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RESEARCH ARTICLE

Localization of neural efficiency of the mathematically gifted brain through a feature subset selection method

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Abstract Based on the neural efficiency hypothesis and task-induced EEG gamma-band response (GBR), this study investigated the brain regions where neural resource could be most efficiently recruited by the math-gifted adolescents in response to varying cognitive demands. In this experiment, various GBR-based mental states were generated with three factors (level of mathematical ability, task complexity, and short-term learning) modulating the level of neural activation. A feature subset selection method based on the sequential forward floating search algorithm was used to identify an "optimal" combination of EEG channel locations, where the corresponding GBR feature subset could obtain the highest accuracy in discriminating pairwise mental states influenced by each experiment factor. The integrative results from multi-factor selections suggest that the right-lateral fronto–parietal system is highly involved in neural efficiency of the math-gifted brain, primarily including the bilateral superior frontal, right inferior frontal, right-lateral central and right temporal regions. By means of the localization method based on single-trial classification of mental states, new GBR features and EEG channel-based brain regions related to mathematical giftedness were identified, which could be useful for the brain function improvement of children/adolescents in mathematical learning through brain–computer interface systems.

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Introduction

The neural efficiency hypothesis has suggested that "intelligence is not a function of how hard the brain works but rather how efficiently it works,… this efficiency may derive from the disuse of many brain areas irrelevant for good task performance as well as the more focused use of specific task relevant areas." (Haier et al. 1992). By integrating a large body of research using different neuroscience measurement methods, Neubauer and Fink (2009) have pointed out that brighter individuals might exhibit lower (i.e., more efficient) brain activity while performing cognitive tasks with low to moderate difficulties, resulting in negative correlationship between brain activation and performance in intelligence test. In rather complex tasks, this relationship would probably be reversed; that is, a positive correlation might be generated between brain activation and intelligence score (Neubauer and Fink 2009). The previous empirical studies have in general concluded that task complexity is an important variable in modulating this negative or positive relationship (Haier et al. 1988; Neubauer and Fink 2003, 2008, 2009; Neubauer et al. 1999, 2002, 2004; Larson et al. 1995; Doppelmayr et al. 2005). Additionally, the effect of short-term learning is also a considerable factor modulating efficient brain activation, under the influence of which a general decrease of brain activation and behavioral advantage might be observed over the duration of a task (Neubauer and Fink 2009). In Wartenburger et al.'s (2009) functional magnetic resonance imaging (fMRI) study, the last third of trials of a geometric analogy task resulted in significant performance improvement (shorter response time and higher accuracy) and decreased blood oxygenation level dependent (BOLD) signals as compared to the first third of trials, reflecting increased processing efficiency of the brain due to rapid learning. It is important to note that the brain activation decrease in dependence on learning has been found correlating with performers' intelligence level, that is, the brighter the individual, the more the activation is reduced after learning (Neubauer et al. 2002, 2005; Neubauer and Fink 2003, 2009; Xu and Wang 2014).

In educational and psychological researches, math-gifted adolescents/children are characterized by above-normal intelligence and domain-specific ability in mathematics. This ability is composed of "computation, symbolic manipulation, memory for number facts, …, reasoning, and so on" (Gardner 1985; Sternberg 2003; Livne and Milgram 2006). The previous neuroimaging studies revealed that math-gifted adolescents had stronger functional activation in prefrontal, frontal, parietal and temporal cortices while performing reasoning or mental imagery tasks (Desco et al. 2011; O'Boyle et al. 2005), which indicated higher usage of neural resource in cognitive processes. Conversely, some electroencephalogram and event-related potential (EEG/ERP) studies confirmed the occurrence of neural efficiency in the math-gifted brain. For example, during problem-solving and stimulus–response processes, math-gifted adolescents exhibited faster response speed and less event-related EEG power change in comparison with average-ability ones (Jaušovec 1996; Liu et al. 2011), suggesting lower brain activity and more efficient neural functions. In the past studies, the regions where neural resource can be efficiently recruited in the math-gifted brain have not been explicitly or consistently localized. The integration of empirical evidence indicates that the frontal lobe, especially the prefrontal cortex (PFC), has been mostly pronounced in the studies on neural efficiency (Jaušovec and Jaušovec 2004; Hoppe et al. 2012; Liu et al. 2011; Neubauer and Fink 2009). Not just in the frontal lobe, the parieto-occipital cortex, parietal lobe, precuneus, thalamus, temporal and frontal gyrus are also involved in efficient brain activation (Jaušovec 1996; Wartenburger et al. 2009). Besides, neural efficiency might be reflected by the difference in functional connectivity between discrete brain areas (Neubauer and Fink 2009). During a simple speeded-processing task, except for less neural activity in some PFC regions, the behaviorally faster-performing subjects showed lower functional interactions between the PFC and parietal cortex, reflecting higher processing efficiency of the task-relevant information (Rypma et al. 2006; Qu et al. 2014).

The aim of the present EEG study is to identify the regions highly involved in efficiency-related activation pattern of the math-gifted brain. Task-induced gamma-band response (GBR) was employed to quantify the level of brain activation. The past studies have demonstrated that widespread 40 Hz GBR is induced by cognitive tasks through augmented band power, with its amount modulated by learning, memory, task complexity, phase synchrony, etc. (Howard et al. 2003; Gruber et al. 2001; Simos et al. 2002; Herrmann et al. 2010; Fitzgibbon et al. 2004; Schoenberg and Speckens 2015). Moreover, task-induced GBR is suggested to be strongly interwoven with sensation, attention, decision-making, high-order cognition, etc. (Gaetz et al. 2013; Li et al. 2011; Muller et al. 2000; Ray et al. 2008; Tanji et al. 2005). Because of the neural correlations with cognitive functions, the difference in event-related GBR is expected to be found between the math-gifted and average-ability subjects. The evidence reviewed so far allows associating the GBR with the modulating factors of neural efficiency tested in this study, i.e., level of mathematical ability of the subjects, task complexity, and the effect of short-term learning. Importantly, Lachaux et al.'s study (2007) using simultaneously recorded fMRI and intra-cranial EEG has discovered that the recording sites of GBR modulations show close spatial correspondence with the brain regions of fMRI activation. Accordingly, the task-induced GBRs distributed on the scalp can be viewed as the appearance of functionally activated brain areas.

To analyze the GBR-based mental states from the brain activities affected by the three factors, we recruited both math-gifted and average-ability adolescents performing two numerical inductive reasoning tasks in our experimental study. Based on the reviewed evidence, our study proposes and tests the following research hypotheses: at the initial stage of the experiment, when confronted with novel reasoning items with high- or low-complexity, the mathematical ability level of the gifted and non-gifted subjects will affect their usage of neural resource, resulting in different mental states. Within each entire reasoning task, while the subjects are processing the sequential items, the task difficulty will subjectively decline because of the effect of short-term learning, leading to changing mental states with generally reduced activation over the duration of the task. Under the influence of each factor, the mental states with distinguishable GBRs are expected to be generated in this experiment. While one mental state is transitioned into another state with more efficient activation pattern (i.e., a general reduction in GBR changes), a combination of EEG channels with the largest difference in GBRs between the two states could be identified. Because of the most inhibited neural activity during this transition process, the identified EEG channel sites represent the brain regions highly involved in the efficiency of cortical processing.

