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Influence of Specimen Size on Compression Behavior of Cement Paste and Mortar under High Strain Rates

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Abstract: Static and dynamic compression tests were carried out on mortar and paste specimens of three sizes (ϕ68 mm×32 mm, ϕ59 mm×29.5 mm and ϕ32 mm×16 mm) to study the influence of specimen size on the compression behavior of cement-based materials under high strain rates. The static tests were applied using a universal servo-hydraulic system, and the dynamic tests were applied by a spilt Hopkinson pressure bar (SHPB) system. The experimental results show that for mortar and paste specimens, the dynamic compressive strength is greater than the quasi-static one, and the dynamic compressive strength for specimens of large size is lower than those of small size. However, the dynamic increase factors (DIF) has an opposite trend. Obviously, both strain rate and size effect exist in mortar and paste. The test results were then analyzed using Weibull, Carpinteri and Bažant's size effect laws. A good agreement between these three laws and the test results was reached on the compressive strength. However, for the experimental results of paste and cement mortar, the size effect is not evident for the peak strain and elastic modulus of paste and cement mortar.

Key words: size effect; cement-based materials; dynamic loading; compressive behavior

1 Introduction

Terrorist attack, explosion scenarios in tunnels, and the potential hazards from the storage of highly energetic materials can cause high strain rate in materials. Widely employed in buildings, cement-based materials are rate-dependent materials which indicate that the mechanical response of structures exposed to explosive loading can only be predicted properly with material models that include this rate effect. Thus, knowledge about the response of concrete structures to impact and explosive loading is required for reliable safety assessment and the design of protective structures[1].

The strain-rate sensitive behavior of cement-based materials has been under investigation for several decades. It is commonly believed that the compressive strength of cement-based materials increases with increasing strain rate. But when it comes to DIF, range of strain rate and dynamic constitutive relation, there exist great debates[2-5].Tests carried out so far have different shape and dimensions. However, analysis of the data seldom or never takes the size effect into account[6-9]. The reliability of the conclusions got in this way cannot be guaranteed.

There are many theories about size effect, such as Weibull's statistical law[10], Bažant's energetic release theory[11], Multifractal scaling law by Carpinteri[12], boundary layer theory[13], singularity of three-dimensional stress[14], time-dependent theory[15], and so on. The first three ones are widely used. Experimentally, many researchers[16-19] conducted compressive and flexural-tensile experiments and drew the conclusion that the strength decreases with the increase of specimen size, also parameters of different models were fitted according to the test results. However, the study of the size effect under dynamic load is still limited. Bindiganavile and Banthia[20] carried out compressive and bending tensile experiments under impact load. The results showed that there exists size effect in cement-based materials under dynamic loading. Jikai Zhou[21] found that dynamic increase factor (DIF) of flexural-tensile strength also has size

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effect under dynamic loading. Motaz[22] studied the size effect of ordinary and high strength concrete under quasi-static and dynamic load, and both the strength and the elastic modulus were analyzed. It can be found from the literature review that there are some problems to be solved: most tests were performed under low or medium strain rate, and few were tested under high strain rate. Thus, whether the proposed theories are suitable for high strain rate situation should be verified. The existing researches are mostly carried out on the size effect on strength, and the deformation properties such as peak strain and elastic modulus are scarcely reported. Irrelevant variables were not completely controlled, so the results might be inaccurate.

In this paper, cement mortar and paste specimens with different diameters were tested using a split Hopkinson pressure bar (SHPB) system, then the size effect on the impact response of cement-based materials was presented. Weibull's statistical law, Bažant's energetic release theory and Multifractal scaling law developed for quasi-static loading were examined in the context of high strain rates. This paper also discussed the size effect on DIF, peak strain and elastic modulus.

