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ORIGINAL ARTICLE

The advancement of a mathematical model of sustainable development

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Abstract Earth System Analysis was postulated as a theory by Hans-Joachim Schellnhuber in 1998 as a way to characterise the Earth System—the coupled relationship between the environment and humans. Within this theory is the notion of Geocybernetics—management of the Earth System in order to achieve strategies and mechanisms of co-evolution between the environment and humans. This is regarded as the concept and application of sustainable development. However, whilst fundamental definitions in Earth System Analysis are presented for the coupled relationship between the environment and humans, no such definitions exist for sustainable development within the Earth System context. Consequently, this paper presents a mathematical model of sustainable development that provides for the fundamental abstraction of the key concepts and parameters necessary for sustainable development to occur. The model utilises basic mathematics to detail these concepts and parameters, as well as the conditions required for sustainable development to occur. The model presented is, in some regards, a work in progress, and further refinements will be made given the nature of the research performed to this point, i.e. the fundamental mathematical definition of sustainable development and its application. However, the research conducted thus far has made it reasonable to communicate the findings made up to the present point. The paper also provides a brief example of the application of the model to an environmental impact assessment of a metro rail scheme in India, for the purpose of evaluating the level of sustainable development (if appropriate) for the project under consideration.

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Introduction

Nath and Tahay (1996) stated that: "it would not be an exaggeration to say that, …we appear to have arrived at an important cross-roads of history with regard to global environmental problems and issues, and it is becoming increasingly urgent for us to decide where we go from here".

The reason why some form of sustainable development is needed is not in question, as Schmidheiny (1992) explains: "We cannot continue in our present methods of using energy, managing forests, farming, protecting plant and animal species, managing growth and producing industrial goods". Chichilnisky (1997) outlines another valid reason: "for the first time in history, human activity has reached levels at which it could alter the planet's climate and its biological mix".

The crux of sustainable development is the understanding of the fundamental dynamic relationships between the environment and humans, and how to apply such knowledge. What is the role played by the environment in the development of the anthroposphere and vice versa? In particular, part of the fundamental weakness within the concept and development of "sustainable development" is the failure to ask the most basic question of all: what is the environment? There can be no sustainable development without an effective answer to this question, as everything regarding the role and relationship of the anthroposphere, and thus how sustainable development is to be achieved and maintained, follows from this question. This paper aims to provide the conceptual understanding of these fundamental concepts through the support of Earth System Analysis (Schellnhuber 1998, 1999, 2001; Schellnhuber and Kropp 1998). Earth System Analysis, contrary to the predominantly word-based arguments that have dominated the field, uses mathematics to define the nature of the dynamic between the environment and humans.

The recent development of understanding of the systematic interconnectivity and interactions between the environment and humans in Earth System Analysis has provided a coherent framework for understanding the feedbacks and synergisms between the ecosphere (**N**) and the anthroposphere (**A**). However, whilst Earth System Analysis does provide a fundamental grounding in the dynamics of the Earth System in the historical past and the present day, the question of defining the fundamental dynamics of the co-evolutionary state that implies sustainable development remains unresolved. Therefore, it is necessary to determine the fundamental dynamics of co-evolution between **N** and **A**, which in turn can determine the nature and level of sustainable development.

Therefore, by using Earth System Analysis as the fundamental basis for supporting the conceptual development demonstrated here, and undertaken through independent research, this paper will propose and present a mathematical model in order to attempt to contribute to the debate: 'What is sustainable development?' This paper consequently intends to present the following:

1. An outline of the concept of Earth System Analysis (Schellnhuber 1998, 1999, 2001; Schellnhuber and Kropp 1998), which describes the coupled dynamic relationship between the environment and humans, and the fundamental paradigms of sustainable development.
2. Using the previous point as a basis for conceptual development—a new approach for the conceptualisation and determination of sustainable development in the form of a mathematical model that describes the fundamental parameters and dynamics of sustainable development.
3. A brief demonstration of an example of the model's application to a quantitative methodology of environmental impact assessment (EIA), as a prelude to a follow-up paper that will fully discuss the initial application aspects of the model conducted during the research.

Research background and context: Earth System Analysis

Introduction to Earth System Analysis

Earth System Analysis (Schellnhuber 1998) seeks to answer three questions posed by Clark (1989) and Blackburn (1992) concerning the coupled dynamic relationship between the environment and humans:

1. What kind of a world do we have?
2. What kind of a world do we want?
3. What must we do to get there?

To help answer these questions, there are three principal components to Earth System Analysis: Global Change; Global Environmental Management; and Geocybernetics.

Global Change

The notion of 'Global Change' addresses the first question: 'what kind of world do we have?' This notion is concerned with effects upon nature, as well as the magnitude of such impacts. Therefore, it requires an understanding of the historical and present behaviour of humans regarding the environment, adopting a global perspective.

Global change in the historical past

In the historical past,[1] the impacts of humans upon the environment tended to be local in magnitude (Schellnhuber 1998). This meant that such impacts could be almost compensated for over time by the environment and, consequently, no long-lasting impact was caused. Therefore, this dynamic relationship is defined by Schellnhuber (1998) as follows:

$$\dot{\mathrm{N}}(t) = F_0(\mathrm{N}; t),$$
$$\dot{\mathrm{A}}(t) = G_0(\mathrm{N}, \mathrm{A})$$

Eq. 1 The dynamic relationship between the ecosphere and the anthroposphere in the historical past as stated by Schellnhuber (1998)

Equation 1 states that the ecosphere (**N**) is independent of any influence from the anthroposphere (**A**) over time. On the other hand, the development of humans was influenced by their own actions as well as those of the environment. However, as Schellnhuber (1998) admits, this is an extremely compacted version of the true dynamics of the Earth System.

[1] The historical past is defined here as the pre-industrial age—the period before the beginnings of the Industrial Revolution of the late eighteenth century (Schellnhuber 1998).

Global change in the present day

The influence and actions of humans on the environment in the present day cannot be compared to those of the historical past (Schellnhuber 1998). This is because human actions and influences are significantly greater in both their intensity and geographical extent (Schellnhuber 1998). This concurs with Schellnhuber's (1998, 1999, 2001) observation that, since the Industrial Revolution, humans intervene at "the scale of the system". Consequently, humans affect the operation of the Earth System by impacting on the processes of the environment.

By using the same approach as for the historical past, a coupled dynamic relationship between **N** and **A** for the present day can be determined. Schellnhuber (1998) defines this relationship as follows:

$$\dot{\mathrm{N}}(t) = F_1(\mathrm{N}, \mathrm{A}; t),$$
$$\dot{\mathrm{A}}(t) = G_1(\mathrm{N}, \mathrm{A})$$

Eq. 2 The current dynamic relationship between the ecosphere and the anthroposphere, as stated by Schellnhuber (1998)

Equation 2 indicates that, in the present day scenario, the relationship has changed so that **N** and **A** are now strongly coupled in the evolutionary development of the planet. This is because human actions and by-products[2] now directly influence the systems and processes of the environment. This means that impacts on one component of the environmental system will have knock-on effects on other components at the same geographical level and further up.

Global environmental management

Because humans are changing the Earth's habitat on a global scale, the question 'what kind of a world do we want?' is dependent upon how humans perceive the Earth System (Schellnhuber 1998, 1999, 2001). However, by the same token, science and technology are capable of channelling human development along acceptable environmentally sound pathways, so achieving sustainable development within the Earth System (Schellnhuber 2001). In order to achieve this, as global actors in the Earth System, humans obtain a new identity—the "Global Subject" (*S*). This is a self-conscience attempt to "conceive the planetary system in its entirety" (Schellnhuber 1998, 1999, 2001). This means that the design of nature is shaped and influenced by the characteristics of the perception system of humans—the construction and maintenance of a set of laws/rules for an image of the outside world based on various and specific impressions. However, an individual person's senses are insufficient to perceive, for example, any global changes to the environment. Instead, *S* perceives the Earth System, as Schellnhuber (1998) observes, through the senses of the global "scientific-medial complex", i.e. monitoring devices, computers and data storage, and electronic networks (e.g. the Internet).