In this study, a feature subset selection method based on the sequential forward floating search (SFFS) algorithm was applied to the channel-based identification of efficiency-related brain regions. By iteratively promoting the accuracy in single-trial classification of mental states, the method can obtain a set of brain locations that optimally discriminates neural responses under different cognitive conditions. Since single-trial analysis of EEG data has been suggested as an effective way to address the problem of trial-to-trial variability of brain activity (Blankertz et al. 2011), it is expected that the method can result in more informative brain locations than traditional trial-averaged analysis while isolating different brain responses. However, single-trial analysis suffers from the superposition of task-relevant signals by task-unrelated brain activities, which leads to a low signal-to-noise rate (SNR) and lacks classification accuracy (Blankertz et al. 2011). In our study, due to seeking the highest accuracy in binary-classification of mental states, the identified combination of brain locations can best distinguish the response areas of interest from the interfering noises, thereby maintaining relatively high SNR in the single-trial analysis.

On the other hand, the activation measurements used in the feature subset selection method are derived from the scalp EEG recordings, which are actually produced by a set of electric diploes in the brain. Thus, a cortical current estimation procedure was performed to obtain the locations corresponding to the underlying event-related source processes. According to the convergent results of the scalp channel localization and the cortical current distribution, the brain regions with efficiently recruited neural resource in the math-gifted brain are suggested, whereby the relevant psychological mechanisms are analyzed and discussed.

Methods

Subjects

Two groups of adolescent subjects were enrolled in this experiment, without left handedness, neurological illness, and history of brain injury. The EEG study was approved by the Academic Committee of the Research Center for Learning Science, Southeast University, China. With the permission of their guardians, all the subjects read and signed the informed consent about this EEG experiment.

The math-gifted group included eight adolescents (five males and three females) aged 16.5 ± 0.7 (mean ± SD) from the Science and Engineering Experimental Class at Southeast University (Nanjing, China), who had passed the special college entrance examination aiming at gifted students with exceptional abilities in mathematics and natural sciences. Moreover, they all had been awarded prizes in nationwide or provincial mathematical competitions. The control group consisted of seven students (five males and two females) aged 16.3 ± 0.8 (mean ± SD) from the Fourth High School in Nanjing, who had average-level performance in their school mathematical tests. The Raven Advanced Progressive Matrices (RAPM) test was employed to further determine the intelligence difference between the two groups. The scores must be higher than 32 for the math-gifted subjects, and lower than 32 for the control subjects.

Experimental paradigm

The experiment included a three-number inductive reasoning task and a four-number induction task (Lu et al. 2010; Jia et al. 2011; Fig. 1). On the left of Fig. 1a, three numbers are located at different angles of a triangle, which are associated with a certain calculation rule, such as ‘A + B = C’ or ‘A + C = B’. On the right side of Fig. 1a, there are four numbers at four angles of a square, which are associated with a calculation rule of relatively high complexity, such as ‘A + B = C + D’ or ‘B = A+C + D’. In the two sample tasks (Fig. 1b), three triangles/squares illustrate two essential processes, i.e., rule induction and rule application. The rule induction process is located on the left side of an arrow, which aims to find a common calculation rule from two triangles/squares, and the validity of this calculation rule is verified by the third triangle/square on the right side of the arrow, i.e., rule application. In this experiment, the calculation rule only involved ‘+’ and ‘−’, and the numbers were within 0–9. Subjects were asked to judge whether the rules in the application and induction processes were consistent or not.

Each task consists of 70 trials, forming a valid block (30 trials), an invalid block (30 trials), and an interferential block (10 trials). The valid trials were constructed by the congruent calculation rule between rule induction and rule application. On the contrary, in the invalid trials, the rule of application process was not consistent with the rule from induction process. The interferential trials with incongruent rules between the first two figures were designed in this experiment to avoid the possibility that subjects could obtain the calculation rule only from the first figure without the consideration of the second figure. However, the trials of the interferential block were not used in the data analysis.

To avoid the effect of rapid skill development on the neural response, there was no practice session designed in this experiment. Before the formal experimental procedure, the subjects read a written instruction including the detailed rule, the procedure, and an illustration. After the subjects had confirmed that they understood the experimental tasks, they were asked to begin the formal procedure directly. The total 140 trials of the two tasks were incorporated in a session and presented randomly in the E-Prime 2.0

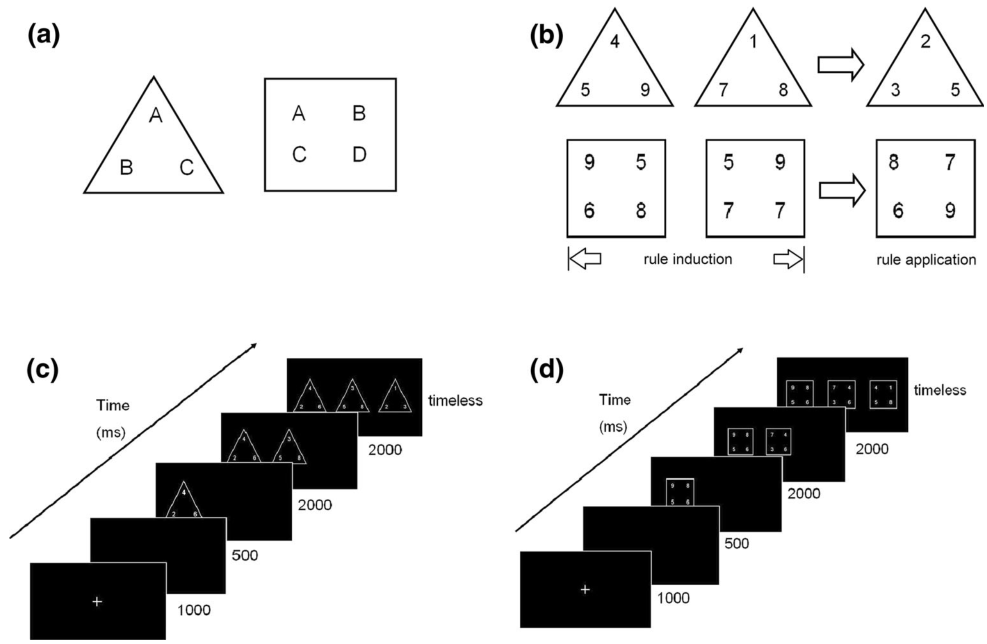

Fig. 1 Experiment protocol: **a** a *triangle* with three numbers and a *square* with four numbers; **b** samples of numerical inductive reasoning tasks with two levels of complexity; timelines of stimuli presentations of **c** low-complexity task and **d** high-complexity task. (Zhang et al. 2013b)

experimental procedure. The numbers and figures were in white with a black background to avoid visual fatigue. At the preparation stage, the subjects put their left index finger on key "D" and right index finger on key "K". The three figures of each trial were presented sequentially along the timeline, as shown in Fig. 1c, d. When the first two triangles/squares were presented, subjects were asked to infer a calculation rule. During the presentation of the third triangle/square, they should judge whether the inferred calculation rule was consistent or inconsistent to that of the third figure as accurately as possible, by pressing "D" for "inconsistent" or "K" for "consistent". Because the correlation of neural efficiency can be found only when the processing time of a task is unrestricted, the rule application process was timeless in our experiment and the end of each trial was determined by button-pressing of subjects.

