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RESEARCH ARTICLE

Delay-decomposing approach to robust stability for switched interval networks with state-dependent switching

Ning Li · Jinde Cao · Tasawar Hayat

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Abstract This paper is concerned with a class of nonlinear uncertain switched networks with discrete time-varying delays. Based on the strictly complete property of the matrices system and the delay-decomposing approach, exploiting a new Lyapunov–Krasovskii functional decomposing the delays in integral terms, the switching rule depending on the state of the network is designed. Moreover, by piecewise delay method, discussing the Lyapunov functional in every different subintervals, some new delay-dependent robust stability criteria are derived in terms of linear matrix inequalities, which lead to much less conservative results than those in the existing references and improve previous results. Finally, an illustrative example is given to demonstrate the validity of the theoretical results.

Keywords Switched interval networks · Robust asymptotic stability · Delay-decomposing approach · Interval time-varying delay · Parameters uncertainty

N. Li
Department of Mathematics, Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096, China

J. Cao (✉)
Department of Mathematics, Southeast University, Nanjing 210096, China
e-mail: jdcao@seu.edu.cn

J. Cao · T. Hayat
Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

T. Hayat
Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan

Introduction

Recently, a class of hybrid systems (Ye et al. 1998) have attracted many researchers' significant attentions as they can model several practical control problems that involve the integration of supervisory logic-based control schemes and feedback control algorithms. As a special class of hybrid systems, switched networks (Brown 1989; Liberzon 2003) consist of a set of individual subsystems and a switching rule, play an important role in research activities, since they have witnessed the successful applications in many different fields such as electrical and telecommunication systems, computer communities, control of mechanical, artificial intelligence and gene selection in a DNA microarray analysis and so on. Therefore, the stability issues of switched networks have been investigated (Huang et al. 2005; Li and Cao 2007; Lian and Zhang 2011; Zhang and Yu 2009; Niamsup and Phat 2010). By using common Lyapunov function method and linear matrix inequality (LMI) approach, authors considered the problem of global stability in switched recurrent neural networks with time-varying delay under arbitrary switching rule in (Li and Cao 2007). However, common Lyapunov function method requires all the subsystems of the switched system (Liu et al. 2009) to share a positive definite radially unbounded common Lyapunov function. Generally, this requirement is difficult to achieve. The average dwell time method is proposed to deal with the analysis and stability of switched networks, which is regarded as an important and attractive method to find a suitable switching signal to guarantee switched system stability or improve other performance, and has been widely applied to investigate the analysis and stability for switched system with or without time-delay. In (Lian and Zhang 2011), employing the average dwell time approach (ADT), novel multiple

Lyapunov functions were employed to investigate the stability of the switched neural networks under the switching rule depending on time. Generally speaking, switching rule is a piecewise constant function dependent on the state or time, most of existing works focus on stability for switched networks with switching rule dependent on time. Perhaps it is limited by existing method and technique, to the best of our knowledge, there are few scholars to deal with the robust stability (He and Cao 2008; Xu et al. 2012) for switched uncertain networks under state-dependent switching rule (Thanha and Phat 2013; Ratchagit and Phat 2011), despite its potential and practical importance.

Due to the finite switching speed of amplifiers, time delay especially time-varying delay is inevitably encountered in many engineering applications and hardware implementations of networks, it is often the main cause for instability and poor performance of system. Consequently, the stability of networks with time-varying delay is a meaningful research topic (Liu and Chen 2007). What the most we concern is how to choose the appropriate Lyapunov–Krasovskii functional, derive the better stability criteria, which can be shown that the results has less conservativeness. To reduce the conservatism of the existing results, new analysis methods such as free weighting matrix method, matrix inequality method, input–output approach are proposed. However, it is impossible to derive a less conservative result by using the common Lyapunov–Krasovskii functional, the delay central-point (DCP) method was firstly proposed in (Yue 2004), to solve the problem for robust stabilization of uncertain systems with unknown input delay. In this approach, introducing the central point of variation of the delay, the variation interval of the delay is divided into two subintervals (Zhang et al. 2009) with equal length. The main advantage of the method is that more information on the variation interval of the delay is employed, and the idea of delay-decomposing (Zhang et al. 2010; Zeng et al. 2011; Wang et al. 2012; Hu and Wang 2011; Wang et al. 2008) has been successfully applied in investigating the H_∞ control and the delay-dependent stability analysis for discrete-time or continuous-time systems with time-varying delay, which significantly reduced the conservativeness of the derived stability criteria. In (Zhang et al. 2010), the delay interval $[0, d(t)]$ was divided into some variable subintervals by employing weighting delays, the stability results based on the weighting delay method were related to the number of subintervals, and the size of the variable subintervals or the position of the variable points. Authors considered the exponential stability analysis for a class of cellular neural networks, constructed a more general Lyapunov–Krasovskii functional by utilizing the central point of the lower and upper bounds of delay, since more information was involved and no useful item was ignored throughout the estimate of upper bound of the derivative of Lyapunov functional, the developed conditions were expected to be less conservative than the previous ones (Wang et al. 2012). Up to now, there no results have been proposed for the switched uncertain systems with discrete time-varying delay based on the delay-decomposing approach. Therefore, it is of great importance to study robust stability of switched uncertain networks with interval time-varying delay.

