Optim Lett (2012) 6:1737–1751
DOI 10.1007/s11590-011-0367-2

ORIGINAL PAPER

Mehar's method to find exact fuzzy optimal solution of unbalanced fully fuzzy multi-objective transportation problems

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Received: 9 February 2011 / Accepted: 2 July 2011 / Published online: 20 July 2011

Abstract To the best of our knowledge, till now there is no method described in literature to find exact fuzzy optimal solution of balanced as well as unbalanced fully fuzzy multi-objective transportation problems. In this paper, a new method named as Mehar's method, is proposed to find the exact fuzzy optimal solution of fully fuzzy multi-objective transportation problems (FFMOTP). The advantages of the Mehar's method over existing methods are also discussed. To show the advantages of the proposed method over existing methods, some FFMOTP, which cannot be solved by using any of the existing methods, are solved by using the proposed method and the results obtained are discussed. To illustrate the applicability of the Mehar's method, a real life problem is solved.

Keywords Multi-objective linear programming · Trapezoidal fuzzy number · *JMD* type trapezoidal fuzzy number · Fully fuzzy multi-objective transportation problem

1 Introduction

It is quite challenging to find better ways to create and deliver goods to customers in today's highly competitive market. How and when to send the products, to the customers in quantities they desire, in a cost-effective manner becomes more and more demanding. Transportation models provide a powerful framework to meet this

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requirement. These ensure the efficient movement and timely availability of raw materials and finished goods. The basic transportation problem was originally developed by Hitchcock [8].

In classical form, the transportation problem minimizes the cost of transporting a product which is available at some sources and is required at various destinations. However, in most real world problems, the complexity of the social and economic environment requires the explicit consideration of objective functions other than cost. For example, the objectives may be minimization of total cost, total time, total deterioration of goods during transportation. These objectives are frequently in conflict with each other, are measured in different scales and are difficult to combine in one overall utility function. Several researchers [2,5,6,9,12] have proposed different methods for solving linear multi-objective transportation problems.

In conventional multi-objective transportation problems it is assumed that decision maker is sure about the precise values of co-efficients of the objective functions, availability and demand of the product. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. For example transportation time will be uncertain with the change of weather, transportation ways and condition of transportation ways. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh [15] may represent this data. So, fuzzy decision making method is needed here. Bellman and Zadeh [3] first introduced the fuzzy sets theory into multi-criteria analysis for effectively dealing with the imprecision, vagueness and subjectiveness of the human decision making. Since then, significant advances have been made in developing numerous methodologies and their applications to various decision problems [13,14,16,18].

Several authors [1,4,7,11] have proposed different methods for solving fuzzy multi-objective transportation problems by representing the co-efficients of the objective functions, availability, demand as fuzzy numbers and the decision variables as real numbers.

To the best of our knowledge, till now there is no method described in literature to find exact fuzzy optimal solution of balanced as well as unbalanced fully fuzzy multi-objective transportation problems (FFMOTP). In this paper, a new method named as Mehar's method, is proposed to find the exact fuzzy optimal solution of FFMOTP. The advantages of the Mehar's method over existing methods are also discussed. To show the advantages of the proposed method over existing methods, some FFMOTP, which cannot be solved by using any of the existing methods, are solved by using the proposed method and the results obtained are discussed. To illustrate the applicability of the Mehar's method, a real life problem is solved.

This paper is organized as follows: In Sect. 2, basic definitions and arithmetic operations are presented. In Sect. 3, formulation of FFMOTP is presented. In Sect. 4, the limitations of the existing methods are pointed out. In Sect. 5, a new method, named as Mehar's method is proposed to find exact fuzzy optimal solution of FFMOTP. In Sect. 6, the advantages of the Mehar's method over existing methods are discussed and illustrated by two examples. To show application of Mehar's method, a real life problem is solved in Sect. 7. The conclusions are discussed in Sect. 8.

2 Preliminaries

Kumar and Kaur [10] proposed JMD representation of trapezoidal fuzzy numbers and pointed out that it is better to use JMD representation instead of existing representation of trapezoidal fuzzy numbers. In this section, basic definitions and arithmetic operations of JMD type trapezoidal fuzzy number are presented [10].

2.1 Basic definitions

In this section, some basic definitions are presented.

Definition 1 Let (m, n, α, β) be a trapezoidal fuzzy number then its JMD representation is $(x, \alpha, \gamma, \beta)_{JMD}$ where $x = m - \alpha, \alpha = \alpha \geq 0, \gamma = n - m \geq 0, \beta = \beta \geq 0$.

Definition 2 A trapezoidal fuzzy number $\widetilde{A} = (x, \alpha, \gamma, \beta)_{JMD}$ is said to be zero trapezoidal fuzzy number if and only if $x = 0, \alpha = 0, \gamma = 0, \beta = 0$.

Definition 3 A trapezoidal fuzzy number $\widetilde{A} = (x, \alpha, \gamma, \beta)_{JMD}$ is said to be non-negative trapezoidal fuzzy number if and only if $x \geq 0$.

Definition 4 Two trapezoidal fuzzy numbers $\widetilde{A} = (x_1, \alpha_1, \gamma_1, \beta_1)_{JMD}$ and $\widetilde{B} = (x_2, \alpha_2, \gamma_2, \beta_2)_{JMD}$ are said to be equal, i.e., $\widetilde{A} = \widetilde{B}$ if and only if $x_1 = x_2, \alpha_1 = \alpha_2, \gamma_1 = \gamma_2, \beta_1 = \beta_2$.

Definition 5 A ranking function is a function $\Re : F(R) \to R$, where $F(R)$ is a set of JMD type trapezoidal fuzzy numbers, defined on set of real numbers, which maps each fuzzy number into a real number.

Let $(x, \alpha, \gamma, \beta)_{JMD}$ be a trapezoidal fuzzy number then $\Re((x, \alpha, \gamma, \beta)_{JMD}) = \frac{4x+3\alpha+2\gamma+\beta}{4}$.

