and applies to the condition of roughly smashed rock fragments. Only Bond's theory is applicable in the natural rock block fragmentation and applies to the condition of moderately smashed rock fragments. Furthermore, application of Kick's and Rittinger's theories has been met with varied success and is not realistic for consideration of size reduction. In spite of the empirical basis of Bond's theory, it is the most widely used method for the sizing of fragment and has become more likely a standard. Furthermore, Gupta and Yan (2006) and Chandar et al. (2016) even carried out a systematic test to predict Bond's crushing work index using three types of rock and validating with fourth type of rock and to determine the grinding efficiency and also to calculate the energy requirement of rock when in fragmentation. According to many field investigations, the fragmentation of moving rock blocks under impact loading in actual landslide disasters always produces pieces with an average size of 5 mm to 500 mm. Thus, Bond's theory may be considered the most suitable for fragment size analysis of moving rock blocks.

1.2.2 Estimating model for average fragment size

As the above section noted, when a rock block under impact loading is broken into fragments, the necessary work per volume is related to the change of dimension, with different equations based on various theories. Here, Bond's theory is chosen to analyze the fragmentation of a moving rock block. Clearly, the average size can be determined by rearranging Eq. (6) as follows:

$$d = \frac{K_B^2}{\left(W_B\sqrt{D} + K_B\right)^2} \cdot D \tag{7}$$

where W_B is the work done per volume of the rock block and can be given as:

$$W_B = \frac{W}{V} \tag{8}$$

where W is the total energy done on the rock block and V is the total volume of the rock block. Then, Eq. (7) can be transformed as follows:

$$d = \frac{K_B^2 V^2}{\left(W\sqrt{D} + K_B V\right)^2} \cdot D \tag{9}$$

As this equation implies, the determination of total energy (W) is an essential requirement to calculate the average size of rock fragments after fragmentation. In general, the fragmentation process of a rock block includes two steps under dynamic loading. As shown in Figure 2, when impact loading affects the rock body, the interior microcracks may propagate until coalesce through the rock. In this stage, a part of the impact energy is consumed to accomplish the extension of the initial cracks. Then the residual impact energy which means that the total impact energy after deducting the consumed energy in the crack propagation of the first step continues to dissipate until the rock is completely broken. Only the second stage is consistent with the energy consumption theory. As the fragments generated from a complete rock mass in the fragmentation may have a certain speed after fragmenting, the impact energy may not be consume completely in the fragmentation and some parts of it had translated into the kinetic energy of generated fragments. So when using Eq. (9) to calculate the average size, the total work is not equal to the impact energy. Define absorptive energy is the consumed energy in the fragmentation which is the different value between the impact energy and the total kinetic energy of fragments after impaction. Then the total energy (W) has the same value as the residual absorptive energy (W_r), which is the absorptive energy after deducting the consumed energy in the first step, so that the total work in Eq. (9) can be expressed as follows:

$$W = W_r = \Delta E - W_{SE} \tag{10}$$

where ΔE is the absorptive energy, which means the reduction of kinetic energy after the impact, and W_{SE} is the surface energy, which means the energy consumed in the process of extending the initial cracks in the first stage of fragmentation. The latter can be determined as:

$$W_{SE} = 2\gamma \sum_{i=1}^{n} A_i \tag{11}$$

where A_i is the area of new surface generated from the extension of crack i and γ is the surface energy per unit area of the rock block. This parameter is related to the fracture and other mechanical properties of the rock such as fracture toughness and elastic modulus (Mecholsky et al. 1974; Cherepanov 1979; Wu et al. 2010), and it can be determined as follows (Dai et al. 2011; Kopp et al. 2014; Xie et al. 2015),

$$\gamma = \frac{K_c^2\left(1-\mu^2\right)}{2E} \tag{12}$$

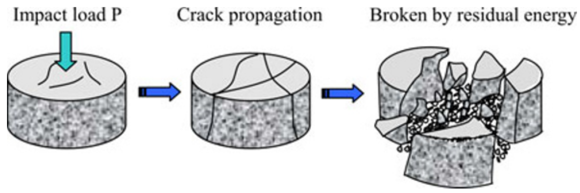

Figure 2 Fragmenting process of brittle rock under an impact loading.

where K_c is the fracture toughness of the rock block and E and μ are the elastic modulus and Poisson's ratio of the rock block, respectively.