Humanity, through scientists translating their results to the best of their ability (e.g. the 4th IPCC Climate Change report; http://www.ipcc.ch/), consequently undertakes appropriate measures to modify its course of action, either of its own volition or as part of an organised approach. This may be achieved through individuals pursuing their own course of action (recycling household waste); or through community-based initiatives (e.g. car-sharing); or through political mechanisms to introduce appropriate responses to alter the current situation towards a more considered and responsible path for co-evolution (e.g. the Kyoto protocols).

With all this in mind, Schellnhuber (1998, 1999, 2001) states it is possible to conceive the Earth System at the most basic level as follows:

$$E = (N, H)$$

where:

$N = (a, b, c\ldots)$ $H = (A, S)$

Eq. 3 Composition of the Earth System, as stated by Schellnhuber (1998)

This conceptualises the Earth System as having two primary components, namely ***N*** and the human factor ***H*** (Schellnhuber 1998, 1999, 2001). ***N*** represents the various planetary sub-spheres, which can be "spelled" alphabetically: *a*, atmosphere; *b*, biosphere; *c*, cryosphere, etc. ***H*** consists of two key sub-components: the anthroposphere (*A*), which is the total sum of all individual lives, actions and products of humanity; and the Global Subject (*S*).

Due to the existence of the Global Subject, the dynamic relationship between the ecosphere and anthroposphere in the present day era changes to include a new factor—**M**(*t*). This represents the management strategy chosen to steer sustainable development along a desired path. The new dynamics associated with the management aspects of the Global Subject (Schellnhuber 1998, 1999, 2001) are as follows:

$$\dot{\mathrm{N}} = F_2(\mathrm{N}, \mathrm{A}; t; \mathrm{M}(t)),$$
$$\dot{\mathrm{A}} = G_2(\mathrm{N}, \mathrm{A}; \mathrm{M}(t))$$

Eq. 4 Dynamic relationship between the ecosphere and the anthroposphere controlled by the selection of strategy **M**(*t*), as stated by Schellnhuber (1998)

[2] That is pollution, infrastructure, industry, urbanisation, waste, etc.

Therefore, the choice of management strategy will determine the path and nature of sustainable development. It is this issue that Geocybernetics addresses.

Geocybernetics and sustainable development

Geocybernetics is concerned with answering the final question, i.e. "what must we do to get there?" Hence, geocybernetics is "the art of controlling the complex dynamic Earth System under uncertainties of all kinds" (Schellnhuber and Kropp 1998). This would tend to infer that if humans are to continue to live on this planet, then there needs to be prudent and effective use of resources—sustainable development. In order to achieve this, there must be co-evolution between the environment and humans. This is achieved by utilising strategies to achieve co-evolution paths over a specified period of time. This can be categorised into one or more of five fundamental geocybernetic paradigms of sustainable development, as stated by Schellnhuber (1998), and which shall be discussed shortly. However, it is important to first discuss the concept of co-evolutionary states and zones, which in turn influences the nature of the strategy undertaken to achieve sustainable development.

Co-evolution states and zones

In order to understand the different geocybernetic paradigms of sustainable development within the context of the Earth System (as presented in Table 1), it is necessary to understand the ways in which co-evolutionary paths can develop according to the actions and motives of humans. Gallopin (2003) provides an effective description of the structure, characteristics and outcomes of various paradigms within co-evolution space—the state of the co-evolution system of **N** and **A**, which can be represented within a multi-dimensional 'state space', and which is defined by all of the potential values of the set of variables that define **N** and **A** (Schellnhuber 1998; Gallopin 2003). Thus, as the state of the system changes over time, the succession of states determines the path that the system takes within this co-evolutionary space.

Table 1 Notation and qualification of the five fundamental paradigms of sustainable development as described by Schellnhuber (1998)

Symbol	Name of paradigm	Positive goal	Negative motive
P_0	Standardisation	Order	Despotism
P_1	Optimisation	Prosperity	Greed
P_2	Pessimisation	Security	Cowardice
P_3	Equitisation	Fairness	Jaundice
P_4	Stabilisation	Reliability	Indolence

Using Gallopin's (2003) approach to simplify the co-evolution system and paths that can occur, the co-evolution system will be condensed to ***N*** and ***H***. ***N*** will refer to the state of the natural subsystem (e.g. global mean temperature, local aggregate index of environmental conditions), and ***H*** will refer to the state of the human subsystem (e.g. degree of development of civilisation, condition of the local community).

In the simplified example shown in Fig. 1, co-evolutionary space is defined as being where all potential and possible values of ***N*** and ***H*** can occur. However, as can be seen in the areas coloured red and blue, no human life can occur within these areas due to their being outside the range of tolerances or conditions for human life to survive unaided.

However, as Schellnhuber (1998, 1999) and Gallopin (2003) outline, two important 'catastrophe domains' exist within co-evolutionary space, which are:

1. Areas within the co-evolutionary space in which humans can survive, but the quality of the socio-ecological system falls below a tolerable level; and
2. Areas that are outside the boundaries for the existence of human life.

There is also the potential existence of 'inaccessible regions'—combined values of ***N*** and ***H*** that cannot be reached by any path, intentionally or impulsively, from the present or initial starting point of P_0 (as shown in Fig. 2b). Figure 2a illustrates a generic state of co-evolution space.

Geocybernetic paradigms of sustainable development

Schellnhuber (1998, 1999, 2001) identifies five fundamental paradigms of sustainable development:

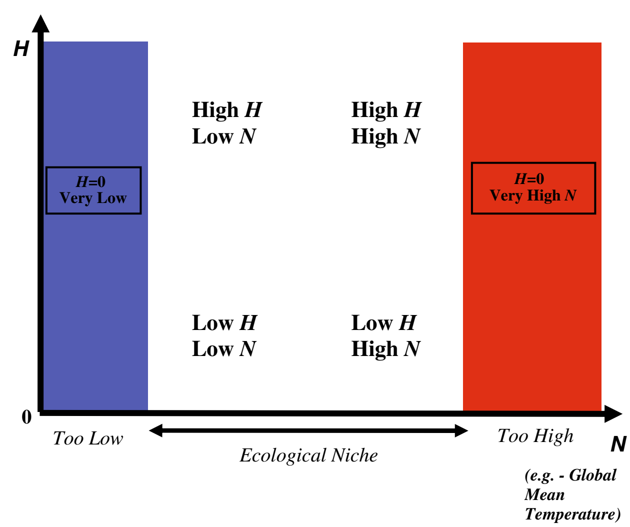


Fig. 1 An idealised representation of the space of a co-evolutionary system, adapted from Gallopin (2003) and based on Schellnhuber (1998)

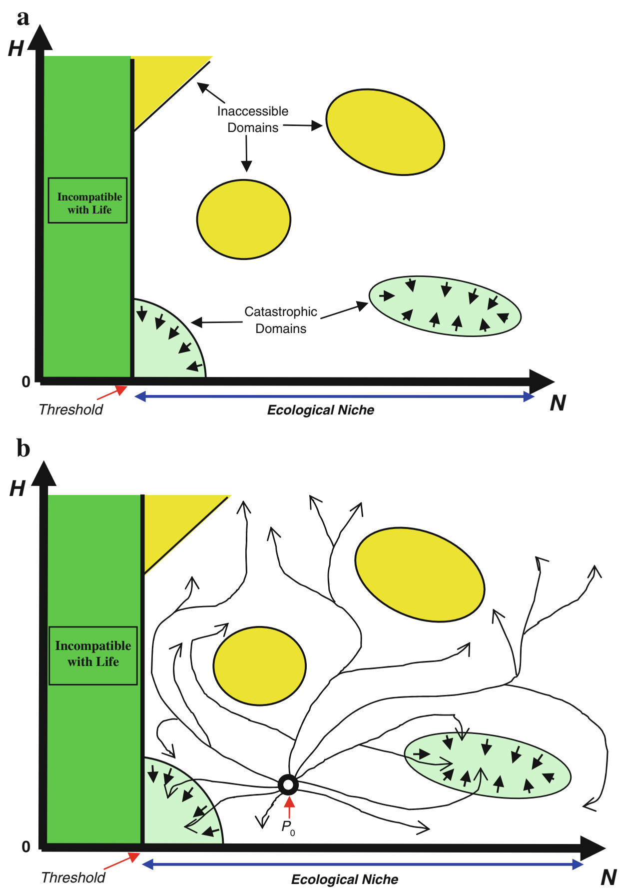


Fig. 2 **a** An elaboration based on, and adapted from, Gallopin (2003, p31, Fig. 12) of the generic state of **N** and **H** in co-evolution space as stated and illustrated by Schellnhuber (1998). **b** The trajectory paths that occur in co-evolution space from a current state P_0 (Standardisation), as alluded to by Gallopin (2003, p31, Fig. 13)

