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ORIGINAL PAPER

Supply chain coordination with auctions

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Abstract Supply chain is a decentralized system where material, financial and information flows connect economic agents. There is much inefficiency in supply chain behavior. Recently, considerable attention of researchers is drown to provide some incentives to adjust the relationship of supply chain agents to coordinate the supply chain, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. There is a vast literature on supply chain coordination recently. Most of coordination mechanisms are based on game theory models and contracts between agents of the supply chain. However, little work has been done on using auctions for supply chain coordination. Auctions are important market mechanisms for the allocation of goods and services. A complex trading model between layers of the supply chain is proposed in the paper. The model is based on so called multidimensional auctions. There is possible to formulate multidimensional auctions as mathematical programming problems. Iterative methods are used to solve the problems.

Keywords Supply chain · Coordination · Auctions · Complex trading model · Iterative methods

JEL Classification L14 · D44 · C61

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1 Introduction

Supply chain management is about matching supply and demand with inventory management. When one or more agents of the supply chain try to optimize their own profits, system performance may be hurt. Developing strategies to decrease the risk faced by the retailer is becoming more and more critical in a supply chain, especially in the global marketplace where firm-to-firm competition is being replaced by supply-chain-to-supply-chain competition. There is much inefficiency in supply chain behavior. Recent years have seen a growing interest among researchers and practitioners in the field of supply chain management. Most of coordination mechanisms are based on game theory models and contracts between agents of the supply chain. However, little work has been done on using auctions for supply chain coordination. The paper proposes a complex trading model for coordination of agents in supply chain. The model is based on, so called, multidimensional auctions.

Auctions are important market mechanisms for the allocation of goods and services. Multidimensional auctions arise by extensions of standard auction models. Multi-item auctions can place bids on combinations of items, so called combinatorial auctions. The advantage of combinatorial auctions is that the bidder can more fully express his preferences. Multi-type auction model includes forward, reverse and double auctions. Multi-round, so called iterative, methods are used for analysis of combinatorial auctions and for negotiation process. Allowing bidders more fully to express preferences often leads to improved economic efficiency and greater auction profit. The multi-item model for multi-type auction is modeled and solved by multi-round approach. The proposed model illustrates the possibility to formulate and solve multidimensional auctions as mathematical programming problems. The model is based on a linear programming model and its extensions. A solution procedure is presented. The procedure is based on primal-dual algorithms.

The rest of the paper is organized as follows. Section 2 presents the supply chain coordination problem and the possibility to solve the problem by games, contracts and auctions. Section 3 summarizes the basics of auctions. In Sect. 4, a complex trading model based on multidimensional auctions is formulated. Multi-round iterative auctions as a solution approach are presented in Sect. 5. Finally, Sect. 6 provides conclusions.

2 Supply chain coordination

Supply chain is a decentralized system composed from layers of potential suppliers, producers, distributors, retailers and customers etc., where agents are interconnected by material, financial and information flows. A supply chain is the collection of steps that a system takes to transform raw components into the final product. There is much inefficiency in supply chain behavior. When one or more agents of the supply chain try to optimize their own profits, system performance may be hurt.

Supply chain management is a process used by companies to ensure that their supply chain is efficient and cost-effective. Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers. Supply chain management has generated a substantial amount of interest both by managers and by researchers. There are many concepts and strategies applied in designing and managing supply chains (see Simchi-Levi et al. 1999; Harrison et al. 2003). The expanding importance of supply chain integration presents a challenge to research to focus more attention on supply chain modelling (see Tayur et al. 1999; Simchi-Levi et al. 2004; Snyder and Shen 2011).

Analysis and modeling of supply chains goes through the following phases:

- designing,
- managing,
- performance measurement,
- performance improvement.

Designing of supply chains involves establishing the most appropriate system elements and their relationships in time. Managing phase covers the entire hierarchy of management activities, from planning through production scheduling and distribution of products to real-time control. Phases of performance measurement and improvement focus on assessing the current state of the system and propose best approaches to improve its performance.

The most important part of managing phase is the coordination of individual activities to be optimal in terms of the whole system. Supply chains are decentralized systems. A centralized system can be taken as a benchmark. The question is: How to coordinate the decentralized supply chain to be efficient as the centralized one?