Motivated by the aforementioned discussions, the purpose of this paper is to deal with the robust asymptotic stability problem for switched interval networks with interval time-varying delays and general activation functions, the activation function can be unbounded and the lower bound of time-varying delay do not need to be zero. Inspired by the (DCP) method in (Yue 2004), constructing new Lyapunov–Krasovskii functional decomposing the delays in integral terms, based on the strictly complete property of the matrices system the delay-decomposing approach, some new delay-dependent robust stability criteria are derived in terms of LMIs, which can be efficiently solved by the interior point method (Boyd et al. 1994). The main novelty of this paper can be summarized as following: (1) switching signal in the paper depends on state of networks. (2) consider the parameters fluctuation, a new mathematical model of the switched networks with parameters in interval is established, it become much closer to the actual model. (3) introduce the delay-decomposing idea and piecewise delay method, analyzing the variation of the Lyapunov functional in every different subintervals, some new delay-dependent robust stability criteria are derived. Note that the delay-decomposing approach has proven to be effective in reducing the conservatism.

The rest of this paper is organized as follows: In "Switched networks model and preliminaries" section, the model formulation and some preliminaries are presented. In "Main results" section, some delay-dependent robust stability criteria for switched interval networks are obtained. An numerical example is given to demonstrate the validity of the proposed results in "An illustrative example" section. Some conclusions are drawn in "Conclusion" section.

Notations Throughout this paper, R denotes the set of real numbers, R^n denotes the n-dimensional Euclidean space, $R^{m \times n}$ denotes the set of all $m \times n$ real matrices. For any matrix A, A^T denotes the transpose of A, $A > 0$ ($A < 0$) means that A is positive definite (negative definite), * represents the symmetric form of matrix. $\dot{x}(t)$ denotes the derivative of $x(t)$. Matrices, if their dimensions not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

Switched networks model and preliminaries

Consider the interval network model with discrete time-varying delay described by the following differential equation in the form:

$$\begin{cases} \dot{y}(t) = -Ay(t) + B_1 g(y(t)) + B_2 g(y(t-\tau(t)) + u, \\ A \in A_I, \quad B_k \in B_I^{(k)}, \quad k = 1,2, \end{cases} \tag{1}$$

where $y(t) = (y_1(t), \ldots, y_n(t))^T \in R^n$ denotes the state vector associated with n neurons; $g(y) = (g_1(y_1), \ldots, g_n(y_n))^T{:}R^n \to R^n$ is a vector-valued neuron activation function; $u = (u_1, \ldots, u_n)^T$ is a constant external input vector. $\tau(t)$ denotes the discrete time-varying delay. $A = \mathrm{diag}(a_1, \ldots, a_n) > 0$ is an $n \times n$ constant diagonal matrix, denotes the rate with which the cell i resets its potential to the resting state when being isolated from other cells and inputs; $B_k = (b_{ij}^{(k)}) \in R^{n\times n}, k = 1,2,$ represent the connection weight matrices, and $A_I = [\underline{A}, \overline{A}] = \{A = \mathrm{diag}(a_i){:}\, 0 < \underline{a}_i \le a_i \le \overline{a}_i, i = 1,2,\ldots,n\}$ $B_I^{(k)} = [\underline{B}_k, \overline{B}_k] = \{B_k = (b_{ij}^{(k)}){:}\, \underline{b}_{ij}^{(k)} \le b_{ij}^{(k)} \le \overline{b}_{ij}^{(k)}, i,j = 1,2,\ldots,n\}$ with $\underline{A} = \mathrm{diag}(\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n), \overline{A} = \mathrm{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n), \underline{B}_k = (\underline{b}_{ij}^{(k)})_{n\times n}, \overline{B}_k = (\overline{b}_{ij}^{(k)})_{n\times n}$.

