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RESEARCH ARTICLE - SOLID EARTH SCIENCES

The earthquake network: the best time scale for network construction

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Abstract

Scientists mapped the seismic time series into networks by considering the geographical location of events as nodes and establishing links between the nodes with different rules. Applying the successively defined models to construct the networks of seismic data, a variety of features of earthquake networks are detected (scale-free and small-world structures). Network construction models had changed in detail to optimize the performance of the verification of the minimum geographical size defined for the node. In all the studies, people try to use large data sets like years of data to ensure their results are good enough. In this work, by proposing the temporal network construction and employing the small-worldness property for data from Iran and California, we could achieve the minimum time scale needed for the best results. We verified the importance of this scale by analyzing two significant centrality measures (degree centrality and PageRank) introduced in the concept of earthquake network.

Keywords Complex networks · Temporal networks · Earthquake networks

Introduction

An earthquake is a sudden motion of a fault that releases an enormous amount of energy and is considered a complex spatiotemporal phenomenon occurring in the earth's crust (Kanamori and Brodsky 2001). Transferring the stress of the movement of one fault to the others results in triggering subsequent events (King et al. 1994; Belardinelli et al. 2003; Freed 2005). The Omori law (Omori 1894) and the Gutenberg–Richter law (Gutenberg and Richter 1944) are empirical laws to characterize the temporal pattern of aftershocks and frequency-energy, respectively. Besides the visible properties, complex interaction exists in the internal of the seismic system (Bak et al. 2002; Baiesi and Paczuski 2004; Gutenberg 2013).

While seismicity is assumed to be a complex phenomenon, the network approach offers a powerful tool for analyzing the dynamic structures of it (Abe and Suzuki 2004a, b, 2005, 2006). Over the last decade, different models proposed to construct the earthquake network (Abe and Suzuki 2004b; Lacasa et al. 2008; Rezaei et al. 2017; Lotfi et al. 2018). In the simple but basic model introduced by (Abe and Suzuki 2004b), the geographical region is divided into small square (cubic) cells, and seismic events with time sequences get connected. Later, (Lacasa et al. 2008) proposed a model to construct the network with a visibility graph. They converted the time series into a graph by inheriting the properties of the series in its structure. They explored periodicity, fractality, chaoticity, and nonlinearity of the seismic time series (Lacasa et al. 2009; Lacasa and Toral 2010; Donges et al. 2013). (Rezaei et al. 2017) introduced the hybrid model, which inherits the bases of the Abe-Suzuki model mixed with a visibility graph. To better capture the evolution of the earthquake network through time, a multiplex network was employed (Lotfi et al. 2018). Analyzing the seismic data with a network approach through different models helped reveal many features of the seismic activity just by knowing the basic information of magnitude, time of occurrence, and the location of seismic events (Baiesi and Paczuski 2004; Abe and Suzuki 2004a; Lotfi and Darooneh 2012; Abe et al. 2011; Lotfi and Darooneh 2013). It had been verified that the earthquake networks that constructed from the seismic data taken from California and Japan (Abe and Suzuki 2004a, b, 2005), Iran (Lotfi and Darooneh 2012, 2013),

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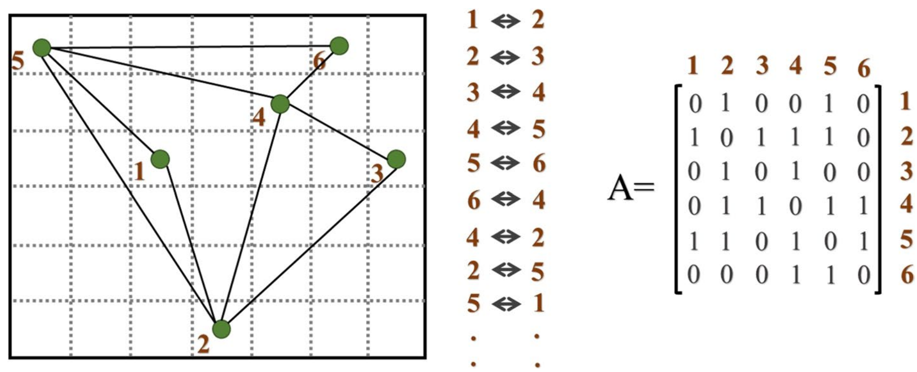