Then, Eq. (10) can be written as follows:

$$W = W_r = \Delta E - \frac{K_c^2\left(1-\mu^2\right)}{E} \cdot \sum_{i=1}^{n} A_i \tag{13}$$

The reduction of kinetic energy (absorptive energy ΔE) is related to the impact form because the stress forms vary for different conditions, such as direct impact, oblique impact or impact with a rotational speed when the moving rock block rolls along the slope to impact against the obstruction. If the rock blocks directly impacts the obstruction with no rotational speed, the reduction of kinetic energy after fragmentation can be obtained from Eq. (14) as:

$$\Delta E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 = \frac{1}{2}m\left[v^2 - \left(e_n v\right)^2\right] = \frac{1}{2}mv^2\left(1-e_n^2\right) \tag{14}$$

where v is the velocity of the moving rock block before impact; v' is the bulk velocity of all of the fragments generated from the moving rock block after fragmentation; and e_n is the normal coefficient of restitution of the rock block when impacted against the obstruction.

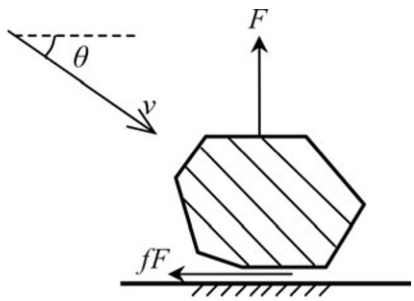

Figure 3 Impacting process of a rock block moving with an angle.

If the moving rock block impacts the obstruction at a certain angle, the tangential stress must be considered. Figure 3 shows a schematic diagram of the moving rock block impacting a horizontal plane with an angle. The rock body is obviously under a tangential friction force as well as the normal impact force. Under this condition, the normal velocity after impact can be determined as:

$$v_n' = -e_n v_n = -e_n \cdot v\sin\theta \tag{15}$$

where θ is the angle between the impact surface and the impact direction.

As in the previous analysis, the impact process in Figure 3 also consists of two stages (deformation stage and recovery stage). Based on the theorem of impulse, the normal impulses of the deformation stage and the recovery stage can be obtained as:

$$\begin{cases} I_n = \int F(t)dt = -mv\sin\theta \\ I_n' = \int F'(t)dt = mv_n' = -e_n \cdot mv\sin\theta \end{cases} \tag{16}$$

where I_n is the normal impulse of the deformation stage and I'_n is the normal impulse of the recovery stage.

Furthermore, the tangential impulses of the two stages can be given as:

$$\begin{cases} I_\tau = \int fF(t)dt = -fmv\sin\theta \\ I_\tau' = \int fF'(t)dt = fmv_n' = -e_n \cdot fmv\sin\theta \end{cases} \tag{17}$$

where I_τ is the tangential impulse of the deformation stage; I'_τ is the tangential impulse of the recovery stage; and f is the sliding friction coefficient between the rock block and the impact surface.

To calculate the tangential velocity after impact, the theorem of impulse is used again here:

$$I_\tau + I_\tau' = mv_\tau' - mv\cos\theta \tag{18}$$

The tangential velocity after impact can be determined as:

$$v_\tau' = v\sin\theta\left(\frac{1}{\tan\theta} - f - fe_n\right) \tag{19}$$

Then, the reduction of kinetic energy can be changed into:

$$\begin{aligned} \Delta E &= \frac{1}{2}mv^2 - \frac{1}{2}m\left(v_n'^2 + v_\tau'^2\right) \\ &= \frac{1}{2}mv^2\left\{1 - \left[e_n^2 + \left(\frac{1}{\tan\theta} - f - fe_n\right)^2\right]\sin^2\theta\right\} \end{aligned} \tag{20}$$

Eq. (14) and Eq. (20) give the reduction of kinetic energy after fragmentation of a moving rock block with no rotational velocity. However, if the moving rock block is approximately spherical and rolls along the slope before impact, it may have a high rotational velocity generated by the rolling movement, as shown in Figure 4. The kinetic energy consists of rotational kinetic energy and translational kinetic energy, and both of them must be considered when calculating the reduction of kinetic energy. Based on the theorem of moment of momentum, the angular impulse is equal to the variation value of angular momentum as follows:

$$\begin{aligned} J\omega' - J\omega &= \int fF(t)Rdt + \int fF'(t)Rdt \\ &= fR\left(I_n + I_n'\right) = -fmvR\sin\theta\left(1 + e_n\right) \end{aligned} \tag{21}$$

where R is the equivalent radius of the rock block which is equal to the radius of a sphere which has a same volume with the rock block; ω is the rotational velocity of the rock block before impact, which is determined as $\omega = v/R$ for pure rolling movement; ω' is the rotational velocity of the whole system after impact; and J is the rotational inertia, which can be determined as: $J = 2/5mR^2$.