- *Standardisation*: this paradigm provides for the setting of clear values, targets and indicators to control the ***N–H*** system. Therefore, it sets thresholds for the safe conduct of the system over the long term (Gallopin 2003).
- *Optimisation*: this paradigm is concerned with choosing the "best" design for ***N–H*** co-evolution by choosing the optimal path/strategy over a fixed period of time (Gallopin 2003; Schellnhuber 1999).
- *Pessimisation*: this paradigm is concerned with the minimum amount of damage for the maximum amount of potential benefit. Thus, avoiding bad management is crucial (Schellnhuber 1999; Gallopin 2003)
- *Equitisation*: this paradigm is about the preservation of options for future generations (Gallopin 2003). Therefore, the paradigm's notion of "equity" is associated with equality of the environmental and development options for future generations (Gallopin 2003).
- *Stabilisation*: this paradigm brings the ***N–H*** complex into the desired state of sustainable development, and then maintains it through good management (Gallopin 2003).

From this list, humanity can select the appropriate master principle for its strategy to control and achieve sustainable development, or it can develop a strategy that utilises a suitable combination of paradigms to achieve the same purpose (Schellnhuber 2001). Where a combination of fundamental geocybernetic paradigms are used, this is defined as a *complex* paradigm (Schellnhuber 1998).

Results

Mathematical model of sustainable development

Defining sustainable development

Sustainable development, as suggested by the co-evolution of the **N–A** system (Schellnhuber 1998, 1999, 2001), and for any specific point in time, can be described as follows:

$$S(t) = E(t) - H_{NI}(t) \qquad (5)$$

where S is sustainable development, E the environment, H_{NI} human needs and interests, and t time

This suggests that using determined or attributed values[3] for **E** and $\mathbf{H_{NI}}$, the level and nature of sustainable development is dependent on the nature and amount of impact that humans have upon the environment—the more humans consume from the environment in order to satisfy their own needs and interests as a species, along with the associated by-products of consumption, the lower the level of sustainable development, and unsustainable development can even occur at that specified time. This results in the depletion and degradation of resources and services of the Environment (**E**). If related to Earth System Analysis with respect to Eqs. 1 and 2, then the fact is that humans now have the ability to change **N**, whether intentionally or unintentionally, in the 'present day' era (Schellnhuber 1998). Unlike in the 'historical past', this would tend to suggest that the overall environment–human relationship in the 'present day' era is a negative one.

However, Eq. 5 does propose that determined or attributed values of **E** and $\mathbf{H_{NI}}$ can be obtained. This is on condition that a consistent determined range of values for both is used, and suggests that both the level and nature of sustainability can be determined. This is achieved in respect of the notion of weak and strong sustainable development, which describes the relationship between

[3] Determined values refers to real-time data collected through experiments, observations or quantitative measures such as indicators or quantitatively based EIA; attributed values are data obtained using a value judgement approach, such as in the case of a qualitative EIA.

natural and human capital. Natural capital is typically considered to be the stock of natural resources or environmental assets that yields a flow of useful goods and services, now and in the future (Pearce and Turner 1990; Daly 1994). Such resources or assets can be differentiated into two broad categories: (1) renewable and (2) non-renewable (Costanza and Daly 1992; Goodland and Daly 1996; Gowdy and O'Hara 1997; Comolli 2006). Gowdy and O'Hara (1997) provide examples of each, such as economically valuable biological species for renewable resources, and minerals and fossil fuels for non-renewable resources. Human (or manufactured) capital includes all human-made machines, tools and buildings that are used in economic production (Daly 1994; Victor et al. 1995; Goodland and Daly 1996). Hence, this requires the physical transformation of natural capital and the use of human labour to produce (Daly 1994; Victor et al. 1995).

Consequently, the notion of weak and strong sustainability is based upon the concept of natural capital, and how much of such capital should be preserved in perpetuity or set aside for use for present and future generations (Bowers 1997). Figure 3 shows an illustrative example of the differences in the use of capital in respect to weak and strong sustainability along the lines discussed.

Therefore, the level of **E** (re: natural capital) in respect to its level of substitution to $\mathbf{H_{NI}}$ (re: human capital) is critical to determining the level and nature of sustainable development that occurs at all spatial–temporal scales—the less impact $\mathbf{H_{NI}}$ has on **E**, the better the level and nature of sustainable development. However, this means maintaining **E** as high as possible in order to achieve this.

The components of sustainable development **E** and $\mathbf{H_{NI}}$ comprise separate and integral relationships within them that define the nature and extent of each. These components of sustainable development within the model shall now be examined in a little more depth.

Defining the environment

The environment (**E**) is composed of the four primary and integral sub-spheres necessary for planetary operation at all spatial–temporal scales. The four primary sub-spheres are: Atmosphere (*Atmo*), Biosphere (*Bio*), Hydrosphere (*Hydro*),[4] and Lithosphere (*Litho*). Without all four of these sub-spheres working as a cohesive and integral system, there can be no environment as we understand the term. Of course, each of these sub-spheres contain many different smaller sub-systems within them, for example, the Gulf Stream, the tropical rainforest biome, the hydrological cycle and plate tectonics, all of which have an impact on or play a role in the development and characterisation of the whole system. Such processes take time to develop and evolve to a perceived end result, or they continuously adapt to change either within the sub-spheric system or to event(s) in other sub-sphere(s). Consequently, the environment (**E**) can, at any point in time, be characterised as follows:

$$\mathrm{E}(t) = (\mathrm{Atmo} + \mathrm{Bio} + \mathrm{Hydro} + \mathrm{Litho}) \quad (6)$$

This definition is more direct in its determination of the composition of the Environment, and its construction is comparable to $\boldsymbol{N} = (a, b, c, \ldots)$ as proposed by Schellnhuber (1998, 1999, 2001; see Eq. 3). Hence, this makes the application of such equations at any spatial–temporal scale more reasonable.

Therefore, a determined or attributed value for the environment (**E**) can be obtained from the sum of determined or attributed values of the sub-spheres, at any spatial level for a specified point in time, using real-time data or prediction techniques (i.e. models).

However, the operation of any system requires a pre-emptive maximum threshold for safe conduct. The Environment (**E**) as a natural integrated system is no different in this respect. It has clear limits of safe operation before the step-by-step failure of the various sub-spheric systems. Such a failure will perform in a similar way to a 'cascade effect'[5] given the interdependence of operations within and between sub-spheres. The sub-spheres of **E** as a whole, may be able to compensate for a period of time before attempting to correct the situation of a critical level or overload within a sub-spheric system(s). This is achieved through responses to mitigate the effects, or to return it to its original state. So, **E** at any point in time has clear boundaries of operational parameters of what it can and cannot do, within the context of limits available to the system in regard to spatial–temporal considerations. These can therefore be expressed as follows:

$$\mathrm{E}(t) = [\mathrm{E}_0 \leq \mathrm{E} \leq \mathrm{E}_{max}] \quad (7)$$

Consequently, **E** is dependent on the space taken or needed by the various sub-spheres to operate in; *and* the time necessary for evolution, adaptation, mitigation and repair of the system(s) in relation to other sub-spheres within the geographical space and scale as appropriate. This means that any determined or attributed values for **E** for any point in time must be evaluated in the context of the significance and magnitude of potential and/or actual effects/impacts with respect to the current state of environmental conditions, in order to provide a proper context for such evaluations.

[4] All water that is liquid, solid or gaseous.

[5] Both Schellnhuber (1998, 1999, 2001) and Lovelock (2000) support the idea of this type of effect.

