How Important Is It to Get the Lot Size Right?

by Hartmut Stadtler

Overview

- It is well-known that moderate deviations from the classical economic order quantity (EOQ) only marginally increase an item's setup and holding costs. The aim of this paper is to extend this analysis with respect to *total variable costs* and to provide a simple formula for its increase.
- The increase of total variable costs now depends on three parameters: The relative deviation from the EOQ, the carrying charge and the optimal time between orders (TBO) of a product.
- Sensitivity analysis shows that production decisions should *not* be constrained by predefined fixed lot sizes based on the EOQ (as is often done in today´s ERP systems), since deviating from the EOQ is often a low cost measure to cope with limited capacities.
- We will introduce the term *lot-sizing flexibility* as a *multiple M* of the optimal TBO where the increase of the total variable costs stays within given limits (e.g. if the accepted increase of total variable costs is limited to 1%, assuming a carrying charge of 10% p.a. and considering an item with an optimal TBO of 5 weeks, then a multiple $M = 4$ results, i.e. the planner can choose a lot-size corresponding to a TBO from 1.25 weeks up to 20 weeks).

(Economic order quantity, lot-sizing flexibility, sensitivity analysis)

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Prof. Dr. Hartmut Stadtler, Universität Hamburg, Institut für Logistik und Transport, Fakultät Wirtschafts- und Sozialwissenschaften, Von-Melle-Park 5, 20 146 Hamburg Tel.: +49 (0) 40-428 38 2609, Fax.: +49(0) 40-428 38 6283, e-mail: hartmut.stadtler@uni-hamburg.de.

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A. Introduction and literature review

The economic order quantity (EOQ) developed by Harris in 1913 (Harris, 1913) is regarded an early classic (Erlenkotter, 1989) of Operations Management and Operations Research and has found its way into numerous textbooks (e.g. Hadley, Whitin, 1963; Silver, Pyke, Petersen, 1998; Vollman, Berry, Whybark, 1997). It has been the starting point of a number of noteworthy extensions (e.g. the economic lot scheduling problem (ELSP)).

The robustness of the EOQ model with respect to variations of its parameters – demand rate, holding cost and setup cost – has been addressed in the literature (Juckler, 1970; Lowe and Schwarz, 1983; Zangwill, 1987; Dobson, 1988). Due to the widespread use of Enterprise Resource Planning (ERP) systems it became possible to specify an item´s order quantity in its data set. A simple choice then is the EOQ. As a result planners are inclined (and even may be instructed) to make use of the EOQ when generating production or procurement plans although it is well-known that relatively small deviations from the EOQ result in an even smaller increase in setup and holding costs (Silver, Pyke, Petersen, 1998). As an example the increase of setup and holding costs δ^{sh} is less than 5% if the deviation Δ from the EOQ is within the range [–27%, +37%].

However, the indicator proposed here is not the relative increase of setup and holding costs but the "relative increase of an item's *total variable costs*" δ^{v} consisting of all variable cost components like setup and holdings costs as well as raw material and direct labour costs (the latter if paid as a piece wage). We regard studying the impact of a certain tool or method on total variable costs a more appropriate indicator for describing the effects on a company's competitive position than only looking at a subset of its cost components. Obviously, the relative cost increase is smaller if the denominator includes further cost components. The interesting point here is that the relative increase of total costs now not only depends on the relative deviation Δ from the EOQ (as in the sensitivity analysis of Hadley, Whitin, 1963 or Silver, Pyke, Petersen, 1998) but also on the carrying charge r and the optimal time between orders (TBO).

Referring to the numerical example above we further assume a carrying charge of 10% per year and an optimal TBO of 4 weeks (1/13 year). Now a deviation of +37% (–27%) from the EOQ increases total variable costs by only 0.04% (see equation (8)). Consequently, we argue that the EOQ should be regarded only as a very rough rule of thumb. Even more, deviations from the EOQ should be actively used as a means to achieve flexibility for the planner at low cost. E.g. it may be advantageous to adapt lot sizes of planned orders such that a rush order can be fulfilled in due time.