Throughout this paper, the following assumptions are made on the activation functions $g_j(\bullet), j = 1, 2, \ldots, n$ and discrete time-varying delay $\tau(t)$:

$(\mathcal{H}_1)$: There exist known constant scalars $\check{l}_i$ and $\hat{l}_i$, such that the activation function $g_j(\bullet)$ are continuous on R and satisfy:

$$\check{l}_i \le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le \hat{l}_i \quad \forall s_1 \ne s_2 \in R, \quad j = 1,2\ldots n$$

$(\mathcal{H}_2)$: The time-varying delay $\tau(t)$ is differentiable and bounded with constant delay-derivative bounds:: $\tau_n \le \tau(t) \le \tau_N$, $\dot{\tau}(t) \le \mu < 1$, where τ_n, τ_N, μ are positive constants.

$(\mathcal{H}_3)$: The time-varying delay $\tau(t)$ satisfies: $\tau_n \le \tau(t) \le \tau_N$, where τ_n, τ_N are positive constants.

Remark 1 In assumption $(\mathcal{H}_2)$, the time-varying delay $\tau(t)$ is differentiable with the derivative less than 1, it is called 'slow delay'; when removing the derivability, $\tau(t)$ maybe show a large rate of change, hence, we call it as 'fast delay'. In this paper, we will discuss interval network model with slow delay and fast delay respectively.

The initial value associated with (1) is assumed to be $y(s) = \psi(s)$, $\psi(s)$ is a continuous function on $[-\tau_N, 0]$. Similar with proof of Theorem 3.3 in (Balasubramaniam et al. 2011), we can show that system (1) has one equilibrium point y^* under the above assumptions, the equilibrium y^* will be always shifted to the origin by letting $x(t) = y(t) - y^*$, and the network system (1) can be represented as follows:

$$\begin{cases} \dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t-\tau(t)), \\ A \in A_I, \quad B_k \in B_I^{(k)}, \quad k = 1,2, \end{cases} \tag{2}$$

where $f_j(x_j(t)) = g_j(x_j(t) + y_j^*) - g_j(y_j^*)$, and $f_j(0) = 0, j = 1,2,\ldots,n$.

The initial condition associated with (2) is given in the form $x(s) = y(s) - y^* = \varphi(s) = \psi(s) - y^*$, $s \in [-\tau_N, 0]$. It is easy to see $f(x(t))$ satisfy the assumption $(\mathcal{H}_1)$.

Based on some transformations, the system (2) can be written as an equivalent form:

$$\begin{aligned} \dot{x}(t) = &-[A_0 + E_A \Sigma_A F_A]x(t) + [B_{10} + E_1 \Sigma_1 F_1] f(x(t)) \\ &+ [B_{20} + E_2 \Sigma_2 F_2] f(x(t-\tau(t)), \end{aligned} \tag{3}$$

where $\Sigma_A \in \Sigma$, $\Sigma_k \in \Sigma$, $k = 1,2$.

$$\Sigma = \Big\{\mathrm{diag}[\delta_{11}, \ldots, \delta_{1n}, \ldots, \delta_{n1}, \ldots, \delta_{nn}] \in R^{n^2 \times n^2}: \\ |\delta_{ij}| \le 1, \quad i,j = 1,2,\ldots,n\}..$$

$$A_0 = \frac{\overline{A} + \underline{A}}{2}, \quad H_A = [\alpha_{ij}]_{n\times n} = \frac{\overline{A} - \underline{A}}{2}. \quad B_{k0} = \frac{\overline{B}_k + \underline{B}_k}{2}, \\ H_B^{(k)} = [\beta_{ij}]_{n\times n} = \frac{\overline{B}_k - \underline{B}_k}{2}.$$

$$E_A = [\sqrt{\alpha_{11}} e_1, \ldots, \sqrt{\alpha_{1n}} e_1, \ldots, \sqrt{\alpha_{n1}} e_n, \ldots, \sqrt{\alpha_{nn}} e_n]_{n\times n^2},$$

