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SHORT COMMUNICATION

A note on two-machine no-wait flow shop scheduling with deteriorating jobs and machine availability constraints

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Abstract This paper is concerned with two-machine no-wait flow shop scheduling problems in which the actual processing time of each job is a proportional function of its starting time and each machine may have non-availability intervals. The objective is to minimize the makespan. We assume that the non-availability intervals are imposed on only one machine. Moreover, the number of non-availability intervals, the start time and end time of each interval are known in advance. We show that the problem with a single non-availability interval is NP-hard in the ordinary sense and the problem with an arbitrary number of non-availability intervals is NP-hard in the strong sense.

Keywords Scheduling · Flow shop · Deteriorating jobs · Makespan

1 Introduction

The common assumptions in the classical scheduling problems are that the processing times of jobs are constant and the machines are available at all the time. However, there are many situations where machines are with availability constraints and the processing time of the job may be subject to change due to job deterioration.

Scheduling with deteriorating jobs was introduced by Browne and Yechiali [2]. Since then, scheduling problems with time-dependent processing times have received increasing attention, see for example [1,3,6,8,9,12,14–16,19,20].

In many realistic situations, machines may be unavailable for processing jobs after a breakdown or during a preventive maintenance activity. Two cases are usually considered for scheduling problem with availability constraints. If a job cannot be finished

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before the next down period of a machine and the job can continue after the machine becomes available again, it is called *resumable*. On the other hand, it is called *non-resumable* if the job has to restart rather than continue. Lee [13] studied the two-machine flow shop scheduling with the resumable assumption and an availability constraint imposed on only one machine. He developed a dynamic programming algorithm and heuristic algorithms to solve the problem. Espinouse et al. [5] considered the two-machine no-wait flow shop scheduling with availability constraints. They showed that the problem with resumable assumption is NP-hard in the ordinary sense. Wang and Cheng [17] studied the same problem and provided two heuristic algorithms. Cheng and Liu [4] considered two-machine no-wait flow shop scheduling with availability constraints. No-wait means that each job must be processed from start to finish, without any interruption on or between machines. They presented a polynomial time approximation scheme for the problem.

There are also a few studies on scheduling with deteriorating jobs and machine availability constraints. Ji et al. [10] considered the problem of scheduling deteriorating jobs on a single machine with an availability constraint. The objectives are to minimize the makespan and the total completion time. They showed that both problems with nonresumable assumption are NP-hard in the ordinary sense and presented dynamic programming algorithms. Gawiejnowicz [7] studied single machine scheduling problems with deteriorating jobs and availability constraints. He proved that these problems with nonresumable assumption are NP-hard in the ordinary sense or in the strong sense, depending on the number of non-availability periods. Wu and Lee [18] considered single machine scheduling with deteriorating jobs and an availability constraint. They showed that the makespan minimization problem with resumable assumption can be solved by transformation of it into 0-1 integer programming problem.

In this paper we consider two-machine no-wait flow shop scheduling under resumable assumption with deteriorating jobs and machine availability constraints.

2 Notation and preliminaries

We first introduce the notation used in the sequel.

$t_0 > 0$: the common starting time of the machines;
M_1, M_2: machine 1 and machine 2;
$J_1, \dots, J_n$: the set of deteriorating jobs to be processed;
$\alpha_{i,j}$: deterioration rate for J_j on $M_i (i = 1, 2)$, $\alpha_{i,j} \geq 0$ for $1 \leq j \leq n$ and $1 \leq i \leq 2$;
$I_q = (s_q, t_q)(q = 1, \dots, m)$: non-availability intervals, i.e., M_1 (or M_2) is unavailable from s_q to t_q, where m is a positive integer;
$\pi = [J_{[1]}, J_{[2]}, \dots, J_{[n]}]$: a schedule, where $J_{[j]}$ is the jth job in π;
$C_j(\pi)$: completion time of J_j on M_2 in π;
$C_{\max}(\pi)$: makespan of π.

In our problem, we assume that (1) all jobs have to be processed continuously without waiting between consecutive machines, (2) the non-availability intervals are known in advance and one machine is always available, (3) jobs are resumable, i.e., if a job cannot be finished before the non-availability interval of a machine then the job can continue after the machine becomes available again, (4) the actual processing time of a deteriorating job J_j on M_i is $p_{i,j} = \alpha_{i,j}t$, where $t \geq t_0$ is the starting time of J_j, (5) all parameters are positive integers.

