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Improved stress release model: Application to the study of earthquake prediction in Taiwan area*

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Abstract

Stress release model used to be applied to seismicity study of large historical earthquakes in a space of large scale. In this paper, we improve the stress release model, and discuss whether the stress release model is still applicable or not in the case of smaller spatio-temporal scale and weaker earthquakes. As an example of testing the model, we have analyzed the $M \geq 6$ earthquakes in recent about 100 years. The result shows that the stress release model is still applicable. The earthquake conditional probability intensity in Taiwan area is calculated with the improved stress release model. We see that accuracy of earthquake occurrence time predicted by the improved stress release model is higher than that by Poisson model in the test of retrospect earthquake prediction.

Key words: improved stress release model; conditional probability intensity; Poisson model; Taiwan area
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Introduction

Stress release model (SRM) was proposed by Vere-Jones (1978) for statistical study of seismicity. Physically it is a stochastic version of the elastic rebound theory of earthquake genesis. The classical elastic rebound model suggests that the stress has been slowly accumulating until the burst of an earthquake occurrence for stress release. This can be simulated by the jump Markov process in stochastic field, and SRM was developed on the basis of Knopoff's Markov model (Knopoff, 1971). Vere-Jones (1988) applied SRM to historical earthquake catalog for North China and obtained some interesting results. Zheng and Vere-Jones (1991, 1994) further studied SRM in detail and provided a detailed computational algorithm, and achieved good results in practical use. Although Zheng and Vere-Jones (1991) divided North China into 4 seismic zones and calculated the parameters of SRM with the zones combined, they did not take into account the interaction between seismic zones and still adopted the simple SRM. On the basis of their study, SHI, *et al* (1998) and LIU, *et al* (1998) investigated the application of SRM to synthetic earthquakes, and found that the SRM is a good model when being applied to the entire system, but it behaves de-

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graded when applied to a region as only a part of the entire system because of neglecting the influences of stress changes produced by the earthquakes occurred outside the region. They therefore proposed the coupled stress release model (CSRM) as an improvement with the inclusion of terms accounting for the influences of stresses interaction between the earthquakes occurred in different regions. LIU, *et al* (1998) applied CSRM to historical M>6.0 earthquakes from 1480 to 2000 in North China, and compared the results obtained from both CSRM and SRM models by AIC criterion. They found that CSRM is superior to SRM, ad hoc significantly raised the earthquake occurrence probability before some main earthquakes.

So far, application of SRM and CSRM are limited to analyzing historical large earthquakes only. Can we apply the SRM to a case of smaller area and shorter period of time? For example, can we apply it to analysis of earthquakes occurred within a period of a hundred years or less and inside a region of several hundred kilometers in size, to make mid-term earthquake prediction? In this paper, we will improve the method and inspect the scope of its applicability, trying to make earthquake prediction with the calculated earthquake occurrence conditional probability intensity in the area.

1 Brief introduction to SRM

1.1 Simple stress release model

Vere-Jones (1988) pointed out that, if regional stress level is simplified as a scalar function $X(t)$, its variation with time can be expressed as

$$X(t)=X(0)+\rho t-S(t) \tag{1}$$

where $X(t)$ is a scalar stress function in the studied area, $X(0)$ is the initial stress level, ρ is a constant loading rate under external tectonic action, and $S(t)$ is the sum of released stress due to all earthquakes in the time interval (0, t).

$$S(t)=\sum_{0<t_i<t} S_i \tag{2}$$

Suppose stress drop is related to earthquake magnitude only, then the stress S_i released by the i-th earthquake can be expressed as

$$S_i=10^{0.75(m_i-m_0)} \tag{3}$$

where m_i is the earthquake magnitude, and m_0 is a constant for normalized magnitude. We assume that the conditional probability intensity λ is proportional to the exponential of stress as

$$\lambda=\exp\{a+b[t-cS(t)]\} \tag{4}$$

where a, b, and c are model parameters, which can be estimated by maximizing the log-likelihood $\ln L(0, t)$ over the time interval (0, t).

$$\ln L=\sum_{i=1}^{N(T)}\ln\lambda(t_i)-\int_0^T\lambda(u)\mathrm{d}u \tag{5}$$

where T is the observation period of time (suppose t=0 at beginning), $N(T)$ is the number of earthquakes occurred in this period of time, and t_i is the occurrence time of the i-th earthquake.