We made some experiments with evaluation of different supply network structures. The supplier rarely has complete information about customer's cost

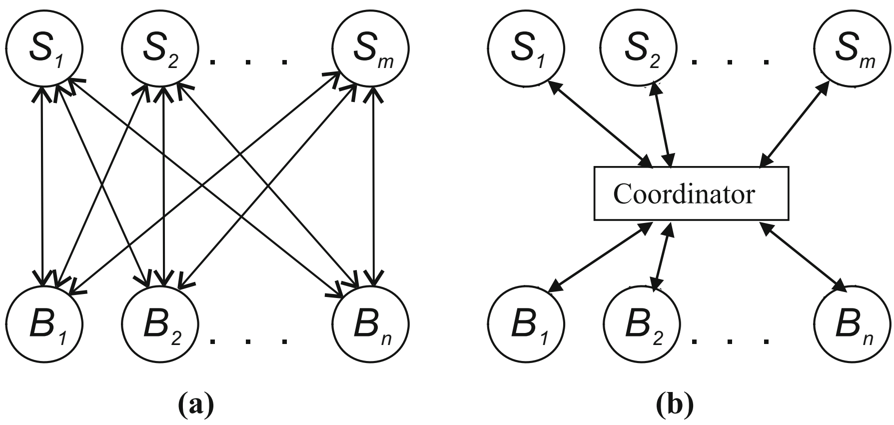

Fig. 1 Decentralized (**a**) and centralized (**b**) seller-buyer relations

structure. However, the quantity the customer will purchase and therefore supplier's profit depend on that cost structure. Somehow, the supplier will have to take this information asymmetry into account. The numbers of suppliers and customers are denoted by m, n, respectively. The symbol S_i represents i-th seller while the symbol B_j represents j-th buyer. The seller-buyer relations in supply chain can be taken as decentralized or centralized (see Fig. 1).

Most supply networks are composed of independent agents with individual preferences. It is expected that no single agent has the power to optimize the supply network. Each agent will attempt to optimize his own preference, knowing that all of the other agents will do the same. This competitive behavior does not lead the agents to choose polices that optimize overall supply chain performance due to supply chain externalities. The agents can benefit from coordination and cooperation. The typical solution is for the agents to agree to a set of transfer payments that modifies their incentives, and hence modifies their behavior. Many types of transfer payments are possible.

The problem of coordination in supply chains involves multiple agents with multiple goals. Coordination between suppliers and customers can be provided through information sharing (schematically see Fig. 2).

A seller S_i and a buyer B_j have information and analytical tools for their problem representations. A coordinator helps by information sharing and by formulation of a joint problem representation (see Fiala 2005).

2.1 Bullwhip effect

The so-called bullwhip effect (see Tayur et al. 1999), describing growing variation upstream in a supply chain, is probably the most famous demonstration of inefficiency in supply chains. There are some known causes of the bullwhip effect: information asymmetry, demand forecasting, lead-times, batch ordering, supply shortages and price variations. Information sharing of customer demand has an impact on the bullwhip effect and other inefficiencies in supply chains (Fiala 2005).

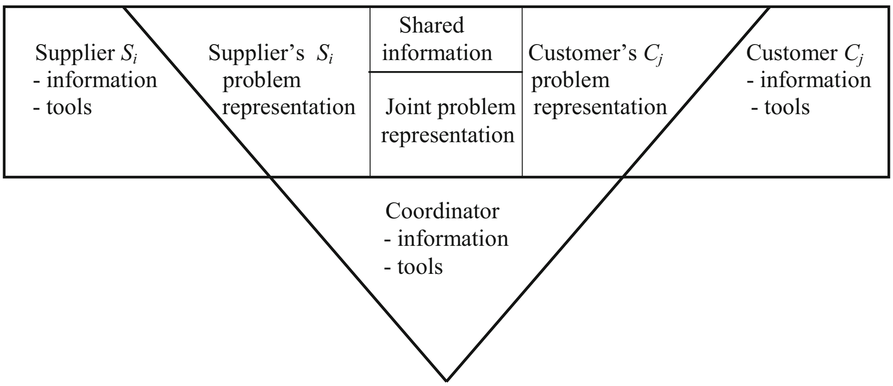

Fig. 2 Coordination through information sharing

The bullwhip effect causes larger inventories along the supply chain and thus increased costs. The analyses of causes and suggestions for reducing the bullwhip effect in supply chains are challenges to modelling techniques. We consider a *k*-layers supply chain. The customer demands are independent and identically distributed random variables. The last layer observes customer demand D and places an order q to previous stage. All layers place orders to the previous layer in the chain. The orders are received with lead-times L_i between layers i and $i+1$. The stages use the moving average forecast model with p observations. To quantify increase in variability, it is necessary to determine the variance of orders q^k relative to the variance of demands D.