In the next section we will recall the usual sensitivity analysis showing the impact of deviations from the EOQ on setup and holding costs. Section 3 extends the sensitivity analysis with respect to total variable costs. Lot-sizing flexibility is defined in Section 4. A numerical analysis of the lot-sizing flexibility for some typical parameter values is also included (Section 5). Conclusions will be stated in Section 6.

B. Sensitivity anlaysis based on setup and holding costs

In order to judge the impact of deviating from the EOQ Q^* by a multiple $(1 + \Delta)$, i.e.

(1) $$Q = (1+\Delta)\cdot Q^*$$

a company is interested in the cost increase incurred. Deviations may become necessary in situations, where the stringent assumptions of the EOQ are violated (e.g. at a certain point in time items compete for a scarce resource and thus are not independent). Silver, Pyke, Peterson (1998, pp. 156 and the literature cited there) present the following formula to calculate the *additional* costs: $C^{sh+}(Q)$

(2) $$C^{sh+}(Q) = \frac{1}{2}\cdot\frac{\Delta^2}{(1+\Delta)}\cdot C^{sh}(Q^*)$$

The authors consider an item's setup and holding costs C^{sh} as total relevant costs "… that is, the sum of those costs per unit time which can be influenced by the order quantity …". The relative increase of setup and holding costs δ^{sh} when deviating from the optimal lot size is

(3) $$\delta^{sh} = \frac{1}{2}\cdot\frac{\Delta^2}{(1+\Delta)}$$

The following notation will be used throughout this paper:
Data:
- d demand rate of the item [units/unit time]
- f fixed cost of ordering or setup cost [monetary units]
- r carrying charge, i.e. the cost of having one monetary unit tied up in inventory for a unit time (e.g. one year) [1/unit time]
- v unit variable cost of the item (excluding setup and holding costs) [monetary units/unit]

Variables:
- τ time between orders (and $t^* = Q^*/d$) [unit time]
- Q order quantity (and Q^* for the EOQ) [units]
- Δ relative deviation from the EOQ (more specific measures to be introduced later are Δ_{LL}^{sh}, Δ_{UL}^{sh}, Δ_{LL}^{v}, Δ_{UL}^{v} where sh (v) refer to stock holding (variable) costs while LL (UL) indicate lower (upper) limits)
- δ^{sh} relative cost increase w.r.t. minimal setup and holding costs
- δ^{v} relative cost increase w.r.t. minimal total variable costs

There are at least two important results. One is that small deviations from the EOQ only marginally in-crease setup and holding costs. The other important result is that the cost increase only depends on the deviation Δ from the EOQ. Hence, there is no need to analyse item specific parameters (e.g. the relation between an item's setup and holding costs).

Now we would like to illustrate the range of deviations that will be accepted if a specific cost increase δ^{sh} is accepted. These lower Δ_{LL}^{sh} and upper limits Δ_{UL}^{sh} on the relative

Table 1: Ranges of relative deviations from the EOQ for some relative increases of setup and holdings costs δ^{sh}

δ^{sh}	0.001	0.005	0.01	0.05	0.10
Δ^{sh}_{UL}	0.046	0.105	0.152	0.370	0.558
Δ^{sh}_{LL}	-0.044	-0.095	-0.132	-0.270	-0.358

deviation from the EOQ are derived from equation (3) by solving the resultant quadratic equation:

$$\Delta^{sh}_{UL,LL} = \delta^{sh} \pm \sqrt{\delta^{sh^2} + 2 \cdot \delta^{sh}} \tag{4}$$

Table 1 exhibits results for some arbitrary values of δ^{sh}. The lower and upper limits calculated indicate the well-known characteristic that relatively small deviations from the EOQ result in a much smaller relative increase in setup and holding costs. Furthermore, falling short of the EOQ is more costly than exceeding the EOQ by the same amount.

In the following we will show that the impact of deviating from the EOQ is even less critical than the above sensitivity analysis suggests if total variable costs are considered and that this effect can also be evaluated by a simple formula.