Fig. 1 Schematic representation of the earthquake network construction from Abe-Suzuki model with the related adjacency matrix

Chile (Abe et al. 2011), Greece (Chorozoglou et al. 2019), and Italy (Rezaei et al. 2017) are scale-free and small-world.

Most recent works focused on improving the proposed models to capture the best minimum resolution of the cell size needed for network construction. It means the cell size should be smaller than the specified limit to be trustable. The main question is how we ensure that the time window, in the scale of dates, months, or years, is large enough for constructing the network. In all the studies done till now, scientists considered the time on such a big scale of years. And the concept of the minimum necessary time window for achieving the best results is missing. In this work, we employ the definition of temporal network construction and capture the lowest time window essential for network construction. Considering the simplest model for earthquake network construction (Abe-Suzuki model), we found that depending on the region of consideration, the value of the time window threshold would change. We verified the trustiness of this time window size by analyzing two important centrality parameters, degree centrality, and PageRank. If the time window is small, we miss the information in centrality, and if it is bigger than the threshold, we do not gain extra knowledge than in the threshold time region.

The rest of the paper is organized as follows. In “Database” section, we provide information about the data sets we employ, and “Results” section is devoted to our results.

Database

We applied our model for the latest four years of data, Jan 01, 2018, to Dec 31, 2021, for Iran in the range of $24N - 44N$ latitude and $40E - 62E$ longitude with 14062 total events obtained from Iranian Seismological Center,[1] and California in the range of $32N - 42N$ latitude and $114W - 124W$ longitude with 7575 total events gained from the Northern California Earthquake Catalog.[2] In both of the considered data sets, we examined only events with a magnitude larger than 2.5–7. Referring to the analysis done to define the best cell size for network construction Lotfi and Darooneh (2013), we considered $N_{cell} = 12.5$ for Iran and $N_{cell} = 10$ for California.

Results

Through different models introduced for earthquake network construction, we used the simple model introduced by (Abe and Suzuki 2004b). As it is plotted in Fig. 1, the geographical region is divided into small square cells and having seismic events data ordered by the occurrence time, each square is regarded as one node if an earthquake with any magnitude occurred, and two nodes with consecutive events are connected.

We also divided the seismic data of four years length into small time windows in the following way; in the first step, we construct the Abe-Suzuki network for the data for the length of one month and study the characteristics of interest. Then, we added the data from the second month to the previous one and reconstructed the network. The process of adding data by time windows of the size of one month continues until the whole 48 months of data are covered. The schematic representation of the temporal network construction is plotted in Fig. 2a–c for Iran and Fig. 3a–c for California. Figures 2a and 3a belong to the data of the length of one month for Iran and ten months for California. As the number of events is low, having a sparse network is anticipated. The second Figs. 2b and 3b are in the middle time when the network is not sparse as the first month and is not too connected as the last, and Figs. 2c and 3c represent the networks for the whole four-year data which have very dense connections.

The second step would be building the adjacency matrix A for facilitating the analysis; $a_{ij} = 1$ if nodes i and j are