2.2 Arithmetic operations of JMD type trapezoidal fuzzy numbers

In this section, addition and multiplication operations of JMD type trapezoidal fuzzy numbers are presented [10].

Let $\widetilde{A}_1 = (x_1, \alpha_1, \gamma_1, \beta_1)_{JMD}$ and $\widetilde{A}_2 = (x_2, \alpha_2, \gamma_2, \beta_2)_{JMD}$ be two JMD type trapezoidal fuzzy numbers, then

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (x_1 + x_2, \alpha_1 + \alpha_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2)_{JMD}$
(ii) $\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (x, \alpha, \gamma, \beta)_{JMD}$

where,

$$
\begin{aligned}
x = {} & \text{minimum}\{x_1x_2, x_1(x_2 + \alpha_2 + \gamma_2 + \beta_2), (x_1 + \alpha_1 + \gamma_1 + \beta_1)x_2, (x_1 + \alpha_1 + \gamma_1 \\
& + \beta_1)(x_2 + \alpha_2 + \gamma_2 + \beta_2)\} \\
\alpha = {} & \text{minimum}\{(x_1 + \alpha_1)(x_2 + \alpha_2), (x_1 + \alpha_1)(x_2 + \alpha_2 + \gamma_2), (x_1 + \alpha_1 + \gamma_1)(x_2 \\
& + \alpha_2), (x_1 + \alpha_1 + \gamma_1)(x_2 + \alpha_2 + \gamma_2)\} - x \\
\gamma = {} & \text{maximum}\{(x_1 + \alpha_1)(x_2 + \alpha_2), (x_1 + \alpha_1)(x_2 + \alpha_2 + \gamma_2), (x_1 + \alpha_1 + \gamma_1)(x_2 \\
& + \alpha_2), (x_1 + \alpha_1 + \gamma_1)(x_2 + \alpha_2 + \gamma_2)\} - x - \alpha \\
\beta = {} & \text{maximum}\{x_1x_2, x_1(x_2 + \alpha_2 + \gamma_2 + \beta_2), (x_1 + \alpha_1 + \gamma_1 + \beta_1)x_2, (x_1 + \alpha_1 + \gamma_1 \\
& + \beta_1)(x_2 + \alpha_2 + \gamma_2 + \beta_2)\} - x - \alpha - \gamma
\end{aligned}
$$

3 Fuzzy linear programming formulation of FFMOTP

Several authors [1,4,7,11] have proposed different methods for solving fuzzy multi-objective transportation problems by representing the co-efficients of the objective functions, availability, demand as fuzzy numbers and decision variables as real numbers. But in real life situations, decision variables may also be fuzzy in nature.

The balanced FFMOTP where all the parameters are represented by fuzzy numbers may be formulated as follows:

$$\begin{aligned} \text{Minimize} \quad & \sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij}, \quad k = 1, 2, \ldots, K \\ \text{subject to} \quad & \sum_{j=1}^{q} \widetilde{x}_{ij} = \widetilde{a}_i, \quad i = 1, 2, \ldots, p, \\ & \sum_{i=1}^{p} \widetilde{x}_{ij} = \widetilde{b}_j, \quad j = 1, 2, \ldots, q, \\ \text{with} \quad & \sum_{i=1}^{p} \widetilde{a}_i = \sum_{j=1}^{q} \widetilde{b}_j \end{aligned} \tag{P_1}$$

$\widetilde{x}_{ij}$ is a non-negative *JMD* type trapezoidal fuzzy number
where,

$p =$ total number of sources,
$q =$ total number of destinations,
$\widetilde{a}_i = (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD}$: the fuzzy availability of the product at ith source,
$\widetilde{b}_j = (x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD}$: the fuzzy demand of the product at jth destination,
$\widetilde{c}_{ij}^{k} = (x_{ij}^k, \alpha_{ij}^k, \gamma_{ij}^k, \beta_{ij}^k)_{JMD}$: the penalty criteria for kth objective function,
$\widetilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$: the fuzzy quantity of the product that should be transported from ith source to jth destination (or fuzzy decision variable) in order to minimize K objective fuctions,
$\sum_{i=1}^{p} \widetilde{a}_i =$ total fuzzy availability of the product,
$\sum_{j=1}^{q} \widetilde{b}_j =$ total fuzzy demand of the product,
$\sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij} =$ fuzzy value of kth objective function.

Remark 1 If $\sum_{i=1}^{p} \widetilde{a}_i = \sum_{j=1}^{q} \widetilde{b}_j$ then the FFMOTP is said to be balanced, otherwise it is called unbalanced.

3.1 Fuzzy optimal solution of FFMOTP

The fuzzy optimal solution of (P_1) is a set of JMD type trapezoidal fuzzy numbers $\{\widetilde{x}_{ij}\}$ which satisfies the following characteristics:

(i) $\widetilde{x}_{ij}$ is a non-negative JMD type trapezoidal fuzzy number.

Table 1 Fuzzy penalties for 1st objective function

Destination→ Source↓	D_1	D_2	D_3	D_4
S_1	$(0, 0.5, 0.5, 1.5)_{JMD}$	$(0, 1, 1, 3)_{JMD}$	$(4, 3, 1, 1)_{JMD}$	$(5, 1, 2, 1)_{JMD}$
S_2	$(0, 0.5, 0.5, 1.5)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(1, 1, 2, 1)_{JMD}$	$(2, 1, 2, 1)_{JMD}$
S_3	$(6, 1, 1, 3)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(2, 1, 2, 1)_{JMD}$	$(4, 1, 1, 3)_{JMD}$