In the case of decentralized information the variance increase is multiplicative at each layer of the supply chain

$$\frac{Var(q^k)}{Var(D)} \geq \prod_{i=1}^{k}\left(1+\frac{2L_i}{p}+\frac{2L_i^2}{p^2}\right). \tag{1}$$

In the case of centralized information, i.e. the last layer provides every layer with complete information on customer demand, the variance increase is additive:

$$\frac{Var(q^k)}{Var(D)} \geq 1+\frac{2\left(\sum_{i=1}^{k} L_i\right)}{p}+\frac{2\left(\sum_{i=1}^{k} L_i\right)^2}{p^2} \tag{2}$$

The centralized solution can be used as a benchmark, but the bullwhip effect is not completely eliminated.

A large number of papers have been published that proposed analyze mechanisms for supply chain coordination.

- Games
- Contracts
- Auctions

Researchers in supply chain management now use tools from game theory to help managers to make strategic operational decisions in complex multi-agent supply chain systems. Game theory models situations where players make decisions to maximize their own utility, while taking into account that other players are doing the same, and that decisions made by players, impact others utilities. There is a broad division of game theory into two approaches: the cooperative and the non-cooperative approach. These approaches, though different in their theoretical content and the methodology used in their analysis, are really just two different ways of looking at the same problem. The field of supply chain management has seen, in recent years, a wide variety of research papers that employ game theory to model interaction between players. Cachon and Netessine (2004) provide an excellent survey and state of art especially non-cooperative game techniques. The concept of using non-cooperative agents to formulate allocation mechanisms in a game theoretical setting is closer to the classical market concept than solutions employing cooperative strategies. Nagarajan and Sošić (2008) review the existing literature on applications of cooperative games to supply chain management. They also deal with certain methodological issues when modeling supply chain problems.

The coordination actions in the supply chain are a mix of cooperative and non-cooperative behavior of the agents. Brandenburger and Stuart (2007) introduced a concept of biform games as a combination of non-cooperative and cooperative games. The biform game is a two-stage game: in the first stage, players choose their strategies in a non-cooperative way, thus forming the second stage of the game, in which the players cooperate. The biform game approach can be used for modeling general seller-buyer relationships in supply chains (Hennet and Mahjoub 2008).

Among the solutions, supply chain contracts, which have drawn much attention from the researchers recently (for review Cachon 2003; Tsay et al. 1999; Corbett and Tang 1999), are used to provide some incentives to adjust the relationship of supply chain agents to coordinate the supply chain, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. The format of supply chain contracts varies in and across industries. The particular contract adopted by the firms is the outcome of some negotiation process that could be also modeled. Coordination of individual activities is very important to be optimal in terms of the whole system. Mechanisms based on non-cooperative game theory usually propose establishment of coordinating contracts. A retailer can usually collect demand information easier than a producer and he has a better motivation for optimally determining sales quantities and prices. There are many types of contracts. Contracts are evaluated by desirable features: coordination of the supply chain, flexibility to allow any division of the supply chain's profit, and easy to use.

The basic type is a wholesale price contract. With a wholesale price contract (Lariviere 1999) the supplier charges the retailer w per unit purchased. The producer knows exactly what retailer will order at every wholesale price and bears no responsibility for the product. All uncertainty regarding the producer profit is foisted onto the retailer. The wholesale price contract coordinates the chain only if the producer earns a non-positive profit. So the producer clearly prefers a higher wholesale price. As a result, the wholesale price contract is generally not considered a coordinating contract. The richer contracts differ from wholesale price contracts by allowing the producer to assume some of the risk arising from stochastic demand. As an example we introduce buy back contracts as an extension of wholesale price contracts. With a buy back contract (Pasternack 1985) the producer charges the retailer w per unit purchased, but pays the retailer b per unit remaining at the end of the season. The retailer should not profit from left over inventory, so assume $b \leq w$. There is assumed that a returns policy on the decentralized chain introduces no additional cost beyond that incurred by the centralized system.