EEG recording and preprocessing

The EEG data were recorded by a 60-channel Neuroscan international 10–20 system with a sampling rate of 1000 Hz (Fig. 2). Additionally, two reference channels were located at the bilateral mastoids of subjects. Electro-oculographic (EOG) signals were simultaneously recorded by four surface channels to monitor ocular movements and eye blinks.

The continuous EEG signals were band-pass filtered with passband being 1–100 Hz. To construct the temporal pattern of brain activation, the sequential trials of each task were divided into three stages depending on the time periods to which they belong, i.e., early (the first third of trials), middle (the middle third of trials), and late (the last third of trials) periods over the course of a task. The EEG epochs of trials with correct responses were extracted by using a time window comprising 500 ms pre-stimulus and 4000 ms post-stimulus periods. Baseline-correction was conducted according to the pre-stimulus interval, and ocular artifacts were removed based on the simultaneously recorded EOG signals. The artifact rejection procedure was used to exclude the trials contaminated by eye blinks, muscle and electrocardiogram noises. As a result, 195–231 trials were retained for each group under each task condition, with each subject having 26–43 trials. After that, the retained trials from the same cognitive condition were concatenated to form a sample set representing a specific mental state. The number of retained trials of each mental state is listed in Table 1. Besides, the independent component analysis (ICA) in the EEGLAB toolbox was used to further clear the visible artifacts, such as the components of possible ocular and muscle movements.

Task-induced GBR activation and feature set construction

For each mental state, GBR-based feature extraction was conducted according to event-related synchronization/desynchronization (ERS/ERD) in the 35–45 Hz frequency interval of the brain activity, through computing the percentage of power increase/decrease in the task period as compared to resting state:

$$ERS/ERD(f, \Delta t) = [A(f, \Delta t) - R(f)]/R(f)] \times 100\,\% \quad (1)$$

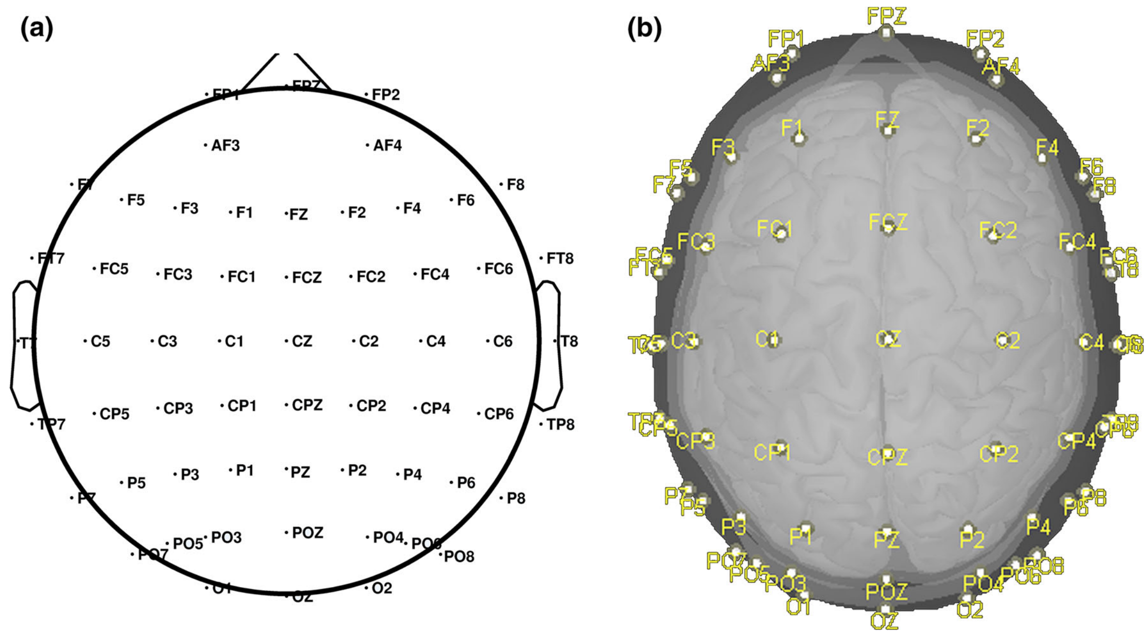

Fig. 2 EEG channel placement of NeuroScan international 10–20 system: **a** EEG channel locations; **b** head model

Table 1 Number of trials (samples) of each mental state

	Four-number induction task Early, middle, and late periods	Three-number induction task Early, middle, and late periods
Math-gifted group	71, 70, 76	75, 76, 71
Control group	64, 63, 67	72, 80, 79

where Δt represents a time window, $A(f, \Delta t)$ is the power spectral density (PSD) of an EEG signal at a specified frequency band, $R(f)$ is the PSD at the same frequency in the pre-stimulus interval of the signal. In this study, the PSD was estimated by using the Burg algorithm with a seventh order autoregressive (AR) model.

Based on the total 60 EEG channels, the spatial GBR distribution constituted a GBR feature set for each mental state, with its dimension equal to (number of channels) × (number of trials).

Feature subset selection algorithm

For determining the efficiency-related brain regions across the subjects, the SFFS algorithm (Pudil et al. 1994) was employed to conduct channel-based localization for discrimination between pairwise GBR feature sets of mental states. The SFFS algorithm can obtain an optimum feature subset (i.e., a combination of spatially distributed EEG channels) by promoting the accuracy in classifying the samples from different feature sets (Zhang et al. 2013a, b). For the localization of task-activated brain areas, the EEG channel sites selected by the method can well correspond to the source positions resulted from the standardized low-resolution electromagnetic tomography (sLORETA) (Dyson et al. 2010).

Let Y be the feature space composed of D features:

$$Y = \{y_j | j = 1, \ldots, D\} \tag{2}$$

where $D = 60$, representing the number of EEG channels. Let X_k be the feature subset that consists of k features selected from feature space Y:

$$X_k = \{x_j | j = 1, \ldots, k, x_j \in Y\}, \quad k = 0, 1, \ldots, D \tag{3}$$

At first, X_k is initialized as an empty set with 0 feature, i.e., $X_0 = \phi$, $(k = 0)$. The feature selection procedure is conducted in "growing" and "pruning" phases alternatively. During the growing phase, the best feature x^+ is added to the selected feature subset as follows:

$$X_{k+1} = X_k + x^+, \quad k = k + 1 \tag{4}$$

which makes the feature subset X_{k+1} have the highest discrimination, i.e.,

$$x^+ = \arg\max_{x \in Y - X_k} J(X_k + x) \tag{5}$$

where J is the mean classification accuracy achieved by linear discriminant analysis (LDA) with cross-validation.

