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Single-machine scheduling with past-sequence-dependent setup times and general effects of deterioration and learning

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Abstract Scheduling with learning effect and deteriorating jobs has become more popular. However, most of the research assume that the setup time is negligible or a part of the job processing time. In this paper, we propose a model where the deteriorating jobs, the learning effect, and the setup times are present simultaneously. Under the proposed model, the setup time is past-sequence-dependent and the actual job processing time is a general function of the processing times of the jobs already processed and its scheduled position. We provide the optimal schedules for some single-machine problems.

Keywords Scheduling · Learning effect · Deteriorating jobs · Past-sequence-dependent setup times

1 Introduction

In classical scheduling models, job processing times are assumed to be known and fixed. However, there are situations where job processing times might be prolonged due to the deterioration effect or shortened due to the learning effect [18]. Alidaee and Womer [1], Cheng et al. [4], and Gawiejnowicz [7] gave a detailed review of scheduling problems with deteriorating jobs. More recent papers with deteriorating jobs include Ji and Cheng [10,11], Lai et al. [15], Voutsinas and Pappis [21], Wang and Wang [28], Zhao and Tang [32], etc. On the other hand, Biskup [2] presented a comprehensive review of scheduling models and problems with learning effects. More recent papers with learning effect include Janiak and Rudek [9], Lai and Lee [14], Lee [17], Rudek [19], Wang et al. [26], Xu et al. [29], etc.

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The phenomena of job deterioration and learning effect might exist simultaneously. For example, Wang [22] claimed that the learning and forgetting that workers undergo in this environment have thus become increasingly important as workers tend to spend more time in rotating among tasks. Wang and Cheng [25] gave a practical example in the production of porcelain crafts where raw material becomes harder with the lapse of time and results in more time to shape a craftwork but the productivity of the craftsmen improve through increasing their proficiency in designs and operations. Cheng et al. [6] provided another example in which silicon-based raw material is first heated up until it becomes a lump of malleable dough from which the craftsman cuts pieces and shapes them according to different designs into different glass craft products. On the other hand, the pieces that are shaped later require shorter shaping times because the craftsman's productivity improves as a result of learning. The research with both the effects of deterioration and learning is rather limited. Lee [16] was probably the first author to discuss them at the same time. Wang [23], Sun [20], Wang and Guo [24], and Wang et al. [27] derived the optimal schedules for some single-machine problems under specific functions of deterioration and learning effects. Haung et al. [8] considered two resource constrained single-machine group scheduling problems with both deterioration and learning effects. They presented polynomial solutions for the makespan minimization problem under the constraint that the total resource consumption does not exceed a given limit, and the total resource consumption minimization problem under the constraint that the makespan does not exceed a given limit, respectively. Yin and Xu [31] considered some single-machine problems with both the learning effect and deteriorating jobs where the effects are expressed as a general function of the scheduled position and the sum of processing times of jobs already processed.

However, most of the research with deteriorating jobs and learning effect treats the setup times as parts of the job processing times. Recently, Koulamas and Kyparisis [12] presented the concept of "past-sequence-dependent" (p-s-d) setup times. They provided an example in high-tech manufacturing that the setup time is proportional to the processing times of jobs already processed. In addition, Biskup and Herrmann [3] provided another example of wear-out of equipment in which the sum of the processing times of the prior jobs adds to the processing time of the actual job. In the examples above, the worker skills might improve while the quality of the materials might worsen during the manufacturing process. Cheng et al. [5] considered some single machine problems with setup times. Yin et al. [30] proposed a scheduling model with the consideration of the p-s-d setup times, the learning and deterioration effects. They derived the optimal solutions for the makespan, the total completion time problems. Moreover, they showed that the total weighted completion time, the maximum lateness and the number of tardy jobs problems are polynomially solvable under certain agreeable conditions. Motivated by this, we propose a general scheduling model with deteriorating jobs, learning effect and p-s-d setup times in this paper. Under the proposed model, the actual job processing time is expressed as a general function of the normal processing time of jobs already processed and its scheduled position at the same time. The model is general in sense that the function form is unspecified.