C. Sensitivity analysis based on total variable costs

We will start our analysis by expressing the setup and holding costs per unit time as function of the EOQ

$$C^{sh}(Q^*) = \sqrt{2 \cdot f \cdot d \cdot v \cdot r} = v \cdot r \cdot \sqrt{\frac{2 \cdot f \cdot d}{v \cdot r}} = v \cdot r \cdot Q^* \tag{5}$$

as well as of the corresponding optimal time between orders (TBO) (variable τ*)

$$C^{sh}(\tau^*) = v \cdot r \cdot d \cdot \tau^* \tag{6}$$

Often the EOQ formula is stated in terms of the unit holding cost per unit time h while here it is substituted by the carrying charge r ($r > 0$) times the unit variable cost v of the item (in accordance with Silver, Pyke, Petersen, 1998, p. 154).

The relative increase of total variable costs δ^v resulting from a deviation Δ from the EOQ now becomes:

$$\delta^v = \frac{C^{sh+}((1+\Delta) \cdot Q^*)}{C^{sh}(Q^*) + v \cdot d} = \frac{\delta^{sh} \cdot C^{sh}(Q^*)}{C^{sh}(Q^*) + v \cdot d} = \frac{\delta^{sh} \cdot v \cdot r \cdot d \cdot \tau^*}{v \cdot r \cdot d \cdot \tau^* + v \cdot d}$$
$$= \frac{\delta^{sh} \cdot v \cdot r \cdot d \cdot \tau^*}{v \cdot d \cdot (r \cdot \tau^* + 1)} = \frac{\delta^{sh} \cdot r \cdot \tau^*}{(r \cdot \tau^* + 1)} \tag{7}$$

The numerator represents the *increase* of setup and holding costs depending on the lot size Q while the denominator is made of the sum of minimal setup and holdings cost per unit time plus variable costs for a demand rate d. By rewriting equation (7) and replacing δ^{sh} by equation (3) we obtain:

$$(8) \qquad \delta^{v} = \delta^{sh} \cdot \frac{1}{(1+\frac{1}{r \cdot \tau^*})} = \frac{1}{2} \cdot \frac{\Delta^2}{(1+\Delta)} \cdot \frac{1}{(1+\frac{1}{r \cdot \tau^*})}$$

<u>Observation 1:</u>
Reducing the TBO from its optimal value is more costly than an increase by the same percentage $|\Delta|$.

This observation corresponds to the well-known sensitivity analysis of C^{sh}. It holds because δ^{sh} is multiplied by a positive constant $(\frac{1}{(1+\frac{1}{r \cdot \tau^*})})$.

<u>Observation 2:</u>
The relative increase of total variable costs depends on three parameters Δ, r and τ^*.

This observation is in contrast to previous sensitivity analyses where the impact of deviations from the optimal TBO on setup and holding costs has been studied (which is independent of r and τ^*). Although calculated for each item individually τ^* can also be used to group products having an optimal TBO within a given range. Consequently, formula (8) can be used to evaluate typical parameter combinations of Δ, r and τ^* without resorting to individual items.

<u>Observation 3:</u>
The relative increase of total variable costs is at most half the relative increase of setup and holding costs assuming a carrying charge r of less than 100% per unit time (e.g. one year) and an optimal TBO less than unit time. In most practical situations *r and* τ^* will be considerably smaller (than “1”) multiplying the effect on the relation between δ^v *and* δ^{sh}*, i.e.* $\delta^v << \delta^{sh}$ for a given Δ.

An simple approximation of the relative increase of total variable costs (equation (8)) results if one skips costs C^{sh} in the denominator of (7):

$$(9) \qquad UB(\delta^{v}) = \delta^{sh} \cdot r \cdot \tau^{*}$$

This approximation is an upper bound on δ^v and may be sufficiently accurate for industrial practice. Equation (9) is intuitive because the portion of setup and holding costs with respect to total variable costs becomes smaller the smaller the carrying charge is and the less lot size stock is held in inventory (corresponding to a small optimal TBO). Fixed cost do not occur explicitly because for the EOQ the setup and holding costs per unit time are equal.

In the following we will restrict our sensitivity analysis to the exact formula (8).