When $k > 2$, the selection procedure enters into pruning phase after growing in each iteration. During this phase, some features in X_k will be removed in turn. If the removal of x^- in X_k results in better discrimination, i.e.,

$$J(X_k - \{x^-\}) > J(X_k) \tag{6}$$

$$x^- = \arg\max_{x \in X_k} J(X_k - x) \tag{7}$$

this feature will be deleted from X_k as follows:

$$X_{k-1} = X_k - x^-, \quad k = k - 1 \tag{8}$$

when k is up to the preset maximum number of selected features (channels), the selection procedure will end. In the final feature subset X_k, the corresponding k EEG channels can be found. Because features (channels) producing the highest classification accuracy were selected, X_k is viewed as an "optimum" feature subset (channel combination) in identifying the trial samples from different mental states.

In this investigation, feature selection was conducted for discrimination between pairwise mental states with efficient activation change. Given each experimental factor tested in our study, the pairwise feature sets of mental states and the related class labels were determined as follows:

1. Level of mathematical ability (class labels: "gifted" vs. "non-gifted"): The feature selection was performed between the two groups in the early period of the four-number induction task, when the mental states were viewed as the initially effortful response of the subjects for addressing novel problems;
2. Task complexity (class labels: "low" vs. "high"): In each group, the feature selection was conducted in the early periods of the two tasks, since the mental states in the two periods could reflect the differential cortical effort that the subjects invested for responding to the tasks with different levels of complexity;
3. Short-term learning over task course (class labels: "early" vs. "late"): To ensure enough difference between the mental states, the trials in the middle period were excluded from the discrimination analysis. In each group, the mental states in the early and late periods of each task were analyzed to discover the brain regions influenced by rapid learning.

For each selection, the maximum number of acceptable EEG channels was set to 15 (a quarter of the total EEG channels). Finally, an optimal scalp channel group composed of all the channels resulted from the multi-factor selections could be determined.

ANOVA statistical test

The between-groups difference in the RAPM score was tested by the one-way analysis of variance (ANOVA). Besides, to examine possible differences in task performances and induced neural responses, three-way ANOVA was performed for testing response accuracy, response time of correct responses, and average global GBRs respectively, with group (gifted vs. non-gifted) serving as the between-subject factor and task complexity (low vs. high) and task period (early vs. late) serving as the within-subject factors.

Cortical current estimation of task-induced GBR activation

In order to obtain the source activities underlying the task-induced GBRs, the gamma-band EEG signals were transformed into the cortical currents by using a source estimation procedure in the Brainstorm toolbox (http://neuroimage.usc.edu/brainstorm; Tadel et al. 2011).

The source analysis was composed of the forward and inverse problem-solving processes. In the forward process, the EEG signals were assumed to be primarily determined by a block of electric dipoles located at the cortical surface. Based on an averaged realistic head model "Colin 27" constituted by four layers (scalp, outer skull, inner skull, and cortex), the symmetric boundary element method (BEM) in the open-source software (http://www-sop.inria.fr/athena/software/OpenMEEG/; Gramfort et al. 2010) was applied to the EEG channel locations on the head model, to obtain the volume conductor modeling of the subjects, i.e., the forward model matrix. After that, the noise of the scalp sensors was removed through computing the noise covariance matrix of the signals in the pre-stimulus time interval. In the inverse process, an inverse kernel matrix was produced by the forward model and the whitened and depth-weighted linear L2-minimum norm estimation algorithm, based on which the raw EEG signals were transformed into the cortical source currents.

After removing the pre-stimulus baseline activity of the gamma-band cortical currents, the Student t tests of equal variance were used to compare the difference in each pair of selected mental states. For each comparison, the threshold of statistical significance was set to 0.05. The multiple comparisons for all cortical vertices generate a statistical map with significant difference in cortical currents between mental states.

Results

Behavioral performances and significant differences

Significant between-groups difference is firstly found in the RAPM test ($p = 0.0000$), in which the average score of the math-gifted group is 33.6 ± 0.9 (mean ± SD) and the control group has an average score of 23.5 ± 4.6 (mean ± SD; Table 2).

Table 2 One-way ANOVA for between-groups difference in RAPM score

Source	SS	*dF*	MS	F	*p*
Groups	377.34	1	377.34	39.06	0.0000
Error	125.59	13	9.66		
Total	502.93	14			

SS Sum of squares, *dF* degrees of freedom, *MS* mean square

p Significance level of the AVOVA

In the tests for the behavioral performances, task complexity shows significant main effect on both response accuracy and response time in the two tasks. In the low-complexity task, the subjects have obtained higher response accuracy [three-number task vs. four-number task: 89.89 ± 3.92 vs. 85.33 ± 6.61 % (mean ± SD); $p = 0.0047$; Table 3a], and shorter response time [three-number task vs. four-number task: 694 ± 290 ms vs. 881 ± 440 ms (mean ± SD); $p = 0.0000$; Table 3b].

Besides, the ANOVA reveals significant main effect of mathematical ability level on response time, in which the math-gifted adolescents show faster response time compared to the control subjects [math-gifted group vs. control

Table 3 Three-way ANOVAs

Source	SS	*dF*	MS	F	*p*
(a) Response accuracy					
Task period	0.0041	1	0.0041	1.88	–
Task complexity	0.0190	1	0.0190	8.77	0.0047
Math level	0.0030	1	0.0030	1.40	–
Task period × task complexity	0.0042	1	0.0042	1.92	–
Task period × math level	0.0001	1	0.0001	0.03	–
Task complexity × math level	0.0030	1	0.0030	1.40	–
Task period × task complexity × math level	0.0019	1	0.0019	0.85	–
Error	0.1133	52	0.0022		
Total	0.1498	59			
(b) Response time					
Task period	2.2815E+05	1	2.2815E+05	1.73	–
Task complexity	5.3872E+06	1	5.3872E+06	40.87	0.0000
Math level	1.2152E+06	1	1.2152E+06	9.22	0.0025
Task period × task complexity	1.6562E+06	1	1.6562E+06	12.56	0.0004
Task period × math level	632.4	1	632.4	0	–
Task complexity × math level	7.5182E+04	1	7.5182E+04	0.57	–
Task period × task complexity × math level	6.2885E+05	1	6.2885E+05	4.77	0.0294
Error	7.4739E+07	567	1.3182E+05		
Total	8.3440E+07	574			
(c) Mean value of global GBRs					
Task period	13.489	1	13.489	205.35	0.0000
Task complexity	0.3445	1	0.3445	5.26	0.0222
Math level	0.0624	1	0.0624	0.95	–
Task period × task complexity	0.0013	1	0.0013	0.02	–
Task period × math level	0.6277	1	0.6277	9.56	0.0021
Task complexity × math level	0.3681	1	0.3681	5.6	0.0183
Task period × task complexity × math level	0.604	1	0.604	9.19	0.0025
Error	37.2442	567	0.0657		
Total	52.8971	574			