Table 2 Fuzzy penalties for 2nd objective function

Destination→ Source↓	D_1	D_2	D_3	D_4
S_1	$(2, 1, 2, 1)_{JMD}$	$(1, 1, 3, 3)_{JMD}$	$(1, 1, 2, 1)_{JMD}$	$(1, 1, 1, 3)_{JMD}$
S_2	$(3, 1, 1, 3)_{JMD}$	$(6, 1, 1, 3)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(8, 1, 1, 3)_{JMD}$
S_3	$(4, 1, 1, 3)_{JMD}$	$(0, 1, 1, 3)_{JMD}$	$(3, 1, 1, 3)_{JMD}$	$(0, 1, 0.25, 0.50)_{JMD}$

(ii) $\sum_{j=1}^{q} \widetilde{x}_{ij} = \widetilde{a}_i,\ i = 1, 2, \ldots, p$ and $\sum_{i=1}^{p} \widetilde{x}_{ij} = \widetilde{b}_j,\ j = 1, 2, \ldots, q.$

(iii) If there exist any other set of non-negative JMD type trapezoidal fuzzy numbers $\{\widetilde{x}'_{ij}\}$ such that $\sum_{j=1}^{q} \widetilde{x}'_{ij} = \widetilde{a}_i,\ i = 1, 2, \ldots, p$ and $\sum_{i=1}^{p} \widetilde{x}'_{ij} = \widetilde{b}_j,\ j = 1, 2, \ldots, q.$

then

$$\Re\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij}\right) \le \Re\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}'_{ij}\right) \qquad \forall\, k = 1, 2, \ldots, K$$

$$\text{and}\ \Re\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij}\right) < \Re\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}'_{ij}\right) \qquad \text{for at least one } k.$$

4 Limitations of the existing methods

In this section, the limitations of existing methods are pointed out.

1. Existing methods [1,4,7,11] can be applied for solving those fuzzy multi-objective transportation problems where decision variables are represented by real numbers and all other parameters are represented by fuzzy numbers. However, none of the existing methods can be used for solving such FFMOTP where all the parameters are represented by fuzzy numbers, e.g., the FFMOTP, chosen in Examples 4.1 and 4.2.

Example 4.1 A company has three sources S_1, S_2, S_3 and four destinations D_1, D_2, D_3, D_4. The fuzzy penalties for supplying a unit quantity of the product from ith source to jth destination for 1st and 2nd objectives are given in Tables 1 and 2 respectively. The fuzzy availability of the product at sources S_1, S_2, S_3 are $(6, 1, 2, 2)_{JMD}$, $(16, 1, 2, 4)_{JMD}$ and $(14, 2, 1, 4)_{JMD}$ respectively and the

Table 3 Fuzzy penalties for 1st objective function

Destination→ Source↓	D_1	D_2	D_3	D_4
S_1	$(0, 0.5, 0.5, 1.5)_{JMD}$	$(0, 1, 1, 3)_{JMD}$	$(4, 3, 1, 1)_{JMD}$	$(5, 1, 2, 1)_{JMD}$
S_2	$(0, 0.5, 0.5, 1.5)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(1, 1, 2, 1)_{JMD}$	$(2, 1, 2, 1)_{JMD}$
S_3	$(6, 1, 1, 3)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(2, 1, 2, 1)_{JMD}$	$(4, 1, 1, 3)_{JMD}$

Table 4 Fuzzy penalties for 2nd objective function

Destination→ Source↓	D_1	D_2	D_3	D_4
S_1	$(2, 1, 2, 1)_{JMD}$	$(1, 1, 3, 3)_{JMD}$	$(1, 1, 2, 1)_{JMD}$	$(1, 1, 1, 3)_{JMD}$
S_2	$(3, 1, 1, 3)_{JMD}$	$(6, 1, 1, 3)_{JMD}$	$(7, 1, 1, 3)_{JMD}$	$(8, 1, 1, 3)_{JMD}$
S_3	$(4, 1, 1, 3)_{JMD}$	$(0, 1, 1, 3)_{JMD}$	$(3, 1, 1, 3)_{JMD}$	$(0, 1, 0.25, 0.50)_{JMD}$

fuzzy demand of the product at destinations D_1, D_2, D_3, D_4 are $(9, 1, 1, 3)_{JMD}$, $(1, 1, 1, 3)_{JMD}$, $(12, 1, 1, 3)_{JMD}$ and $(14, 1, 2, 1)_{JMD}$ respectively. The company wants to determine the fuzzy quantities of the product to be transported from each source to various destinations in order to minimize each objective function.

Example 4.2 A company has three sources S_1, S_2, S_3 and four destinations D_1, D_2, D_3, D_4. The fuzzy penalties for supplying a unit quantity of the product from ith source to jth destination for 1st and 2nd objectives are given in Tables 3 and 4 respectively. The fuzzy availability of the product at sources S_1, S_2, S_3 are $(6, 1, 1, 3)_{JMD}$, $(17, 1, 1, 3)_{JMD}$ and $(14, 2, 1, 4)_{JMD}$ respectively and the fuzzy demand of the product at destinations D_1, D_2, D_3, D_4 are $(9, 1, 1, 3)_{JMD}$, $(1, 1, 1, 3)_{JMD}$, $(12, 1, 1, 3)_{JMD}$ and $(14, 1, 1, 3)_{JMD}$ respectively. The company wants to determine the fuzzy quantities of the product to be transported from each source to various destinations in order to minimize each objective function.

5 Mehar's method

In this section, to overcome the limitations of existing methods discussed in Sect. 4, a new method, named as Mehar's method, is proposed to find the exact fuzzy optimal solution of FFMOTP occurring in real life situations by representing all the parameters as JMD type trapezoidal fuzzy numbers. The steps of the proposed method are as follows:

Step 1 Find the total fuzzy availability $\sum_{i=1}^{p} \widetilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{q} \widetilde{b}_j$. Let $\sum_{i=1}^{p} \widetilde{a}_i = (x, \alpha, \gamma, \beta)_{JMD}$ and $\sum_{j=1}^{q} \widetilde{b}_j = (y, \lambda, \delta, \mu)_{JMD}$. Examine that the problem is balanced or not, i.e., $\sum_{i=1}^{p} \widetilde{a}_i = \sum_{j=1}^{q} \widetilde{b}_j$ or $\sum_{i=1}^{p} \widetilde{a}_i \neq \sum_{j=1}^{q} \widetilde{b}_j$.

