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ERRATUM

Erratum to: On convex optimization without convex representation

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The proof of Theorem 3 in the original publication of the article contains an incorrect statement that we fix below.

Theorem 3 *Let* $\mathbf{K}$ *in* (1.2) *be compact and let Assumption* 1 *hold true. For every fixed* $\mu > 0$, *choose* $\mathbf{x}_\mu \in \mathbf{K}$ *to be an arbitrary stationary point of* ϕ_μ *in* $\mathbf{K}$.

Then every accumulation point $\mathbf{x}^* \in \mathbf{K}$ *of such a sequence* $(\mathbf{x}_\mu) \subset \mathbf{K}$ *with* $\mu \to 0$, *is a global minimizer of* f *on* $\mathbf{K}$, *and if* $\nabla f(\mathbf{x}^*) \neq 0$, $\mathbf{x}^*$ *is a KKT point of* $\mathbf{P}$.

Proof Let $\mathbf{x}_\mu \in \mathbf{K}$ be a stationary point of ϕ_μ, which by Lemma 2 is guaranteed to exist. So

$$\nabla \phi_\mu(\mathbf{x}_\mu) = \nabla f(\mathbf{x}_\mu) - \sum_{j=1}^{m} \frac{\mu}{g_j(\mathbf{x}_\mu)} \nabla g_j(\mathbf{x}_\mu) = 0. \tag{0.1}$$

As $\mu \to 0$ and $\mathbf{K}$ is compact, there exists $\mathbf{x}^* \in \mathbf{K}$ and a subsequence $(\mu_\ell) \subset \mathbb{R}_+$ such that $\mathbf{x}_{\mu_\ell} \to \mathbf{x}^*$ as $\ell \to \infty$. We need consider two cases:

Case when $g_j(\mathbf{x}^*) > 0$, $\forall j = 1, \ldots, m$. Then as f and g_j are continuously differentiable, $j = 1, \ldots, m$, taking limit in (0.1) for the subsequence (μ_ℓ), yields $\nabla f(\mathbf{x}^*) = 0$ which, as f is convex, implies that $\mathbf{x}^*$ is a global minimizer of f on $\mathbb{R}^n$, hence on $\mathbf{K}$.

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Case when $g_j(\mathbf{x}^*) = 0$ *for some* $j \in \{1, \ldots, m\}$. Let $J := \{j \,:\, g_j(\mathbf{x}^*) = 0\} \neq \emptyset$. We next show that for every $j \in J$, the sequence of ratios $(\mu_\ell / g_j(\mathbf{x}_{\mu_\ell}), \ell = 1, \ldots,$ is bounded. Indeed let $j \in J$ be fixed arbitrary. As Slater's condition holds, let $\mathbf{x}_0 \in \mathbf{K}$ be such that $g_j(\mathbf{x}_0) > 0$ for all $j = 1, \ldots, m$; then $\langle \nabla g_j(\mathbf{x}^*), \mathbf{x}_0 - \mathbf{x}^* \rangle > 0$. Indeed, as $\mathbf{K}$ is convex, $\langle \nabla g_j(\mathbf{x}^*), \mathbf{x}_0 + \mathbf{v} - \mathbf{x}^* \rangle \geq 0$ for all $\mathbf{v}$ in some small enough ball $\mathbf{B}(0, \rho)$ around the origin. So if $\langle \nabla g_j(\mathbf{x}^*), \mathbf{x}_0 - \mathbf{x}^* \rangle = 0$ then $\langle \nabla g_j(\mathbf{x}^*), \mathbf{v} \rangle \geq 0$ for all $\mathbf{v} \in \mathbf{B}(0, \rho)$, in contradiction with $\nabla g_j(\mathbf{x}^*) \neq 0$. Next,

$$\begin{aligned} \langle \nabla f(\mathbf{x}_{\mu_\ell}), \mathbf{x}_0 - \mathbf{x}^* \rangle &= \underbrace{\sum_{k \notin J}^{m} \frac{\mu_\ell}{g_k(\mathbf{x}_{\mu_\ell})} \langle \nabla g_k(\mathbf{x}_{\mu_\ell}), \mathbf{x}_0 - \mathbf{x}^* \rangle}_{A_\ell} \\ &\quad + \underbrace{\sum_{k \in J}^{m} \frac{\mu_\ell}{g_k(\mathbf{x}_{\mu_\ell})} \langle \nabla g_k(\mathbf{x}_{\mu_\ell}), \mathbf{x}_0 - \mathbf{x}^* \rangle}_{B_\ell} \end{aligned} \tag{0.2}$$

Observe that

- Every term of the sum B_ℓ is nonnegative for sufficiently large ℓ, say $\ell \geq \ell_0$, because $\mathbf{x}_{\mu_\ell} \to \mathbf{x}^*$ and $\langle \nabla g_k(\mathbf{x}^*), \mathbf{x}_0 - \mathbf{x}^* \rangle > 0$ for all $k \in J$.
- $A_\ell \to 0$ as $\ell \to \infty$ because $\mu_\ell \to 0$ and $g_k(\mathbf{x}_{\mu_\ell}) \to g_k(\mathbf{x}^*) > 0$ for all $k \notin J$.

Therefore $|A_\ell| \leq A$ for all sufficiently large ℓ, say $\ell \geq \ell_1$, and so for every $j \in J$:

$$\langle \nabla f(\mathbf{x}_{\mu_\ell}), \mathbf{x}_0 - \mathbf{x}^* \rangle + A \geq \frac{\mu_\ell}{g_j(\mathbf{x}_{\mu_\ell})} \langle \nabla g_j(\mathbf{x}_{\mu_\ell}), \mathbf{x}_0 - \mathbf{x}^* \rangle, \quad \ell \geq \ell_2 := \max[\ell_0, \ell_1],$$

which shows that for every $j \in J$, the nonnegative sequence $(\mu_\ell / g_j(\mathbf{x}_{\mu_\ell}))$, $\ell \geq \ell_2$, is bounded from above.

So take a subsequence (still denoted (μ_ℓ), $\ell \in \mathbb{N}$, for convenience) such that the ratios $\mu_\ell / g_j(\mathbf{x}_{\mu_\ell})$ converge for all $j \in J$, that is,

$$\lim_{\ell \to \infty} \frac{\mu_\ell}{g_j(\mathbf{x}_{\mu_\ell})} = \lambda_j \geq 0, \quad \forall\, j \in J,$$

and let $\lambda_j := 0$ for every $j \notin J$, so that $\lambda_j g_j(\mathbf{x}^*) = 0$ for every $j = 1, \ldots, m$. Taking limit in (0.1) as $\ell \to \infty$, yields:

$$\nabla f(\mathbf{x}^*) = \sum_{j=1}^{m} \lambda_j \nabla g_j(\mathbf{x}^*), \tag{0.3}$$

which shows that $(\mathbf{x}^*, \lambda) \in \mathbf{K} \times \mathbb{R}^m_+$ is a KKT point for $\mathbf{P}$. Finally, invoking Theorem 1, $\mathbf{x}^*$ is also a global minimizer of $\mathbf{P}$. □