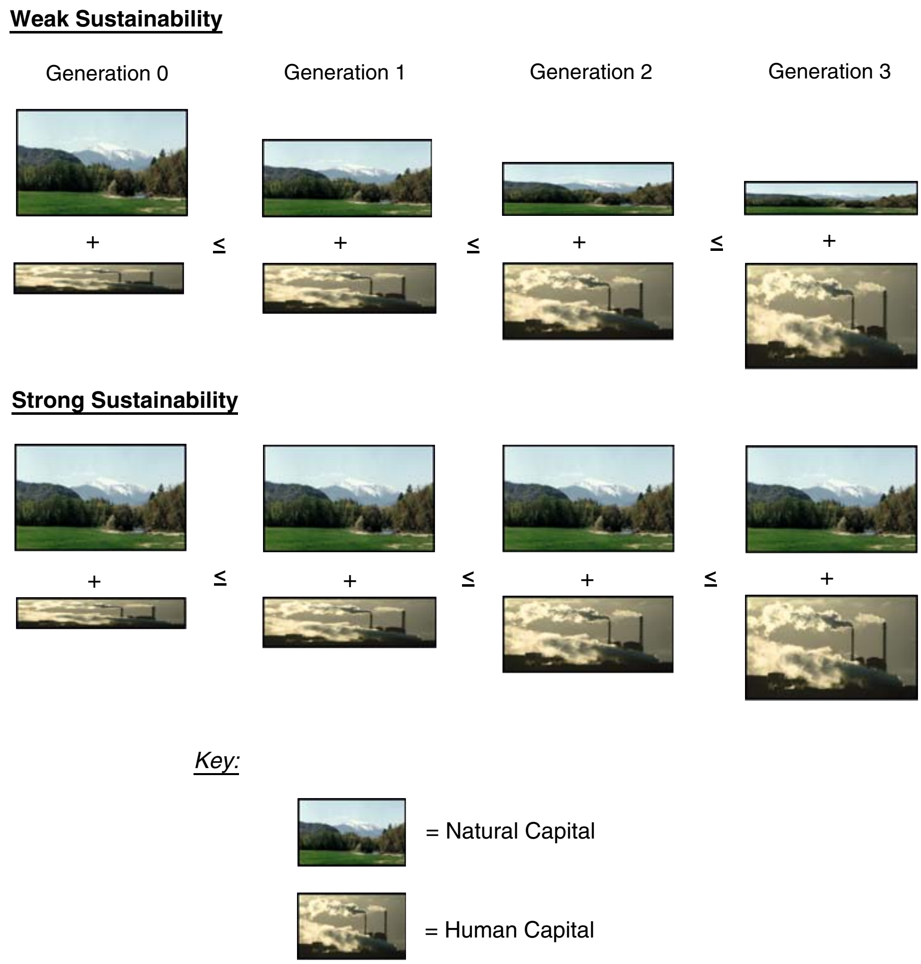


Fig. 3 A diagram based on, and adapted from, Roberts (2004, p 82) illustrating differences in the conversion of natural capital to human capital in weak and strong sustainability approaches

Defining human needs and interests

Using a Darwinian, and even an anthropocentric perspective, humans as a species have developed further than any other species. This is because we not only meet our own basic needs, but also have gone beyond to satisfy needs and interests that are both tangible and intangible in nature. This is comparable to the definition of the Human Factor $\boldsymbol{H}$ as outlined in Eq. 3 earlier. This would seem to suggest that each generation of humans continually desires more than the past generation. Further, there is the desire to succeed and improve the human condition through acts of inspiration (e.g. the Apollo moon landings); or to deteriorate the human condition through acts of self-destruction by human desires, needs or interests (e.g. war). $\mathbf{H_{NI}}$ would appear to be infinite and vary according to the level of human hierarchy at which it occurs.

However, is $\mathbf{H_{NI}}$ actually infinite in reality? $\mathbf{H_{NI}}$ is dependent upon the resources available and provided by **E**, in order for humans to live. If $\mathbf{H_{NI}}$ increases at a rate that is at the increasing detriment of **E**, this then suggests that there is actually a maximum limit for $\mathbf{H_{NI}}$ based upon the resources and services of **E** left available—i.e. oil and supplies, clean air, etc. As a consequence, when **E**, at whatever geographical scale, is degraded beyond the point of no return, then humans would have to seek alternative places to live. Therefore, there is a limit to the potential determined or attributed value of $\mathbf{H_{NI}}$ that can be obtained, depending upon the availability of the determined or attributed value of **E** for any specified point in time. This is characterised as follows:

$$\mathrm{H_{NI}}(t) = [\mathrm{H_{NI0}} \leq \mathrm{H_{NI}} \leq \mathrm{H_{NI\,max}}] \quad (8)$$

The parameters that are the determinant factors for the degree and/or value of $\mathbf{H_{NI}}$ are as follows:

$$\mathrm{H_{NI}}(t) = [\mathrm{I(NI), Comm(NI), Soc(NI), Sp(NI)}] \quad (9)$$

where:

$$\mathrm{NI}(t) = [\mathrm{QL, Ec, So, BN}] \quad (10)$$

and:

$$\mathrm{BN}(t) = [\mathrm{Sh, F, En, Rep}] \quad (11)$$

The parameters represented in Eqs. 9–11 characterise the nature and components that informs the use of **E** in the maintenance of $\mathbf{H_{NI}}$.

Equation 9 defines $\mathbf{H_{NI}}$ within the context of the human hierarchy that, at each level, adopts the needs and interests (*NI*) relevant to that level. **NI** itself is defined in Eq. 10. Starting with Individual (*I*), the hierarchy then moves to Community (*Comm*), Social (*Soc*), and finally Species (*Sp*). At each level, there will be differing requirements of **NI**, based on their status and development at the time. As the human hierarchy changes in response to environmental and human pressures of evolution and development, so **NI** would corresponding change to meet the aspirations of each level of the hierarchy. Each successive generation of the hierarchy seeks improvements in their condition from the previous generation. However, each hierarchal level still fundamentally requires to meet basic needs (**BN**). **BN** ensures that the absolute minimum necessary conditions for human survival and development at any hierarchal level are met. As humans become more sophisticated in respect to **NI** over time, their wants and desires become more intangible with social development. Consequently, **BN** is by this stage of human development is almost automatically achieved at the Society level. This is because the Society level will ensure that everyone within a society has a satisfactory level in meeting the parameters of **NI** to sustain them (e.g. the 'welfare state'). However, where human conditions meet only the requirements of **BN** (Eq. 11), this can be due to environmental and/or human factors that have restricted the development of the human hierarchy further—e.g. war, changing or poor environmental conditions, excessive carrying capacity, natural disasters, famine, poor political structure, lack of education, poor social structure and cohesion, lack of technological development, etc.

Human development and the needs and interests that fuel it, is therefore dependent upon the level of three factors at a time, and the extent to which they are available: Social Development (*SD*), Technology, (*T*)[6] and Knowledge (*K*). This can be suggested by the following:

$$\mathrm{H_{NI}}(t) = f[\mathrm{SD,T,K}] \to \infty \tag{12}$$

The boundaries of these factors are potentially infinite, as proposed in Eq. 12, as is the human potential to meet its needs and interests. So, now that **E** and $\mathbf{H_{NI}}$ have been defined, what does this mean for sustainable development?

[6] Social Development and Technology factors are comparable to those described in 'Our Common Future' (WCED 1987) in one of the two caveats of the well-known definition: 'development that meets the needs of the present without compromising the ability of future generations to meet their own needs'. The caveat states that 'the idea of limitations imposed by the state of technology and social organisation on the environment's ability to meet present and future needs' (WCED 1987). However, because of the fact that the report failed to properly recognise the role of knowledge as a factor in the development of Social Development and Technology as part of the development of $\mathbf{H_{NI}}$, it provided an incomplete picture of the anthropogenic component of human needs and interests.

Determining the level and nature of sustainable development

For a level of **S** to occur at any point in time, any determined or attributed value of **E** must be greater than the determined or attributed value of $\mathbf{H_{NI}}$. This can be expressed by the following:

$$\mathrm{E}(t) > \mathrm{H_{NI}}(t) \Leftrightarrow \mathrm{S}(t) > 0 \tag{13}$$

However, if any determined or attributed value of **E** is less than or equal to any determined or attributed value of $\mathbf{H_{NI}}$, then **S** would not occur, as there must be a continuous source of **E** for $\mathbf{H_{NI}}$ to utilise that does not endanger the safe operation of **E**. This can be described as follows:

$$\mathrm{E}(t) \leq \mathrm{H_{NI}}(t) \Leftrightarrow \mathrm{S}(t) \leq 0 \tag{14}$$

Considering the equations described in totality, the dynamic relationship between **E** and $\mathbf{H_{NI}}$ can be determined by assuming an unmitigated situation to the scenarios described in Eqs. 1 and 2, and utilising the relationships and rules described in Eqs. 5, 13 and 14. This can be achieved through adopting an approach similar to a Newtonian thought experiment. The results of this are illustrated in Fig. 4.