Quantity flexibility contracts define terms under which the quantity a retailer ultimately orders from the producer may deviate from a previous planning estimate (Tsay 1999; Lariviere 1999). Backup agreements (Eppen and Iyer 1997) state that if a retailer commits to a number of units for the season, the producer will hold back a fraction of the commitment and the retailer can order up to this backup quantity at the original purchase price after observing early demand. Option contracts (Barnes-

Schuster et al. 2002) specify that in addition to a firm order at a regular price, the retailer can also purchase options at an option price at the beginning of the selling season. Price protection (Lee et al. 2000) states, that the supplier pays the retailer a credit applying to the retailer's unsold goods when the wholesale price drops during the life cycle. The newsvendor problem with pricing is used to model contracts (Yao et al. 2006). Chen and Cheng (2012) presented price-dependent revenue sharing mechanism.

However, little work has been done on using auctions for supply chain coordination. Using of combinatorial auctions is promising for solving the supply chain formation problem (Walsh et al. 2000; Walsh and Wellman 2003). Combinatorial auction mechanism is used as one-shot mechanism. Agents submit bids reporting costs and values, and then the auction computes an allocation that maximizes the reported value and informs the agents of results. An agent pays the price it bids for the allocation it receives. If the auction receives more money than it pays out, the proceeds are distributed evenly among all consumers. Another approach based on auctions is presented in this paper.

3 Auctions

Auctions are important market mechanisms for the allocation of goods and services. Auctions are preferred often to other common processes because they are open, quite fair, and easy to understand by agents, and lead to economically efficient outcomes. Many modern markets are organized as auctions. Design of auctions is a multidisciplinary effort made of contributions from economics, operations research, informatics, and other disciplines. Auction theory has caught tremendous interest from both the economic side as well as the Internet industry. An auction is a competitive mechanism to allocate resources to buyers based on predefined rules. These rules define the bidding process, how the winner is determined, and the final agreement.

3.1 Auction mechanism

The auction mechanism is a process that transforms bids on allocation of objects to winners and determining the payments that must be paid by the buyer and the seller receives (see Fig. 3).

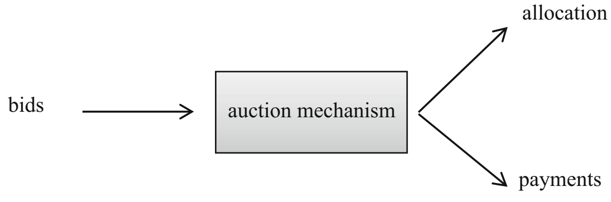

Fig. 3 Auction mechanism

Classification of auctions is based on some specific characteristics as:

- *the numbers of sellers and buyers,*
- *traded items (indivisible, divisible, pure commodities, structured commodities),*
- *the number of items,*
- *the number of units of items,*
- *type of auctions (forward, reverse, double),*
- *evaluating criteria,*
- *preferences of the agents,*
- *complexity of bids (simply, related bids),*
- *organization of auctions (single-round, multi-round, sequential, parallel, price schemes).*

It is distinguished between open and sealed-bid auctions. In open auctions all the bids are observable, while in sealed-bid auctions the bids are not seen. The bids are sent in sealed envelopes or via Internet.

We can also differentiate between ascending and descending price auctions. The ascending auction starts at a low price and bids have to be increasing until the moment with only one bid remaining, whereas the descending auction starts at a high that continuously declines until one of the bidders stops the process by acquiring the object.

There are four basic auction formats: English, Dutch, first-price and second-price (Vickrey) auctions.

The English format is open ascending price auction. The Dutch format is open descending price auction. The first-price format is sealed-bid auction. The person submitting the highest bid wins and pays what he bid. The second-price (Vickrey) format is sealed-bid auction. The person submitting the highest bid wins and pays not what he bid, but the second highest bid.

An auction provides a mechanism for negotiation between buyers and sellers. Multidimensional auctions are a generalization of standard auctions. These auctions can be classified:

- multi-item auction,
- multi-type auction,
- multi-round auction.

Multi-item auctions can place bids on combinations of items, so called combinatorial auctions. Combinatorial auctions (see Rothkopf et al. 1998; de Vries and Vohra 2003; Cramton et al. 2006) are those auctions in which bidders can place bids on combinations of items, so called bundles. The advantage of combinatorial auctions is that the bidder can more fully express his preferences. This is particular important when items are complements. The auction designer also derives value from combinatorial auctions. Allowing bidders more fully to express preferences often leads to improved economic efficiency and greater auction revenues.