D. Lot-sizing flexibility

Similarly to Section 2 we will present lower and upper limits but not with respect to relative deviations from the EOQ but from the optimal TBO. Our sensitivity analysis requires restating equation (8) such that upper Δ^v_{UL} and lower limits Δ^v_{LL} become dependent variables:

$$\text{(10)} \quad \Delta^v_{UL,LL} = \theta \cdot \delta^v \pm \sqrt{\theta^2 \cdot {\delta^v}^2 + 2 \cdot \theta \cdot \delta^v}$$

where

$$\text{(11)} \quad \theta := 1 + \frac{1}{r \cdot \tau^*}$$

Observation 4:
The interval of accepted deviations $[\Delta^v_{LL}, \Delta^v_{UL}]$ from the optimal TBO for a given relative cost increase is (much) larger when total variable costs are considered than when only setup and holding costs are taken into account.

Proof:
Equations (4) and (10) yield the same result provided $\theta \cdot \delta^v = \delta^{sh}$. Since $\theta > 1$ the same upper Δ^v_{UL} and lower Δ^v_{LL} limits result if δ^v is smaller than δ^{sh}. Consequently, lower and upper limits can be increased from their starting values ($\Delta^v_{LL} = \Delta^{sh}_{LL}$ and $\Delta^v_{UL} = \Delta^{sh}_{UL}$) until $\delta^v = \delta^{sh}$.

As an example we allow $\delta^{sh} = \delta^v = 0.5\%$ yielding an interval $\Delta^{sh}_{LL,UL} = [-9.5\%, +10.5\%]$ while for an optimal TBO of one year and $r = 0.1$ p.a. the interval based on total variable costs is $\delta^{sh}_{LL,UL} = [-28\%, +38\%]$. Roughly speaking the allowable deviations in percent of the optimal TBO have tripled by taking total variable costs instead of setup and holding costs alone. This multiple increases if smaller optimal TBO are taken for a comparison.

Referring to the definition of Δ (equation (1)) we are now interested in the range of acceptable TBO which is given by the interval $[(1 + \Delta^v_{LL}) \cdot \tau^*, (1 + \Delta^v_{UL}) \cdot \tau^*]$. A simplification results if we define a multiple M as

$$\text{(12)} \quad M := 1 + \Delta^v_{UL} = 1 + \theta \cdot \delta^v + \sqrt{\theta^2 \cdot {\delta^v}^2 + 2 \cdot \theta \cdot \delta^v}$$

One can prove by simple arithmetic that

$$\text{(13)} \quad \frac{1}{M} = 1 + \Delta^v_{LL}$$

Now the range of acceptable TBO becomes

$$\text{(14)} \quad [\frac{1}{M} \cdot \tau^*, M \cdot \tau^*]$$

Multiple M indicates the range of acceptable relative deviations from the optimal TBO (EOQ) and can guide the planner to adapt lot sizes according to the opportunities and constraints of a given planning situation. Hence, multiple M is defined here as the *lot-sizing* *flexibility* of an item. It depends on three parameters: An item's optimal TBO, the carrying charge r and the acceptable relative cost increase δ^v (to be provided by management). We would like to add that a symmetry similar to [14] also exists when restricting the analysis to the cost increase of setup and holding costs (see Hadley, Whitin, 1963, p. 70).

E. A numerical analysis of lot-sizing flexibility

Now, we will illustrate the impact of deviating from the optimal TBO (or EOQ) on total variable costs for a carrying charge $r = 0.1$ p.a. and an acceptable cost increase $\delta^v = 1.0\%$. Resulting lot-sizing flexibility is documented in Table 2 for a few 'specific' optimal TBO. A broader range of values is shown graphically in Figure 1 for optimal TBO ranging from 1 to 52 weeks and $\delta^v \in \{0.1\%, 0.5\%, 1.0\%\}$.

Table 2: Specific multiples M (rounded) and corresponding optimal TBO for $\delta^v = 1.0\%$ and $r = 0.1$

M	12	7	4	3	2	1.5
τ^* [in weeks]	1	2	5	8	22	71

Multiple M is 2 (see Table 2) for an optimal TBO of 22 weeks resulting in an interval of acceptable TBO of [11, 44] weeks. Likewise the interval of acceptable TBO becomes [2 days, 14 weeks] for an item with an optimal TBO of 2 weeks (assuming seven days a week). A range of TBO from 2 days to 14 weeks incurs an enormous degree of freedom for the planner to devise feasible and near optimal (production or procurement) plans.