$$F_A = [\sqrt{\alpha_{11}} e_1, \ldots, \sqrt{\alpha_{1n}} e_n, \ldots, \sqrt{\alpha_{n1}} e_1, \ldots, \sqrt{\alpha_{nn}} e_n]^T_{n^2\times n},$$

$$E_k = \left[\sqrt{\beta_{11}^{(k)}} e_1, \ldots, \sqrt{\beta_{1n}^{(k)}} e_1, \ldots, \sqrt{\beta_{n1}^{(k)}} e_n, \ldots, \sqrt{\beta_{nn}^{(k)}} e_n\right]_{n\times n^2},$$

$$F_k = \left[\sqrt{\beta_{11}^{(k)}} e_1, \ldots, \sqrt{\beta_{1n}^{(k)}} e_n, \ldots, \sqrt{\beta_{n1}^{(k)}} e_1, \ldots, \sqrt{\beta_{nn}^{(k)}} e_n\right]^T_{n^2\times n},$$

where $e_i \in R^n$ denotes the column vector with *ith* element to be 1 and others to be 0.

System (3) can be changed as

$$\dot{x}(t) = -A_0 x(t) + B_{10} f(x(t)) + B_{20} f(x(t-\tau(t))) + E\Delta(t) \tag{4}$$

where $E = [E_A, E_1, E_2]$,

$$\Delta(t) = \begin{bmatrix} -\Sigma_A F_A x(t) \\ \Sigma_1 F_1 f(x(t)) \\ \Sigma_2 F_2 f(x(t-\tau)) \end{bmatrix} \\ = \mathrm{diag}\{\Sigma_A, \Sigma_1, \Sigma_2\} \begin{bmatrix} -F_A x(t) \\ F_1 f(x(t)) \\ F_2 f(x(t-\tau(t))) \end{bmatrix},$$

and $\Delta(t)$ satisfies the following matrix quadratic inequality:

$$\Delta^T(t)\Delta(t) \le \begin{bmatrix} x(t) \\ f(x(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} F_A^T \\ F_1^T \\ F_2^T \end{bmatrix} \begin{bmatrix} F_A^T \\ F_1^T \\ F_2^T \end{bmatrix}^T \times \begin{bmatrix} x(t) \\ f(x(t)) \\ f(x(t-\tau(t))) \end{bmatrix}.$$

In this paper, our main purpose is to study the switched interval networks, it consists of a set of interval network with discrete time-varying delays and a switching rule. Each of the interval networks regards as an individual subsystem. The operation mode of the switched networks is determined by the switching rule. According to (2), the switched interval network with discrete interval delay can be described as follows:

$$\begin{cases} \dot{x}(t) = -A^\sigma x(t) + B_1^\sigma f(x(t)) + B_2^\sigma f(x(t-\tau(t))), \\ A^\sigma \in A_{I_\sigma}, \quad B_k^\sigma \in B_{I_\sigma}^{(k)}, \quad k = 1,2, \end{cases} \tag{5}$$

where $A_{I_\sigma} = [\underline{A}^\sigma, \overline{A}^\sigma] = \{A^\sigma = \mathrm{diag}(a_{i_\sigma}) : 0 < \underline{a}_{i_\sigma} \le a_{i_\sigma} \le \overline{a}_{i_\sigma}, i = 1,2,\ldots,n\}$ $B_{I_\sigma}^{(k)} = [\underline{B}_k^\sigma, \overline{B}_k^\sigma] = \{B_k^\sigma = [b_{ij_\sigma}^{(k)}] : 0 < \underline{b}_{ij_\sigma}^{(k)} \le b_{ij_\sigma}^{(k)} \le \overline{b}_{ij_\sigma}^{(k)}, i,j = 1,2,\ldots,n\}$ with $\underline{A}^\sigma = \mathrm{diag}(\underline{a}_{1_\sigma}, \underline{a}_{2_\sigma}, \ldots, \underline{a}_{n_\sigma})$ $\overline{A}^\sigma = \mathrm{diag}(\overline{a}_{1_\sigma}, \overline{a}_{2_\sigma}, \ldots, \overline{a}_{n_\sigma})$
$\underline{B}_k^\sigma = [\underline{b}_{ij_\sigma}^{(k)}]_{n\times n}$, $\overline{B}_k^\sigma = [\overline{b}_{ij_\sigma}^{(k)}]_{n\times n}$.