SS Sum of squares, *dF* degrees of freedom, *MS* mean square

p Significance level of the AVOVA

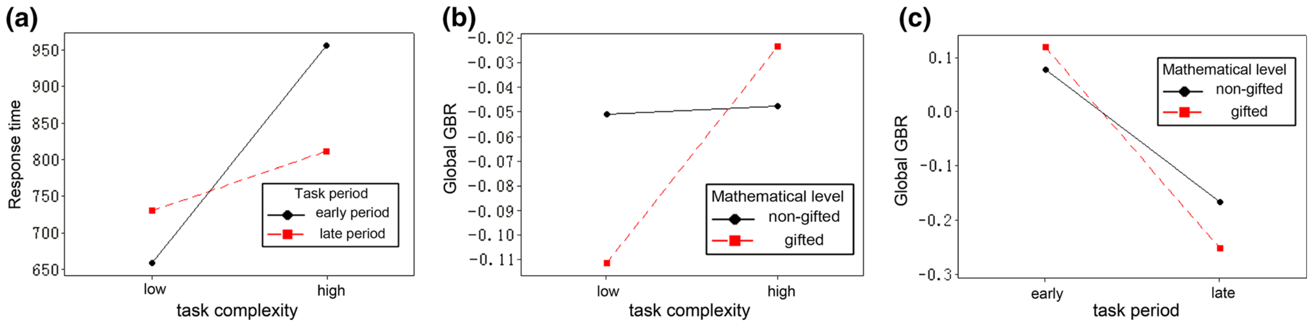

Fig. 3 Significant interaction effect plots based on three-way AVOVA tests with factors of mathematical ability level, task complexity, and task period ($p < 0.05$): **a** response time affected by the interaction between task complexity and task period; **b** mean value of global GBR affected by the interaction between task complexity and mathematical ability level; **c** mean value of global GBR affected by the interaction between task period and mathematical ability level

group: 658 ± 229 vs. 730 ± 336 ms (three-number task); 827 ± 373 vs. 941 ± 499 ms (four-number task; mean ± SD); $p = 0.0025$; Table 3b].

The behavioral effect of short-term learning has been tested by the comparison of response accuracy and response time within the trials in the early and late periods of the tasks. The ANOVA shows that there is no significant main effect of task period on the behavioral performances, but the statistical result reveals significant effect of interaction between task period and task complexity on response time (Table 3b). As shown in Fig. 3a, the response time is significantly shortened from early to late period of the four-number induction task [Early period: 956 ± 480 ms; Late period: 812 ± 378 ms (mean ± SD); $p = 0.0004$]. The behavioral performance improvement in the complex induction task shows the expected learning effect when there are high task demands.

Furthermore, for response time, there is statistically significant three-way interaction among mathematical ability level, task complexity, and task period ($p = 0.0294$; Table 3b).

Significant GBR-based mental states related to neural efficiency

In the brain electric activity maps of the gamma-band ERS/ERD, the global GBR is manifested as the widely distributed power change with respect to the baseline activity in the frontal, parietal, temporal, and occipital regions (Fig. 4). The three-way ANOVA test for average global GBRs shows a significant three-way interaction among mathematical ability level, task complexity, and task period ($p = 0.0025$; Table 3c).

The brain maps and the results of the ANOVA test indicate that multiple modulating factors of neural efficiency can be reflected by the GBR-based mental states, as show in Table 3c, Fig. 3b–c, and 4. At first, neural efficiency influenced by task complexity appears to occur in the four GBR maps in the early periods of the two tasks (Fig. 4a1, b1, c1, d1). At this stage, as compared with the non-gifted subjects, the math-gifted adolescents show stronger cortical response in the four-number induction items, but less cortical activity in processing the three-number items. From complex to easy reasoning items, this reversed relation between mathematical ability level and neural response is rather consistent with the findings of the previous neural efficiency studies. Furthermore, the ANOVA confirms significant main effect of task complexity on global GBR (Table 3c). As shown in Fig. 3b, the two groups of subjects show higher GBR in the four-number induction task. Meanwhile, there is significant effect of interaction between mathematical ability level and task complexity on global GBR. In Fig. 3b, the math-gifted adolescents show higher neural response in complex induction task but lower activity in easy task than the control subjects.

In addition, the temporal pattern expressed as the sequential GBR maps also shows the globally decreasing neural responses over the early–middle–late periods of the tasks (Fig. 4a–d). The neural activities modulated by short-term learning were tested by the comparisons of the GBRs in the early and late periods of the tasks. The ANOVA shows significant main effect of task period on global GBR (Table 3c). It should be noted that there is significant effect of interaction between mathematical ability level and task period on global GBR. As shown in Fig. 3c, the math-gifted group exhibits more significantly decreasing GBR from the early to late period of the tasks than the control group, especially over the course of the four-number induction task (Fig. 4a1, a2, a3). These results collectively suggest that the short-term learning exerts stronger influence on the brain activity of the math-gifted adolescents.

Optimum channel combination for discrimination between pairwise mental states

In the early period of the four-number induction task, the optimum channel combination with discrimination accuracy