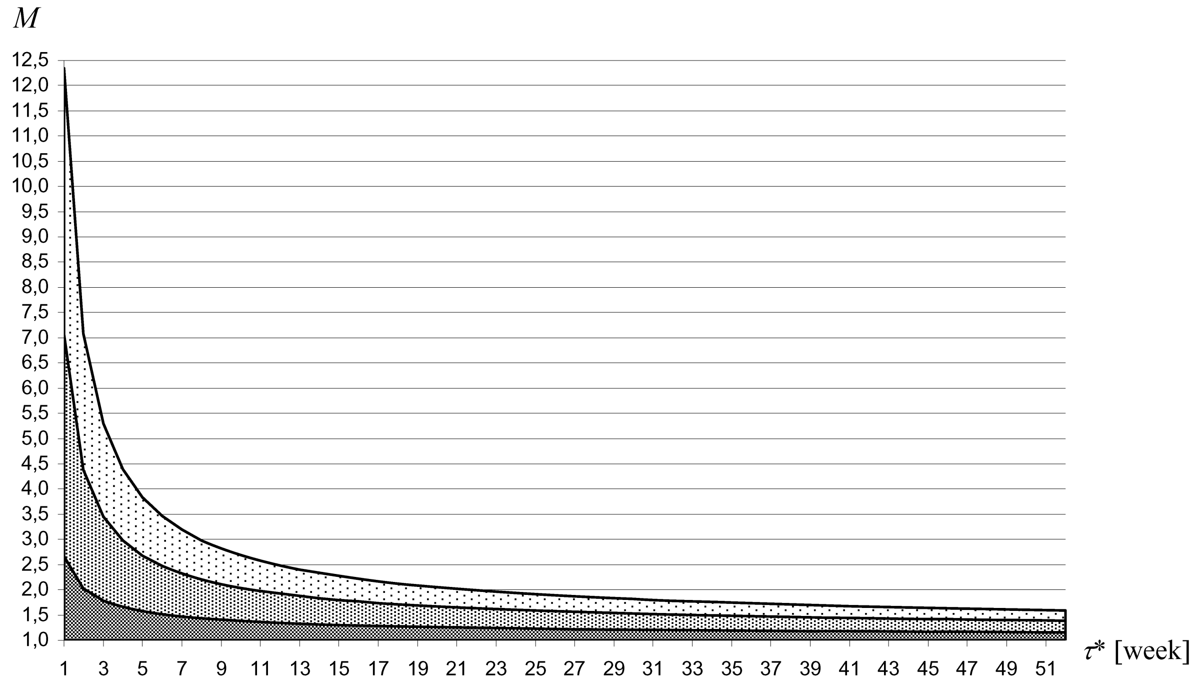
As can be observed from Figure 1 multiples M increase drastically the smaller the optimal TBO becomes. Note, that the x-axis has been transformed into a weekly time scale for convenience although the unit time is one year in our numerical example.

A numerical analysis of various carrying charges r has not been included here, because from equation (9) one can conclude that lot-sizing flexibility is (roughly) proportional to the carrying charge r (i.e. doubling the carrying charge will (nearly) halve lot-sizing flexibility).

F. Summary and conclusions

Previous sensitivity analyses have shown that small deviations from the EOQ only marginally increase setup and holding costs. This result is even more pronounced if an item's relative increase of total variable costs is considered. A simple formula has been derived showing the effect of deviations from the EOQ on the relative increase of total variable costs. Furthermore, the multiple by which the optimal TBO can be varied without exceeding a given relative increase of total variable costs has been introduced here as the *lot-sizing flexibility*. It increases drastically the smaller the optimal TBO becomes.