Adopting the notation introduced by [13], we denote the problem by $F2|\alpha_{ij}t$, *no-wait*, $r - a(M_i)|C_{\max}$, i.e., to minimize the makespan in a two-machine no-wait flow shop under resumable assumption with deteriorating jobs and availability constraints imposed on M_i, here $r - a$ in the second field denotes *resumable availability constraints*.

3 A single non-availability interval

In this section, the first main result derived is presented. The following lemma will be used in the sequel.

Lemma 1 ([14]) *For the single machine scheduling problem* $1|\alpha_j t|C_{\max}$*, if* $\pi = [J_{[1]}, J_{[2]}, \ldots, J_{[n]}]$*, the starting time of job* $J_{[1]}$ *is* t_0*, then makespan is*

$$C_{\max}(\pi) = t_0 \prod_{j=1}^{n} (1 + \alpha_{[j]}).$$

Theorem 1 *The problem* $F2|\alpha_i t$*, no-wait,* $r - a(M_2)|C_{\max}$ *with a single non-availability interval is NP-hard in the ordinary sense.*

Proof We show the result by reducing the Subset Product problem (SP), which is NP-hard in the ordinary sense [11], to our problem in polynomial time. An instance I of the Subset Product problem can be formulated as follows.

Given a finite set $S = \{1, 2, \ldots, k\}$, a size $x_j \in Z^+$ for each $j \in S$, and a positive integer A, does there exist a subset $T \subseteq S$ such that the product of the sizes of the elements in T satisfies $\prod_{j \in T} x_j = A$?

We set $D = \prod_{j \in S} x_j = AB$. Without loss of generality, it can be assumed that $\min\{A, B\} \geq \max\{x_j\}$ and $x_j \geq 2$ for every $j \in S$ (since element $j \in S$ with $x_j = 1$ will not affect the product of any subset).

For any given instance I of the SP, we construct the corresponding instance II of $F2|\alpha_i t$, *no-wait*, $r - a(M_2)|C_{\max}$ as follows:

- The common starting time of machines: $0 < t_0 \leq \min\{A, B\}$.
- Number of jobs: $n = k + 1$.
- Jobs' deterioration rates: $\alpha_{1,j} = x_j - 1, \alpha_{2,j} = 1, 1 \leq j \leq n - 1; \alpha_{1,n} = A - 1, \alpha_{2,n} = 1$.
- non-availability interval: $I = (2t_0 A, t_0 A^2)$.
- Threshold: $G = 2t_0 A^2 B$.

Obviously, the reduction can be done in polynomial time. Now we prove that the instance I has a solution if and only if the instance II has a solution.

If I has a solution, there exists a subset $T \subseteq S$ such that $\prod_{j \in T} x_j = A$. Denote by $\mathcal{J}_T(\mathcal{J}_{S \setminus T})$, the set of jobs corresponding to the elements of set $T(S \setminus T)$. As a result, a schedule $\pi = [\mathcal{J}_T, J_n, \mathcal{J}_{S \setminus T}]$ is obtained, we denote the last job in $\mathcal{J}_{S \setminus T}$ as $J_k, k \in \{1, 2, \ldots, n-1\}$. From Lemma 1 the makespan is

$$\begin{aligned} C_{\max} &= t_0 \prod_{J_j \in \mathcal{J}_T} (1 + \alpha_{1,j})(1 + \alpha_{1,n}) \prod_{J_j \in \mathcal{J}_{S \setminus T}} (1 + \alpha_{1,j})(1 + \alpha_{2,k}) \\ &= 2t_0 A^2 B = G. \end{aligned}$$

Hence, instance II has a solution.

Conversely, if II has a solution, there exists a schedule π such that $C_{\max}(\pi) \le G$. Since $\prod_{j=1}^{n}(1 + \alpha_{1,j}) = A^2 B$ and $\alpha_{2,j} = 1, 1 \le j \le n$, equality $C_{\max}(\pi) = G = 2t_0 A^2 B$ must hold. Hence, all jobs are processed without idle time on M_1 from t_0. Suppose $\pi = [\mathcal{J}_T, J_n, \mathcal{J}_{S \setminus T}]$ where T is a subset of S.