There are several types of auctions (forward, reverse, and double). Forward auctions are oriented to the sale, with one seller and multiple buyers. Reverse auctions are oriented to purchase, with only one buyer and multiple sellers. Double auctions combine the two previous types and mediate an exchange between multiple sellers and multiple buyers. There is an effort to propose a general multi-type auction that covers all the types.

In the iterative approach, there are multiple rounds of bidding and allocation and the problem is solved in an iterative and incremental way. Iterative combinatorial auctions are attractive to bidders because they learn about their rivals' valuations through the bidding process, which could help them to adjust their own bids. There are possible combinations of the multidimensional characteristics. However, alongside their advantages, combinatorial auctions raise a host of questions and challenges.

The literature concerning auctions is quite rich. The standard models are based on game theory (Klemperer 2004; Krishna 2002; Milgrom 2004). The popularity of auctions and the requirements of e-business have led to growing interest in the development of complex trading models (see Bellosta et al. 2004; Bichler 2000). The literature concerning applications of auctions is versatile. For example, auctions have been proposed for the distribution of airport arrival and departure time slots (Rassenti et al. 1982), have been used for allocating radio spectrums for wireless communications services (Cramton 2002), truckload transportation (Caplice and Sheffi 2006), bus routes (Cantillon and Pesendorfer 2006), and industrial procurement (Bichler et al. 2006).

4 Complex trading model

We propose a complex trading model based using of iterative process for combinatorial multi-type auctions.

4.1 Multi-item auctions

Many types of combinatorial auctions can be formulated as mathematical programming problems. From different types of combinatorial auctions we present a forward auction of indivisible items with one seller and multiple buyers. Let us suppose that one seller S offers a set R of r items, $j = 1, 2, \ldots, r$, to n potential buyers $B_1, B_2, \ldots, B_n$.

Items are available in single units. A bid made by buyer B_i, $i = 1, 2, \ldots, n$, is defined as

$$b_i = \{C, p_i(C)\},$$

where $C \subseteq R$, is a combination of items, $p_i(C)$, is the offered price by buyer B_i for the combination of items C.

The objective is to maximize the revenue of the seller given the bids made by buyers. Constraints establish that no single item is allocated to more than one buyer.

Binary variables are introduced for model formulation:

$x_i(C)$ is a binary variable specifying if the combination C is assigned to buyer B_i ($x_i(C) = 1$).

The forward auction can be formulated as follows

$$\begin{aligned} & \sum_{i=1}^{n}\sum_{C\subseteq R} p_i(C)x_i(C) \to \max \\ \text{subject to} \quad & \sum_{i=1}^{n}\sum_{C\subseteq R} x_i(C) \leqq 1, \quad \forall j \in R, \\ & x_i(C) \in \{0, 1\}, \quad \forall C \subseteq R, \quad \forall i, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{3}$$

The objective function expresses the revenue. The constraints ensure that overlapping sets of items are never assigned. The problem (3) is called the winner determination problem.

Complexity is a fundamental question in combinatorial auction design. There are some types of complexity:

- computational complexity,
- valuation complexity,
- strategic complexity,
- communication complexity.

Computational complexity covers the expected computation amount of the mechanism to compute an outcome given the bid information of the bidders (Sandholm 2002). This is an extremely important question because winner determination problem is an NP-complete optimization problem. Valuation complexity deals with the required computation amount to provide preference information within a mechanism. Estimating every possible bundle of items requires exponential space and hence exponential time. Bidders need to determine valuations for $2^m - 1$ possible bundle. Strategic complexity concerns the best strategy for bidding. Which of the $2^m - 1$ bundles to bid on? Must bidders model behavior of other bidders and solve problems to compute an optimal strategy? Communication complexity concerns the required communication amount to exchange between bidders and auctioneer until an equilibrium price is reached. The problem of communication complexity can be addressed through the design of careful bidding languages that provide expressive but concise bids. Many researchers consider iterative auctions as an alternative.