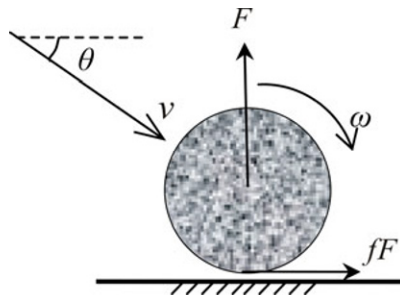

Figure 4 Impacting process of a moving rock block with a rotational velocity.

Furthermore, the rotational velocity of the whole system after impact can be given as:

$$\omega' = \left[1 - \frac{5f\sin\theta(1+e_n)}{2}\right] \cdot \frac{v}{R} \tag{22}$$

In reality, it is impossible for the rotational velocity of the whole system after impact (ω') to be directed opposite to the initial rotational velocity (ω), which means that the value of ω' obtained through Eq. (22) must satisfy $\omega' \geq 0$. If ω' is less than 0, the moving rock block loses all of the initial rotational kinetic energy; thus, Eq. (20) can be written as:

$$\begin{aligned} \Delta E &= \frac{1}{2}mv^2\left\{1 - \left[e_n^2 + \left(\frac{1}{\tan\theta} - f - fe_n\right)^2\right]\sin^2\theta\right\} + \frac{1}{2}J\omega^2 \\ &= \frac{1}{2}mv^2\left\{\frac{7}{5} - \left[e_n^2 + \left(\frac{1}{\tan\theta} - f - fe_n\right)^2\right]\sin^2\theta\right\} \end{aligned} \tag{23}$$

When the value of ω' satisfies the condition $\omega' \geq 0$, the reduction of kinetic energy may be changed into the following expression:

$$\Delta E = \frac{1}{2}mv^2\left\{1-\left[e_n^2+\left(\frac{1}{\tan\theta}-f-fe_n\right)^2\right]\sin^2\theta\right\}+\frac{1}{2}J\omega^2-\frac{1}{2}J\omega'^2$$
$$=\frac{1}{2}mv^2\left\{\frac{7}{5}-\left[e_n^2+\left(\frac{1}{\tan\theta}-f-fe_n\right)^2\right]\cdot\sin^2\theta-\frac{2}{5}\left[1-\frac{5f(1+e_n)\sin\theta}{2}\right]^2\right\} \quad (24)$$

Based on the analysis above, it is clear that the fragmentation of a moving rock block is a complex and varied process involving many conditions, such as the mechanical properties of the rock block and the obstruction, the shape of the rock block and the type of motion. If the moving rock block directly impacts the obstruction with no rotational velocity, the estimating model for average fragment size is computed by the simplest method using Eqs. (9), (13) and (14). If the moving rock block impacts the obstruction obliquely with no rotational velocity, the estimating model is changed to Eqs. (9), (13) and (20). Finally, when the moving rock block impacts the obstruction with a rotational velocity, the estimating model has two different forms. If the value of the rotational velocity of the whole system after impact (ω') is less than 0, the estimating model consists of Eqs. (9), (13) and (23). On the other hand, when ω' is greater than or equal to 0, the model uses Eqs. (9), (13) and (24).

2 Results and Sensitivity Analyses

In this section, the model proposed above is applied to analyze the fragmentation results of a moving rock block that falls from a great height or slides along a steep slope and dashes against the rigid ground surface. Furthermore, different motion forms and different conditions are discussed, such as the falling height and slope angle.

2.1 Determination of velocity and trajectory

Before the estimation, motion parameters including velocity, motion direction and position before the impact must be determined to provide the data input for the estimating model. All of the needed parameters can be obtained by the calculation model of velocity and trajectory for a moving rock block that has been proposed and widely used by researchers when studying the disaster of a rock fall (Broili 1973; Dorren 2003). In general, the motion forms of a falling rock block include free falling, elastic bouncing, rolling and sliding (Guzzetti et al. 2002; Song et al. 2006; Labiouse and Heidenreich 2009; Asteriou et al. 2012). Figure 5 shows a schematic diagram of different motion forms, the motion characteristics of the rock block are determined by the shape of the slope and the rock block, as well as the initial motion state. Figure 5a represents a rock block avalanched from the top of a cliff and falling freely to the ground; the velocity before impacting on the ground under this condition can be calculated as:

$$v = \sqrt{2gH} \quad (25)$$

where H is the falling height.