Since there is no-wait constraint and the deterioration rates are integers, then starting times and completion times on both machines are integers. From this it follows that there cannot be idle time on M_1, no job can start on M_2 during the interval $[2t_0 A + 1, t_0 A^2 - 1]$. Hence, there must exist one job which starts at or before $t_0 A$ and finishes at or after $t_0 A^2$ on M_1, and this is job J_n. By the no idle and no-wait constraints, job J_n must start exactly at $t_0 A$ and finish at $t_0 A^2$ on M_1. We assume the last job is $J_k, k \in \{1, 2, \ldots, n-1\}$. Note that $\alpha_{2,k} = 1$ and $C_{\max}(\pi) = G = 2t_0 A^2 B$, job J_k must start at time $t_0 A^2 B$ on M_2, i.e., the completion time of job J_k on M_1 is $t_0 A^2 B$. Based on the above discussion, the starting time of job J_n on M_1 is $t_0 A$, and there exist two time intervals $[t_0, t_0 A]$ and $[t_0 A^2, t_0 A^2 B]$ on M_1, such that jobs $J_1, J_2, \ldots, J_k$ are processed in these intervals. Denote by $\mathcal{J}_T$ the set of jobs processed in time interval $[t_0, t_0 A]$ on M_1, we have

$$\prod_{J_j \in \mathcal{J}_T} (1 + \alpha_{1,j}) = \prod_{j \in T} x_j \le A.$$

If $\prod_{J_j \in \mathcal{J}_T}(1 + \alpha_{1,j}) < A$, then there is idle time between jobs in $\mathcal{J}_T$ and J_n and $\prod_{J_j \in \mathcal{J}_{S \setminus T}}(1+\alpha_{1,j}) > B$. Therefore, $C_{\max}(\pi) > t_0 A^2 \prod_{J_j \in \mathcal{J}_{S \setminus T}}(1+\alpha_{1,j})(1+\alpha_{2,k}) > 2t_0 A^2 B$. Hence $\prod_{J_j \in \mathcal{J}_T}(1 + \alpha_{2,j}) = \prod_{j \in T} x_j = A$, and we get a solution for I (see Fig. 1). □

Theorem 2 *The problem $F2|\alpha_i t$, no-wait, $r - a(M_1)|C_{\max}$ with a single non-availability interval is NP-hard in the ordinary sense.*

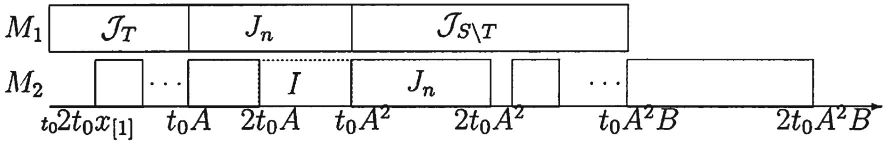

Fig. 1 Example schedule in proof of Theorem 1

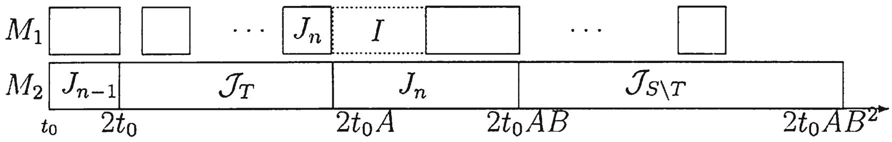

Fig. 2 Example schedule in proof of Theorem 2

Proof Similar to the proof for Theorem 1, we can show the result by reducing the SP problem to our problem. The instance used is as follows:

- The common starting time of machines: $0 < t_0 \leq \min\{A, B\}$.
- Number of jobs: $n = k + 2$.
- Jobs' deterioration rates: $\alpha_{1,j} = 1, \alpha_{2,j} = x_j - 1$, $1 \leq j \leq n-2$; $\alpha_{1,n-1} = 0$, $\alpha_{2,n-1} = 1$; $\alpha_{1,n} = 1, \alpha_{2,n} = B - 1$.
- non-availability interval: $I = (2t_0A, t_0AB)$.
- Threshold: $G = t_0AB^2$.

A schedule which corresponds to Subset Product problem is shown in Fig. 2. We omit the detailed proof here. □

4 An arbitrary number of non-availability intervals

In this section we consider the problem with an arbitrary number of non-availability intervals.

Theorem 3 *The problem $F2|\alpha_i t, no\text{-}wait, r - a(M_2)|C_{\max}$ with an arbitrary number of non-availability intervals is NP-hard in the strong sense.*

Proof We show the result by reducing the 4-Product problem (4-P), which is NP-hard in the strong sense, to our problem in polynomial time. An instance *I* of the 4-P can be formulated as follows.