4.2 Multi-type auctions

An auction provides a mechanism for negotiation between buyers and sellers. In forward auctions a single seller sells resources to multiple buyers [model (8)]. In a reverse auctions, a single buyer attempts to source resources from multiple suppliers, as is common in procurement. Auctions with multiple buyers and sellers are called double auctions. Auctions with multiple buyers and sellers are becoming increasing popular. It is well known that double auctions in which both sides submit demand or supply bids are much more efficient than several one-sided auctions combined. Attention is devoted to double combinatorial auctions (Xia et al. 2005). Combinatorial double auctions can be transformed to combinatorial single-sided auctions and solved by methods for these auctions. Special case of double auction for one seller is the forward auction and special case of double auction for one buyer is the reverse auction.

We present a reverse auction of indivisible items with one buyer and several sellers. This type of auction is important for supplier selection problem. Let us suppose that m potential sellers $S_1, S_2, \ldots, S_m$ offer a set R of r items, $j = 1, 2, \ldots, r$, to one buyer B.

A bid made by seller S_h, $h = 1, 2, \ldots, m$, is defined as

$$b_h = \{C, c_h(C)\},$$

where $C \subseteq R$, is a combination of items, $c_h(C)$, is the offered price by seller S_h for the combination of items C.

The objective is to minimize the cost of the buyer given the bids made by sellers. Constraints establish that the procurement provides at least set of all items.

Binary variables are introduced for model formulation:

$y_h(C)$ is a binary variable specifying if the combination C is bought from seller S_h ($y_h(C) = 1$).

The reverse auction can be formulated as follows

$$\begin{aligned} & \sum_{h=1}^{m} \sum_{C \subseteq R} c_h(C) y_h(C) \to \min \\ \text{subject to} \quad & \sum_{h=1}^{m} \sum_{C \subseteq R} y_h(C) \geq 1, \quad \forall j \in R, \\ & y_h(C) \in \{0, 1\}, \quad \forall C \subseteq R, \quad \forall h, \quad h = 1, 2, \ldots, m. \end{aligned} \tag{4}$$

The objective function expresses the cost. The constraints ensure that the procurement provides at least set of all items.

Double auctions (auctions with multiple buyers and multiple sellers) are becoming increasing popular in electronic commerce. The numerous applications in electronic commerce, including stock exchanges, business-to-business commerce, bandwidth allocation, etc. have led to a great deal of interest in double auctions (see Bellosta et al. 2004). For double auctions, the auctioneer is faced with the task of matching up a subset of the buyers with a subset of the sellers. The profit of the auctioneer (supply chain) is the difference between the prices paid by the buyers and the prices paid to the sellers. The objective is to maximize the profit of the auctioneer given the bids made by sellers and buyers. Constraints establish the same conditions as in single-sided auctions.

We present a double auction problem of indivisible items with multiple sellers and multiple buyers. Let us suppose that m potential sellers $S_1, S_2, \ldots, S_m$ offer a set R of r items, $j = 1, 2, \ldots, r$, to n potential buyers $B_1, B_2, \ldots, B_n$.

A bid made by seller S_h, $h = 1, 2, \ldots, m$, is defined as $b_h = \{C, c_h(C)\}$, a bid made by buyer B_i, $i = 1, 2, \ldots, n$, is defined as $b_i = \{C, p_i(C)\}$, where $C \subseteq R$, is a combination of items, $c_h(C)$, is the offered price by seller S_h for the combination of items C, $p_i(C)$, is the offered price by buyer B_i for the combination of items C.

Binary variables are introduced for model formulation: $x_i(C)$ is a binary variable specifying if the combination C is assigned to buyer B_i ($x_i(C) = 1$), $y_h(C)$ is a binary variable specifying if the combination C is bought from seller S_h ($y_h(C) = 1$).

$$\begin{aligned} & \sum_{i=1}^{n}\sum_{C\subseteq R} p_i(C)x_i(C) - \sum_{h=1}^{m}\sum_{C\subseteq R} c_h(C)y_h(C) \to \max \\ \text{subject to} \quad & \sum_{i=1}^{n}\sum_{C\subseteq R} x_i(C) \leqq \sum_{h=1}^{m}\sum_{C\subseteq R} y_h(C), \quad \forall j \in R, \\ & x_i(C) \in \{0,1\}, \quad \forall C \subseteq R, \quad \forall i, \quad i = 1, 2, \ldots, n, \\ & y_h(C) \in \{0,1\}, \quad \forall C \subseteq R, \quad \forall h, \quad h = 1, 2, \ldots, m. \end{aligned} \tag{5}$$

The objective function expresses the profit of the auctioneer. The constraints ensures for buyers to purchase a required item and that the item must be offered by sellers.