2 Experimental

2.1 Materials and specimens

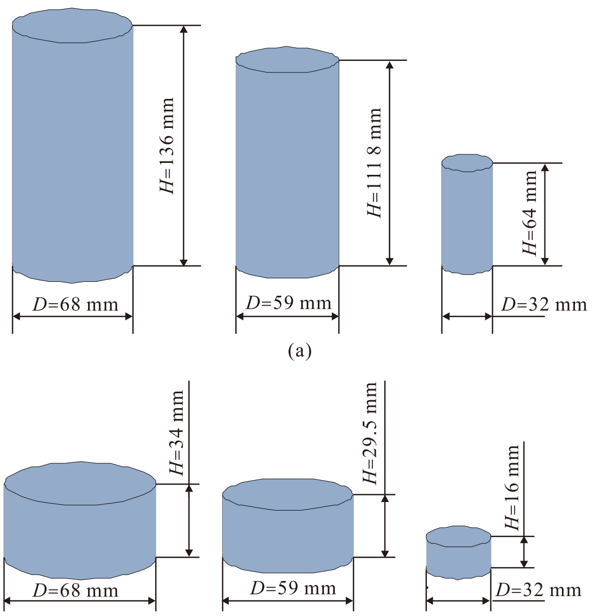

Fig.1 Schematic plot of specimens: (a)static tests; (b)dynamic tests

Ordinary portland cement (OPC) was used in the production of cement mortar and paste specimens. This cement is the most widely used cement in general concrete construction work in China. The fine aggregate was river sand and water was from the tap. All the specimens had the water-cement ratio, *w/c*=0.4 and the sand-cement ratio, *s/c*=1, was used for cement mortar.

The standard specimens were cast in steel molds with dimensions of 150 mm × 150 mm × 550 mm. Following casting, the specimens were covered with a plastic membrane to prevent moisture from evaporating. After curing for 90 days, specimens were cored from the standard specimens. Mortar and paste cores were cut and ground smooth to produce 68 mm, 59 mm and 32 mm diameter cylindrical specimens. Specimens for static tests had a height-diameter ratio of 2, and that for dynamic tests of 0.5, as shown in Fig.1.

2.2 Split Hopkinson pressure bar test system

Quasi-static tests were preceded by electro hydraulic servo compressive testing machine and dynamic tests by SHPB system. A split Hopkinson pressure bar consists of a striker bar, an incident bar, and a transmitter bar, as shown in Fig.2.

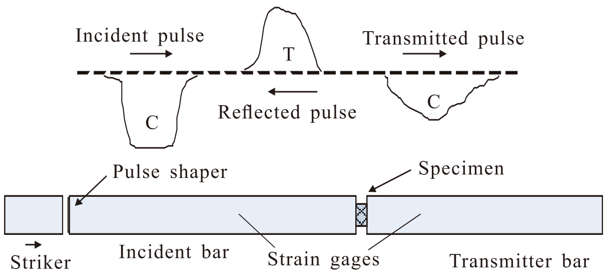

Fig.2 Split Hopkinson pressure bar (SHPB)

The specimen under investigation was placed between the incident and transmitter bars. The striker bar was launched at a known velocity toward the incident bar. The impact generated a stress pulse in the incident bar that travels toward the specimen. The wave gets partly reflected back and partly transmitted into the specimen upon reaching the incident bar-specimen interface. Performing one-dimensional wave analysis on the strain signal obtained from the incident and transmitted bar, the stress-strain curve in the specimen can be obtained.

The resulting stress $\sigma_s(t)$, strain $\varepsilon_s(t)$, and strain rate $\dot{\varepsilon}$, of the specimen were obtained by the following equations [23]:

$$\sigma_s(t)=E_b\left(\frac{A_b}{A_s}\right)\varepsilon_T(t) \tag{1}$$

$$\varepsilon_s(t)=-\frac{2C_0}{l}\int_0^t\varepsilon_R(t)\mathrm{d}t \tag{2}$$

$$\dot{\varepsilon} = \frac{d\varepsilon_s(t)}{dt} \tag{3}$$

where, A_b, E_b, and C_0 are the cross-sectional area, the Young's modulus, and the wave velocity of the bar material, respectively; H and A_s are the length and cross-sectional area of the specimen, respectively.