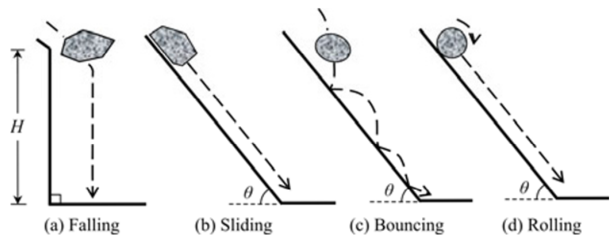

Figure 5 Four motion forms for a moving rock block.

Figure 5b shows a rock block sliding along the slope, with an obvious plane of contact between the block and the slope. Based on the kinematic principle, the velocity of the moving block before fragmentation can be obtained as follows (Giani et al. 2004):

$$v = \sqrt{v_0^2 + 2gH\left(1-\frac{f}{\tan\theta}\right)} \quad (26)$$

where v_0 is initial velocity and θ is the slope angle.

If the rock block is airborne at the beginning, it bounces downward after hitting the slope. Therefore, this motion form includes flight and collision, and the velocity of flight can be determined as:

$$v = \sqrt{v_0^2 + 2g\Delta H} \quad (27)$$

where ΔH is the vertical falling height from the initial position to the calculated position.

Undoubtedly, a determination of the trajectory during the bouncing stage that reflects the actual position of the rock block can effectively determine the impact point. In the flight stage (Figure 5c), the movement trajectory equation can be expressed as Eq. (28). After colliding with the slope surface, the motion state of rock block inevitably changes, and the velocity after collision can be written as Eq. (29):

$$y = y_0 + \frac{x - x_0}{v_{0x}} v_{0y} - \frac{1}{2} g \left(\frac{x - x_0}{v_{0x}} \right)^2 \quad (28)$$

$$\begin{cases} v'_n = e_n v_{n0} \\ v'_\tau = e_\tau v_{\tau 0} \end{cases} \quad (29)$$

where (x_0, y_0) are the initial coordinates of the rock block; (x, y) are the coordinates of the calculated point; v_{0x} and v_{0y} are the components of initial velocity in the x and y directions, respectively; e_n and e_τ are the normal and tangential coefficients of restitution of the rock block; v_{n0} and $v_{\tau 0}$ are the components of the initial velocity in the normal and tangential directions with regard to the impact surface, respectively; and v'_n and v'_τ are the components of velocity in the normal and tangential directions after collision, respectively.

When the rock block is approximately spherical and rolls down the slope as shown in Figure 5d, velocity can be determined as follows:

$$v = \sqrt{v_0^2 + \frac{2gH(1 - f' \cos\theta / \tan\theta) R^2}{R^2 + d^2}} \quad (30)$$

where f' is the coefficient of rolling friction between the rock block and the slope surface and d is the radius of inertia of the rock block, which is determined by $d^2 = 2R^2/5$.

2.2 Example analysis

Combining the calculation method for kinematic parameters and the estimating model for average fragment size of a moving rock block can theoretically analyze the fragmentation of the rock block after colliding with the hard ground, either free-falling from a cliff (Figure 6a) or sliding or rolling along a slope (Figures 6b and 6c).

Figure 6a shows a rectangular rock block falling freely from a great height and breaking into fragments after impacting the ground. Suppose the rock is a square block of granite with side length of 1 m and two initial interior cracks. When striking the ground with huge impact energy, the two initial cracks inevitably extend throughout the whole rock block, assuming that the two inner cracks extend along the diagonals, which breaks the initial rock block into four parts, and each part receives a quarter of the residual absorptive energy in the subsequent fragmentation. Therefore, the area of new surface produced by impact is 2.83 m² and D=0.78 m calculated by $D=(1.5V/\pi)^{1/3}$ (where V is the total volume of the initial block). Some of the mechanical parameters of the rock block are taken from related research and listed in Table 1 (Hou et al. 2015a). Otherwise, the material of the slope and the ground surface is assumed to be a kind of hard granite which has same property with the natural slope of Changheba Hydropower Station in China, and its mechanical parameters and other correlation coefficients with the rock block are obtained by referring to related research (Hou et al. 2015b) and listed in Table 2. To estimate the fragment size more accurately, the value of Bond's constant is taken as 188.5 kW·h/t (Refahi et al.