The formulated combinatorial double auction can be transformed to a combinatorial single-sided auction. Substituting $y_h(C)$, $h = 1, 2, \ldots, m$, with $1 - x_i(C)$, $i = n + 1, n + 2, \ldots, n + m$, and substituting $c_h(C)$, $h = 1, 2, \ldots, m$, with $p_i(C)$, $i = n + 1, n + 2, \ldots, n + m$, we get a model of a combinatorial single-sided auction.

$$\begin{aligned} & \sum_{i=1}^{n+m}\sum_{C\subseteq R} p_i(C)x_i(C) - \sum_{i=n+1}^{n+m}\sum_{C\subseteq R} p_i(C) \to \max \\ \text{subject to} & \sum_{i=1}^{n+m}\sum_{C\subseteq R} x_i(C) \leqq m, \quad \forall j \in R, \\ & x_i(C) \in \{0,1\}, \forall C \subseteq R, \quad \forall i, \quad i = 1, 2, \ldots, n+m. \end{aligned} \tag{6}$$

The model (6) can be solved by methods for single-sided combinatorial auctions. The specific forward (3) or reverse (4) auctions can be modeled as special cases of the model (6).

4.3 Multi-round auctions

The key challenge in the iterative combinatorial auctions design (Parkes 2005) is to provide information feedback to the bidders after each iteration (Pikovsky and Bichler 2005). Pricing was adopted as the most intuitive mechanism of providing feedback. In contrast to the single-item single-unit auctions, pricing is not trivial for iterative combinatorial auctions. The main difference is the lack of the natural single-item prices. With bundle bids setting independent prices for individual items is not obvious and often even impossible. Different pricing schemes are introduced and discussed their impact on the auction outcome.

A set of prices $p_i(C)$, $i = 1, 2, \ldots, n$, $C \subseteq R$ is called:

- linear, if $\forall i$, C: $p_i(C) = \sum_{j \in S} p_i(j)$,
- anonymous, if $\forall$ k, l, C: $p_k(C) = p_l(C)$.

Prices are linear if the price of a bundle is equal to the sum of the prices of its items, and anonymous if the prices of the same bundle are equal for every bidder. The simple pricing scheme with linear anonymous prices will be used. Linear anonymous prices are easily understandable and usually considered fair by the bidders. The communication costs are also minimized, because the amount of information to be transferred is linear in the number of items.

A set of prices $p_i(S)$ is called compatible with the allocation $x_i(C)$ and valuations $v_i(C)$, if

$$\forall i, C : x_i(C) = 0 \Leftrightarrow p_i(C) > v_i(C) \text{ and } x_i(C) = 1 \Leftrightarrow p_i(C) \leq v_i(C)$$

The set of prices is compatible with the given allocation at the given valuations if and only if all winning bids are higher than or equal to the prices and all loosing bids are lower than the prices (assuming the bidders bid at their valuations).

Compatible prices explain the winners why they won and the losers, why they lost. In fact, informing the bidders about the allocation $x_i(C)$ is superfluous, if compatible prices are communicated. However, not every set of compatible prices provides the bidder with meaningful information for improving bids in the next auction iteration. Another important observation is the fact that linear compatible prices are harder and often even impossible to construct, when the bidder valuations are super- or sub-additive.

A set of prices $p_i(C)$ is in competitive equilibrium with the allocation $x_i(C)$ and valuations $v_i(C)$, if

1. The prices $p_i(C)$ are compatible with the allocation $x_i(C)$ and valuations $v_i(C)$.
2. Given the prices $p_i(C)$, there exists no allocation with larger total revenue than the revenue of the allocation $x_i(C)$.

The idea behind this concept is to define prices characterizing the optimal allocation. The prices may not be too low to violate the compatibility condition 1, but they may not be too high to violate the condition 2. In general, one can show that the existence of competitive equilibrium prices implies optimality of the allocation.

5 Solving the trading model

Multi-round iterative auctions can be taken as a solution approach. There is a strong interrelationship between the iterative auctions and the primal-dual linear programming algorithms.

5.1 Primal-dual algorithms

One way of reducing some of the computational burden in solving the winner determination problem is to set up a fictitious market that will determine an allocation and prices in a decentralized way. In the iterative approach, there are multiple rounds of bidding and allocation and the problem is solved in an iterative and incremental way. Iterative combinatorial auctions are attractive to bidders because they learn about their rivals' valuations through the bidding process, which could help them to adjust their own bids.