3 Results and discussion

3.1 Stress-strain curves

The static compressive tests were conducted according to the ASTM C192 standard[24]. Four specimens were designed for each diameter, and the representative compressive strength should be the average. Test results of mortar and paste are presented in Table 1.

When loaded on the SHPB, inhomogeneity in the internal structures of the specimens might cause deviation in the incident wave. Therefore, pulse shaper was stuck on the end face of the incident bar to ensure constant strain rate and even stress. There were eight specimens for each diameter of mortar and paste and the representative stress-strain curves are shown in Figs.3 and 4.

Table 1 Representative compressive strength of mortar and paste/MPa

Dimension/mm	$\phi68\times32$	$\phi59\times29.5$	$\phi32\times16$
Mortar	28.96	29.85	34.46
Paste	27.79	30.90	33.76

It is obvious from the curves that both the dimension of specimens and the load velocity would influence the compressive properties of mortar and paste. How does size affect some of the properties (*eg*, compressive strength, DIF, peak strain and elastic modulus) would be discussed in detail and presented in the following section.

3.2 Compressive strength

It could be concluded from the stress-strain curves that the compressive strength of mortar and

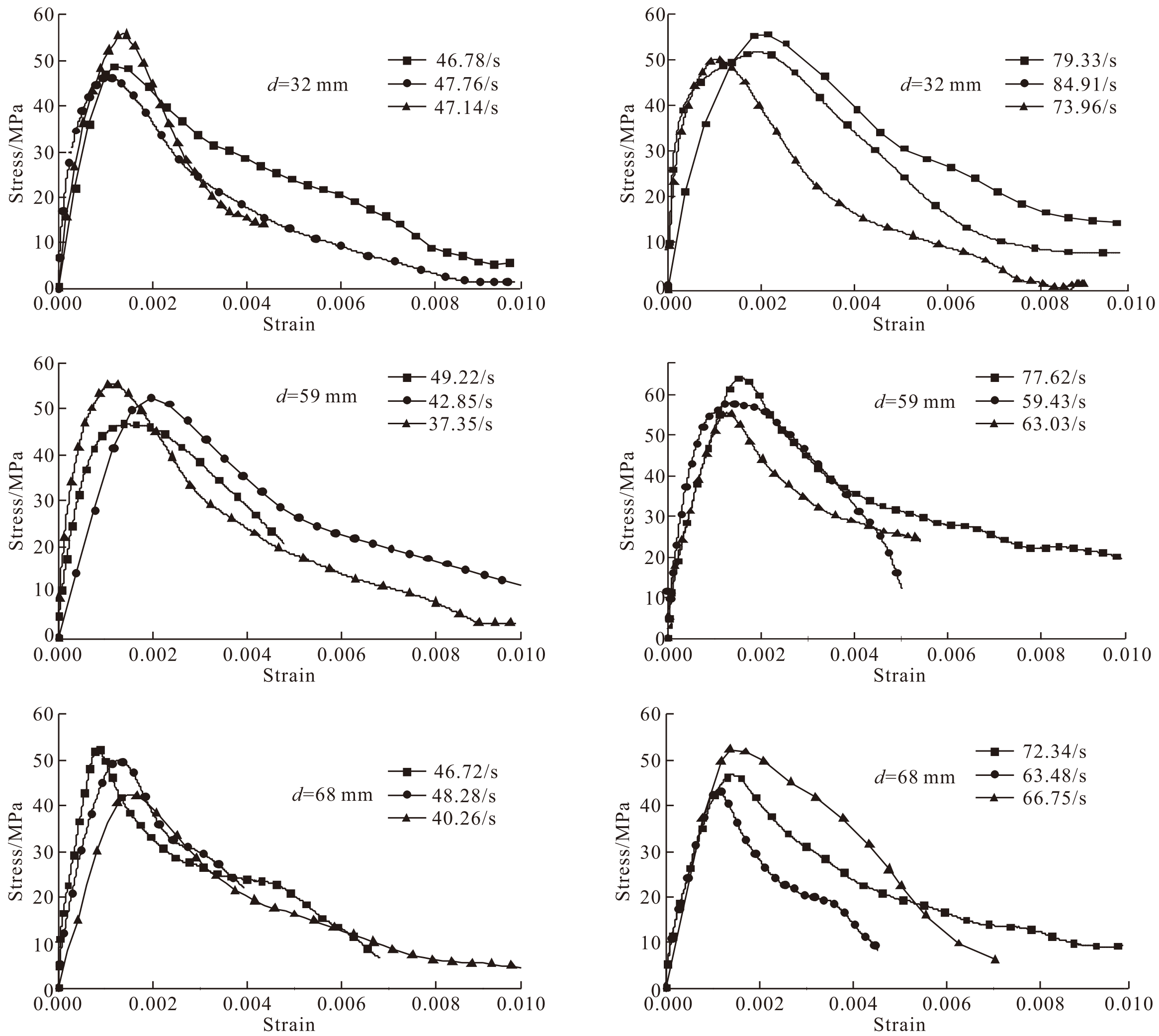

Fig.3 Stress-strain curves of mortar

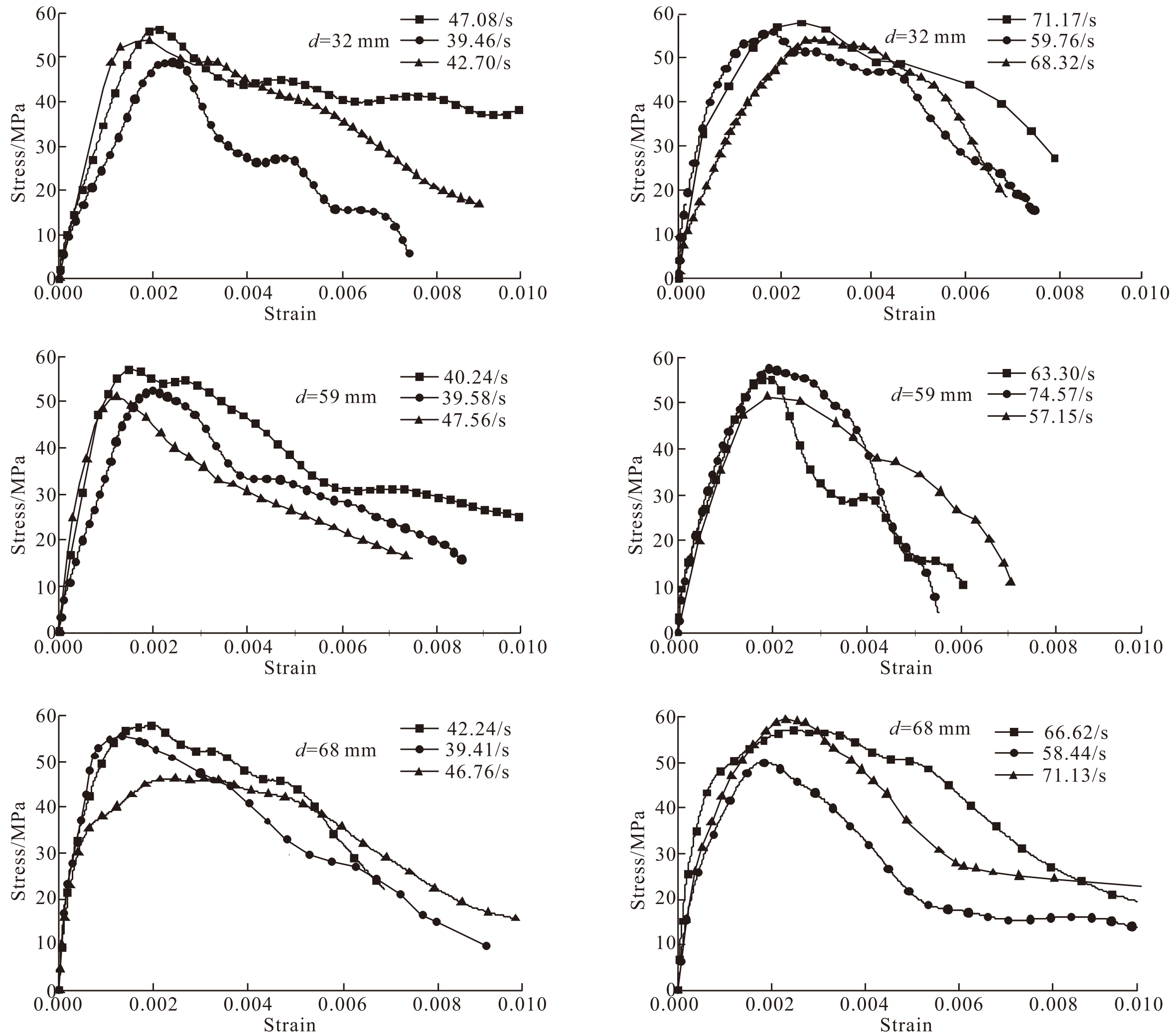

Fig.4 Stress-strain curves of paste

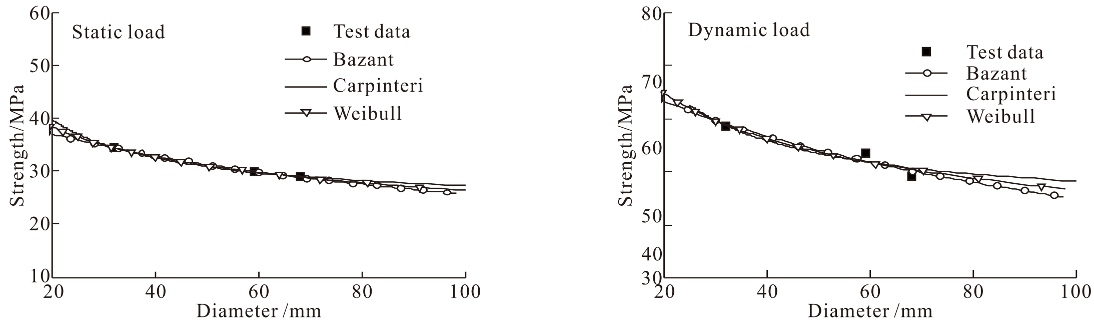

Fig.5 Relationship between compression strength of mortar and size

paste were both affected by the size of specimens. The compressive strength was fitted by using Weibull's statistical law, Bažant's energetic release theory and Multifractal scaling law.

Fig.5 shows the relationship between compression strength of mortar and dimension under static/dynamic load. The fitted equations are presented in Table 2.

Fig.6 shows the relationship between compressive strength of paste and dimension under static/dynamic load. The fitted equations are presented in Table 3.

From the fitted results, it is obvious that the Weibull, Carpinteri and Bažant's size effect laws can be used to describe the compressive strength of cementitious materials under high strain rate.