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{p} \widetilde{a}_i = \sum_{j=1}^{q} \widetilde{b}_j$, then Go to Step 2.

Case (ii) If $\sum_{i=1}^{p} \widetilde{a}_i \neq \sum_{j=1}^{q} \widetilde{b}_j$ then convert the unbalanced problem $\sum_{i=1}^{p} \widetilde{a}_i \neq \sum_{j=1}^{q} \widetilde{b}_j$ into balanced problem $\sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} \widetilde{b}_j$, $m = p$ or $p + 1$ and $n = q$ or $q + 1$ by using the existing method [10].

Step 2 The balanced FFMOTP, obtained from Step 1, may be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij}, \qquad k = 1, 2, \ldots, K$$

$$\text{subject to} \sum_{j=1}^{n} \widetilde{x}_{ij} = \widetilde{a}_i, \qquad i = 1, 2, \ldots, m;\, m = p \text{ or } p + 1,$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij} = \widetilde{b}_j, \qquad j = 1, 2, \ldots, n;\, n = q \text{ or } q + 1,$$

$$\text{with} \sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} \widetilde{b}_j$$

$\widetilde{x}_{ij}$ is a non-negative JMD type trapezoidal fuzzy number.
where,
$\widetilde{c}_{ij}^{k} = (x_{ij}^{k}, \alpha_{ij}^{k}, \gamma_{ij}^{k}, \beta_{ij}^{k})_{JMD}$; $\widetilde{a}_i = (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD}$; $\widetilde{b}_j = (x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD}$; $\widetilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$

Step 3 Now our objective is to find $\widetilde{x}_{ij}$ such that

$$\text{Minimize } \Re\Big(\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij}\Big), \quad k = 1, 2, \ldots, K$$

$$\text{subject to} \sum_{j=1}^{n} \widetilde{x}_{ij} = \widetilde{a}_i, \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij} = \widetilde{b}_j, \quad j = 1, 2, \ldots, n,$$

$$\text{with} \sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} \widetilde{b}_j$$

$\widetilde{x}_{ij}$ is a non-negative JMD type trapezoidal fuzzy number.

Step 4 Let $\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}^{k} \otimes \widetilde{x}_{ij} = (x_0^k, \alpha_0^k, \gamma_0^k, \beta_0^k)_{JMD}$, then the fuzzy multi-objective linear programming problem (FMOLPP), obtained in Step 3, may be written as:

$$\text{Minimize } \Re((x_0^k, \alpha_0^k, \gamma_0^k, \beta_0^k)_{JMD}), \quad k = 1, 2, \ldots, K$$

$$\text{subject to} \left(\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} \alpha_{ij}, \sum_{j=1}^{n} \gamma_{ij}, \sum_{j=1}^{n} \beta_{ij}\right)_{JMD} = (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD},$$
$$i = 1, 2, \ldots, m,$$

$$\left(\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} \alpha_{ij}, \sum_{i=1}^{m} \gamma_{ij}, \sum_{i=1}^{m} \beta_{ij}\right)_{JMD} = (x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD},$$
$$j = 1, 2, \ldots, n$$
$$\text{with } \sum_{i=1}^{m} (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD} = \sum_{j=1}^{n} (x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD}$$

$(x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$ is a non-negative JMD type trapezoidal fuzzy number.

Step 5 The FMOLPP, obtained in Step 4, is converted into following crisp multi-objective linear programming problem:

Minimize $\frac{1}{4}\left(4x_0^k + 3\alpha_0^k + 2\gamma_0^k + \beta_0^k\right)$, $\quad k = 1, 2, \ldots, K$

subject to

$$\sum_{j=1}^{n} x_{ij} = x_i,\ i = 1, 2, \ldots, m \qquad \sum_{j=1}^{n} \alpha_{ij} = \alpha_i,\ i = 1, 2, \ldots, m$$
$$\sum_{j=1}^{n} \gamma_{ij} = \gamma_i,\ i = 1, 2, \ldots, m \qquad \sum_{j=1}^{n} \beta_{ij} = \beta_i,\ i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} x_{ij} = x'_j,\ j = 1, 2, \ldots, n \qquad \sum_{i=1}^{m} \alpha_{ij} = \alpha'_j,\ j = 1, 2, \ldots, n$$
$$\sum_{i=1}^{m} \gamma_{ij} = \gamma'_j,\ j = 1, 2, \ldots, n \qquad \sum_{i=1}^{m} \beta_{ij} = \beta'_j,\ j = 1, 2, \ldots, n$$
$$x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij} \geq 0\ \forall\ i, j.$$

Step 6 Solve the crisp multi-objective linear programming problem obtained in Step 5 by using any classical multi-objective linear programming approach.

Step 7 Find the fuzzy optimal solution $\widetilde{x}_{ij}$ by putting the values of $x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij}$ in $\widetilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$.

Step 8 Find the fuzzy optimal value of each objective function by putting the values of $\widetilde{x}_{ij}$ in $\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}^k \otimes \widetilde{x}_{ij}$.

6 Advantages of proposed method over existing methods

In this section, the advantages of the proposed method over existing methods are discussed.

(i) Existing methods [1,4,7,11] can be applied for solving those fuzzy multi-objective transportation problems where decision variables are represented by real numbers and all other parameters are represented by fuzzy numbers. However, in real life situations, there may exist fuzzy multi-objective transportation problems where all the parameters are represented by fuzzy numbers and none of the existing methods can be used for solving such FFMOTP wherein all the parameters are represented by fuzzy numbers. The main advantage of the proposed method over existing methods is that it can be used for solving such FFMOTP.

(ii) The existing methods may be used for solving balanced fuzzy multi-objective transportation problems. However, these cannot be used for solving unbalanced fuzzy multi-objective transportation problems. On the other hand, the proposed method can be used for solving balanced as well as unbalanced fuzzy multi-objective transportation problems.