[1] http://irsc.ut.ac.ir

[2] http://www.usgs.gov/

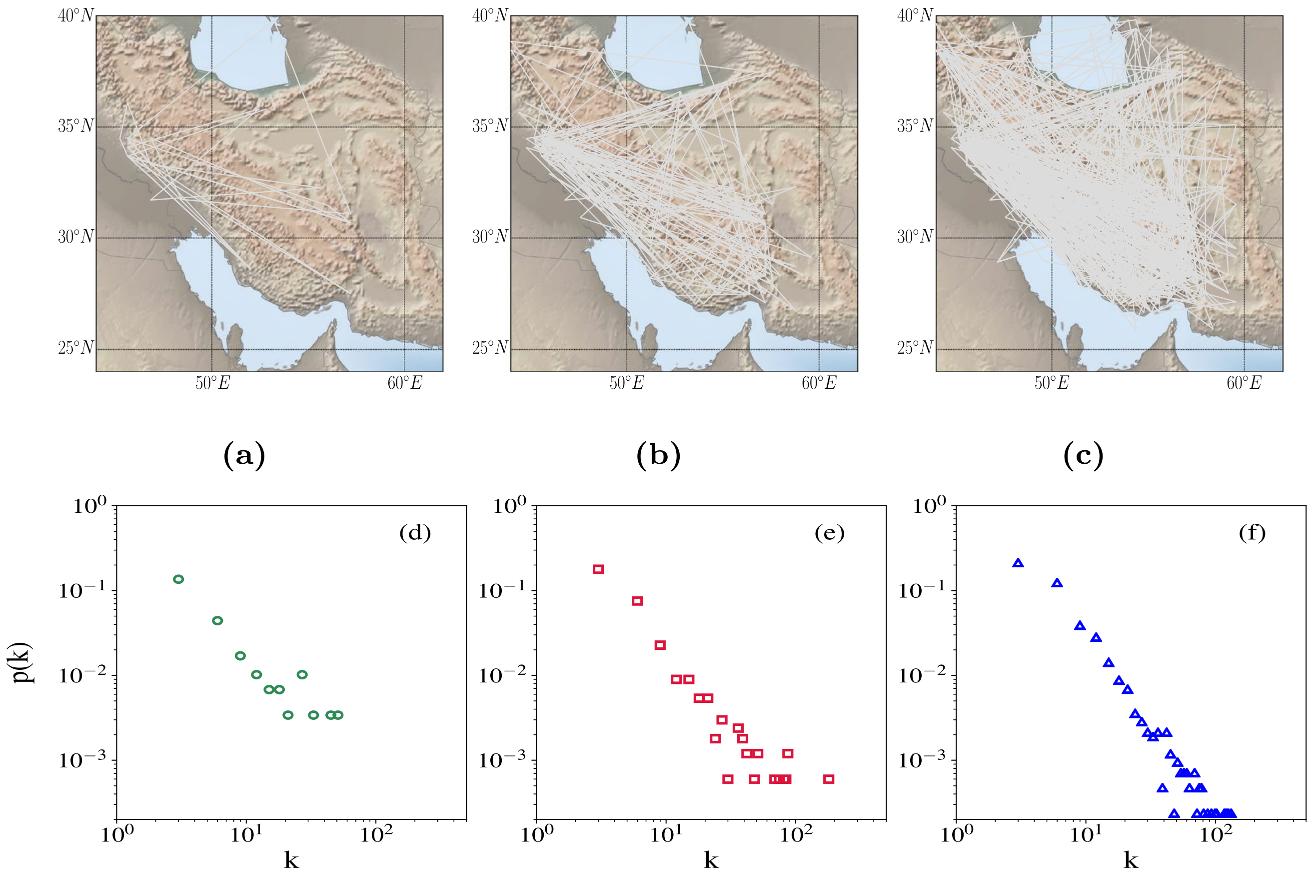

Fig. 2 **a–c** The schematic representation of temporal earthquake networks of Iran for three different time steps with time windows of **a** one month, **b** 10 months, and **c** 46 months with filtered data magnitudes > 4.0, **d–f** are the degree distribution of the networks for the three defined networks, respectively, for data with magnitudes > 2.5)

connected, and 0 otherwise (Fig. 1). In the network definitions, the degree of the node is the number of connections a node could have and is calculated from the adjacency matrix $k_i = \sum_j a_{ij}$. The degree distribution of the earthquake network of different regions obeys a power law indicating the scale-free network characteristics (Abe and Suzuki 2004a, 2005; Lotfi and Darooneh 2012). In the scale-free networks, the power law decay of the degrees means that the vast majority of nodes have very few connections, while a few important nodes have a huge number of connections. To check the validity of this characteristic, for each of the above-mentioned networks (Figs. 2 and 3), we plot the degree distribution in Figs. 2d and 3d. One could see that no matter the time length, we would have approximately the power-law distribution with power of γ. This γ varies between 2 and 2.2 for Iran and 1.5 to 1.8 for California.