3.3 DIF analysis

According to the controlled variable method, the size of specimens should be the only one variable in order to study the size effect and other variables such as

Table 2 Fitting results of mortar

Size effect	Static: Formula for strength	Static: Correlation coefficien, R^2	Dynamic: Formula for strength	Dynamic: Correlation coefficient, R^2
Bažant's law	$\sigma_B = \dfrac{3.14\times10^{-3}}{\sqrt{1+\dfrac{D}{0.1366}}}$	0.99	$\sigma_B = \dfrac{1.75\times10^{-3}}{\sqrt{1+\dfrac{d}{0.1277}}}$	0.79
Weibull's law	$\sigma_W = 71.52\times V^{-\frac{1}{12.94}}$	0.99	$\sigma_W = 112.17\times V^{-\frac{1}{14.66}}$	0.78
Multifractal sealing law	$\sigma_M = 23.08\times\left(1+\dfrac{39.39}{D}\right)^{1/2}$	0.99	$\sigma_M = 42.45\times\left(1+\dfrac{29.38}{D}\right)^{1/2}$	0.77

Note: D stands for diameter; V stands for volume, and $V=\dfrac{\pi D^3}{8}$

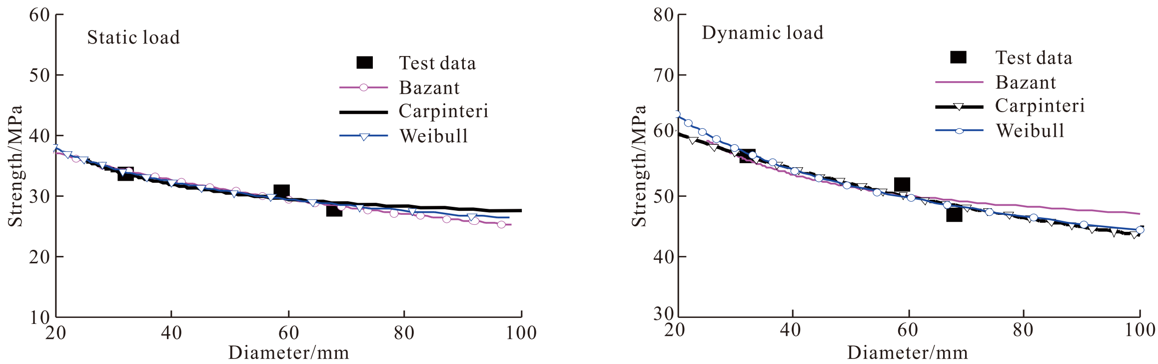

Fig.6 Relationship between compression strength of paste and size: (a) static load; (b) dynamic load

Table 3 Fitting results of paste

Size effect	Static: Formula for strength	Static: Correlation coefficien, R^2	Dynamic: Formula for strength	Dynamic: Correlation coefficient, R^2
Bažant's law	$\sigma_B = \dfrac{3.26\times10^{-3}}{\sqrt{1+\dfrac{D}{0.1446}}}$	0.71	$\sigma_B = \dfrac{1.77\times10^{-3}}{\sqrt{1+\dfrac{D}{0.1212}}}$	0.87
Weibull's law	$\sigma_W = 73.69\times V^{-\frac{1}{13.16}}$	0.82	$\sigma_W = 113.30\times V^{-\frac{1}{13.69}}$	0.92
Multifractal sealing law	$\sigma_M = 23.58\times\left(1+\dfrac{34.24}{D}\right)^{1/2}$	0.63	$\sigma_M = 42.45\times\left(1+\dfrac{23.71}{D}\right)^{1/2}$	0.77

quasi-static strength and strain rate should be constant. However, because of the inhomogeneity of cement-based materials, the static strength and strain rate could not be exactly the same. Thus, the parameter dynamic increase factor (DIF) was introduced. The DIF was defined as the ratio of dynamic to quasi-static strength. DIF could eliminate the influence of static strength and strain rate.

Based on theoretical analysis and experimental study, many researchers proposed DIF models which indicated the effect of strain rate on uniaxial strength. In order to compare different models, ϕ59mm×29.5 mm and ϕ32 mm×16 mm mortar specimens and existing DIF models[8,9,25-31] were put together in one single figure , as shown in Fig.7.