As a result, this would suggest that a mode of assessing and implementing the sustainable development of a project is necessary to correct the detrimental situation suggested in Fig. 4. This would be satisfied by implementing a pool of management options (Schellnhuber 1998, 1999, 2001; i.e. EIA, EMS, SEA, Global Reporting Initiative, etc.) as proposed in Eq. 4,

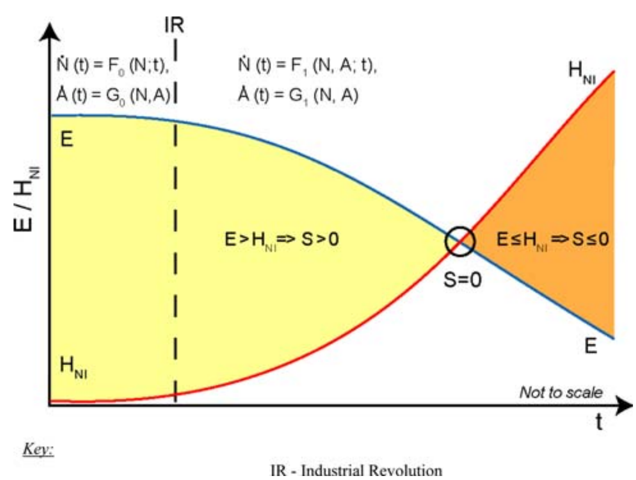


Fig. 4 A simple conceptualisation of the potential unmitigated and unmanaged relationship between the environment (**E**) and human needs and interests ($\mathbf{H_{NI}}$) in respect to the governing dynamics of the geocybernetic model of sustainable development, related to the coupled dynamics of the Earth System as outlined by Schellnhuber (1998, 1999, 2001) and using ideas of Thomas Malthus. The E/$\mathrm{H_{NI}}$ label refers to values for both represented on the diagram

The potential determination of values for **E** and $\mathbf{H_{NI}}$ means that a determination of **S** using Eq. 5 can occur. This is assuming that a suitable quantitative approach to the valuation of **E** and $\mathbf{H_{NI}}$ is adopted, and both are calculated in a final value range of $0 < x \leq 1$.

The less impact $\mathbf{H_{NI}}$ has on **E**, the better the nature and level of sustainable development that is occurring. However, this means maintaining **E** as high as possible (natural capital) in order to achieve sustainable development. Sustainable development is therefore deemed to occur at a value greater than or equal to 0.001, using an accuracy of three decimal places, and which satisfies the requirements of Eq. 14.

The level and nature of sustainable development is consequently dependent on the design and implementation of the strategy of management undertaken in relation to the geocybernetic paradigms. As most strategies implemented for sustainable development tend to conform to the complex paradigm, then such a strategy would suggest that careful judgement and expertise to balance the delicate needs of the Earth System is necessary. Adopting the same approach as used in respect to Figs. 4 and 5 suggests that a potential new dynamic relationship exists to implement sustainable development. The choice of strategy for **N–A** management will determine the nature and extent of sustainable development occurring within the designated spatial–temporal scale of the Earth System.

In the case illustrated in Fig. 5, the introduction of a pool of management option (**M**) for the mitigation and

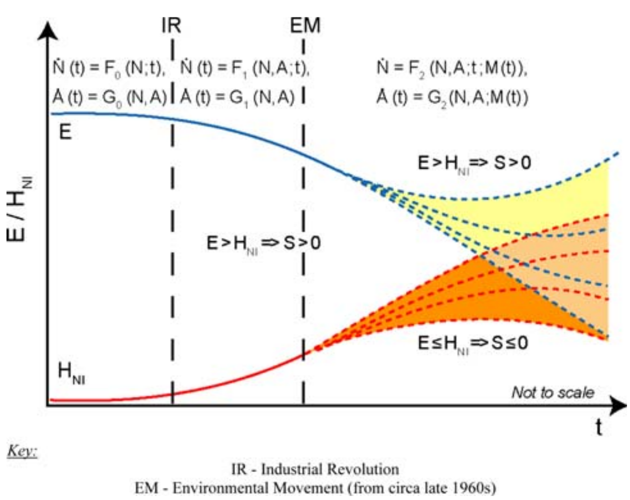


Fig. 5 A simple conceptualisation of the revised potential relationship between the Environment (**E**) and human needs and interests ($\mathbf{H_{NI}}$) with respect to the governing dynamics of the geocybernetic model of sustainable development, and related to the coupled dynamics of the Earth System as outlined by Schellnhuber (1998). The $\mathbf{E/H_{NI}}$ label refers to values for both represented on the diagram. The emergence of environmental consciousness and awareness, starting from the Environmental Movement, has developed the political, economic and social necessity and will to develop alternative paths for environment–human relationships through policy, law, assessment and new scientific understanding

control of the anthroposphere (**A**), leads to numerous new potential paths for sustainable development to be made available after the emergence of the environmental consciousness of the Global Subject (*S*). The number of paths for **E** and $\mathbf{H_{NI}}$ could be significantly greater or fewer than those shown, depending on the potential options available at the time. The degree of success or failure in implementation and maintenance of the chosen path is dependent on the factors such as the strategy undertaken, the personnel employed, the desire to succeed, the resources and time available, progress in knowledge and technology, the social development of environmental awareness, etc. Management options will include the obtaining and interpretation of data concerning the project or issue being reviewed [i.e. the use of an EIA or indicators; the development of an appropriate strategy for achieving sustainable conduct (i.e. through strategic environmental assessment or policy development mechanisms); the implementation of the chosen strategy; and the maintenance of the chosen strategy through appropriate mechanisms for assessing performance (i.e. ISO 14001 or indicators).

Values of E and H_{NI} over time

The proposal of sustainable development occurring or not in respect to **E** and $\mathbf{H_{NI}}$ over time, as outlined in Eqs. 13 and 14, is a reflection of the idea of co-evolutionary paths and zones as discussed earlier. The nature of the strategy used to achieve co-evolution (sustainable development) will be reflected in the 'values' obtained or suggested for **E** and $\mathbf{H_{NI}}$ over time through scientific monitoring and observations. Consequently, the level of the determined or attributed values of **E** and $\mathbf{H_{NI}}$ will strongly influence whether sustainable development is occurring and to what extent, and how successful the strategy for co-evolution will be. So, the question that is raised subsequently is "what would certain 'values' of **E** and $\mathbf{H_{NI}}$ indicate at any point or period of time?"

In the case of $\mathbf{H_{NI}}$, any determined or attributed values at any point in time will tend to be greater than zero, on condition that there some form of resources of **E** to meet basic needs (**BN**) are available. This is due to the fact that the natural desire to survive becomes prevalent and dominant within humans. Further, the human ability to develop solutions for new habitats outside our natural niche will allow for continual survival elsewhere than Earth through colonisation of other planets, once the technology and knowledge becomes available. Such endeavours may be amongst the first crucial steps towards reversing the issues facing the planet Earth currently—the confinement within one sphere, and the strains placed upon its humanity through pollution, over-extraction of resources, overpopulation and destruction of critical environmental systems are now resulting in global changes at all spatial scales.

Therefore, the effective management of the Earth System is becoming unavoidable if humans are to continue to survive and prosper. The impact and level of $\mathbf{H_{NI}}$ must be altered to a state that provides for some form of comfort for humanity over time, whilst allowing for its continuous development. However, and more critically, this must not cause excessive damage to the 'value' of environment (**E**) to the extent that, over time, would endanger the Earth System as a whole, as seen potentially with climate change.