The other famous characteristic of earthquakes small-worldness (Abe and Suzuki 2004b; Lotfi and Darooneh 2012). In small-world networks, although most of the nodes are not neighbor together, they could be reached from every other nodes by a small number of links. In other words, in these networks, with N nodes and M links, the value of the shortest path is similar to the random network with the same numbers of nodes and links, while the clustering coefficient has a larger value. The clustering coefficient of a node i is the fraction of connection existing among its nearest neighbor nodes to the maximum number of possible links among them. The clustering coefficient of the network would be the average clustering of all nodes:

$$C_i = \frac{1}{k_i(k_i - 1)} \sum_{j,k} a_{ij}a_{jk}a_{ki} , \quad C = \frac{1}{N} \sum_{i=1}^{N} C_i, \tag{1}$$

where N is the total number of nodes in the network. In other words, the clustering coefficient is the tendency of the nodes in the graph to cluster together and has a value $0 \le C \le 1$. On the other hand, the shortest path is the minimum path length needed to traverse to get from one node to another. The average over all nodes would result in the shortest path of the network:

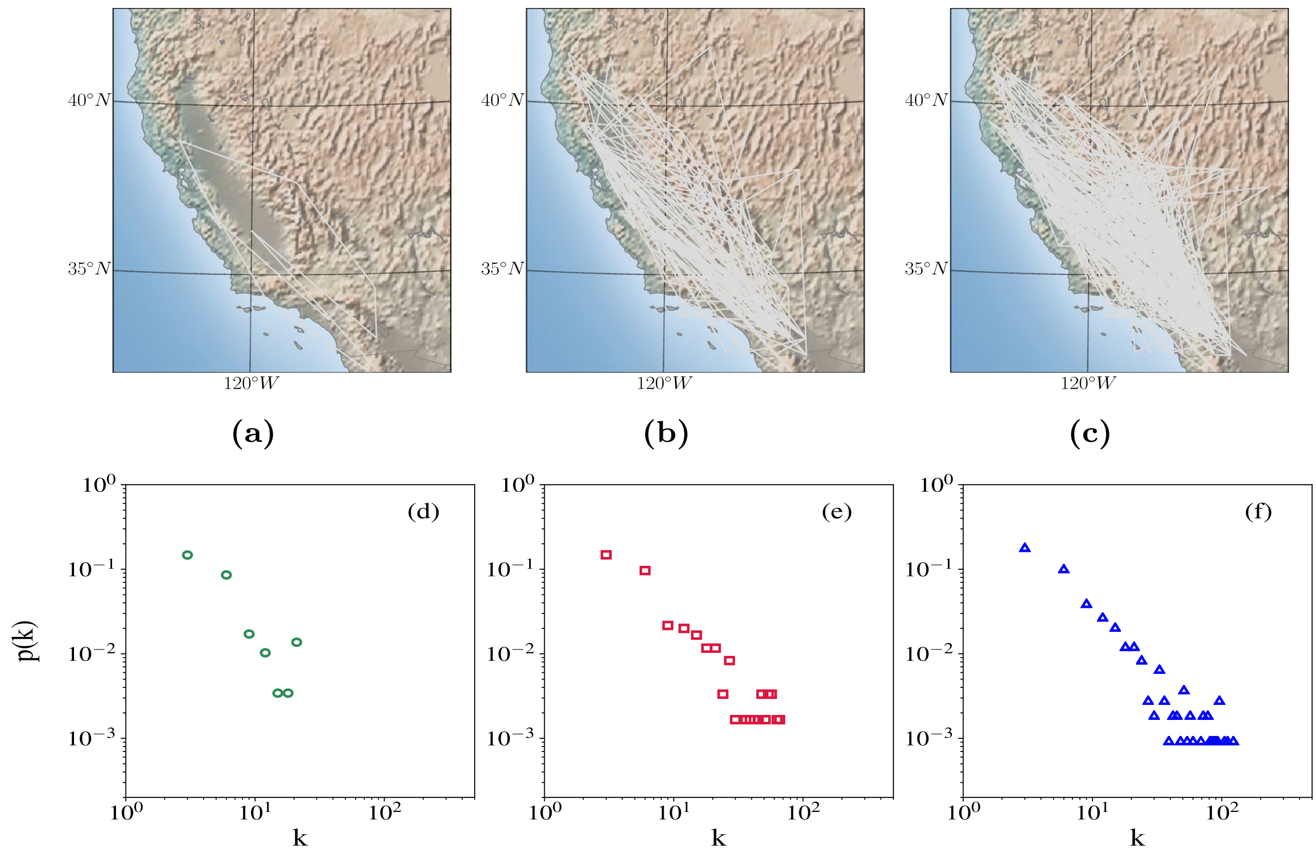