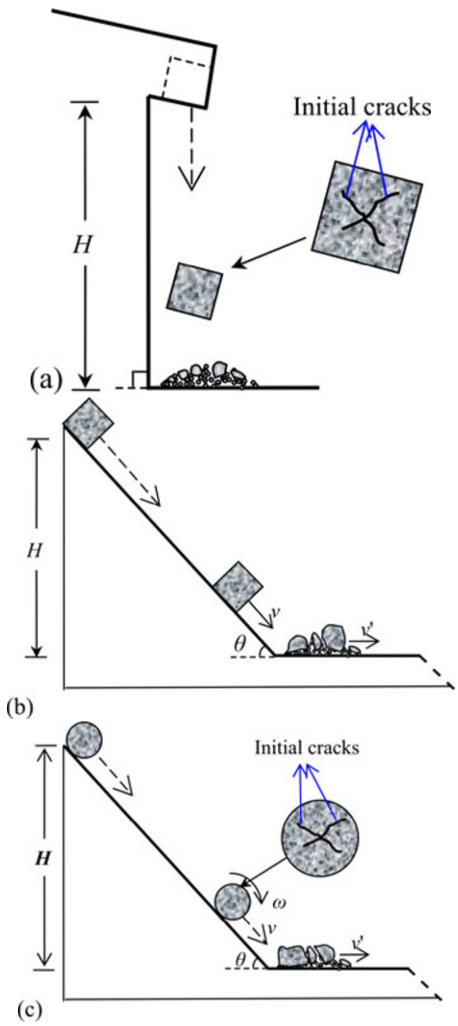

Figure 6 Schematic diagram of a rock block impacting on the ground: (a) free-fall; (b) sliding; (c) rolling.

2007). Then, fragmentation and average fragment size corresponding to different falling heights can be analyzed by the kinematic model Eq. (25) and the average fragment size estimating model Eqs. (9), (13) and (14) because of the direct impact. The calculation result is shown in Figure 7a, and it is clear that the curve of average fragment size declines from 0.542 m to 0.013 m as the drop height increases from 10 m to 200 m. In addition, fragment size decreases sharply as height increase from 10 m to 50 m and decreases slowly as the drop height increases further when exceeding 50 m.

Table 1 Mechanical parameters of the rock block

Parameter	E_1 (GPa)	μ_1	σ_c (MPa)	K_c (MPa·m$^{1/2}$)	ρ (kg/m^3)
Value	4.4	0.23	47	0.5	2574

Note: E_1 is the elastic moduli of rock block; μ_1 is the Poisson's ratios of rock block; σ_c is the compressive strength of the rock block; K_c is the fracture toughness of the rock block; ρ is the density of the rock block.

Table 2 Mechanical parameters of the ground and other correlation coefficients

Parameters	Ground		Correlation coefficients between slope and rock		
	E_2 (GPa)	μ_2	f	e_n	e_τ
Value	3.6	0.2	0.4	0.3	0.92

Note: E_2 is the elastic moduli of ground; μ_2 is the Poisson's ratios of ground; f is the sliding friction coefficient between the rock block and the impact surface; e_n and e_τ are the normal and tangential coefficients of restitution of the rock block when impacted against the obstruction, respectively.