Being different from Poisson model, in which earthquake risk is regarded as a constant regardless of stress change, SRM suggests that probability of earthquake occurrence varies with time as stress changes, and assumes that earthquake occurrence probability increases with stress exponentially.

1.2 Improved stress release model including stress interactions

On the basis of simple SRM, SHI, *et al* (1998) and LIU, *et al* (1998) proposed the coupled stress release model (CSRM), the physical idea of which is taking into account the interaction between earthquakes occurred in different regions. Therefore, supposing a simple case of only two regions in the system, the conditional probability intensity λ_1 and λ_2 in region 1 and region 2 can be described as:

$$\begin{cases} \lambda_1 = \exp\{a_1 + b_1[t - c_{11}S_1(t) - c_{12}S_2(t)]\} \\ \lambda_2 = \exp\{a_2 + b_2[t - c_{21}S_1(t) - c_{22}S_2(t)]\} \end{cases} \tag{6}$$

where $S_1(t)$ and $S_2(t)$ are accumulation of released stress in the region 1 and region 2, respectively, and can be calculated using equations (2) and (3). a_1, b_1, c_{11}, c_{12}, a_2, b_2, c_{21} and c_{22} are model parameters, where c_{12} is the influence coefficient of the earthquakes in regions 2 on region 1, and can be either positive or negative; while c_{21} is that of the earthquakes in region 1 on region 2. These model parameters can also be calculated by maximum likelihood from earthquake catalogue.

Here we propose an improved stress release model (ISRM), which makes use of the essence of the coupled stress released model and divides the entire system into an inner region and an outer region. ISRM only studies earthquake probability intensity in the inner region, but considers the effects of stresses released in outer region. Here

$$\lambda = \exp\{a_1 + b_1[t - c_1S_1(t) - c_2S_2(t)]\} \tag{7}$$

where S_1 is the sum of stress released in the inner region, usually $c_1 \geq 0$; and S_2 is the sum of stress released from outer region, c_2 is not restricted in sign, being either positive or negative. All parameters in formula (7) can be calculated in the same way as in (6). Generally, a circle region is taken as the inner region, the radius of which is empirically chosen to be about 300 km through practical optimization trials for Taiwan earthquakes with $M \geq 6.0$.

1.3 AIC criterion

How well the stochastic model is judged by AIC (Akaike Information Criterion) proposed by Akaike (1977). Suppose $\ln L$ is log-likelihood of the model, then

$$\text{AIC} = -2\ln L + 2k \tag{8}$$

where k is the number of parameters used in the model. The smaller the AIC calculated for a model, the better the model is. In general, if the difference between two models is above 5% confidence level, with the corresponding difference of AIC being 1.5~2, we can tell which one is better between two models.

As for ISRM, the total parameters are 4, including 1 for inner region and 3 for outer region, that is $k=4$,

$$\text{AIC} = -2\ln L + 2\times 4 \tag{9}$$

2 Prediction method and calculated results

2.1 Sub-region division and earthquake selection

From physical point of view, SRM is applicable to large earthquakes, which can change the system stress significantly. Analysis of earthquake data has verified it (LIU, *et al*, 1998). Then, is SRM still applicable to smaller areas and moderate strong earthquakes? We try to estimate earthquake risk with SRM for the case of moderate strong earthquakes in a smaller area and shorter period. In this paper, we further improve the model based on the idea of ISRM. According to latest studies on stress triggering and stress shadow (Harris, 1998), we take a circular area with 300 km radius as the inner region, within which, an earthquake of magnitude 6 can produce detectable stress change in the system. The areas out of the circle are regarded as the outer region. Earthquakes in the outer region, if magnitude is great enough, can also affect stress change in the inner-region. Apparently, for earthquakes in the outer region, the nearer the epicenters to the center of the inner region, the larger the influence that they produce on the inner region is, and *vice versa*. As analysis data we select earthquakes with M>6.0 in inner region and $M\geq 6.0\times(r/300)^{1/3}$ in outer region from 1900 through 2000, where r is distance between the epicenter and the center of the inner region. Choice of the model parameters is made through trial for optimization.