$$A_0^\sigma = \frac{\overline{A}^\sigma + \underline{A}^\sigma}{2}, \quad H_A^\sigma = [\alpha_{ij_\sigma}]_{n\times n} = \frac{\overline{A}^\sigma - \underline{A}^\sigma}{2}.$$

$$B_{k0}^\sigma = \frac{\overline{B}_k^\sigma + \underline{B}_k^\sigma}{2}, \quad H_{B_\sigma}^{(k)} = [\beta_{ij_\sigma}]_{n\times n} = \frac{\overline{B}_k^\sigma - \underline{B}_k^\sigma}{2}.$$

$$E_A^\sigma = \left[\sqrt{\alpha_{11_\sigma}}e_1, \ldots, \sqrt{\alpha_{1n_\sigma}}e_1, \ldots, \sqrt{\alpha_{n1_\sigma}}e_n, \ldots, \sqrt{\alpha_{nn_\sigma}}e_n\right]_{n\times n^2}.$$

$$F_A^\sigma = \left[\sqrt{\alpha_{11_\sigma}}e_1, \ldots, \sqrt{\alpha_{1n_\sigma}}e_n, \ldots, \sqrt{\alpha_{n1_\sigma}}e_1, \ldots, \sqrt{\alpha_{nn_\sigma}}e_n\right]_{n^2\times n}^T.$$

$$E_k^\sigma = \left[\sqrt{\beta_{11_\sigma}^{(k)}}e_1, \ldots, \sqrt{\beta_{1n_\sigma}^{(k)}}e_1, \ldots, \sqrt{\beta_{n1_\sigma}^{(k)}}e_n, \ldots, \sqrt{\beta_{nn_\sigma}^{(k)}}e_n\right]_{n\times n^2}.$$

$$F_k^\sigma = \left[\sqrt{\beta_{11_\sigma}^{(k)}}e_1, \ldots, \sqrt{\beta_{1n_\sigma}^{(k)}}e_n, \ldots, \sqrt{\beta_{n1_\sigma}^{(k)}}e_1, \ldots, \sqrt{\beta_{nn_\sigma}^{(k)}}e_n\right]_{n^2\times n}^T.$$

$\sigma : R^n \to \Gamma = \{1,2,\ldots,N\}$ is the switching signal, which is a piecewise constant function dependent on state $x(t)$. For any $i \in \{1,2,\ldots,N\}$ $A^i = A_0^i + E_A^i\Sigma_A^i F_A^i$ $B_k^i = B_{k0}^i + E_k^i\Sigma_k^i F_k^i$, and $\Sigma_A^i \in \Sigma$, $\Sigma_k^i \in \Sigma$, $k = 1,2$. This means that the matrices $(A^\sigma, B_1^\sigma, B_2^\sigma)$ are allowed to take values, at an arbitrary time, in the finite set $\{(A^1, B_1^1, B_2^1), (A^2, B_1^2, B_2^2), \ldots, (A^N, B_1^N, B_2^N)\}$.

By (4), the system (5) can be written as

$$\dot{x}(t) = -A_0^\sigma x(t) + B_{10}^\sigma f(x(t)) + B_{20}^\sigma f(x(t-\tau(t)) + E^\sigma \Delta^\sigma(t), \tag{6}$$

where $E^\sigma = [E_A^\sigma, E_1^\sigma, E_2^\sigma]$and $\Delta^\sigma(t)$ satisfies the following quadratic inequality:

$$(\Delta^\sigma(t))^T\Delta^\sigma(t) \le \begin{bmatrix} x(t) \\ f(x(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} (F_A^\sigma)^T \\ (F_1^\sigma)^T \\ (F_2^\sigma)^T \end{bmatrix} \begin{bmatrix} (F_A^\sigma)^T \\ (F_1^\sigma)^T \\ (F_2^\sigma)^T \end{bmatrix}^T \times \begin{bmatrix} x(t) \\ f(x(t)) \\ f(x(t-\tau(t))) \end{bmatrix}. \tag{7}$$

To derive the main results in the next section, the following definitions and lemmas are introduced.

Definition 2.1 The switched interval neural network model (5) is said to be globally robustly asymptotically stable if there exists a switching function $\sigma(\cdot)$ such that the neural network model (5) is globally asymptotically stable for any $A^\sigma \in A_{I_\sigma}$, $B_k^\sigma \in B_{I_\sigma}^{(k)}$, $k = 1,2$.