Fig. 3 **a**–**c** The schematic representation of temporal earthquake networks of California for three different time steps with time windows of **a** 10 months, **b** 19 months, and **c** 46 months with filtered data magnitudes > 3.0, **d** to **f** are the degree distribution of the networks for the three defined networks, respectively, for data with magnitudes > 2.5)

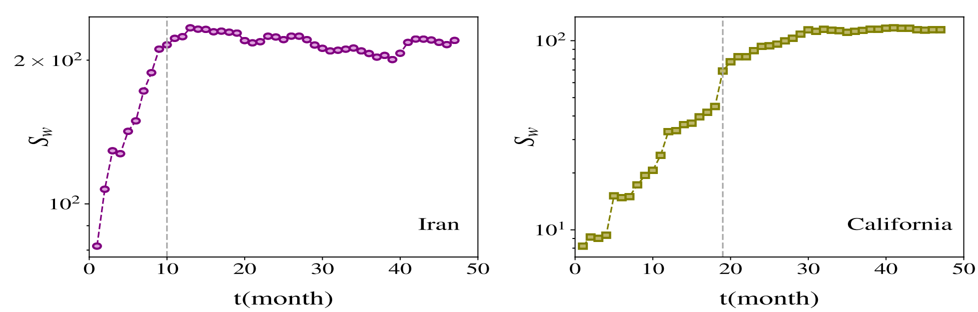

Fig. 4 The variation of small-worldness in the scale of time windows computed for earthquake networks of Iran and California

$$L = \frac{1}{N(N-1)} \sum_{i,j=1,N;i\neq j} d_{ij}, \tag{2}$$

in which d_{ij} is the minimum length of the path between two nodes of i and j.

By having the clustering coefficient and shortest path of the network, Humphries and Gurney (2008) introduced a small-worldness metric defined with the averaged clustering coefficient and path length relative to these metrics for random networks. This metric helps to provide an overview of connectivity in the entire network:

$$S_w = \frac{C/C_{\mathrm{rand}}}{L/L_{\mathrm{rand}}}. \tag{3}$$

$C_{\mathrm{rand}} = \frac{<k>}{N}$ and $L_{\mathrm{rand}} = \frac{ln(N)}{ln(<k>)}$ are the values obtained for random networks by randomizing the connections of each earthquake network by keeping the same number of nodes and links. Here, $<k>$ indicates the mean degree of the networks.