To show the advantage of proposed method over existing methods [1,4,7,11], FFMOTPs chosen in Examples 4.1 and 4.2 which cannot be solved by using any of the existing methods [1,4,7,11] are solved by using Mehar's method.

6.1 Fuzzy optimal solution of 1st FFMOTP

The fuzzy optimal solution of FFMOTP, chosen in Example 4.1, by using Mehar's method is as follows:

Step 1 Total fuzzy availability $= (36, 4, 5, 10)_{JMD}$ and total fuzzy demand $= (36, 4, 5, 10)_{JMD}$. Since total fuzzy availability = total fuzzy demand, so it is a balanced fuzzy transportation problem.

Step 2 Assuming $\widetilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$. Then, the balanced FFMOTP, chosen in Example 4.1, may be formulated into the following FMOLPP:
Minimize $((0, 0.5, 0.5, 1.5)_{JMD} \otimes \widetilde{x}_{11} \oplus (0, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{12} \oplus (4, 3, 1, 1)_{JMD} \otimes \widetilde{x}_{13} \oplus (5, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{14} \oplus (0, 0.5, 0.5, 1.5)_{JMD} \otimes \widetilde{x}_{21} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{22} \oplus (1, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{23} \oplus (2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{24} \oplus (6, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{31} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{32} \oplus (2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{33} \oplus (4, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{34})$
Minimize $((2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{11} \oplus (1, 1, 3, 3)_{JMD} \otimes \widetilde{x}_{12} \oplus (1, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{13} \oplus (1, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{14} \oplus (3, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{21} \oplus (6, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{22} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{23} \oplus (8, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{24} \oplus (4, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{31} \oplus (0, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{32} \oplus (3, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{33} \oplus (0, 1, 0.25, 0.50)_{JMD} \otimes \widetilde{x}_{34})$
subject to

$$\begin{aligned}
\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} \oplus \tilde{x}_{14} &= (6, 1, 2, 2)_{JMD}\\
\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \oplus \tilde{x}_{24} &= (16, 1, 2, 4)_{JMD}\\
\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \oplus \tilde{x}_{34} &= (14, 2, 1, 4)_{JMD}\\
\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} &= (9, 1, 1, 3)_{JMD}\\
\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} &= (1, 1, 1, 3)_{JMD}\\
\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} &= (12, 1, 1, 3)_{JMD}\\
\tilde{x}_{14} \oplus \tilde{x}_{24} \oplus \tilde{x}_{34} &= (14, 1, 2, 1)_{JMD}
\end{aligned}$$

$\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ are non-negative JMD type trapezoidal fuzzy numbers.

Step 3 Using Step 3 to Step 5 of the proposed method, the FMOLPP obtained in Step 2, may be converted into the following crisp multi-objective linear programming problem:

$$\text{Minimize}\frac{1}{4}(4x_{11}+4\alpha_{11}+3.5\gamma_{11}+2.5\beta_{11}+8x_{12}+8\alpha_{12}+7\gamma_{12}+5\beta_{12}+28x_{13}+24\alpha_{13}+17\gamma_{13}+9\beta_{13}+28x_{14}+23\alpha_{14}+17\gamma_{14}+9\beta_{14}+4x_{21}+4\alpha_{21}+3.5\gamma_{21}+2.5\beta_{21}+36x_{22}+29\alpha_{22}+21\gamma_{22}+12\beta_{22}+12x_{23}+11\alpha_{23}+9\gamma_{23}+5\beta_{23}+16x_{24}+14\alpha_{24}+11\gamma_{24}+6\beta_{24}+32x_{31}+26\alpha_{31}+19\gamma_{31}+11\beta_{31}+36x_{32}+29\alpha_{32}+21\gamma_{32}+12\beta_{32}+16x_{33}+14\alpha_{33}+11\gamma_{33}+6\beta_{33}+24x_{34}+20\alpha_{34}+15\gamma_{34}+9\beta_{34})$$

$$\text{Minimize}\frac{1}{4}(16x_{11}+14\alpha_{11}+11\gamma_{11}+6\beta_{11}+16x_{12}+15\alpha_{12}+13\gamma_{12}+8\beta_{12}+12x_{13}+11\alpha_{13}+9\gamma_{13}+5\beta_{13}+12x_{14}+11\alpha_{14}+9\gamma_{14}+6\beta_{14}+20x_{21}+17\alpha_{21}+13\gamma_{21}+8\beta_{21}+32x_{22}+26\alpha_{22}+19\gamma_{22}+11\beta_{22}+36x_{23}+29\alpha_{23}+21\gamma_{23}+12\beta_{23}+40x_{24}+32\alpha_{24}+23\gamma_{24}+13\beta_{24}+24x_{31}+20\alpha_{31}+15\gamma_{31}+9\beta_{31}+8x_{32}+8\alpha_{32}+7\gamma_{32}+5\beta_{32}+20x_{33}+17\alpha_{33}+13\gamma_{33}+8\beta_{33}+4x_{34}+4\alpha_{34}+3\gamma_{34}+1.75\beta_{34})$$