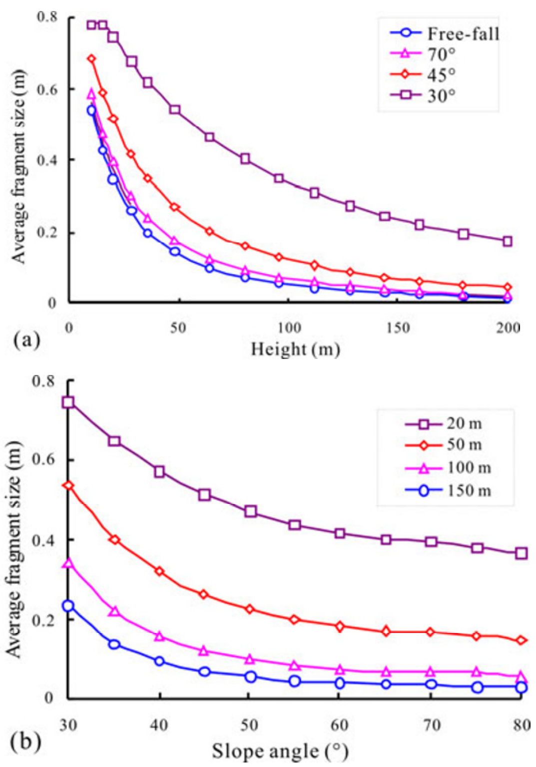

Figure 7 Average fragment size for free-falling and sliding rock blocks: (a) size-height curve; (b) size-angle curve.

Results are very different if the rock block slides along the slope before the impact, as shown in Figure 6b. To compare this situation with the condition of free-fall, consider that the rock type and material of the rock block, the slope and the ground are the same as in the above example, and all of the needed parameters also have the same values. Various slopes with three different angles (70°, 45° and 30°) are analyzed for the study of a sliding rock block. For the condition of sliding, Eq. (26) can be chosen as the kinematic model and the average fragment size estimating model consists of Eqs. (9), (13) and (20); the calculation results are shown in Figure 7a. Undoubtedly, the change law of the average fragment size curve for a sliding rock block has characteristics similar to those of the free-fall rock block: it decreases with increasing slope height, and the rate of decrease also tends to be rapid from 10 m to 50 m and slow for greater heights, like the curve for free-fall. Moreover, the curves for sliding rock blocks are obviously higher than for free-fall and rise further as the slope angle decreases. In addition, the average fragment size of a rock block sliding down a 30° surface is somewhat different from the others, in that the top of the curve, for heights from 10 m to 17 m, stabilizes at approximately 0.78 m, which is equal to the initial average size after the original cracks propagate. This is because the calculated value of the average fragment size in this range is greater than the initial average size D, indicating that the residual absorptive energy after the initial crack propagation cannot break the rock into fragments. Figure 7b reveals further that, with the same drop height, the average fragment size decreases as the slope angle increases, which agrees with the data in Figure 7a.

To explore the distinction between a rolling impact and sliding or free falling one, a spherical rock block rolling along the slope is considered here, and the average fragment size of the moving rock block after fragmentation is analyzed for a certain rotational velocity. To make the comparison more effective, consider that the volume of the spherical rock block is equal to the one in the above analysis (V=1 m^3) and that there are two initial interior cracks. Then, its radius can be determined by $R=(3V/4\pi)^{1/3}$; the result is R=0.62 m, and the area of newly produced surface after the initial cracks propagate is 2.42 m^2. Based on these data, the dissipative surface energy in the process of initial crack propagation can be given as 65 kJ, and the fragment size before impact is 0.78 m, as determined by $D=(1.5V/\pi)^{1/3}$, which is also same as the above. Then, we use the kinematic model Eq. (30) and the average fragment size estimating model of Eqs. (9), (13) and (23) or Eqs. (9), (13) and (24) to calculate the average fragment size of a rolling rock block along slopes with different angles and heights. The results are shown in Figure 8, along with data for a sliding rock block with the same slope angles. Obviously, the calculation result for a rolling rock block has a trend similar that for a sliding rock block, which means that the data values decrease as the slope height increases. Figures 8a, 8b and 8c correspond to different slopes with angles of 45°, 60° and 70°, respectively. Furthermore, it is clear that with the same slope angle and same volume, a rolling rock block has a greater degree of fragmentation than a sliding rock block, as implied by the fact that the curve for a rolling rock block is below that of the sliding rock block. This observation means that a rolling rock block has a much greater kinetic energy before impact. In addition, the three figures reveal that, as the slope angle increases, the curves of average fragment size of the rolling block and the sliding block are more compact and closer to the condition of free-fall.