The variation of S_w in time (in the scale of the length of the month) is shown in Fig. 4. One could see that this

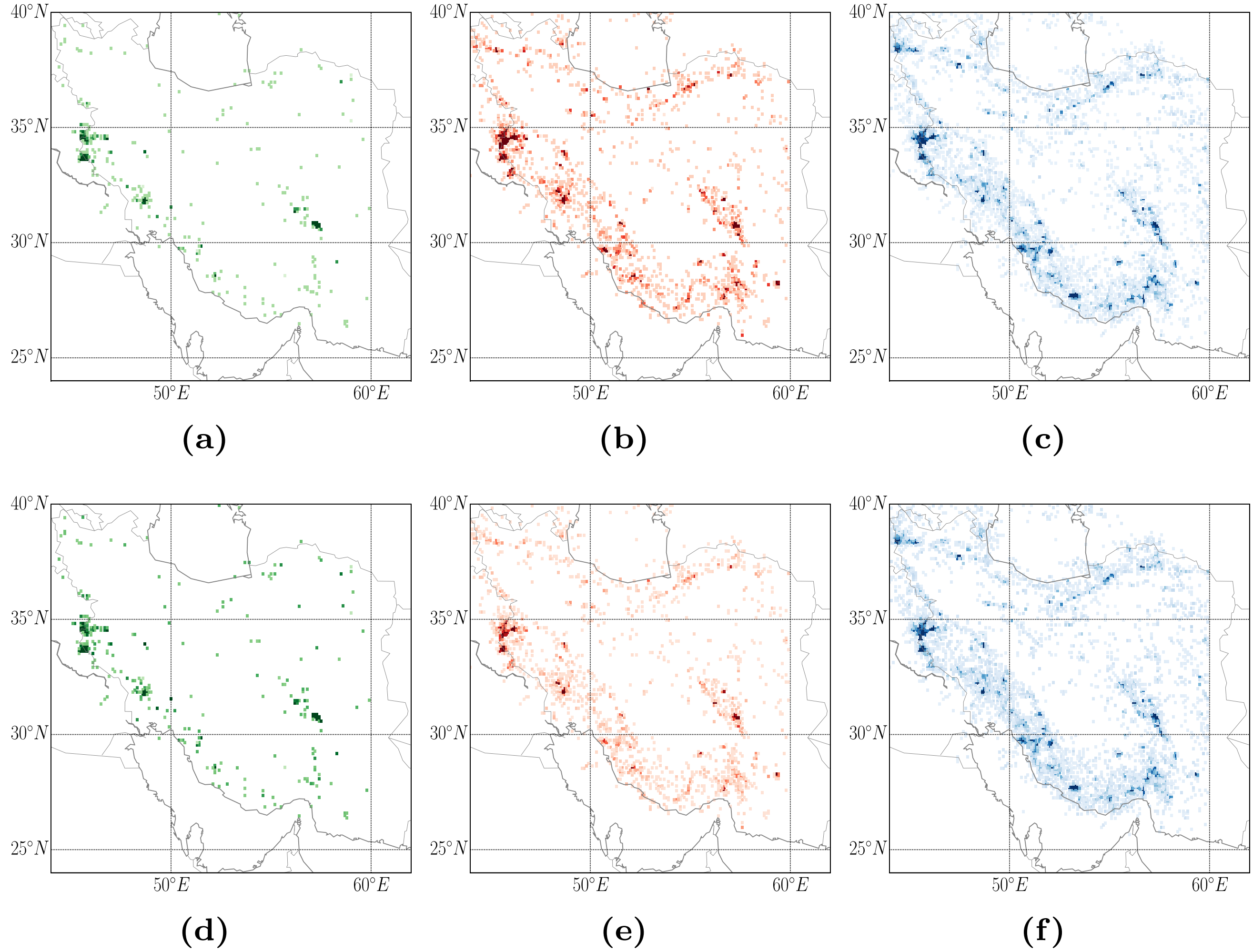

Fig. 5 Degree centrality and PageRank for earthquake network of Iran for time windows of **a**, **d** one month, **b**, **e** 10 months, and **c**, **f** 46 months, respectively

value is small for the first months of consideration. It starts to increase until a threshold and gets stationery later. This behavior could emphasize that until a specific time window, the variation of the parameters is high. The fluctuations disappear, while one considers a time window that is large enough, and the system gets stationary. The geographical region under consideration and frequency of the seismic event could result in observing different values. This value for Iran's data is approximately ten months, while for California it is around 19 months.