As a result, in order to least ensure this does not occur for the present and future generations, the determined or attributed values of **E** should always be greater than $\mathbf{H_{NI}}$ at any specified point in time in order for humanity to survive as well as for a continuing habitat over time. This consequently suggests the need for a planetary system that is diverse in its environments, as well as having sufficient resources to meet human requirements, both basic and advanced, from generation to generation. The better the 'value' of **E** available, the better the opportunity for human survival and meeting its needs at a consistent level. This means that the environment and humans must be balanced in such a way that humans do not use any more resources than necessary to fulfil their requirements at a consistent and sustained level. This effort must not over-exploit the resources available through despotism, greed and avarice at the expense of others or, more importantly, the environment. This is supported in Fig. 1, in the representation of co-evolutionary space provided by Gallopin (2003), using the ideas of Schellnhuber (1998). The representation of high 'values' of ***N***[7] in the diagram suggests one of two fundamental situations. Either:

1. High ***N***, High ***H***[8]: over time, humans could learn to use the environment in a manner that is conducive to the highest form of planetary management: maintain the optimum level for the environment whilst ensuring that human needs are fully satisfied; or
2. High ***N***, Low ***H***: humans decide that it is better to live well within the constraints of the environment, whilst preserving as much as possible for future generations.

Where the value of **E** is regarded as low over time, this is the result of a harsh environment created by natural or anthropogenic means. Whilst it is still habitable, humans will be required to adapt to the conditions in order to survive, as in the Low ***N*** scenarios in Fig. 1.

Where **E** equals zero, this would indicate an environment incapable of sustaining life of any kind. Where values of $\mathbf{H_{NI}}$ are zero, this would then indicate that human needs and interests cannot be supported as no viable resources (environmental and mineral) are available to sustain human existence.

[7] This is comparable to **E** in the model.

[8] This is comparable to $\mathbf{H_{NI}}$ in the model.

Potential application

The mathematical model of sustainable development is more than just a theory, it is a mechanism with the potential of assessing and evaluating sustainable development. It is to this end that the follow-up paper will demonstrate the model's applicability using two quantitatively based EIAs. However, in order to demonstrate the potential of the model and the nature of the follow-up paper, an example of the application is provided here in the form of a case study from the EIA evaluation for the Bangalore Metro Rail Scheme. This example utilises an EIA report conducted by a private environmental consultancy in India, which was publicly available on the Internet. Therefore, this report has not been peer-reviewed, and thus acquisition and interpretation of the data is dependent solely upon the competence and opinions of the personnel of the consultancy. Permission to use the data was granted by the Bangalore Metro Rail Corporation, and will be accordingly extended full academic courtesies. This example is used for illustrative purposes only for the purposes of demonstrating application of the model. A fuller description of the relative EIA methodologies and model application processes will be provided in the follow-up paper.

Project outline

Bangalore is the principal capital of Karnataka state in India. At the time of the EIA, the city was experiencing significant economic growth and, as a result, large-scale urbanisation (BMRC 2005). The city's population at the time of the 2001 census was 5.7 million (BMRC 2005). The economic and social growth can be attributed to factors such as good infrastructure, an abundant labour force (skilled and unskilled), a good scientific and industrial base, and a favourable climate (BMRC 2005). However, the public transport network was considered to be very poor and inadequate to meet the demands of a growing city (BMRC 2005). As a consequence, use of private vehicular transport had increased significantly (BMRC 2005).

In response to this situation, and after exploring the potential options available, the Government of Karnataka determined that an elevated light rail transit system (ELRTS) was the best option to relieve the traffic bottlenecks created (BMRC 2005).

The scheme would consist of two dedicated metro rail corridors: East–West and North–South (BMRC 2005). The system uses primarily an elevated system of tracks with sufficient clearance for double decker coaches (BMRC 2005). Both corridors cross underground at Majestic Station, which serves as an interchange station (BMRC 2005). The tracks run along and stop at key business districts of the city (BMRC 2005). The terminal stations of both

Table 2 The original table of results for the Bangalore Metro Rail Scheme using Battelle Environmental Evaluation System (BEES) methodology (adapted from BMRC 2005, Table 16, p. xxxii)

Environmental aspect and components	PIU	Project EIU			
		Without project North–South (NS)	Without project East–West (EW)	With project and EMP North–South (NS)	With project and EMP East–West (EW)
Physical					
Air quality	395	202	158	275	236
Water table/quality	130	122	119	122	119
Land	40	39	35	38	34
Total	565	363	312	435	389
Biological					
Terrestrial ecosystem	150	139	147	149	149
Aquatic ecosystem	15	12	13	14	14
Total	165	151	160	163	163
Socio-economic	270	51	48	228	236
Total	1,000	565	520	826	788

PIU Parameter impact unit, *EIU* environmental impact unit, *EMP* environmental management plan

corridors serve as depots for rolling stock to be stabled and maintained (BMRC 2005). An EIA of the project using the Battelle Environmental Evaluation System (Dee et al. 1973) produced the results outlined in Table 2.

Methodology

The methodology for applying the model to the results obtained using the BEES system is detailed in Fig. 6. The example shows the process for determining the nature of sustainable development occurring (environmental or ecological),[9] as well as a set of calculations performed for determining the North–South corridor without the project (i.e. the baseline).

Example of model application to an EIA

Determination of components of S

(i) Components of E

Primary

A Physical (P): air quality
B Biological (B): terrestrial ecosystem; aquatic ecosystem
H Physical (P): water table/quality
L Physical (P): Land

[9] The specific issue of environmental and ecological parameters in EIAs and their impact upon the model in calculating sustainable development will be addressed fully in the follow-up paper.

As all of the primary components of E are present, the following equation and its procedure will be used: $S = E - H_{NI}$

(ii) Components of H_{NI}

- Socio-Economic (SE)

(iii) Evaluation process
The process will evaluate the level of sustainable development (if appropriate) in the following regards:

- Without project: North–South corridor
- Without project: East–West corridor
- With project: North–South corridor
- With project: East–West corridor

Evaluate S for project: without project-North–South corridor

$S = E - H_{NI}$

(i) Calculate *E*

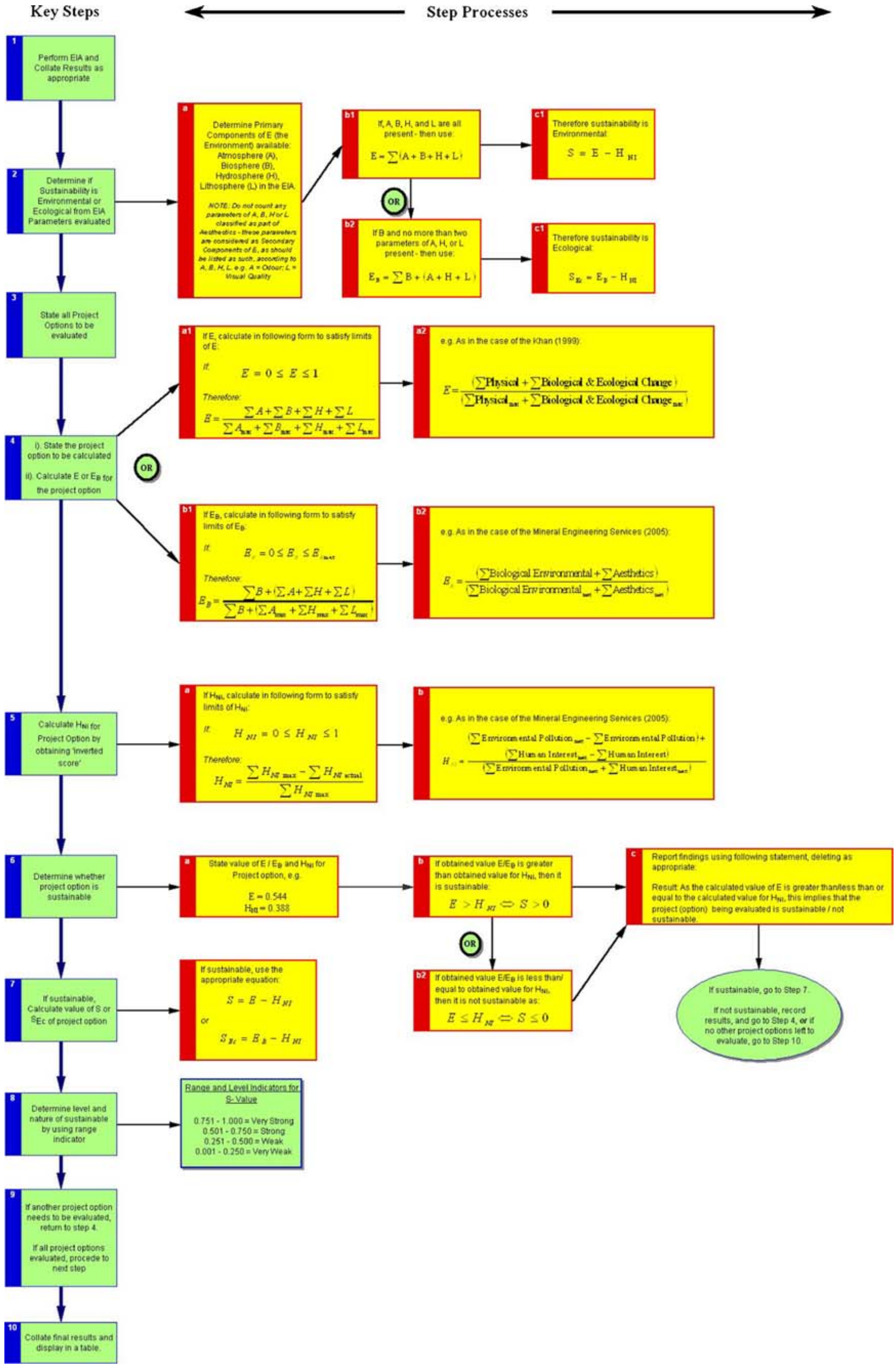
Determine *E* within range $0 \leq E \leq 1$