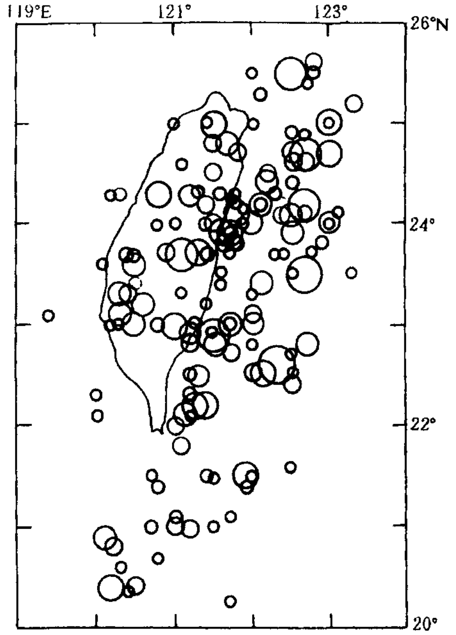

Figure 1 Spatial distribution of earthquakes ($M_S\geq 6.0$) in Taiwan area during 1999~2000

Figure 1 shows earthquake spatial distribution in Taiwan and its adjacent area and Figure 2 is their M-t plot. Based on the spatial distribution of earthquakes, 10 grid points are chosen between the latitude 21°N to 25°N, and longitude 120°E to 122°E. They are (21°N, 120°E), (22°N, 120.3°E), (23°N, 120°E), (24°N, 120°E), (25°N, 120.5°E), (21°N, 122°E), (22°N, 122°E), (23°N, 122°E), (24°N, 122°E), and (25°N, 122°E). Circular areas of 300 km radius centered at these points are regarded as the inner regions, and areas out of the circle are defined as the outer regions. A rule of thumb in selecting an area is to make sure that the number of earthquakes must be large enough for statistics. The 10 circular areas overlap partly and cover the entire studied area for CSRM application. To calculate the influence of the earthquakes in the outer region on the inner region, effect of distance from the epicenter to the inner region must be considered. We introduce a modified $M'=M/(300/r)^{1/3}$ to calculate such effect, the function being similar to that of selecting earthquakes in the outer region according to equation (3).

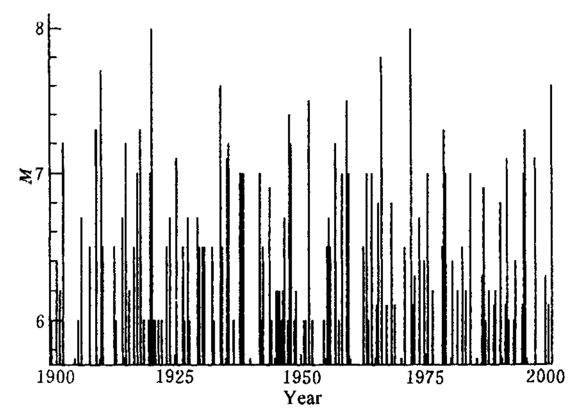

Figure 2 M-t plot of $M\geq 6$ earthquakes in Taiwan area

Figure 3a shows the typical curves of conditional probability intensity variation with time obtained by Poisson model, SRM and ISRM. M-t plot in the inner region and in the outer region is given in Figure 3b, c. The figure shows that the conditinal probability intensity in the inner region usually increases with time, however the earthquake occurred in the inner region can decrease the intensity suddenly. The greater magnitude the earthquake has, the larger the intensity decreases. That means the occurrence of earthquakes in the inner region reduces the earthquake risk there. In the same way, the earthquake occurrence conditional probability intensity in the inner region in ISRM decreases immediately whenever an earthquake occurs in either inner or outer region, although sometimes earthquake in outer region may raise the probability intensity in inner region when $c_{12}<0$. Moreover, Table 1 shows that AIC values obtained for ISRM are smaller than those for SRM in corresponding regions with one exception.

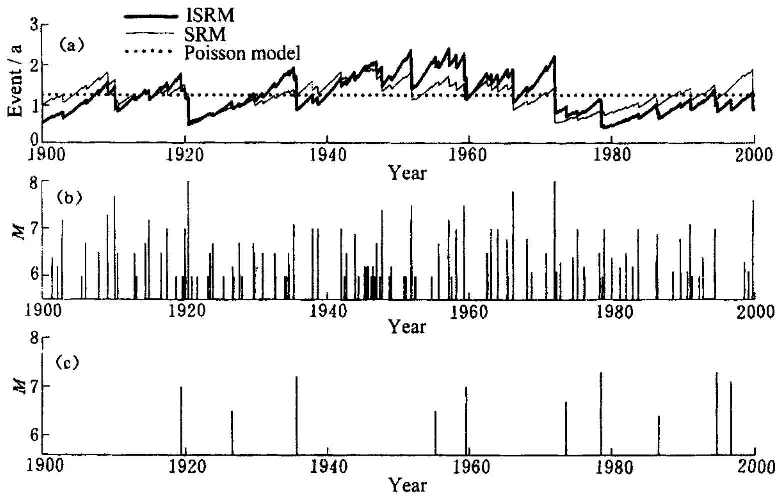

Figure 3 Earthquake M-t plot and variation of conditinal probability intensity with time
(a) Variation of conditional probability intensity with time for Poisson model, SRM and ISRM;
(b) M-t plot in the inner region; (c) M-t plot in the outer region