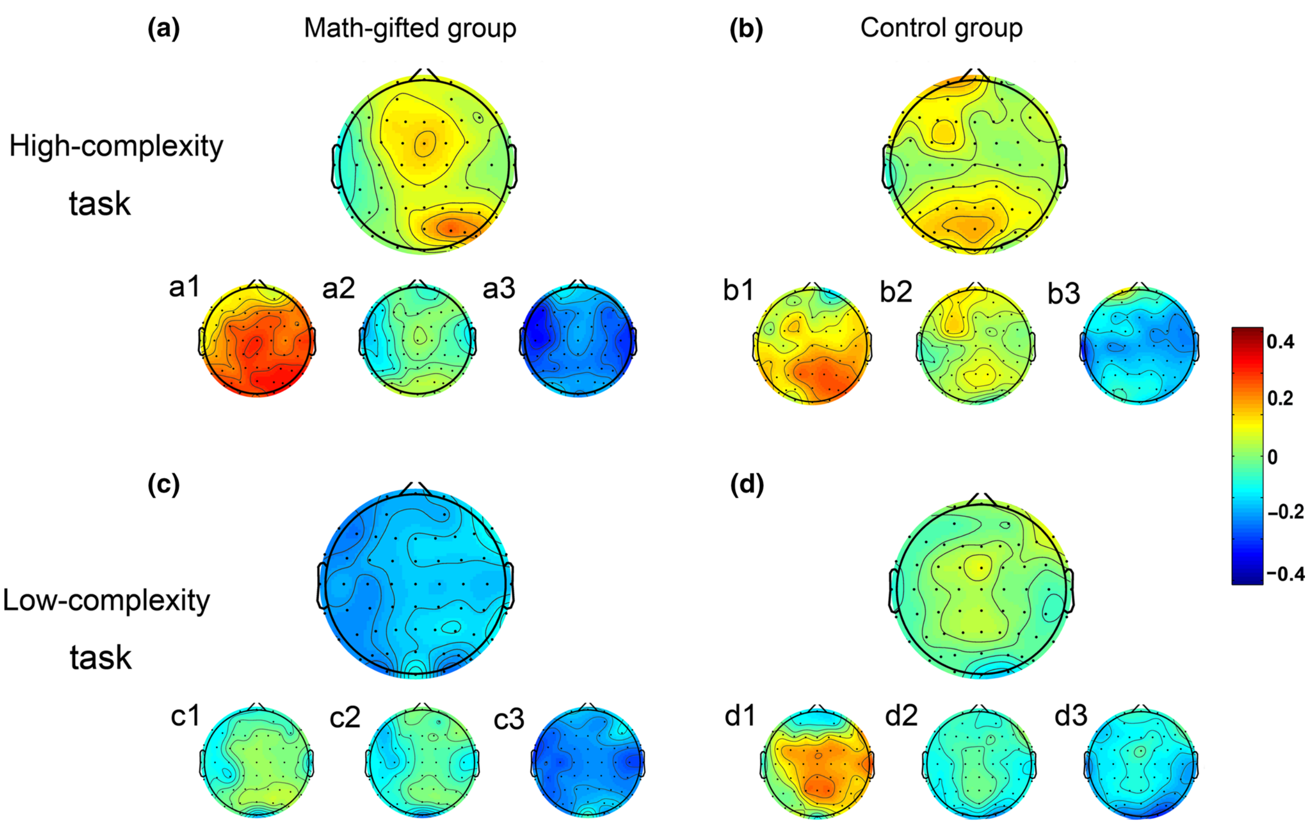

Fig. 4 Brain electric activity mapping of GBR activation, with positive value representing ERS and negative value ERD: **a** math-gifted group and **b** control group in high-complexity task; **c** math-gifted group and **d** control group in low-complexity task. The four larger topological maps are derived from the mean ERS/ERD values over total trials of a task. The three smaller maps below the larger ones are from the trials in the early, middle, and late periods over the task course respectively

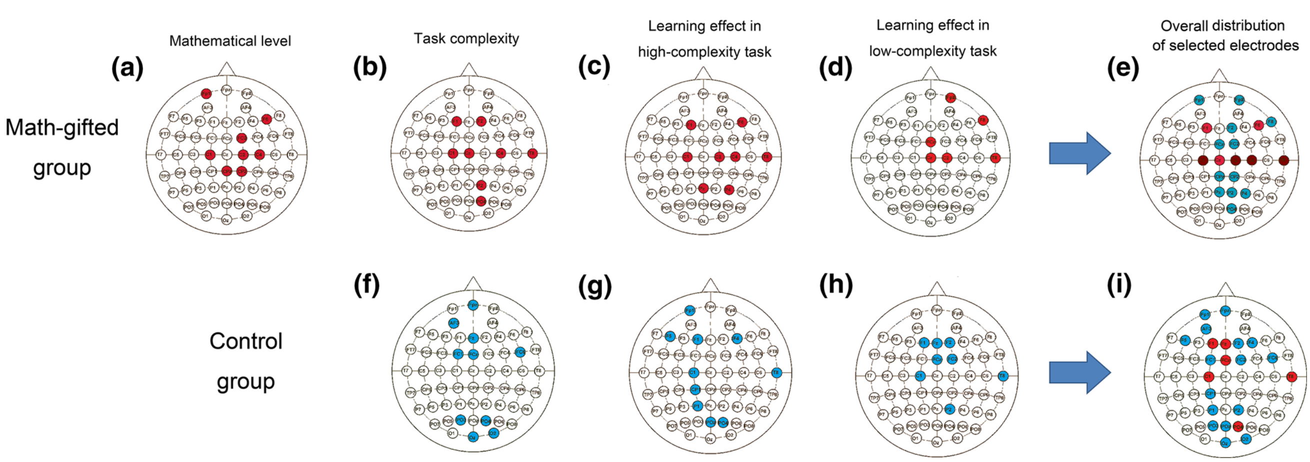

Fig. 5 Optimal EEG channel combinations selected by the SFFS algorithm: *Top row* math-gifted group; *Bottom row* control group. The channels with the highest discrimination accuracy for identifying **a** "gifted" against "non-gifted", **b**, **f** "high-complexity" against "low-complexity", **c**, **g** "early period" against "late period" mental states in high-complexity task, and **d**, **h** "early period" against "late period" mental states in low-complexity task; **e**, **i** overall distribution of the selected channels, in which the *blue channels* were selected once, the *red channels* twice, and the *dark red channels* three times. (Color figure online)

of 0.6808 between the two groups mainly covers the right-lateral frontal–centroparietal regions, including left prefrontal (FP1), right inferior frontal (F6), right superior frontocentral (FC2), right-lateral central (C1, C2, and C4), and right-lateral centroparietal (CPZ and CP2) brain locations (Fig. 5a). The result indicates that, at the early stage of the complex task, the most significant activation difference between the “gifted” and “non-gifted” subjects is located at the anterior fronto–parietal brain regions, where the math-gifted adolescents can recruit more neural resources to solve the novel reasoning problems.

On the other hand, while the math-gifted adolescents are confronted with the high- to low-complexity tasks, their brain regions with the most inhibited neural activity are characterized by the fronto–parietal EEG channels, including the left superior frontal (F1), right superior frontal (F2), right-lateral central (C1, CZ, and C4), right temporal (T8), right parietal (P2), and right parietooccipital (PO4) locations (Fig. 5b). The accuracy is 0.7110 in discriminating the “high-complexity” and “low-complexity” trial samples. The result suggests that the coherent fronto–parietal brain regions might be flexibly recruited by the math-gifted adolescents in response to effortful cognitive processing.

Over the course of the four-number induction task, the brain regions with learning-related decrease of neural activity in the math-gifted adolescents are also illustrated by the fronto–parietal distribution of the EEG channels with discrimination accuracy of 0.7444 between the “early period” and “late period” trial samples, including left superior frontal (F1), right inferior frontal (F6), right-lateral central (C1, C2, and C4), right temporal (T8), and right-lateral parietal (PZ and P4) locations (Fig. 5c). During the three-number induction task performed by the math-gifted subjects, the brain regions with declined activation caused by the rapid learning has obtained the lowest discrimination accuracy of 0.6712 among all the discriminations tested in this experiment. The channel combination that optimally discriminates the “early period” and “late period” trial samples is primarily located at the anterior brain regions of the right-lateral cerebral hemisphere, including the right prefrontal (FP2), right inferior frontal (F8), frontocentral (FCZ), right-lateral central (CZ and C2), and right temporal (T8) locations (Fig. 5d). The result indicates that, when the task is easy enough for the math-gifted subjects, the left hemisphere is not highly involved in the reallocation of neural resources, and the dominant fine tuning occurs in their right-lateral anterior brain regions.