subject to

$$\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}=6, && \alpha_{11}+\alpha_{12}+\alpha_{13}+\alpha_{14}=1,\\
&\gamma_{11}+\gamma_{12}+\gamma_{13}+\gamma_{14}=2, && \beta_{11}+\beta_{12}+\beta_{13}+\beta_{14}=2,\\
&x_{21}+x_{22}+x_{23}+x_{24}=16, && \alpha_{21}+\alpha_{22}+\alpha_{23}+\alpha_{24}=1,\\
&\gamma_{21}+\gamma_{22}+\gamma_{23}+\gamma_{24}=2, && \beta_{21}+\beta_{22}+\beta_{23}+\beta_{24}=4,\\
&x_{31}+x_{32}+x_{33}+x_{34}=14, && \alpha_{31}+\alpha_{32}+\alpha_{33}+\alpha_{34}=2,\\
&\gamma_{31}+\gamma_{32}+\gamma_{33}+\gamma_{34}=1, && \beta_{31}+\beta_{32}+\beta_{33}+\beta_{34}=4,\\
&x_{11}+x_{21}+x_{31}=9, && \alpha_{11}+\alpha_{21}+\alpha_{31}=1,\\
&\gamma_{11}+\gamma_{21}+\gamma_{31}=1, && \beta_{11}+\beta_{21}+\beta_{31}=3,\\
&x_{12}+x_{22}+x_{32}=1, && \alpha_{12}+\alpha_{22}+\alpha_{32}=1,\\
&\gamma_{12}+\gamma_{22}+\gamma_{32}=1, && \beta_{12}+\beta_{22}+\beta_{32}=3\\
&x_{13}+x_{23}+x_{33}=12, && \alpha_{13}+\alpha_{23}+\alpha_{33}=1,\\
&\gamma_{13}+\gamma_{23}+\gamma_{33}=1, && \beta_{13}+\beta_{23}+\beta_{33}=3,\\
&x_{14}+x_{24}+x_{34}=14, && \alpha_{14}+\alpha_{24}+\alpha_{34}=1,\\
&\gamma_{14}+\gamma_{24}+\gamma_{34}=2, && \beta_{14}+\beta_{24}+\beta_{34}=1
\end{aligned}$$

$x_{11},\alpha_{11},\gamma_{11},\beta_{11},x_{12},\alpha_{12},\gamma_{12},\beta_{12},x_{13},\alpha_{13},\gamma_{13},\beta_{13},x_{14},\alpha_{14},\gamma_{14},\beta_{14},$ $x_{21},\alpha_{21},\gamma_{21},\beta_{21},x_{22},\alpha_{22},\gamma_{22},\beta_{22},x_{23},\alpha_{23},\gamma_{23},\beta_{23},x_{24},\alpha_{24},\gamma_{24},\beta_{24},$ $x_{31},\alpha_{31},\gamma_{31},\beta_{31},x_{32},\alpha_{32},\gamma_{32},\beta_{32},x_{33},\alpha_{33},\gamma_{33},\beta_{33},x_{34},\alpha_{34},\gamma_{34},\beta_{34}$ ≥ 0.

Step 4 Using fuzzy programming technique [17] the optimal solution of the crisp multi-objective linear programming problem, obtained in Step 3, is: $x_{11}=4.001157, \alpha_{11}=0, \gamma_{11}=0, \beta_{11}=0, x_{12}=1, \alpha_{12}=1, \gamma_{12}=1,$ $\beta_{12}=2, x_{13}=0.998843, \alpha_{13}=0, \gamma_{13}=0, \beta_{13}=0, x_{14}=0, \alpha_{14}=0,$ $\gamma_{14}=1, \beta_{14}=0, x_{21}=4.998843, \alpha_{21}=1, \gamma_{21}=1, \beta_{21}=3, x_{22}=0,$

$\alpha_{22} = 0, \gamma_{22} = 0, \beta_{22} = 0, x_{23} = 11.001157, \alpha_{23} = 0, \gamma_{23} = 1, \beta_{23} = 1, x_{24} = 0, \alpha_{24} = 0, \gamma_{24} = 0, \beta_{24} = 0, x_{31} = 0, \alpha_{31} = 0, \gamma_{31} = 0, \beta_{31} = 0, x_{32} = 0, \alpha_{32} = 0, \gamma_{32} = 0, \beta_{32} = 1, x_{33} = 0, \alpha_{33} = 1, \gamma_{33} = 0, \beta_{33} = 2, x_{34} = 14, \alpha_{34} = 1, \gamma_{34} = 1, \beta_{34} = 1$.

Step 5 Putting the values of $x_{ij}, \alpha_{ij}, \gamma_{ij}$ and β_{ij} in $\widetilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$, the fuzzy optimal solution is $\widetilde{x}_{11} = (4.001157, 0, 0, 0)_{JMD}, \widetilde{x}_{12} = (1, 1, 1, 2)_{JMD}, \widetilde{x}_{13} = (0.998843, 0, 0, 0)_{JMD}, \widetilde{x}_{14} = (0, 0, 1, 0)_{JMD}, \widetilde{x}_{21} = (4.998843, 1, 1, 3)_{JMD}, \widetilde{x}_{22} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{23} = (11.001157, 0, 1, 1)_{JMD}, \widetilde{x}_{24} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{31} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{32} = (0, 0, 0, 1)_{JMD}, \widetilde{x}_{33} = (0, 1, 0, 2)_{JMD}, \widetilde{x}_{34} = (14, 1, 1, 1)_{JMD}$.

Step 6 Putting the values of $\widetilde{x}_{11}, \widetilde{x}_{12}, \widetilde{x}_{13}, \widetilde{x}_{14}, \widetilde{x}_{21}, \widetilde{x}_{22}, \widetilde{x}_{23}, \widetilde{x}_{24}, \widetilde{x}_{31}, \widetilde{x}_{32}, \widetilde{x}_{33}, \widetilde{x}_{34}$ in $((0, 0.5, 0.5, 1.5)_{JMD} \otimes \widetilde{x}_{11} \oplus (0, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{12} \oplus (4, 3, 1, 1)_{JMD} \otimes \widetilde{x}_{13} \oplus (5, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{14} \oplus (0, 0.5, 0.5, 1.5)_{JMD} \otimes \widetilde{x}_{21} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{22} \oplus (1, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{23} \oplus (2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{24} \oplus (6, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{31} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{32} \oplus (2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{33} \oplus (4, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{34})$ and in $((2, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{11} \oplus (1, 1, 3, 3)_{JMD} \otimes \widetilde{x}_{12} \oplus (1, 1, 2, 1)_{JMD} \otimes \widetilde{x}_{13} \oplus (1, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{14} \oplus (3, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{21} \oplus (6, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{22} \oplus (7, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{23} \oplus (8, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{24} \oplus (4, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{31} \oplus (0, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{32} \oplus (3, 1, 1, 3)_{JMD} \otimes \widetilde{x}_{33} \oplus (0, 1, 0.25, 0.50)_{JMD} \otimes \widetilde{x}_{34})$ the fuzzy optimal values of first and second objective are: $(70.996529, 42.997686, 68.001157, 144)_{JMD}$ and $(102.005785, 47, 61, 159.75)_{JMD}$ respectively.