Table 1 The AIC value and R-score calculated for Poisson model, SRM and ISRM in Taiwan area

Location	Model	AIC	R-score	Location	Model	AIC	R-score
(21°N, 120°E)	Poisson	182.1	0	(21°N, 122°E)	Poisson	194.5	0
	SRM	178.9	0.067		SRM	193.6	0.028
	ISRM	162.8	0.116		ISRM	169.1	0.071
(22°N, 120.3°E)	Poisson	202.1	0	(22°N, 122°E)	Poisson	192.0	0
	SRM	190.8	0.140		SRM	186.2	0.062
	ISRM	187.6	0.185		ISRM	179.7	0.130
(23°N, 120°E)	Poisson	197.8	0	(23°N, 122°E)	Poisson	183.1	0
	SRM	186.7	0.116		SRM	176.2	0.125
	ISRM	178.6	0.171		ISRM	172.9	0.127
(24°N, 120°E)	Poisson	202.6	0	(24°N, 122°E)	Poisson	187.6	0
	SRM	200.1	0.007		SRM	178.8	0.151
	ISRM	189.5	0.126		ISRM	180.6	0.125
(25°N, 120.5°E)	Poisson	198.7	0	(25°N, 122°E)	Poisson	194.6	0
	SRM	193.9	0.133		SRM	189.4	0.094
	ISRM	186.0	0.148		ISRM	184.7	0.097

2.2 Earthquake prediction R-score

Previous work puts emphasis on theoretical study on SRM. Because of the much smaller spatio-temperal scale dealt with in this study than before, it is possible to apply SRM to long-term or even mid-term earthquke prediction. We make retrospect test of R-score with conditional probability intensity in the inner region calculated by using SRM or by ISRM, where R = (The number of earthquakes predicted) / (The total number of actual earthquakes) – (Sum of predicted time interval)/(The total period of time) (SHI, 1992). The period of predicted earthquake is defined if the conditional probability intensity λ obtained by using SRM is greater than that by Poisson model. The results are shown in Table 1. AIC values for ISRM are smaller than those for SRM, and R-scores for ISRM are larger than those for SRM except one. The average R-score for SRM is 0.092, whereas it is 0.13 for ISRM. Although the value is only marginally greater than 0, the result may still be useful, since the average R-score is only 0.18 in China's annual earthquake prediction (1990~1998) (SHI, *et al*, 2002).

2.3 Prediction of occurrence time by SRM

SRM can be used to predict occurrence time of earthquakes in the following way: Suppose we know the model parameters such as a, b, c and the λ_0 ($t=t_0$) in SRM, which were calculated from the data of earthquake series already occurred. The earthquake occurrence probability from t_0 to $t_0+\delta t$ can be calculated by the formula $\lambda=\exp\{a+b[t-cS(t)]\}$, then a random number is produced by computer. Whether an earthquake would occur or not is determined by whether the random number is less than the occurrence probability in that period of time or not. If an earthquake occurs, its magnitude is determined randomly by following frequency-magnitude relation for a given b-value in that region. If earthquake does not occur, the time step is given again. This process will go on until an earthquake occurs. Monte Carlo tests of this kind repeat many times, then the average of these earthquake occurrence times is regarded as the main shock occurrence time after t_0.

As for ISRM we have

$$\begin{aligned}\lambda(t_0+\delta t) &= \exp\{a_1+b_1[(t_0+\delta t)-c_1S_1(t_0+\delta t)-c_2S_2(t_0+\delta t)]\}\\ &= \lambda_0\exp\{b_1[\delta t-c_1S_1(\delta t)-c_2S_2(\delta t)]\}\end{aligned} \tag{10}$$

Being different from SRM, if an earthquake occurs in the outer region after t_0 in ISRM, $S_2(t)$, the sum of stress released in the outer region, is computed according to formula (2) and (3). The stress drop is calculated in the same way as above. The method of predicting earthquakes is also the same as that in SRM.