Definition 2.2 The system of matrices $\{G_i\} quadi = 1,2,\ldots,N$, is said to be strictly complete if for every $x \in R^n \backslash \{0\}$ there is $i \in \{1,2,\ldots,N\}$ such that $x^TG_ix < 0$.

Let us define N regions

$\Omega_i = \{x \in R^n : x^TG_ix < 0\}, \quad i = 1, 2\ldots, N.$

where Ω_i are open conic regions, obvious that the system $\{G_i\}$ is strictly completely if and only if these open conic regions overlap and together cover $R^n \setminus \{0\}$, that is

$$\bigcup_{i=1}^N \Omega_i = R^n \backslash \{0\}.$$

Proposition 2.1 (Uhlig 1979)*The system $\{G_i\}$, $i = 1, 2, \ldots, N$, is strictly complete if there exist $\lambda_i \ge 0$, $i = 1, 2, \ldots, N$, $\sum_{i=1}^N \lambda_i = 1$, such that*

$$\sum_{i=1}^N \lambda_i G_i < 0.$$

Lemma 2.1 (Han and Yue 2007) *Given any real matrix $M = M^T > 0$, for any $t > 0$, function $\tau(t)$ satisfies $\tau_n \le \tau(t) \le \tau_N$, and $x(t) : [-\tau_N, -\tau_n] \longrightarrow R^n$, the following integration is well defined*:

$$-(\tau_N - \tau_n)\int_{t-\tau_N}^{t-\tau_n} \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_N) \end{bmatrix}^T \times \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_N) \end{bmatrix}.$$

Lemma 2.2 (Zhang et al. 2009) *For any constant matrices ψ_1 and ψ_2 and Ω of appropriate dimensions, function $\tau(t)$ satisfies $\tau_n \le \tau(t) \le \tau_N$, then*

$(\tau(t) - \tau_n)\psi_1 + (\tau_N - \tau(t))\psi_2 + \Omega < 0$

holds, if and only if

$(\tau_N - \tau_n)\psi_1 + \Omega < 0, (\tau_N - \tau_n)\psi_2 + \Omega < 0$

In the following section, we use the generalized the DCP method, partition the interval delay into m subintervals with equal length, be some scalars satisfying

$\tau_n = \tau_0 \le \tau_1 \le \tau_2 \le \ldots \tau_m = \tau_N$

Obviously, $[\tau_n, \tau_N] = \bigcup_{j=1}^{m}[\tau_{j-1}, \tau_j]$. For convenience, we denote the length of the subinterval $\delta = \tau_j - \tau_{j-1}$, therefor, for any $t > 0$, there should exist an integer k, such that $\tau(t) \in [\tau_{k-1}, \tau_k]$.

Remark 2 In this paper, we consider the case when $m = 3$, interval delay is decomposed into three subintervals: $[\tau_n, \tau_1]$, $[\tau_1, \tau_2]$, and $[\tau_2, \tau_N]$. Let $\mathcal{S}_1 = \{t|t > 0, \tau(t) \in [\tau_n, \tau_1]\}$ $\mathcal{S}_2 = \{t|t > 0, \tau(t) \in (\tau_1, \tau_2]\}$ $\mathcal{S}_3 = \{t|t > 0, \tau(t) \in (\tau_2, \tau_N]\}$, in the proof of our main results, applying a piecewise analysis method (Zhang et al. 2009) to check the variation of derivative of the Lyapunov functional in $S1$ $S2$ and $S3$ respectively.

$$\Pi^i = \begin{bmatrix} \Pi^i_{11} & * & * & * & * & * & * & * & * \\ R_4 & -Q_1 - R_4 & * & * & * & * & * & * & * \\ 0 & 0 & -Q_2 & * & * & * & * & * & * \\ 0 & 0 & 0 & -Q_3 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -Q_4 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & \Pi^i_{66} & * & * & * \\ \Pi^i_{71} & 0 & 0 & 0 & 0 & 0 & \Pi^i_{77} & * & * \\ \Pi^i_{81} & 0 & 0 & 0 & 0 & \gamma_3 L_2 & \Pi^i_{87} & \Pi^i_{88} & * \\ (E^i)^T P - (E^i)^T \phi A^i_0 & 0 & 0 & 0 & 0 & 0 & \Pi^i_{97} & \Pi^i_{98} & \Pi^i_{99} \end{bmatrix} < 0,$$