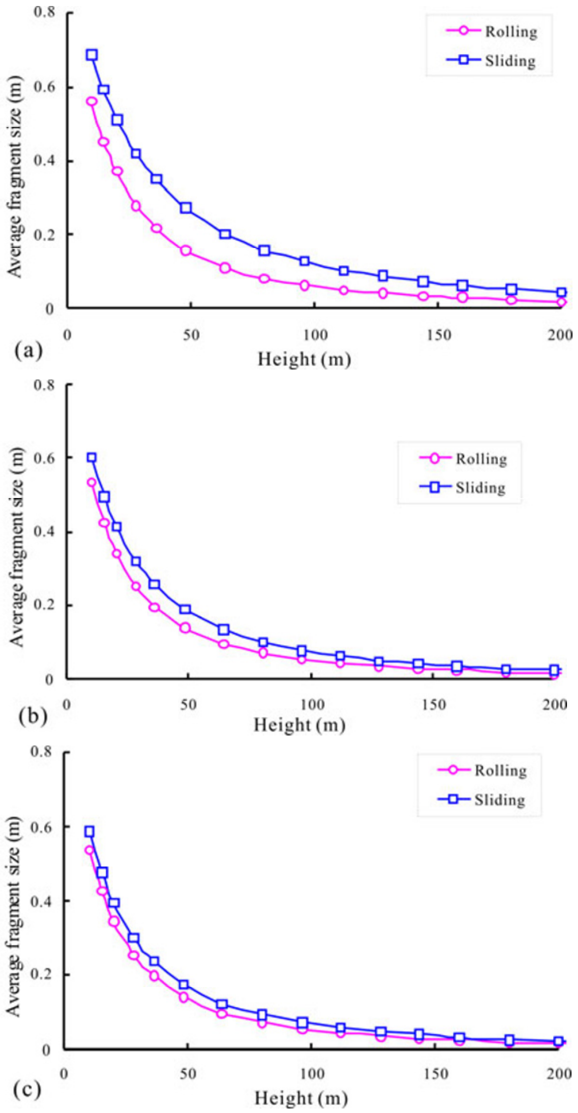

Figure 8 Comparison of average fragment size curves of rolling and sliding rock blocks with different slope angles: (a) 45°; (b) 60°; (c) 70°.

3 Discussions

In the detailed analysis in the previous section, several impact events of a moving rock block under different conditions are described. Some general guidelines are revealed through a comprehensive analysis of all of the calculation results. However, to verify whether these generalizations are reasonable requires more research or comparison with other studies. The determination of average fragment size for a moving rock block or rockfall is an extremely difficult goal, and efforts are mostly focused on the fragment number to approximately reflect the extent of fragmentation with different drop heights or different impact angles.

In studies of a moving rock block such as in a rockfall, the energy at impact is a key parameter, and the interior structure of the block and its impacting angle are two other relevant factors. Giacomini et al. (2009) conducted a series of tests to study the influence of impacting energy and impacting angle on fragmentation of a free-falling rock that impacts obliquely, using two types of hard ornamental stones named Beola and Serizzo, which are granitic orthogneisses having properties similar to the granite in the above calculated examples. Based on their exploration, the influence of the impacting energy and impacting angle shows that the number of fragments has a smoothly curved increasing trend with the increase of impacting energy and impacting angle.

In our paper, only average fragment size is figured, which is difficult to determine but crucial in the study of moving rock blocks. As the fragment has different shapes but approximating a sphere especially for smaller ones, all the fragments are considered as sphere. Then, the number of fragments (which is abbreviated as "No. fragments" in Figure 9) generated during the impact can be approximately estimated as follows:

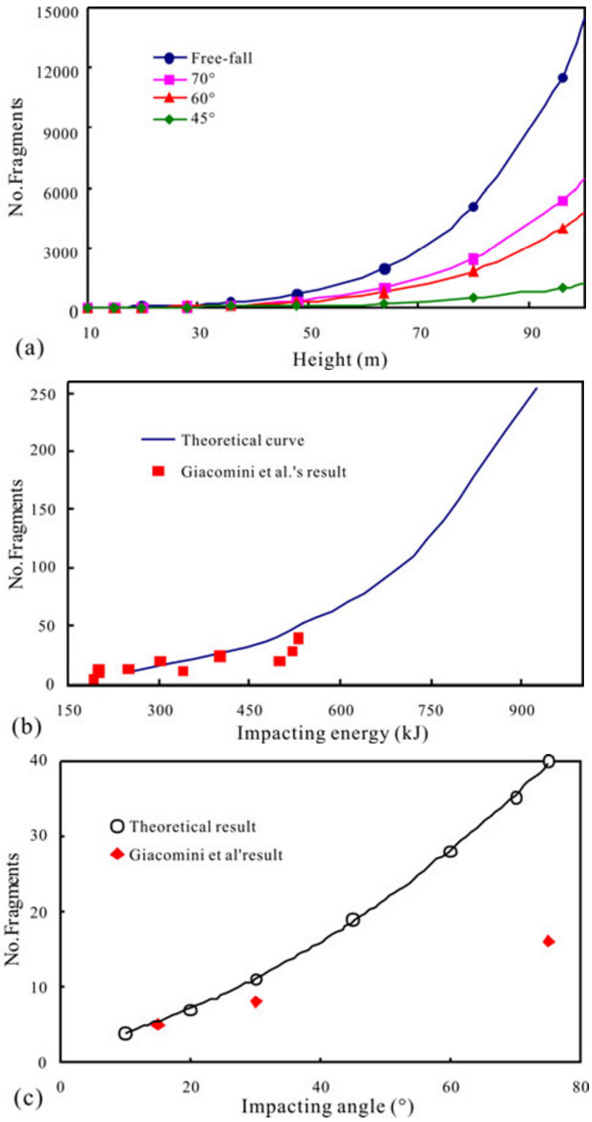

Figure 9 Variation in number of fragments with the change of height or impact energy: (a) Number of fragments with variation in height; (b) Number of fragments with variation in impact energy for free-fall and (c) Number of fragments with variation of impacting angle for an impacting energy around 400 kJ.

$$N = \frac{6V}{\pi d^3} \tag{31}$$

where V is the total volume of the initial rock block and d is the average fragment size calculated above.

Based on the calculation result in the previous section, the number of fragments for each situation can be computed. In Giacomini's paper, the range of drop height in the fragmentation tests is from 10 m to 40 m, which leads the impacting energy to distribute less than 1,000 kJ. Figure 9a shows the curves for estimating the number of fragments for the free-falling and sliding rock blocks, with the variation in drop heights used in the above calculation examples. The calculation results indicated that the curves have an exponentially increasing trend, and may increase very rapidly when the dropping or sliding height is more than 80 m. To simplify the analysis, only heights less than 100 m are considered in the charts. Moreover, curves for smaller impacting angles, which produce fewer fragments, are below the ones for greater impacting angles.

Furthermore, the impact energy for a free-falling rock block can be determined as:

$$E = mgH \tag{32}$$

For a free-falling rock block impacting an inclined surface, the impact energy can be determined as:

$$E = mgH\left(1 - \frac{f}{\tan\theta}\right) \tag{33}$$

Figure 9b shows the curve for the number of fragments with the variation of impact energy for the condition of free-fall. The results of Giacomini et al. are also shown in the figure to support our research. The data are more consistent to some extent for a relatively lower impacting energy. Moreover, Figure 9c shows the comparison of theoretical result and Giacomini et al.'s result about the fragments number variation as the impacting angle increases for which the impacting energy is around 400 kJ. It is clear that there have a similar variation trend, obviously, fragments numbers increase with the angle growing. To some extend, values of the two results are close when the impacting angle is less than 60° as the calculation results indicated. In fact, rock blocks with different interior structures may have various effects on the change curve for the number of fragments, as Giacomini et al. (2009) revealed. For example, if a rock has a large-scale inner structural surface, the corresponding curve may have an opposite trend to the above result. This is because the forced direction, which is seriously affected by both the impacting angle and the slope angle, has a great influence on the stress state of the rock block when under an impact loading, due to the direction of the structural surface. Therefore, for a relatively intact rock block with no large structural surface or a rock block with very little structural surface that can be considered as isotropic, the theoretical estimating model is reliable to a certain extent.

4 Conclusions

The following conclusions can be reached based on the calculated examples and discussion of the above analyses. It is helpful to use the energy consumption theory to establish a theoretical model for the estimation of average fragment size and number for the fragmentation of a moving rock block in a rock fall or avalanche. Through the analysis of different conditions of motion, some general principles can be found for how average size changes with variations in height and slope angle. First, for each motion form, including free-fall, sliding and rolling, the average fragment size decreases as the drop height increases, and the change curves decline rapidly for lower heights but slowly as height continues to increase. Second, the chart of average fragment size vs. slope angle reveals that the average fragment size also decreases as the slope angle increases. Third, compared with the sliding block, the rolling rock block has a greater degree of fragmentation, even for the same slope angle and block volume. Finally, the calculated number of fragments and its change trend further support the applicability of the proposed model.

Acknowledgments

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