2 Problem formulation

There are n jobs to be processed on a single machine. For each job j, there is a normal processing time p_j, a weight w_j and a due date d_j. Due to the learning and the deterioration effects, the actual processing time of job j is

$$p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) \quad \text{for} \quad r = 1, 2, \ldots, n, \tag{1}$$

if it is scheduled in the rth position in a sequence where $p_{[k]}$ denotes the normal processing time of the job scheduled in the kth position and $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n$. It is assumed that $f(x, y) : (0, \infty) \times [1, \infty) \to (0, \infty)$ is a differentiable non-decreasing function with respect to x, non-increasing with respect to y, $f_x(x, y_0) = \frac{\partial}{\partial x} f(x, y_0)$ is non-decreasing with respect to x for every fixed y_0 and $f(0, 1) = 1$. As in [12], the p-s-d setup time of job j if it is scheduled in the rth position of a sequence is as follows:

$$s_{j[1]} = 0 \quad \text{and} \quad s^A_{j[r]} = b \sum_{l=1}^{r-1} p^A_{[l]}, \tag{2}$$

where b is a normalizing constant number with $0 < b < 1$ and $p^A_{[k]}$ denotes the actual processing time of the job scheduled in the kth position.

Note that it is the model in [12] if $f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) = 1$, the model in [13] if $f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) = r^a$ where $a \leq 0$, and the model in [30] if $f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) = \left(p_0 + \sum_{k=1}^{r-1} p_{[k]} / \left(p_0 + \sum_{l=1}^{n} p_l\right)\right)^{a_1} r^{a_2}$ where $a_1 < 0$ and $a_2 < 0$. Throughout the paper, we will use the notation C_j, $L_j = C_j - d_j$ and $T_j = \max\{0, C_j - d_j\}$ to denote the completion time, the lateness and the tardiness of job j.

3 Some single-machine problems

Before presenting the main results, we first state the lemmas that will be used in the proofs in the sequel.

Lemma 1 $F(t) = (\theta - 1)(1 + c) f(u, y_1) - \theta f(u + \lambda t, y_2) + f(u + \lambda\theta t, y_2) \geq 0$ *for* $\theta \geq 1, c > 0, u \geq 0, y_1 \leq y_2, \lambda \geq 0$ *and* $t \geq 0$.

Proof Taking the first derivative of $F(t)$, we have

$$F'(t) = \lambda\theta \frac{\partial}{\partial x} f(u + \lambda\theta t, y_2) - \lambda\theta \frac{\partial}{\partial x} f(u + \lambda t, y_2) \geq 0$$

since $\frac{\partial}{\partial x} f(x, y_0)$ is a non-decreasing function of x and $\theta \geq 1$. It implies that $F(t)$ is a non-decreasing function. Thus,

$$F(t) \geq F(0) = (\theta - 1)[(1+c)f(u, y_1) - f(u, y_2)] \geq 0$$

since $c > 0$. This completes the proof.

Lemma 2 $G(\theta) = \delta_2[f(u+\lambda\theta t, y_2) + c\theta f(u, y_1)] - \delta_1[\theta f(u+\lambda t, y_2) + cf(u, y_1)] + (\theta - 1)f(u, y_1) \geq 0$ *for* $\theta \geq 1, u \geq 0, 0 \leq y_1 \leq y_2, \lambda \geq 0, t > 0, c \geq 0$ *and* $0 < \delta_1 < \delta_2 < 1$.

Proof To show that $G(\theta) \geq 0$, we first claim that

$$G'(\theta) = f(u, y_1) + c\delta_2 f(u, y_1) + \delta_2\lambda t \frac{\partial}{\partial x} f(u + \lambda\theta t, y_2) - \delta_1 f(u + \lambda t, y_2) \geq 0$$

for $0 \leq y_1 \leq y_2, u \geq 0, t > 0, \theta \geq 1, c \geq 0$ and $0 < \delta_1 < \delta_2 < 1$.

To prove the claim, we have

$$\begin{aligned} G'(\theta) &= f(u, y_1) + c\delta_2 f(u, y_1) - \delta_1 f(u, y_2) + \delta_1 f(u, y_2) \\ &\quad + \lambda\delta_2 t \frac{\partial}{\partial x} f(u + \lambda\theta t, y_2) - \delta_1 f(u + \lambda t, y_2) \\ &\geq \lambda\delta_2 t \frac{\partial}{\partial x} f(u + \lambda t, y_2) + \delta_1[f(u, y_2) - f(u + \lambda t, y_2)] \end{aligned} \quad (3)$$

since $\delta_1 < 1$, f is a nonnegative, non-decreasing function with respect to the first variable x and non-increasing with respect to y. By Mean Value Theorem, we have from Eq. (3) that there exists an ξ where $0 < \xi < 1$ such that

$$\begin{aligned} G'(\theta) &\geq \delta_1 \left[\frac{\partial}{\partial x} f(u + \lambda\xi t, y_2)\right](-\lambda t) + \lambda\delta_2 t \frac{\partial}{\partial x} f(u + \lambda t, y_2) \\ &\geq \delta_2\lambda t \left[\frac{\partial}{\partial x} f(u + \lambda t, y_2) - \frac{\partial}{\partial x} f(u + \lambda\xi t, y_2)\right] \geq 0 \end{aligned}$$

since $0 < \delta_1 < \delta_2 < 1$ $\lambda \geq 0, t > 0$ and $\frac{\partial}{\partial x} f(x, y_0)$ is non-decreasing with respect to x for every fixed y_0. This completes the proof of the claim. Thus, we have

$$G(\theta) \geq G(1) = (\delta_2 - \delta_1)[f(u + \lambda t, y_2) + cf(u, y_1)] \geq 0$$

since $0 < \delta_1 < \delta_2 < 1$. This completed the proof of Lemma 2.