Given $D \in Q^+$, a finite set $N = \{1, 2, \ldots, 4k\}$ for some natural number k, a size $y_j \in Q^+$ and $D^{\frac{1}{5}} < y_j < D^{\frac{1}{3}}$ for each $j \in N$, $\prod_{j \in N} y_j = D^k$, does there exist disjoint subsets $N_1, N_2, \ldots, N_k$ such that $\cup_{l=1}^{k} N_l = N$ and the product of the sizes of the elements in N_l satisfies $\prod_{j \in N_l} y_j = D$ for $1 \leq l \leq k$?

Without loss of generality, we assume that $y_j \geq 2$ for every $j \in N$.

For any given instance *I* of the 4-P, we construct the corresponding instance *II* of $F2|\alpha_i t, no\text{-}wait, r - a(M_2)|C_{\max}$ as follows:

- The common starting time of machines: $0 < t_0 \leq \min\{A, B\}$.
- Number of jobs: $n = 5k - 1$.
- Jobs' deterioration rates: $\alpha_{1,j} = y_j - 1, \alpha_{2,j} = 1$, $1 \leq j \leq 4k$; $\alpha_{1,j} = D - 1$, $\alpha_{2,j} = 1$, $4k + 1 \leq j \leq n$.
- non-availability intervals: $I_q = (2t_0D^{2q-1}, t_0D^{2q})$, $1 \leq q \leq k$.
- Threshold: $G = 2t_0D^{2k-1}$.

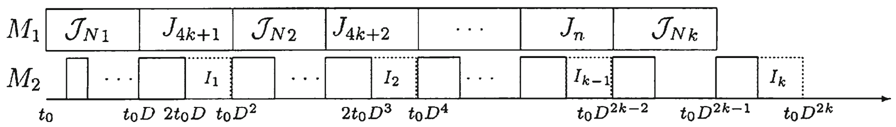

Fig. 3 Example schedule in proof of Theorem 3

Obviously, the reduction can be done in polynomial time. Now we prove that the instance *I* has a solution if and only if the instance *II* has a solution.

If *I* has a solution, there exists disjoint subsets $N_1, N_2, \ldots, N_k$ such that $\cup_{l=1}^{k} N_l = N$ and $\prod_{j\in N_l} y_j = D$ for $1 \le l \le k$. Denote by $\mathcal{J}_{N_l}, 1 \le l \le k$, the set of jobs corresponding to the elements of set N_l. We can construct a schedule $\pi = [\mathcal{J}_{N_1}, J_{4k+1}, \mathcal{J}_{N_2}, J_{4k+1}, \ldots, J_n, \mathcal{J}_{N_k}]$ (see Fig. 3). We denote the last job in $\mathcal{J}_{N_k}$ as $J_k, k \in \{1, 2, \ldots, n\}$.

From Lemma 1, the makespan is

$$\begin{aligned}
C_{\max} &= t_0 \prod_{J_j\in\mathcal{J}_{N_1}} (1+\alpha_{1,j})(1+\alpha_{1,4k+1}) \prod_{J_j\in\mathcal{J}_{N_2}} (1+\alpha_{1,j}) \\
&\quad \ldots (1+\alpha_{1,n}) \prod_{J_j\in\mathcal{J}_{N_k}} (1+\alpha_{1,j})(1+\alpha_{2,k}) \\
&= t_0 \left[\prod_{l=1}^{k} \prod_{J_j\in\mathcal{J}_{N_l}} (1+\alpha_{1,j})\right]\left[\prod_{j=4k+1}^{n} (1+\alpha_{1,j})\right](1+\alpha_{2,k}) \\
&= t_0 D^{k-1} \prod_{l=1}^{k} \prod_{J_j\in\mathcal{J}_{N_l}} (1+\alpha_{2,j}) \\
&= 2t_0 D^{k-1} \prod_{l=1}^{k} \prod_{j\in N_l} y_j \\
&= 2t_0 D^{2k-1} = G.
\end{aligned}$$

Hence, instance *II* has a solution.

Conversely, if *II* has a solution, there exists a schedule π such that $C_{\max}(\pi) \le G$.