To clarify the importance of having the minimum time window, we calculate two of the most important centralities in the concept of earthquake networks and compare them in three different time windows. Looking through the literature, one could find different parameters to calculate the centrality of nodes in the earthquake networks. The simplest and most common centrality that uses the local structure around the nodes is the degree centrality. In Figs. 5 and 6a–c, we plot the degree centrality for three different time scales as the following: Figs. 5a and 6a are for the time window of length 1 month for Iran and 10 months for California. Near the time window of the threshold, we selected the network with the length of 10 months of data for Iran (Fig. 5b) and 19 months for California (Fig. 6b). And Figs. 5c and 6c belong to the largest time window (48 months).

The second famous centrality in the concept of earthquake network is PageRank (Darooneh and Lotfi 2014; Rezaei et al. 2019). PageRank is an algorithm used to assess the ranks of nodes in a network based on their connections' levels used in the Google search engine for ranking web pages for the first time (Brin and Page 1998). PageRank explained through the random walk. The random walker starts from one node and selects the next one randomly. In this definition, PageRank of node i is the asymptotic probability that the walker meets the node. One could infer that the possibility of reaching one important node is higher

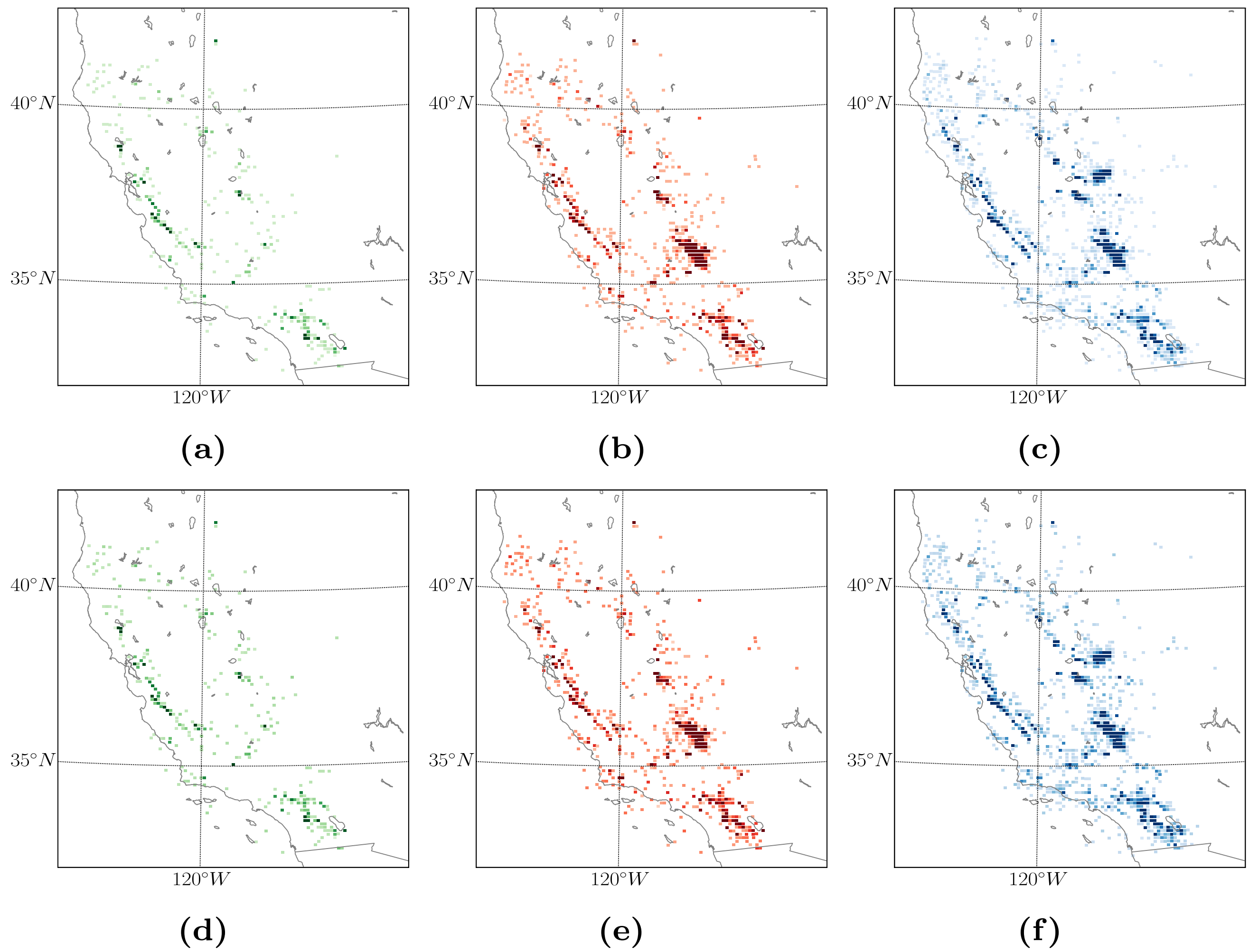

Fig. 6 Degree centrality and PageRank for earthquake network of California for time windows of **a**, **d** 10 month, **b**, **e** 19 months, and **c**, **f** 46 months, respectively

than the unimportant ones. Evaluating this centrality is an iterative procedure in which the PageRank of nodes depends on all its neighbors' PageRanks. The following equation describes such a random walking procedure:

$$\mathrm{PR}_i = \frac{d}{N} + (1-d)\sum_{j\in B_i}\frac{\mathrm{PR}_j}{k_j^{out}}, \tag{4}$$

in which PR_i is the PageRank of node i, B_i is the set of nearest neighbors of node i, and k^{out} is the out-degree of each node. d is a constant value (mostly considered as 0.15) defined as the probability of jumping to any vertex. Figures 5 and 6d–f are representing the PageRank of the networks for the three different time windows. Taking into account both above-introduced centralities, one could see that in the small-time windows, we could not find enough information about the central locations of the regions as it should. If we increase the length of the time window up to the threshold, the results capture the same central regions as the largest time window.