6.2 Fuzzy optimal solution of 2nd FFMOTP

The exact fuzzy optimal solution of FFMOTP chosen in Example 4.2, may be obtained by using Mehar's method as follows:

Total fuzzy availability $= (37, 4, 3, 10)_{JMD}$ and total fuzzy demand $= (36, 4, 4, 12)_{JMD}$. Since total fuzzy availability $\neq$ total fuzzy demand, so it is an unbalanced fuzzy transportation problem. Now as described in the existing method [10], the unbalanced fuzzy transportation problem can be converted into a balanced fuzzy transportation problem, by introducing a dummy source with fuzzy availability $(0, 0, 1, 2)_{JMD}$ and a dummy destination with fuzzy demand $(1, 0, 0, 0)_{JMD}$ so that total fuzzy availability = total fuzzy demand i.e., $(37, 4, 3, 10)_{JMD} \oplus (0, 0, 1, 2)_{JMD} = (36, 4, 4, 12)_{JMD} \oplus (1, 0, 0, 0)_{JMD}$. Assume the fuzzy penalities $(\widetilde{c}^k_{ij})$ from dummy source to all destinations and from all sources to dummy destination as zero JMD type trapezoidal fuzzy numbers for all objectives i.e., $\widetilde{c}^k_{41} = \widetilde{c}^k_{42} = \widetilde{c}^k_{43} = \widetilde{c}^k_{44} = \widetilde{c}^k_{15} = \widetilde{c}^k_{25} = \widetilde{c}^k_{35} = \widetilde{c}^k_{45} = (0, 0, 0, 0)_{JMD}\ \forall\, k = 1, 2$.

Using Steps 2 to 8 of the proposed method, the fuzzy optimal solution of this balanced FMOLPP is $\widetilde{x}_{11} = (4.550552, 0, 0, 0)_{JMD}, \widetilde{x}_{12} = (1, 1, 1, 3)_{JMD}, \widetilde{x}_{13} = (0.449448, 0, 0, 0)_{JMD}, \widetilde{x}_{14} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{15} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{21} = (4.449448, 1, 1, 3)_{JMD}, \widetilde{x}_{22} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{23} = (11.550552, 0, 0, 0)_{JMD}, \widetilde{x}_{24} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{25} = (1, 0, 0, 0)_{JMD}, \widetilde{x}_{31} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{32} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{33} = (0, 1, 0, 1)_{JMD}, \widetilde{x}_{34} = (14, 1, 1, 3)_{JMD}, \widetilde{x}_{35} = (0, 0, 0, 0)_{JMD}, \widetilde{x}_{41} =$

Table 5 Comparison of results obtained by using existing methods and Mehar's method

Example	Existing methods [1,4,7,11]	Mehar's method
4.1	Not applicable	Fuzzy optimal value of 1st objective is = $(70.996529, 42.997686, 68.001157, 144)_{JMD}$ Fuzzy optimal value of 2nd objective is = $(102.005785, 47, 61, 159.75)_{JMD}$
4.2	Not applicable	Fuzzy optimal value of 1st objective is = $(69.348344, 41.898896, 56.550552, 142)_{JMD}$ Fuzzy optimal value of 2nd objective is = $(104.75276, 47, 49, 140.25)_{JMD}$

$(0, 0, 0, 0)_{JMD}$, $\widetilde{x}_{42} = (0, 0, 0, 0)_{JMD}$, $\widetilde{x}_{43} = (0, 0, 1, 2)_{JMD}$, $\widetilde{x}_{44} = (0, 0, 0, 0)_{JMD}$, $\widetilde{x}_{45} = (0, 0, 0, 0)_{JMD}$ and the fuzzy optimal values of first and second objective are $(69.348344, 41.898896, 56.550552, 142)_{JMD}$ and $(104.75276, 47, 49, 140.25)_{JMD}$ respectively.

6.3 Result and discussion

To compare the existing methods [1,4,7,11] and the Mehar's method, the results of FFMOTP chosen in Examples 4.1 and 4.2 obtained by using the existing methods and the Mehar's method are shown in Table 5.

On the basis of results, shown in Table 5, it can be easily seen that FFMOTP, chosen in Examples 4.1 and 4.2, cannot be solved by any of existing methods while same problems can be solved by using the Mehar's method.

7 Application of Mehar's method

To show the application of the Mehar's method, the data is collected from a trader supplying apples to various markets of the country. The best varieties of the apples are grown in the orchards of the Himalayan region in North of India. The trader collects the apples from orchards at Kinnaur in Himachal Pradesh and Almora in Uttar Pradesh and supplies these in the markets of Chandigarh and Delhi. The approximate transportation cost (in dollars) per ton, approximate quantity of deterioration of apples (in kilograms) per ton, approximate availability of the apples (in tons) and approximate demand of the apples (in tons) are shown in Tables 6 and 7. The trader desires to minimize the total fuzzy transportation cost and the total fuzzy quantity of deterioration of apples.