Comparing different models with test data, the DIF model proposed by Hartmann[28] was adopted in this paper. Taking size effect into account, a modified model was present in equation (4):

$$\text{For mortar:}\quad DIF=(0.07\frac{D}{D_0}+0.36)\times\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{0.2}+0.97$$

$$\text{For paste:}\quad DIF=(0.1\frac{D}{D_0}+0.30)\times\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{0.2}+0.97 \tag{4}$$

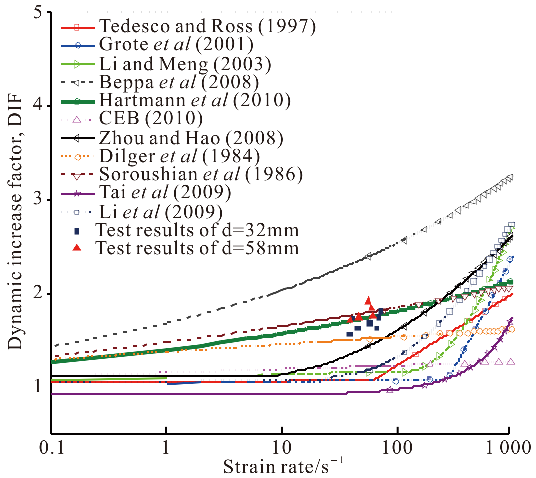

Fig.7 Comparison of experimental and calculated results by existing models

where, D is the diameter of specimen (mm), D_0=32 mm, and $\dot{\varepsilon}_0$=1 s^{-1}.

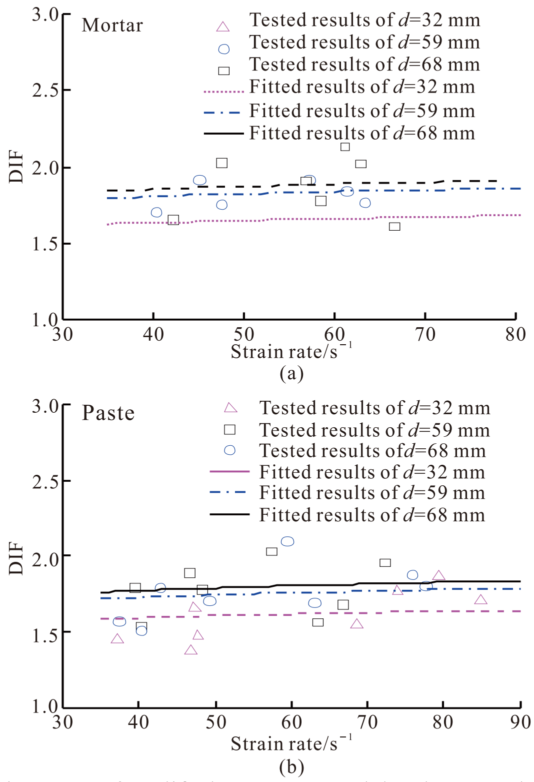

Fig.8 DIF of modified Hartmann model and test results (a) mortar and (b) paste

Theoretical results calculated by the modified Hartmann model and the test data are presented in Fig.8. It is evident that the modified Hartmann model could reflect the influence of size effect, and DIF rises with increasing size. Comparing Figs.8(a) and 8(b), DIF for paste has greater discreteness, which might be due to the higher brittleness of this material.

3.4 Peak strain

Peak strain ε_u is defined as the strain at peak stress σ_u. Assuming that materials have the same strength, the larger peak strain, the larger energy it can absorb during its failure process. The influence of size and strain rate on the peak strain is studied in Fig.9.