There is a connection between efficient auctions for many items, and duality theory. The Vickrey auction can be taken as an efficient pricing equilibrium, which corresponds to the optimal solution of a particular linear programming problem and its dual. The simplex algorithm can be taken as static approach to determining the Vickrey outcome. Alternatively, the primal-dual algorithm can be taken as a decentralized and dynamic method to determine the pricing equilibrium. A primal-dual algorithm usually maintains a feasible dual solution and tries to compute a primal solution that is both feasible and satisfies the complementary slackness conditions. If such a solution is found, the algorithm terminates. Otherwise the dual solution is updated towards optimality and the algorithm continues with the next iteration. The fundamental work (Bikhchandani and Ostroy 2002) demonstrates a strong interrelationship between the iterative auctions and the primal-dual linear programming algorithms. A primal-dual linear programming algorithm can be interpreted as an auction where the dual variables represent item prices. The algorithm maintains a feasible allocation and a price set, and it terminates as the efficient allocation and competitive equilibrium prices are found.

For the winner determination problem we will formulate the LP relaxation and its dual. Consider the LP relaxation of the winner determination problem (3):

$$\begin{aligned} & \sum_{i=1}^{n} \sum_{C \subseteq R} v_i(C) x_i(C) \to \max \\ \text{subject to} \quad & \sum_{C \subseteq R} x_i(C) \leq 1, \quad \forall i, \quad i = 1, 2, \ldots, n, \\ & \sum_{i=1}^{n} \sum_{C \subseteq R} x_i(C) \leq 1, \quad \forall j \in R, \\ & x_i(C) 0, \quad \forall C \subseteq R, \quad \forall i, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{7}$$

The corresponding dual to problem (7)

$$\begin{aligned} & \sum_{i=1}^{n} p(i) + \sum_{j \in C} p(j) \to \min \\ \text{subject to} \quad & p(i) + \sum_{j \in C} p(j) \geq v_i(C) \quad \forall i, C, \\ & p(i), p(j) \geq 0, \quad \forall i, j, \end{aligned} \tag{8}$$

The dual variables $p(j)$ can be interpreted as anonymous linear prices of items, the term $\sum_{j \in S} p(j)$ is then the price of the bundle C and $p(i) = \max_S[v_i(C) - \sum_{j \in S} p(j)]$ is the maximal utility for the bidder i at the prices $p(j)$.

Following two important properties can be proved for the problems (7) and (8):

1. The complementary-slackness conditions are satisfied if and only if the current allocation (primal solution) and the prices (dual solution) are in competitive equilibrium.
2. The formulation (7–8) is weak. For the optimal allocation there no always exist anonymous linear competitive equilibrium prices.

5.2 Auction formats

Several auction formats based on the primal-dual approach have been proposed in the literature. Though these auctions differ in several aspects, the general scheme can be outlined as follows:

1. Choose minimal initial prices.
2. Announce current prices and collect bids. Bids have to be higher or equal than the prices.
3. Compute the current dual solution by interpreting the prices as dual variables. Try to find a feasible allocation, an integer primal solution that satisfies the stopping rule. If such solution is found, stop and use it as the final allocation. Otherwise update prices and go back to 2.

Concrete auction formats based on this scheme can be implemented in different ways. The most important design choices are the following: bid structure, pricing scheme, price update rule, bid validity, feedback, way of computing a feasible primal solution in each iteration, and stopping rule.

6 Conclusions

The proposed trading model has some advantages in comparisons with other approaches. Auctions are the important subject of an intensive economic research. Auctions are very popular mechanisms in practice and it is not necessary to conclude contracts between agents. Restrictions are only rules of the auction process. Agents need to monitor the price only. Auctions can be made via Internet. The approach coordinates layers of agents in supply chain, not only individual agents as by contracts. Repeating the procedure of coordination between layers of agents makes it possible to coordinate the entire supply chain.

A possible flexible approach for modeling and solving such auctions is presented. The analysis of the simple cases gives recommendations for more complex real problem. The combination of such models can give more complex views on auctions. For example, the model can be extended by multiple objectives. Complex problems require consider multiple objectives, not just profit. Multi objective linear programming (MOLP) problem can be used for the extended model. The problem is possible to solve by interactive methods of MOLP problems.

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