Two main conclusions can be drawn for lot-sizing in industrial practice. First, management should *not* fix lot sizes to the EOQ due to the relatively small increase of total



Explanations:

lot-sizing flexibility for a given optimal TBO (τ^*) and $0 < \delta^v \leq 0.1\%$

lot-sizing flexibility for a given optimal TBO (τ^*) and $0.1\% < \delta^v \leq 0.5\%$

lot-sizing flexibility for a given optimal TBO (τ^*) and $0.5\% < \delta^v \leq 1.0\%$

Fig. 1: Multiples M indicating the lot-sizing flexibility for τ^* ranging from 1 to 52 weeks, $\delta^v \in \{0.1\%, 0.5\%, 1.0\%\}$ and $r = 0.1$

variable costs when deviating from the optimal TBO. (A further argument often put forward in textbooks is the difficulty to calculate the parameters of the EOQ correctly.) Instead it seems more appropriate to define an interval of TBO for each item which keeps the relative increase of total variable costs within certain limits (e.g. $\delta^v \leq 1.0\%$). Also, one should bear in mind that new model formulations for dynamic lot-sizing have been developed in recent years (e.g. Billington et al., 1983; Suerie and Stadtler, 2003; Suerie 2005; Wolsey, 2002) which model dynamic lot sizes explicitly as one element of more complex decision situations and which will make use of the existing lot-sizing flexibility implicitly.

Second, management should *not* regard implementing the EOQ as a predominant lever to achieve cost reductions *if* the optimal TBO as well as the carrying charge are relatively low (e.g. an optimal TBO less than 5 weeks for $r = 0.1$ p.a.). The reason is that the portion of setup and holding costs over total variable costs is rather low and hence lot-sizing flexibility becomes large (M ≥ 4) (Table 2).

Our study also stresses that reducing setup cost remains advantageous. It still holds that "... the benefits of cutting set up increase the more we cut ..." (Zangwill, 1987). For a recent case study regarding setup time reductions see Trovinger and Bohn (2005). Cutting setup cost not only result in smaller optimal TBO but also in increased lot-sizing flexibility thus enabling the planner to adapt procurement and production plans (and sequences of orders) according to the needs of a given planning situation at low cost.

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Zur Bedeutung der Wahl der „richtigen" Losgröße

Zusammenfassung

Während in Lehrbüchern gemeinhin die Auswirkungen einer Abweichung von der klassischen wirtschaftlichen Losgröße (EOQ) auf die Rüst- und Lagerkosten analysiert werden, betrachten wir die Auswirkungen auf die *gesamten variablen Kosten*. Die Erhöhung der gesamten variablen Kosten hängt dabei von drei Parametern ab: Der relativen Abweichung von der EOQ, dem Lagerhaltungskostensatz und der optimalen Reichweite eines Produkts.

Als *Losgrößenflexibilität* definieren wir ein *Vielfaches* der optimalen Reichweite, bei dem die relative Erhöhung der gesamten variablen Kosten eine gegebene obere Schranke nicht überschreitet. Es wird gezeigt, dass die Losgrößenflexibilität umso größer wird, je kürzer die optimale Reichweite ist. Für den Fall einer optimalen Reichweite von nicht mehr als vier Wochen und einer Verzinsung des im Lager gebundenen Kapitals von bis zu 10% p.a. wird die Bandbreite akzeptabler Losgrößen so groß, dass die Umsetzung einer optimalen wirtschaftlichen Losgröße aus Kostengesichtspunkten unerheblich ist (insbesondere auch angesichts der Schwierigkeiten, die „richtigen" Rüst- und Lagerkostensätze zu bestimmen).

How Important Is It to Get the Lot Size Right?

Summary

While textbooks concentrate on an analysis of the relative increase of an item's setup and holding costs when deviating from the classical economic order quantity (EOQ) we present an analysis with respect to *total variable costs*. The increase of total variable costs depends on three parameters: The relative deviation from the EOQ, the carrying charge and the optimal time between orders (TBO) of a product.

The term *lot-sizing flexibility* is introduced as a *multiple M* of the optimal TBO where the increase of total variable costs stays within given limits. It turns out that lot-sizing flexibility increases sharply the smaller the optimal TBO is. Considering a product with a TBO ≤ 4 weeks and a carrying charge ≤ 10% p.a. the range of acceptable lot sizes becomes so large that implementing the "optimal" EOQ is irrelevant (especially in light of the difficulty of finding the "correct" setup and holding cost figures).

Keywords: Economic order quantity, lot-sizing flexibility, sensitivity analysis

JEL: M11, C60