Since $\prod_{j=1}^{n}(1+\alpha_{1,j}) = D^{2k-1}$ and $\alpha_{2,j} = 1, 1 \le j \le n$, equality $C_{\max}(\pi) = G = 2t_0 D^{2k-1}$ must hold. Hence, all jobs are processed without idle time on M_1 from t_0. Similar to the analysis in the proof of Theorem 1, there must exist jobs that start at or before $s_q = t_0 D^{2q-1}$ and finish at or after $t_q = t_0 D^{2q}$, $(1 \le q \le k-1)$ on M_1, respectively. Noting that $\alpha_{1,j} = D-1$, $(4k+1 \le j \le n)$, $J_{4k+1}, J_{4k+2}, \ldots, J_n$ must start exactly at moment $s_q = t_0 D^{2q-1}$, $(1 \le q \le k-1)$ on M_1. For convenience, we can also assume that J_{4k+1} starts at $t_0 D$, J_{4k+2} starts at $t_0 D^3, \ldots, J_{n-1}$ starts at $t_0 D^{2k-5}$ and J_n starts at $t_0 D^{2k-3}$ on M_1.

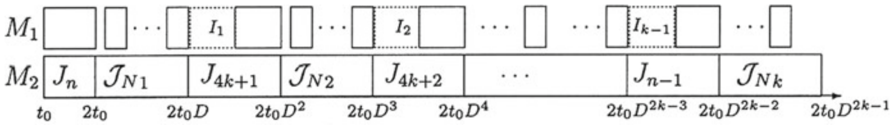

Fig. 4 Example schedule in proof of Theorem 4

Since there is no-wait constraint and $\alpha_{2,j} = 1(4k+1 \le j \le n)$, then J_{4k+q} starts at $t_0 D^{2q-1}(q = 1, 2, \ldots, k-1)$ and finishes at $t_0 D^{2q}$ on M_1.

From $C_{\max}(\pi) = G = 2t_0 D^{2k-1}$, there are k time intervals on M_1 : $[t_0, t_0 D]$, $[t_0 D^2, t_0 D^3], \ldots, [t_0 D^{2k-2}, t_0 D^{2k-1}]$. The jobs $J_1, J_2, \ldots, J_{4k}$ must be processed in these intervals. Denote by $\mathcal{J}_{N_l}$ the set of jobs processed in time interval $[t_0 D^{2l-2}, t_0 D^{2l-1}](1 \le l \le k)$ on M_1. Since $\prod_{i=1}^{4k}(1+\alpha_{2,j}) = D^k(\prod_{j\in N} y_j = D^k)$ and all sets of jobs are disjoint, we have

$$\prod_{J_j \in \mathcal{J}_{N_l}} (1+\alpha_{2,j}) = \prod_{j\in N_j} y_j = D$$

for $1 \le l \le k$. Hence we get a solution for I (see Fig. 3). □

When the availability constraint is only imposed on M_1 and M_2 is always available, we have a similar result.

Theorem 4 *The problem $F2|\alpha_i t, no\text{-}wait, r-a(M_1)|C_{\max}$ with an arbitrary number of non-availability intervals is NP-hard in the strong sense.*

Proof Similar to the proof for Theorem 3, we can show the result by reducing the 4-Product problem to our problem. The instance used is as follows:

- The common starting time of machines: $0 < t_0 \le \min\{A, B\}$.
- Number of jobs: $n = 5k$.
- Jobs' deterioration rates: $\alpha_{1,j} = 1, \alpha_{2,j} = y_j - 1, 1 \le j \le 4k; \alpha_{1,j} = 1, \alpha_{2,j} = D-1, 4k+1 \le j \le n-1; \alpha_{1,n} = 0, \alpha_{2,n} = 1$.
- non-availability intervals: $I_q = (2t_0 D^{2q-1}, t_0 D^{2q}), 1 \le q \le k-1$.
- Threshold: $G = 2t_0 D^{2k-1}$.

A schedule which corresponds to the 4-Product problem is shown in Fig. 4. We omit the detailed proof here. □

5 Conclusions

We considered two-machine no-wait flow shop scheduling to minimize the makespan with deteriorating jobs and machine availability constraints. We assumed that the actual processing time of the job is a simple linear increasing function of its starting time and the non-availability intervals are imposed on only one machine. For the resumable model, we have shown that the problem with a single non-availability interval is NP-hard in the ordinary sense and the problem with an arbitrary number of non-availability intervals is NP-hard in the strong sense, respectively. These results still hold for both non-availability intervals on first machine and second machine.

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