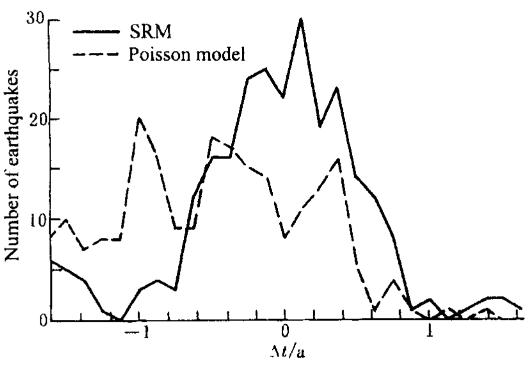

Figure 4 Difference between earthquake occurrence time predicted by ISRM and by Poisson model and the number of earthquakes predicted

In order to test the effectiveness of this method, this paper retrospectively predicts earthquakes in Taiwan area in every 5 years from 1905 to 1995, and compares the predicted with the actual case. At the same time, a similar test is carried out for Poisson model. The results are shown in Figure 4. The abscissa axis in Figure 4 represents Δt, the difference between predicted and actual earthquake occurrence time, and ordinate axis is the number of earthquake occurrence time predicted by Poisson model and SRM, respectively, in that period of difference time. The figure shows clearly that the prediction by SRM is better than that by Poisson model. The square root of the difference between earthquake occurrence time predicted by Poisson model and actual occurrence time is 1.40 years, whereas it is 0.74 years by SRM. It is expected that the next earthquake with $M>6$ can be predicted more accurately.

3 Discussion

A key problem in this paper is how to estimate the effect on stress change in inner region produced by a large earthquake in outer region. Physically the stronger the earthquake, the larger influence the earthquake produces. However, the farther the earthquake locates, the less influence it acts on the inner region. This is a reasonable rule. Considering this spatial characteristics, the form of earthquake influence attenuation is selected according to $(r/r_0)^{1/3}$ (Sokolnikoff, 1956). In computation, r_0 is picked through optimization after r_0 is taken as 300 km, 400 km, and 500 km. Also, the attenuation coefficient 1/3 is the optimum result after trial of 1/2, 1/2.5, 1/3, and 1/4.

Theoretically, earthquakes with different type of faults (normal, thrust, strike-slip), but the same magnitude, produce different stress release in the studied area. Different location and strike of the fault also affect the stress change. By no means can SRM evaluate the influence of a single earthquake because it is a statistical model. This is its limitation. The advantage of using statistical model lies in that we can study the overall tectonic activity without knowing the mechanics in detail. Stochastic model and mechanical model each has its own advantage. It is necessary to study both of them in future and combine the SRM with analysis of mechanical process.

Although we only show application of the Monte Carlo method to predict earthquake occurrence time, magnitude of earthquakes can be predicted in similar way. After further study on earthquake spatial distribution, the epicenter can also be predicted statistically. All of these open a new way to quantify the estimation of strong earthquake risk.

The conclusion above is so far based on the application to Taiwan area only, but this study is worth to be extended to other areas. There are many details to be settled down to make practical earthquake prediction with ISRM. For ISRM in dealing with influence of the outer region on the inner region, it is worthwhile to utilize knowledge related to active tectonics and the real stress distribution. Meanwhile, mathematical expressions for earthquake occurrence time and magnitude should be worked out from conditional probability intensity provided by statistical method.

4 Conclusions

From this study we conclude that improved stress release model (ISRM) is applicable to the case of smaller spatio-temporal scale and moderate strong earthquakes. The model parameters can be estimated from analyzing earthquake catalog by maximum likelihood method. The variation of conditional probability intensity curve is consistent with that given by previous researchers, suggesting that the physical essence of the ISRM fits with reality. The AIC values calculated by ISRM are smaller than those by SRM in the case of smaller spatio-temporal scale and moderate strong earthquakes. This indicates that ISRM is superior to SRM in this case. Moreover, R-score obtained by ISRM is greater than that by SRM, suggesting that the efficacy of earthquake prediction in ISRM is higher than that in SRM and Poisson model. In a word, ISRM is superior to SRM. Besides, the efficiency of earthquake prediction in ISRM is higher than that in SRM and Poisson model. In retrospective test of earthquake prediction, the earthquake occurrence time predicted by ISRM is closer to the actual occurrence time than by Poisson model. The accuracy of prediction by ISRM almost doubles that by Poisson model.

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