Main results

In this section, the global robust asymptotic stability of the proposed model (5) will be discussed. By delay fractioning approach, designing a effective switch rule and constructing a suitable Lyapunov functional, a new robust delay-dependent criterion for the global asymptotic stability of switched network system (5) is derived in terms of LMIs.

$$\text{Set } G_i(A^i_0, Q_1, Q_2, Q_3) = -(A^i_0)^T P - PA^i_0 + Q_1 + Q_2 + Q_3,$$
$$\Omega_i = \{x \in R^n : x^T(t) G_i(A^i_0, Q_1, Q_2, Q_3)x(t) < 0\},$$
$$\bar{\Omega}_1 = \Omega_1, \quad \bar{\Omega}_i = \Omega_i \backslash \bigcup_{j=1}^{i-1} \bar{\Omega}_j, \quad i = 2, 3, \ldots, N.$$
$$L_1 = diag\{\check{l_1}\hat{l_1}, \check{l_2}\hat{l_2}, \ldots, \check{l_n}\hat{l_n}\}$$
$$L_2 = diag\{\check{l_1} + \hat{l_1}, \check{l_2} + \hat{l_2}, \ldots, \check{l_n} + \hat{l_n}\}$$

$$\begin{aligned} N &= [N_1\, N_2\, N_3\, N_4\, N_5\, N_6\, N_7\, N_8\, N_9]M \\ &= [M_1\, M_2\, M_3\, M_4\, M_5\, M_6\, M_7\, M_8\, M_9]S \\ &= [S_1\, S_2\, S_3\, S_4\, S_5\, S_6\, S_7\, S_8\, S_9]Z = [Z_1\, Z_2\, Z_3\, Z_4\, Z_5\, Z_6\, Z_7\, Z_8\, Z_9]X \\ &= [X_1\, X_2\, X_3\, X_4\, X_5\, X_6\, X_7\, X_8\, X_9]Y \\ &= [Y_1\, Y_2\, Y_3\, Y_4\, Y_5\, Y_6\, Y_7\, Y_8\, Y_9]\delta = \frac{1}{3}(\tau_N - \tau_n), \tau_1 = \tau_n + \delta, \tau_2 \\ &= \tau_n + 2\delta \end{aligned}$$

Theorem 3.1 *Under the assumption* $(\mathcal{H}_1)$ *and* $(\mathcal{H}_2)$, *if there exist matrices* $P > 0$, $T_1 > 0$, $T_2 > 0$, $Q_j > 0$, $R_j > 0$ $(j = 1, 2, 3, 4)$ *and diagonal matrices* $\gamma_k = diag\{\gamma_{k,1}, \gamma_{k,2}, \ldots, \gamma_{k,n}\} > 0 (k = 1, 2, 3)$, *and matrices* $N_l, M_l, Z_l, S_l (l = 1, 2, \ldots, 9)$ *with appropriate dimensions such that for all m and n, the following conditions hold*:

(i) $\exists \xi^i \ge 0, \quad i = 1, 2, \ldots, N,$
$\sum_{i=1}^{N} \xi^i = 1 : \sum_{i=1}^{N} \xi^i G_i(A^i_0, Q_1, Q_2, Q_3) < 0.$

(ii)

$$\begin{bmatrix} \Pi^i + \Theta^i_m & * \\ \Upsilon^i_{mn} & -R^i_m \end{bmatrix} < 0, \quad m = 1, 2, 3, \quad n = 1, 2 \qquad (8)$$

where

where

$$\Pi^i_{11} = T_1 + (A^i_0)^T_0 i + Q_4 - R_4 - \gamma_2 L_1 - L_1 \gamma_2 + (F^i_A)^T F^i_A$$
$$\Pi^i_{66} = -(1 - \mu)T_1 - \gamma_3 L_1 - L_1 \gamma_3$$
$$\Pi^i_{71} = L_2 \gamma_2 + (B^i_{10})^T P - \gamma_1 A^i_0 - (B^i_{10})^T_0 i + (F^i_A)^T F^i_1$$
$$\Pi^i_{77} = \gamma_1 B^i_{10} + (B^i_{10})^T \gamma_1 + T_2 - \gamma_2 - \gamma_2^T + (B^i_{10})^T_{10} i + (F^i_1)^T F^i_1$$
$$\Pi^i_{81} = (B^i_{20})^T P - (B^i_{20})^T_0 i + (F^i_A)^T F^i_2$$
$$\Pi^i_{87} = (B^i_{20})^T \gamma_1 + (B^i_{20})^T_{10} i + (F^i_1)^T F^i_2$$
$$\Pi^i_{88} = -(1 - \mu)T_2 - \gamma_3 - \gamma_3^T + (B^i_{20})^T_{20} i + (F^i_2)^T F^i_2$$
$$\Pi^i_{97} = (E^i)^T \gamma_1 + (E^i)^T \phi^T (B^i_{10})^T$$
$$\Pi^i_{98} = (E^i)^T \phi^T (B^i_{20})^T$$
$$\Pi^i_{99} = (E^i)^T \phi E^i - I$$