By contrast, the “optimum” channels related to task complexity and short-term learning of the control subjects are mainly distributed in the left-lateral frontal and bilateral parietooccipital brain regions (Fig. 5f–h, classification accuracy: 0.6603, 0.6938, and 0.7059), which are consistent with the results from an fMRI study on normal adolescents (Wartenburger et al. 2009). Especially, unlike the math-gifted adolescents, the channels at left prefrontal (FP1), left frontal pole (AF3), and left inferior frontal (F5) brain locations show the “optimal” efficiency-related activation variations in the average-ability subjects.

Selective recruitment of neural resource in the right-lateral fronto–parietal system

In the integrative results from the multi-factor feature subset selections, the optimal channel group presents a coherent right-lateral fronto–parietal distribution network (Fig. 5e). The channels at the right-lateral frontal lobe, especially the right inferior frontal gyrus, have been selected more than once by the SFFS algorithm in multiple discriminations, which represent the most selectively recruited brain regions with neural activity varying under different cognitive conditions. Meanwhile, the right temporal region has also been selectively used by the math-gifted adolescents, which can be associated with the responsiveness of the temporofrontal system for the changing language processing demands in the tasks.

In the comparisons of the gamma-band cortical source currents (Fig. 6), significant difference in cortical activation can be observed in the left superior and middle frontal gyri and the right superior and inferior frontal gyri (Fig. 6e). Specifically, the strength of cortical currents in the right superior and inferior frontal gyri varies with the four comparison conditions (Fig. 6a–d), which indicates the flexible recruitment of the right frontal cortical resource in the math-gifted brain. Meanwhile, significant differences are also found in the right frontal-parietal cortices and the regions within the right temporal and occipital lobes, which constitute the important fronto–parietal system of information processing.

Discussion and conclusion

Efficiency-related GBR activation pattern

By using task-induced GBR to quantify the level of neural activity, we investigated the brain regions where neural resources were used in a most efficient manner under different cognitive conditions. As a result, in the subjects with two levels of mathematical ability, neural efficiency occurred at the initial stages of the tasks with two levels of complexity, and was manifested as the declining GBRs over the course of tasks.

Previous studies suggest that the emergence of neural efficiency is often associated with decreased mental effort for cognitive load in the working memory (Neubauer and Fink 2009; Hoppe et al. 2012). Because the subjects started the tasks without any practice in this experiment, the strongly induced early GBRs can be associated with the individual effortful processing for the novel information taxed on the working memory system (Neubauer and Fink 2009). As becoming more skilled for the tasks, the subjects can switch from an effortful processing mode to a more

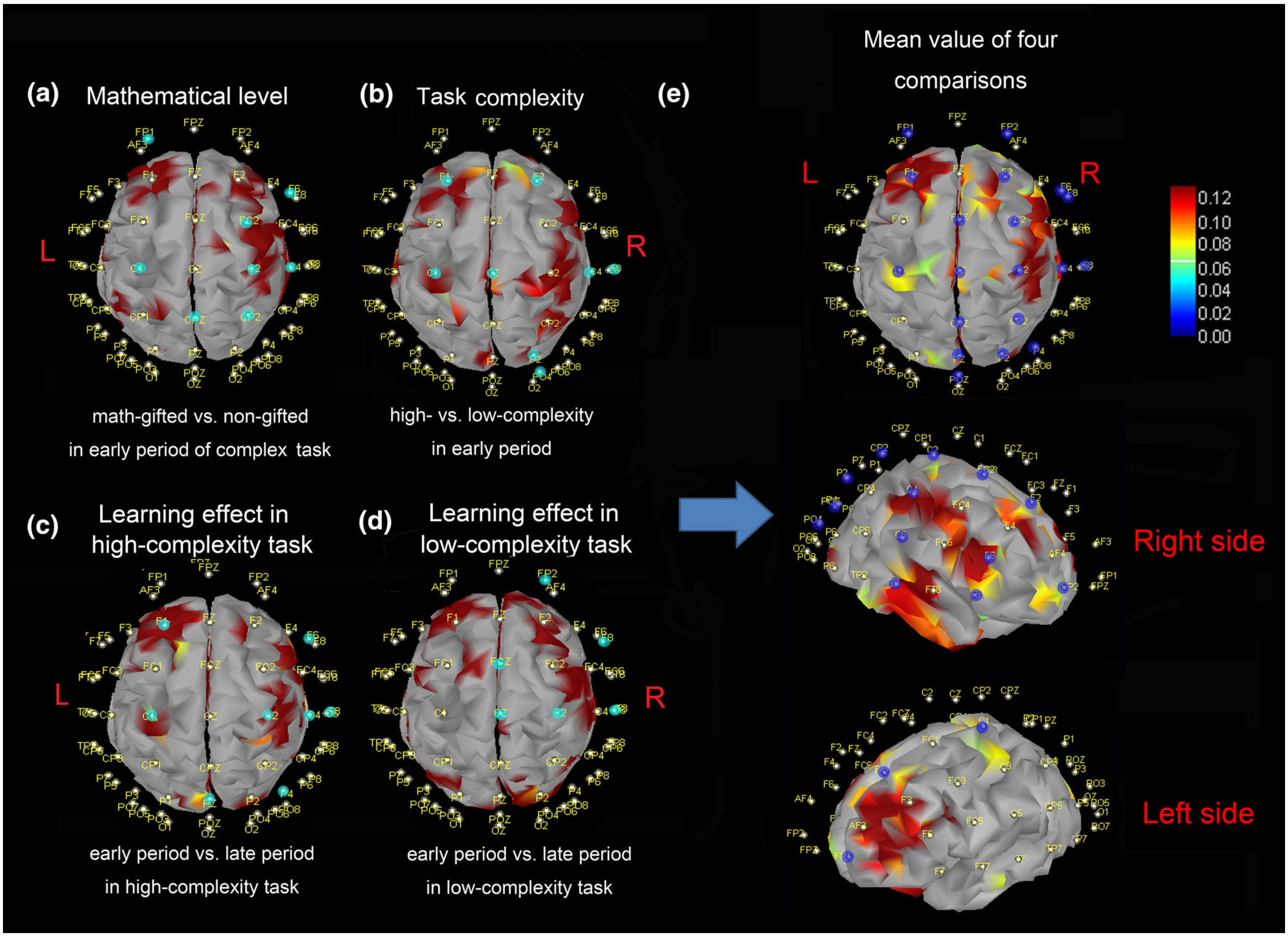

Fig. 6 Significant difference in gamma-band cortical currents between pairwise mental states ($p < 0.05$): After removing baseline currents, the activation difference is estimated by a source analysis procedure, regarding **a** mathematical ability level, **b** task complexity, and short-term learning effects in **c** high-complexity task and **d** low-complexity task; **e** the three maps indicate the mean difference from the comparisons in (**a**–**d**). The *circles* on the cortical surface represent the EEG channels selected by the SFFS algorithm

automated processing mode that needs less working memory capacity and thus consumes less cortical resource (Neubauer and Fink 2009; Rypma et al. 2006).