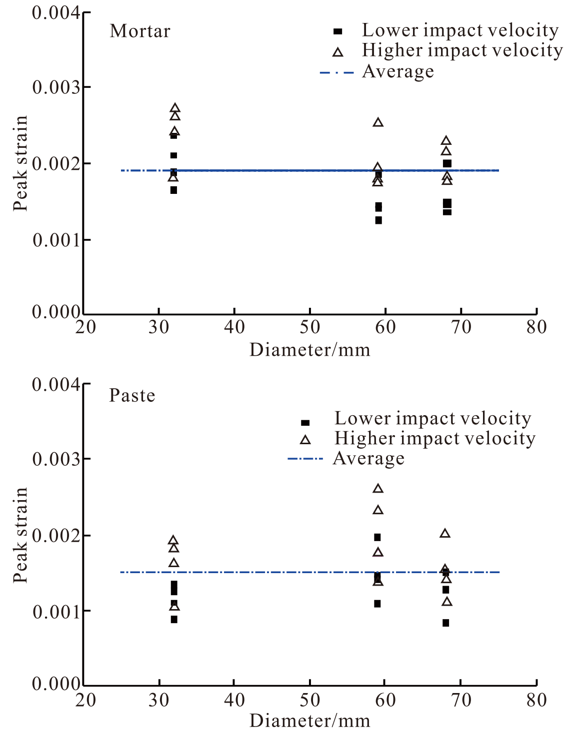

Fig.9 Peak strain of mortar and paste

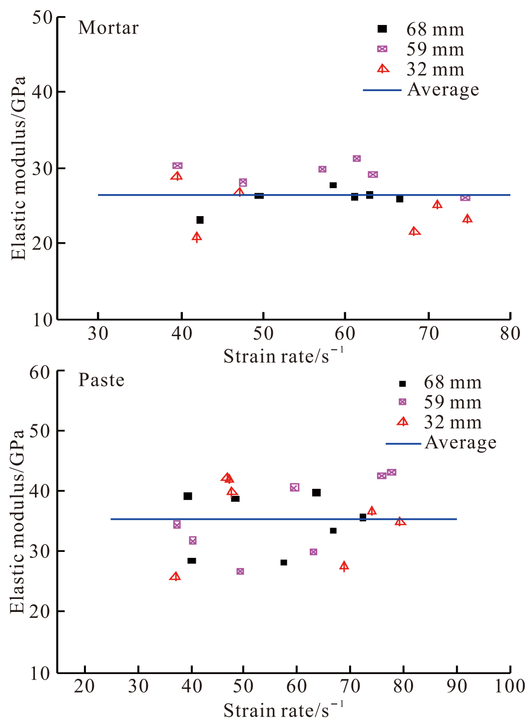

Fig.10 Elastic modulus versus strain rate for cement mortar and paste

From Fig.9, we could find that the peak strains of mortar and paste were both affected by strain rate, and increased with the increase of strain rate.

However, dimension had little effect on the peak strain. Considering the average values, mortar had a relatively higher peak strain than paste had. That is to say, paste had greater brittleness.

3.5 Elastic modulus

Elastic modulus illustrates the relationship between the stress and strain of materials and is an essential parameter for the design of concrete structures. In this study, the elastic modulus takes the ratio of peak stress and peak strain, $E=\sigma_u/\varepsilon_u$. Fig.10 presents the elastic modulus of mortar and paste.

At high strain rate, elastic modulus of mortar and paste had great discreteness. There seemed to be no discipline about the size effect of elastic modulus. This conclusion needs further study since the range of size was not large enough.

4 Conclusions

Quasi-static and dynamic axial compression tests were carried out on mortar and paste specimens with three different sizes. From the analysis of the test results, the following conclusions can be drawn:

Cementitious materials seem to exhibit a size effect under both quasi-static and dynamic strain rates. Both the quasi-static and dynamic compressive strengths of mortar and paste specimens of larger size are smaller than those of smaller size.

Weibull's statistical law, Bažant's energetic release theory and Multifractal scaling law by Carpinteri all can be applied to the size effect analysis of quasi-static and dynamic compressive strength of mortar and paste. The theoretical results can fit the test data well.

The size effect exists on the DIF of mortar and paste. The DIF value increases with increasing specimen size. Peak strain increases with the increase of strain rate, however, the size effect is not evident. Under impact loading, the size effect on the elastic modulus of mortar and paste is not obvious.

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