Conclusion

Recently, different models proposed to study the earthquake phenomena to explore the features of this harmful disaster. Although this phenomenon is very complex from a fault and inside earth interactions point of view, it is possible to study it with the complex network with the minimum information: geographical location, time, and magnitude. Among the most famous models proposed, Abe-Suzuki and visibility models, scientists were trying to improve the model's performances. The main idea that got most of the attention from those studying was how they could introduce the best minimum geographical cell size.

Here, we proposed the temporal earthquake network construction for capturing another essential factor of network analysis, the best time window size. We start with constructing an earthquake network in windows of the month length and adding data with a length of one month in each step. We used the most straightforward model introduced by Abe-Suzuki to build our networks. For each constructed network, the small-worldness is evaluated. Studying how this parameter changes by increasing the time window, we could verify the minimum length of time window needed for network construction. This value is smaller than its value in the threshold time window and gets stationary by enlarging the time lengths. This time threshold is different for each geographical region as the construction of the earth is different. One point of these differences appears in the frequency of the events on the same time scale. Then, it is a delicate factor to study the minimum and efficient time window size for different geographical regions before the rest of the analysis to ensure obtaining the best results. By considering two famous centralities measures in the concept of earthquake networks (degree centrality, and PageRank), we show that if this size is smaller than the threshold, we will miss the information we should have. If the time window is too large, it does not provide extra information.

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Declarations

Conflict of Interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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