$$E = \frac{\Sigma P + \Sigma B}{P_{\max} + B_{\max}} = \frac{(363 + 151)}{(565 + 165)} = \frac{514}{730}$$

$E = 0.704$

(ii) Calculate H_{NI}

Determine H_{NI} within range $0 \leq H_{NI} \leq 1$.



Key Steps
Step Processes
1 Perform EIA and Collate Results as appropriate
2 Determine if Sustainability is Environmental or Ecological from EIA Parameters evaluated
3 State all Project Options to be evaluated
4 i). State the project option to be calculated ii). Calculate E or E_B for the project option
5 Calculate H_NI for Project Option by obtaining 'inverted score'
6 Determine whether project option is sustainable
7 If sustainable, Calculate value of S or S_Ec of project option
8 Determine level and nature of sustainable by using range indicator
9 If another project option needs to be evaluated, return to step 4. If all project options evaluated, procede to next step
10 Collate final results and display in a table.
a Determine Primary Components of E (the Environment) available: Atmosphere (A), Biosphere (B), Hydrosphere (H), Lithosphere (L) in the EIA.
b1 If, A, B, H, and L are all present - then use:
b2 If B and no more than two parameters of A, H, or L present - then use:
c1 Therefore sustainability is Environmental:
c1 Therefore sustainability is Ecological:
OR
a1 If E, calculate in following form to satisfy limits of E:
a2 e.g. As in the case of the Khan (1999):
b1 If E_B, calculate in following form to satisfy limits of E_B:
b2 e.g. As in the case of the Mineral Engineering Services (2005):
a If H_NI, calculate in following form to satisfy limits of H_NI:
b e.g. As in the case of the Mineral Engineering Services (2005):
a State value of E / E_B and H_NI for Project option, e.g. E = 0.544 H_NI = 0.388
b If obtained value E/E_B is greater than obtained value for H_NI, then it is sustainable:
b2 If obtained value E/E_B is less than/ equal to obtained value for H_NI, then it is not sustainable as:
c Report findings using following statement, deleting as appropriate:
Result: As the calculated value of E is greater than/less than or equal to the calculated value for H_NI, this implies that the project (option) being evaluated is sustainable / not sustainable.
If sustainable, go to Step 7. If not sustainable, record results, and go to Step 4, or if no other project options left to evaluate, go to Step 10.
If sustainable, use the appropriate equation:
Range and Level Indicators for S- Value
0.751 - 1.000 = Very Strong
0.501 - 0.750 = Strong
0.251 - 0.500 = Weak
0.001 - 0.250 = Very Weak

◀ **Fig. 6** Flowchart of process for the application of the geocybernetic model of sustainable development to the Battelle Environmental Evaluation System (BEES) (In Step 4a2, equation parameters refer to Khan (1999). In Step 4b2 and 5b, equation parameters refer to Mineral Engineering Services (2005), http://southgoa.nic.in/Caneli%20Donger%20Mine.pdf. Accessed 18 Jan 2008.)

$$H_{NI} = \frac{(SE_{\max} - \Sigma SE)}{(SE_{\max})} = \frac{(270 - 51)}{270} = \frac{219}{270}$$

$$H_{NI} = 0.811$$

(iii) Determine if S occurs

$$E = 0.704$$

$$H_{NI} = 0.811$$

Thus:

$$E \leq H_{NI} \Leftrightarrow S \leq 0$$

Result: As the calculated value of E is less than/equal to the calculated value for H_{NI}, this implies that the project option being evaluated is not sustainable.

Evaluation of S-values Using the methodology described in Fig. 6, the following results were obtained (Table 3)

Discussion of results Table 2 indicates that the projected development's outcomes would be an overall improvement in environmental and human well-being (Socio-Economic) due to the significant improvements in environmental quality achieved, and potential for improvements in socio-economic conditions. There were significant improvements in the physical environmental impact unit (EIU) scores, particularly given the fact that the project is located in an urban area. The improvements in Biological EIU scores, is also of benefit to local environmental conditions. The change in EIU scores is illustrated in Table 4.

With reference to Table 4, the East–West corridor is indicated to be environmentally poorer in quality than the North–South corridor. This is due mainly to the significantly lower quality of the air before the project with environmental management plan (EMP).

However, Table 4 also shows that the project with EMP EIU scores for both corridors would seem to drastically improve air quality: +73 for the North–South corridor and +78 for the East–West corridor. The project also provides for vastly improved environmental quality for Socio-Economic parameters: +177 for North–South corridor and +188 for East–West corridor. There is also a small-to-moderate improvement in biological parameters, predominantly in Terrestrial Ecosystem in the North–South corridor. EIU scores without and with project would therefore seem to indicate a dramatic improvement in environmental quality for the **E** components of the model, as well as a substantial reduction in the impact of the components of $\mathbf{H_{NI}}$ on the environmental quality of the area. Therefore, the initial indications for the sustainability of the project are good.

Table 3 indicates a clear conversion from unsustainability without the project, to sustainability with the project and EMP. Indeed, the values for sustainable development for 'with project and EMP' options are $S = 0.663$ (North–South) and 0.630 (East–West). This indicates a very satisfactory improvement in environment–human co-evolution. The improvements are related to improvements in environmental quality in respect to air quality and socio-economic conditions. This would seem to suggest that socio-economic factors, if properly managed and mitigated for, can result in improvements in environmental quality. This leads to the potential confirmation that significant **N–A** co-evolution can occur at the local level for projects/developments. Further, there is the potential suggestion that EIAs can play a direct role in achieving and ensuring sustainable outcomes for project/development

Table 3 S-values obtained for Bangalore Metro Rail Scheme

	E_B	H_{NI}	S-value
Without Project: North–South	0.704	0.811	N/A
Without Project: East–West	0.647	0.822	N/A
With Project and EMP: North–South	0.819	0.156	0.663
With Project and EMP: East–West	0.756	0.126	0.630

Table 4 The change in EIU scores for the Bangalore Metro Rail Scheme using the BEES methodology (BMRC 2005, p. xxxii)

Environmental aspects and components	PIU	Δ In project EIU (with–without)	
		North–south	East–west
Physical			
Air quality	395	+73	+78
Water table/quality	130	0	0
Land	40	−1	−1
Total	565	+72	+77
Biological			
Terrestrial ecosystem	150	+10	+2
Aquatic ecosystem	15	+2	+1
Total	165	+12	+3
Socio-economic	270	+177	+188
Overall total	1,000	+261	+268

activities. This is providing that an appropriate framework is in place to make judgements with respect to sustainable development issues.