Suppose that $S = (\pi, i, j, \pi')$ and $S' = (\pi, j, i, \pi')$ are two job schedules, where π and π' each denote a partial sequence. Furthermore, we assume that there are $r - 1$ scheduled jobs in π. In addition, let A denote the completion time of the last job in π. Under S, the completion times of jobs i and j are

$$C_i(S) = A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) \quad (4)$$

and

$$C_j(S) = A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]}^A + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)\right) + p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_i, r+1\right). \quad (5)$$

Similarly, the completion times of jobs j and i in S' are

$$C_j(S') = A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) \quad (6)$$

and

$$C_i(S') = A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]}^A + p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)\right) + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_j, r+1\right). \quad (7)$$

Theorem 1 *For the* $1|p_{j[r]}^A = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right), s_{psd}|C_{\max}$ *problem, the optimal schedule is obtained by the shortest processing time (SPT) rule.*

Proof Suppose $p_j \geq p_i$. To show that S dominates S', it suffices to show that $C_j(S) \leq C_i(S')$.

Taking the difference between Eqs. (5) and (7), we have

$$C_i(S') - C_j(S) = (p_j - p_i)(b+1) f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_i, r+1\right). \quad (8)$$

Substituting $u = \sum_{k=1}^{r-1} \beta_k p_{[k]}, \theta = p_j/p_i, c = b, t = p_i, \lambda = \beta_r, y_1 = r$ and $y_2 = r+1$ into Eq. (8), we have from Lemma 1 that $C_j(S') \geq C_i(S)$. This completes the proof.

Theorem 2 *For the* $1|p_{j[r]}^A = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right), s_{psd}|\sum C_i$ *problem, the optimal schedule is obtained by the SPT rule.*

Proof It is similar to that of Theorem 1 and is omitted.

It is known that the weighted shortest processing time (WSPT) rule provides the optimal solution for the classical total weighted completion time problem. We will show that it still provides the optimal solution for the total weighted completion time problem if the processing times and the weights are agreeable, i.e., $p_j/p_i \geq 1 \geq w_j/w_i$ for all jobs i and j.

Theorem 3 *For the* $1|p_{j[r]}^A = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right), s_{psd}|\sum w_i C_i$ *problem, the optimal schedule is obtained by the weighted shortest processing time (WSPT) rule if the processing times and the weights are agreeable.*

Proof Suppose that $p_j/p_i \geq 1 \geq w_j/w_i$. Since $p_i \leq p_j$, we have from Theorem 1 that $C_j(S) \leq C_i(S')$. To show that S dominates S', it suffices to show that $w_i C_i(S) + w_j C_j(S) \leq w_j C_j(S') + w_i C_i(S')$. From Eqs. (4–7), we have

$$
\begin{aligned}
&[w_j C_j(S') + w_i C_i(S')] - [w_i C_i(S) + w_j C_j(S)] \\
&= w_i \left[b \sum_{k=1}^{r-1} p_{[k]}^A + p_j b f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) + p_i f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_j, r+1\right)\right] \\
&\quad - w_j \left[b \sum_{k=1}^{r-1} p_{[k]}^A + p_i b f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) + p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]} + \beta_r p_i, r+1\right)\right] \\
&\quad + f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)(w_i + w_j)(p_j - p_i). \qquad (9)
\end{aligned}
$$

Substituting $u = \sum_{k=1}^{r-1} \beta_k p_{[k]}, \theta = p_j/p_i, t = p_i, \lambda = \beta_r, \delta_1 = w_j/(w_i + w_j), \delta_2 = w_i/(w_i + w_j), c = b, y_1 = r$ and $y_2 = r+1$ into Eq. (9), we have from Lemma 2 that $w_i C_i(S) + w_j C_j(S) \leq w_j C_j(S') + w_i C_i(S')$ since $w_i \geq w_j$ and $p_i \leq p_j$. This completes the proof.