It is notable that the development of efficient cognitive strategies depending on short-term learning is an important factor for reducing cognitive load from task demands (Neubauer and Fink 2009). In this experiment, while the subjects were processing the numerical induction items, a better-honed problem-solving strategy could be rapidly formed to replace the originally required calculations with numbers. For example, for the three-number induction task (i.e., the case of ‘A + B = C’ or ‘A + C = B’ in Fig. 1a), while the three triangles are presented, the subjects can judge whether the largest numbers in the three triangles are at the same locations or not to rapidly verify the consistency of the calculation rules. For the four-number induction task, if the subjects can rapidly gain the insight into the law of the task, they can first sort the four numbers in each square. If the largest one is the sum of the other three numbers (i.e., the case of ‘B = A + C + D’), they can just compare the locations of the largest numbers in the three squares, similar to the three-number induction task; otherwise (i.e., the case of ‘A + B = C + D’), the largest number can be matched with the smallest one in a square, and the other two numbers are matched as well. After that, the subjects just need to compare the locations of the matched number pairs in the three squares to determine the consistency of the rules.

Combining the behavioral performances and the GBR changes, we find that a more significant behavioral advantage (reduced response time and increased accuracy) is accompanied by rapid GBR decrease in the math-gifted subjects, especially over the course of the four-number induction task. The mental state changes indicate that the math-gifted adolescents can devote more neural resource in addressing novel problems. As time goes by, they can use the brain circuitry in a more efficient manner, possibly through developing a better-honed cognitive strategy for the repeated stimuli with the same processing type, whilst the non-gifted subjects' learning speed for the four-number task is relatively slow and the GBR changes are thus less obvious in comparison with the math-gifted subjects. On the other hand, as the three-number induction task was easy enough, even at the initial stage, the processing capacity of the subjects is not taxed strongly by this task. Therefore, smaller change between mental states can be observed over the course of this task.

The brain of a child has more adjustable characteristics through specific training and other environmental variables than the adult's brain. In our study, the math-gifted adolescents show stronger learning-induced changes in functional brain response. Nevertheless, the plasticity of the brain is not restricted to the functional characteristic of brain activation. Under the influence of extensive training and practice, structural gray matter changes might be observed, at least in some brain regions (Neubauer and Fink 2009). Based on the malleability of the brain, cognitive training of mental abilities is expected to affect functional and structural improvements of the child's brain, which helps to further build or strengthen cognitive skills such as processing speed, memory, and reasoning.

Right-lateral fronto–parietal system involved in neural efficiency of the math-gifted brain

As an extended analysis of Zhang et al.'s (2013a, b) study, the present investigation further constructed the temporal pattern of the task-induced neural responses, in which the effect of short-term learning was considered as an important factor influencing GBR changes. Through integrating the results from multiple feature subset selections, more EEG channel sites were identified as the "optimal" brain locations involved in neural efficiency of the math-gifted brain, especially the channels located at central sensorimotor area. As a result, the "optimal" EEG channels identified by this study form a right-lateral fronto–parietal distribution network, not just the right-lateral frontal and bilateral superior parietal cortical regions reported by the previous study (Zhang et al. 2013a, b).

General intelligence and specific aptitude (or talent) may constitute specific giftedness (Gardner 1985). The past neuroscience research has suggested different neural basis common to general intelligence or specific to gifted ability in mathematics (O'Boyle et al. 2005; Prescott et al. 2010). Particularly in the frontal areas identified by this study, the neural resources have been maximally used or inhibited by the math-gifted individuals, which can be viewed as a strong ability of top-down control on processing important task-relevant information and inhibiting retrieval of irrelevant/repeated information (Klimesch et al. 2007). In this experiment, the recruitment of frontal central executive functions of the math-gifted subjects is actually affected by the interplay of subjects' intelligence and acquired expertise in mathematics. Of particular interest is the fact that the distribution of the channel sites presents the right hemisphere lateralization, especially the right frontal lobe. For the math-gifted adolescents, the substantial right hemisphere involvement in information processing and the enhanced reliance on the right hemisphere function have been suggested as the important indications of precocious mathematical ability (Desco et al. 2011; O'Boyle et al. 2005; Prescott et al. 2010). Importantly, the cognitive functions in right-lateral frontal lobe are related to gifted thinking abilities in mathematics, including spatial information processing, reasoning and creative thinking. (O'Boyle et al. 2005). Therefore, the selective engagement of right frontal brain regions might be the functional characteristic of superior mathematical ability of the math-gifted adolescents.

Additionally, we used a source analysis procedure to transform the task-induced GBRs into the cortical dipoles to estimate the cortical distribution of changed gamma-band currents. While comparing the results from the two methods, we find that the major scalp locations determined by the SFFS method are consistent with the regions of cortical current change, including the bilateral superior frontal, right inferior frontal, right-lateral central and right temporal locations (Fig. 6e). However, some regions localized by the SFFS algorithm are inconsistent with the spatial distribution of cortical activation difference. As can be seen in Fig. 6d, although the channels at the midline frontal and precentral gyri have been selected as the optimal learning-dependent sites in the low-complexity task, these brain regions have not shown the highest cortical current variations in the source analysis. In EEG studies, the midline anterior brain locations are usually viewed as the appearance of the activated anterior cingulate (AC). In the past neuroimaging studies, the learning-related activation decrease in the AC has been reported consistently, because of its correlation with cognitive controlling function of the brain (Chein and Schneider 2005). Since the SFFS algorithm allows for a certain degree of trial-to-trial variability, while discriminating the brain responses across all the subjects and trials, the isolated brain response area (e.g., the AC) may be more informative than that of the trial-averaged source analysis.

Practical implication of the brain regions localized by the SFFS feature subset selection method

In the locations involved in neural efficiency of the math-gifted brain, the right frontal regions have great potentials in developing the cortical resource of children and early adolescents in mathematical learning, since the focus of their brain maturation is still on the relationship forged between the frontal lobe function and higher-order cognitions (Alexander et al. 1996). EEG-based brain–computer interfaces (BCI) could provide a feasible approach to functional improvement through on-line neurofeedback for children or early adolescents. For example, attention training and assessment of the PFC through BCI games are useful for enhancing child's attention concentration (Chen et al. 2012). Through a BCI system, neurofeedback training in EEG upper alpha frequency is expected to improve child's performance in working memory (Escolano et al. 2011). In addition, the laterality of the frontal cortical activity can be manipulated by on-line neurofeedback training to improve individual's neural response capacity (Harmon-Jones et al. 2010). According to the findings from our experiment and previous neuroimaging studies, future on-line laterality training with more effective neurofeedback would be expected to mediate the functions of the right frontal lobe that is most strongly associated with gifted mathematical thinking activity and efficient neural manipulations. The GBR pattern and the optimal EEG channel combination identified by our study could provide possibly more effective neural features and channel locations for training the frontal functions of children/adolescents through future BCI systems.

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