Discussion

Context within existing literature

Geocybernetics

This paper has attempted to adopt a geocybernetic approach to make a contribution to answering the question 'what is sustainable development?' The existing literature through Schellnhuber (1998, 1999, 2001; Schellnhuber and Kropp 1998) has detailed, through Earth System Analysis, the dynamics of the Earth System in respect to historical and present relationships between the ecosphere (**N**) and the Anthroposphere (**A**), as well as providing a fundamental definition for the Earth System in the form of $E = (N, H)$. With respect to the existing literature, Schellnhuber (1998, 1999, 2001; Schellnhuber and Kropp 1998) outlines the concept of geocybernetics, at its most basic, as concerned with the control of the Earth System in order to achieve sustainable development between $\boldsymbol{N}$ and $\boldsymbol{H}$. This involves the use of co-evolutionary paths that balance the needs of both to achieve the best possible outcome. Achievement of this goal is based on the use of appropriate strategies and management which, as Schellnhuber (1998, 1999, 2001; Schellnhuber and Kropp 1998) states, fall into one of five fundamental paradigms: standardisation, optimisation, pessimisation, equitisation and stabilisation. Typically, combinations of two or more of these paradigms are used to achieve the co-evolutionary strategy, and Schellnhuber (1998, 1999, 2001; Schellnhuber and Kropp 1998) defines such a paradigm as Complex. Indeed, Schellnhuber (1998) demonstrates the dynamics of all six paradigms in the context of various scenarios and their potential outcomes.

Nevertheless, it is the lack of a fundamental definition of sustainable development within the Earth System context that prompted the primary contribution of this paper to the existing literature concerning geocybernetics and sustainable development theory. There is no doubt that understanding different strategies for sustainable development over time has enormous merit and usefulness to achieving co-evolution. The problem has been, however, that, whilst Schellnhuber very elegantly defines the Earth System and the dynamic relationship between **N** and **A**, no such fundamental definition for sustainable development exists. The application of fuzzy solutions to questions of sustainable development has only perpetuated the confusion within the literature as to what is sustainable development. In the final analysis, whilst the work of Earth System Analysis is of enormous benefit to those involved in research on sustainable development theory and practices, as far as consumption and action for a wider audience in resolving questions of sustainable development is concerned, the work is not as successful. This is where this paper makes a contribution to the existing literature. The geocybernetic model defines the fundamental principles of sustainable development within the context of the Earth System. By defining the role of the environment (comparable to $\boldsymbol{N}$) and human needs and interests (comparable to $\boldsymbol{H}$), the model provides for a clear understanding as to what sustainable development is, and, importantly, why. The sophistication of the equations/expressions allows all to at least benefit from having a clearer understanding as to the nature of sustainable development. Crucially. the model though attempts to place the missing piece that Schellnhuber did not fully address in his work—what is sustainable development? The fact that, for whatever reason, he did not choose to attempt a fundamental definition of sustainable development in the Earth System context is not the concern of this paper. What is important, however, is that the model defines how sustainable development, at a fundamental level, operates within the Earth System at a specified spatial scale at any specific point in time. This extends the notion of geocybernetics presented in the literature to the extent where it is enhanced by such a definition. Instead of determining complex and almost ungainly equations for various potential paths for co-evolution using different strategies, the model provides a more direct mode of operationalisation to determine sustainable development at an appropriate spatial scale and levels of understanding. Most importantly of all perhaps is the fact that the model is related to the equations of Earth System Analysis in the context of the dynamics of sustainable development. This provides the best indication of all as to the extent to which the model defines the nature of sustainable development occurring with the Earth System. Using the existing literature concerning Earth System Analysis and Geocybernetics, in respect of both its successes and failures, this paper has built upon and advanced the existing literature to a desired state of understanding of the question "what is sustainable development"?

The broader context

The broader context of the model within the existing literature is that it is grounded in different fundamental aspects of sustainable development theory—ecocentrism and technocentrism; weak and strong sustainable development and nested sustainable development.

Firstly, within the context of the ecocentric perspective, the model address the notions of: humans being a part of nature rather than a separate entity that uses nature; the concept of human demands for resources and waste within the capabilities and limits of the environment; *and* the need to maintain environmental quality as well as material well-being. From the technocentric perspective, the model underpins the idea of extracting services and needs, with the least possible amount of damage to the environment. Also, the techocentric notion of using science and technology and having faith in them to advance humanity's progress and the resolution of environmental problems posed as a result of progress.

In respect to weak and strong sustainable development, the concept of natural and human capital is reflected in the model as **E** and $\mathbf{H_{NI}}$. Therefore, the less impact $\mathbf{H_{NI}}$ has on **E**, the better the nature and level of sustainable development. However, this means maintaining **E** (i.e. natural capital) as high as possible in order to achieve sustainable development. Also, the use of weak and strong in determining the nature and level of sustainable development reflects the use and impact of natural and human capital, as outlined in Fig. 3.

In respect to nested sustainable development, the model reflects the interdependent relationship between the environment and socio-economic factors, just as in Earth System Analysis. Both state that humans are dependent upon the environment for their needs, and thus draw upon environmental resources and services in order to fulfil social and economic needs.

Conclusion

It is fair and acceptable to state that the mathematical model of sustainable development presented in this paper is still somewhat a work in progress.

Nevertheless, the primary issues concerning what sustainable development is, are more than adequately addressed within the model construct suggested. Primarily, the nature of the environment-human relationship, in relation to the use of and impact upon our planet at any spatial scale at any specified point in time, is addressed. The potential quantification of sustainable development provides great potential to use the 'scientific medial complex' (Schellnhuber 1998) and to utilise the pool of management options (**M**) to calculate and implement sustainable development. It is the intention to demonstrate the model's applicability, in the initial instance, to quantitatively based EIAs in the follow-up paper.

However, the model represents a contribution to the debate concerning sustainable development, as well as providing the opportunity to utilise the model in order to make an assessment of the level and nature of sustainable development of a project/development. Such a model may be viewed with some scepticism. One potential criticism of the model could be that it is merely a re-invention of Earth System Analysis. This is not the case, as when the model was first developed in 2000/2001, the author was completely unaware of the existence of Earth System Analysis, and derived the model through independent research and considerable thought and reflection. The fact that the model has some strong correlations with Earth System Analysis provides validity for the integrity of the model and the ideas conveyed. Further, the model goes somewhat further than Schellnhuber in the definition and role of humans within the Earth System, by defining and simplifying the human hierarchy and motivations for actions that can and do impact upon the environment. This provides an opportunity to re-evaluate the way humans interact with the environment at all spatial scales over time.

It has been a long held view within sustainable development theory that it is not only about the use of resources, but also the impact that humans have upon the environment in terms of production, consumption and waste. Therefore, the model attempts to address these issues, and the factors behind them.

In the follow-up paper, the initial application of the model to two quantitative-based EIA methodologies will be discussed and demonstrated.

Acknowledgments I would like to sincerely thank my supervisors at the University of Exeter: Dr. Stewart Barr of the Department of Geography (Streatham Campus) and Dr. Patrick Foster of the Camborne School of Mines. Most of all, I would like to thank my mother, Mrs. Pamela Phillips, without whom, I could not have achieved what I have. I am forever in her debt.

Appendix

Glossary of mathematical terms

(1). Earth System Analysis

N, 'Macro-state' of the ecosphere (*N*)
A, 'Macro-state' of the anthroposphere (*A*)
t, Time variable ($t = 0$)
F_0, G_0, F_1, G_1, F_2, G_2: Function
M, Coherent voluntary strategies of management that are available to the "global subject" (S)
E, The Earth System contains the ecosphere ***N***, and the human factor ***H***
N Represents the various sub-spheres that can be "spelled" alphabetically: *a*, atmosphere; *b,* biosphere; *c,* cryosphere etc.
H Represents the Human Factor comprised of:
A, The physical sub-components of anthroposphere: all of the individual lives, actions and products; and

S, The "metaphysical" sub-component, reflecting the emergence of a 'global subject'.

(2). Mathematical model of sustainable development

S, Sustainable development
E, (the) Environment
$\mathbf{H}_{\mathbf{NI}}$, Human needs and interests
t, Time
Atmo, Atmosphere
Bio, Biosphere
Hydro, Hydrosphere
Litho, Lithosphere
I, Individual
Comm, Community
Soc, Society
Sp, Species
NI, Needs and interests
QL, Quality of life
Ec, Economic
So, Social
BN, Basic needs
Sh, Shelter
F, Food/water
En, Energy
Rep, Reproduction of species
SD, Social development
T, Technology
K, Knowledge
f, Function

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