Theorem 4 *For the* $1|p_{j[r]}^A = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right), s_{psd}|\sum T_i$ *problem, the optimal schedule is obtained by the EDD rule if the job processing times and the due dates are agreeable, i.e.,* $d_i \leq d_j$ *implies* $p_i \leq p_j$ *for all jobs* i *and* j.

Proof Suppose that $d_i \leq d_j$. It implies $p_i \leq p_j$ since they are agreeable. The total tardiness of jobs in π are the same since they are processed in the same order. By Theorem 1, the makespan is minimized by the SPT rule, thus, the total tardiness of partial sequence π' in S will not be greater than that of π' in S'. To prove that the total tardiness of S is less than or equal to that of S', it suffices to show that $T_i(S) + T_j(S) \leq T_j(S') + T_i(S')$.

To compare the total tardiness of jobs i and j in S and in S', we divide it into two cases. In the first case that $A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right) \leq d_j$, we have from Eqs. (4–7) that the total tardiness of jobs i and j in S and in S' are

$$T_i(S) + T_j(S) = \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right) - d_i, 0\right\}$$
$$+\max\left\{A + 2b\sum_{k=1}^{r-1} p_{[k]}^A + (1+b)p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)\right.$$
$$\left.+ p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_i, r+1\right) - d_j, 0\right\}$$

and

$$T_j(S') + T_i(S') = \max\left\{A + 2b\sum_{k=1}^{r-1} p_{[k]}^A + (1+b)p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)\right.$$
$$\left.+ p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_j, r+1\right) - d_i, 0\right\}.$$

Suppose that neither $T_i(S)$ nor $T_j(S)$ is zero. It is the most restrictive case since it comprises the case that either one or both $T_i(S)$ and $T_j(S)$ are zero. From Theorem 1 and $d_i \leq d_j$, we have

$$[T_j(S') + T_i(S')[-[T_i(S) + T_j(S)] = (1+b)(p_j - p_i) f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)$$
$$+ p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_j, r+1\right)$$
$$- p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_i, r+1\right)$$
$$+ d_j - A - b\sum_{k=1}^{r-1} p_{[k]}^A$$
$$- p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right) \geq 0.$$

Thus, $[T_j(S') + T_i(S')] - [T_i(S) + T_j(S)] \geq 0$ in the first case. In the second case that $A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right) > d_j$, the total tardiness of jobs i and j in S and in S' are

$$T_i(S) + T_j(S) = \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]}^A + p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right) - d_i, 0\right\}$$
$$+ \max\left\{A + 2b\sum_{k=1}^{r-1} p_{[k]}^A + (1+b)p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)\right.$$
$$\left. + p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_i, r+1\right) - d_j, 0\right\}.$$

and

$$T_j(S') + T_i(S') = 2A + 3b\sum_{k=1}^{r-1} p_{[k]}^A + (2+b)p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)$$
$$+ p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_j, r+1\right)$$
$$+ p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_j, r+1\right) - d_i - d_j.$$

Suppose that neither $T_i(S)$ nor $T_j(S)$ is zero. From Theorem 1, $d_i \le d_j$ and $p_i \le p_j$, we have

$$[T_j(S') + T_i(S')] - [T_i(S) + T_j(S)] = (p_j - p_i)(b+2)f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right)$$
$$+ p_i f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]} + \beta_r p_i, r+1\right) \ge 0.$$

Thus, $[T_j(S') + T_i(S')] - [T_i(S) + T_j(S)] \ge 0$ in the second case. This completes the proof.

Theorem 5 *For the* $1|p_{j[r]}^A = p_j f(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r), s_{psd}|L_{\max}$ *problem, the optimal schedule is obtained by the EDD rule if the job processing times and the due dates are agreeable.*

Theorem 6 *For the* $1|p_{j[r]}^A = p_j f\left(\sum_{k=1}^{r-1}\beta_k p_{[k]}, r\right), s_{psd}|T_{\max}$ *problem, the optimal schedule is obtained by the EDD rule if the job processing times and the due dates are agreeable.*

4 Conclusions

In this paper, we propose a new scheduling model in which the job deterioration, the learning effect, and the past-sequence-dependent setup times are considered at the same time. Moreover, the deterioration and learning effects are described by a general function which depends on the actual processing times of the jobs already processed and its scheduled position. Under the proposed model, we showed that the single-machine problems to minimize the makespan and the total completion time are polynomially solvable. We also showed that the total weighted completion time, the total tardiness, the maximum lateness, and the maximum tardiness problems are polynomially solvable under certain conditions. With shorter product cycle times in many production lines, workers must constantly learn new skill and technology. Thus, forgetting effects might occur in these situations. Considering both the effects of learning and forgetting and/or extending the problem to the parallel-machine setting are interesting topics for future research.

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