7.1 Results

On solving the FFMOTP shown in Tables 6 and 7 by using the Mehar's method, the obtained fuzzy optimal solution, minimum total fuzzy transportation cost and minimum total fuzzy quantity of deterioration of apples is $\widetilde{x}_{11} = (13.095, 1, 3, 7)_{JMD}$, $\widetilde{x}_{12} = (36.905, 0, 0, 0)_{JMD}$, $\widetilde{x}_{21} = (14.905, 0, 0, 0)_{JMD}$, $\widetilde{x}_{22} = (26.095, 1, 1, 3)_{JMD}$,

Table 6 Transportation cost (*in dollars*) per ton

Destination→ Source↓	Chandigarh	Delhi	Availability (*in tons*)
Kinnaur (*Himachal Pradesh*)	$(178, 1, 1, 3)_{JMD}$	$(295, 1, 4, 9)_{JMD}$	$(50, 1, 3, 7)_{JMD}$
Almora (*Uttar Pradesh*)	$(257, 2, 1, 4)_{JMD}$	$(137, 3, 1, 1)_{JMD}$	$(41, 1, 1, 3)_{JMD}$
Demand (*in tons*)	$(28, 1, 3, 7)_{JMD}$	$(63, 1, 1, 3)_{JMD}$	

Table 7 Deterioration of apples (*in kg*) per ton

Destination→ Sources↓	Chandigarh	Delhi	Availability (*in tons*)
Kinnaur (*Himachal Pradesh*)	$(25, 1, 1, 3)_{JMD}$	$(17, 2, 1, 4)_{JMD}$	$(50, 1, 3, 7)_{JMD}$
Almora (*Uttar Pradesh*)	$(14, 1, 3, 7)_{JMD}$	$(23, 1, 1, 3)_{JMD}$	$(41, 1, 1, 3)_{JMD}$
Demand (*in tons*)	$(28, 1, 3, 7)_{JMD}$	$(63, 1, 1, 3)_{JMD}$	

$(20623.485, 477.095, 884.715, 2178.145)_{JMD}$ and $(1763.615, 177.905, 228.81, 681.525)_{JMD}$, respectively.

7.2 Physical interpretation of the results

The minimum total fuzzy transportation cost can be physically interpreted as follows:

(1) The least amount of total transportation cost is 20,623.485 dollars.
(2) The usual possible amount of total transportation cost lies between 21,100.58 and 21,985.295 dollars.
(3) The greatest amount of total transportation cost is 24,163.44 dollars.

i.e., the cost will be always >20,623.485 dollars and <24,163.44 dollars and maximum chances are that the cost will lie between 21,100.58 and 21,985.295 dollars. The variation in cost with respect to chances are shown in Fig. 1.

The minimum total fuzzy quantity of deterioration of apples can be physically interpreted as follows:

(1) The least amount of total quantity of deterioration of apples is 1,763.615 kgs.
(2) The usual possible amount of total quantity of deterioration of apples lies between 1941.52 and 2,170.33 kgs.
(3) The greatest amount of total quantity of deterioration of apples is 2,851.855 kgs.

The variation in total quantity of deterioration of apples with respect to chances are shown in Fig. 2.

8 Conclusions

In this paper, the limitations of the existing methods for solving fuzzy multi-objective transportation problems are pointed out and to overcome the limitations of the existing methods, a new method named as Mehar's method is proposed for solving FFMOTP. To show the advantages of the proposed method over existing methods, some FFMOTP

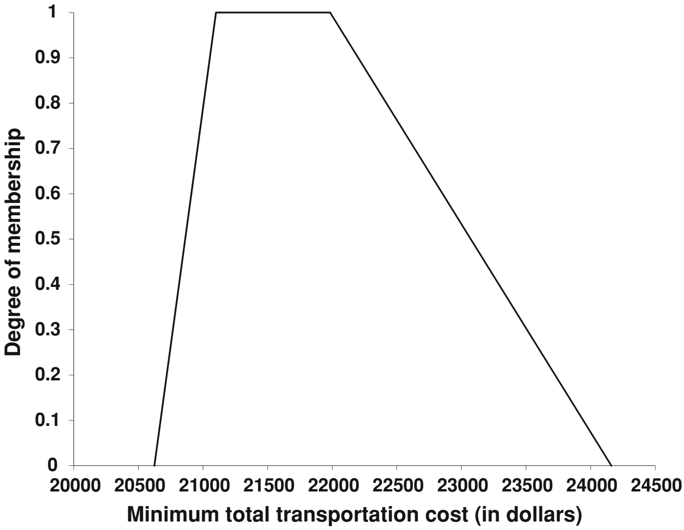

Fig. 1 Membership function of fuzzy number representing the minimum total transportation cost

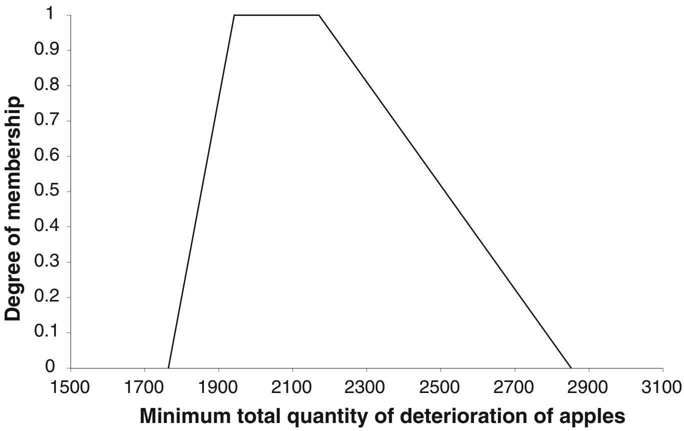

Fig. 2 Membership function of fuzzy number representing the minimum total quantity of deterioration of apples

are solved by using the existing methods and proposed method and the obtained results are compared. Also a FFMOTP occurring in real life situations is solved by using the proposed method.

Acknowledgments The authors would like to thank the Editor-in-Chief "Prof. Pardalos" and anonymous referees for the various suggestions which have led to an improvement in both the quality and clarity of the paper. I, Dr. Amit Kumar, want to acknowledge the innocent blessings of Mehar. I believe that Mehar is an angel for me and without Mehar's blessing it was not possible to think the idea proposed in this paper. Mehar is a lovely daughter of Parampreet Kaur (Research Scholar under my supervision).

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