$$\Theta_1^i = \begin{bmatrix} 0 & * & * & * & * & * & * & * & * \\ \delta N_1^T & \delta N_2^T + \delta N_2^T & * & * & * & * & * & * & * \\ -\delta M_1^T & \delta N_3^T - \delta M_2^T & \Theta_{1_{33}}^i & * & * & * & * & * & * \\ 0 & \delta N_4^T & R_2 - \delta M_4^T & -R_2 - R_3 & * & * & * & * & * \\ 0 & \delta N_5^T & -\delta M_5^T & R_3 & -R_3 & * & * & * & * \\ \delta M_1^T - \delta N_1^T & \Theta_{1_{62}}^i & \Theta_{1_{63}}^i & -\delta N_4^T + \delta M_4^T & \Theta_{1_{65}}^i & \Theta_{1_{66}}^i & * & * & * \\ 0 & \delta N_7^T & -\delta M_7^T & 0 & 0 & -{}_7^T + \delta M_7^T & 0 & * & * \\ 0 & \delta N_8^T & -\delta M_8^T & 0 & 0 & -{}_8^T + \delta M_8^T & 0 & 0 & * \\ 0 & \delta N_9^T & -\delta M_9^T & 0 & 0 & -{}_9^T + \delta M_9^T & 0 & 0 & 0 \end{bmatrix} < 0,$$

$$\Theta_{1_{33}}^i = -R_2 - \delta M_3^T - \delta M_3$$
$$\Theta_{1_{62}}^i = \delta N_6^T - \delta N_2^T + {}_2^T$$
$$\Theta_{1_{63}}^i = -\delta N_3^T + \delta M_3^T - {}_6^T$$
$$\Theta_{1_{65}}^i = -\delta N_5^T + \delta M_5^T$$

$$\Theta_2^i = \begin{bmatrix} 0 & * & * & * & * & * & * & * & * \\ 0 & -R_1 & * & * & * & * & * & * & * \\ \delta Z_1^T & R_1 + \delta Z_2^T & \Theta_{2_{33}}^i & * & * & * & * & * & * \\ -\delta S_1^T & -\delta S_2^T & \delta Z_4^T - {}_3^T & \Theta_{2_{44}}^i & * & * & * & * & * \\ 0 & 0 & \delta Z_5^T & R_3 - \delta S_5^T & -R_3 & * & * & * & * \\ -\delta Z_1^T + \delta S_1^T & -\delta Z_2^T + {}_2^T & \Theta_{2_{63}}^i & \Theta_{2_{64}}^i & \Theta_{2_{65}}^i & \Theta_{2_{66}}^i & * & * & * \\ 0 & 0 & \delta Z_7^T & -\delta S_7^T & 0 & -{}_7^T + \delta S_7^T & 0 & * & * \\ 0 & 0 & \delta Z_8^T & -\delta S_8^T & 0 & -{}_8^T + \delta S_8^T & 0 & 0 & * \\ 0 & 0 & \delta Z_9^T & -\delta S_9^T & 0 & -{}_9^T + \delta S_9^T & 0 & 0 & 0 \end{bmatrix} < 0,$$

$$\Theta_{2_{33}}^i = -R_1 + \delta Z_3 + \delta Z_3^T$$
$$\Theta_{2_{44}}^i = -R_3 - \delta S_4^T - \delta S_4$$
$$\Theta_{2_{63}}^i = \delta Z_6^T - \delta Z_3^T + {}_3^T$$
$$\Theta_{2_{64}}^i = -\delta Z_4^T + \delta S_4^T - {}_6^T$$
$$\Theta_{2_{65}}^i = -\delta Z_5 + \delta S_5$$
$$\Theta_{2_{66}}^i = -\delta Z_6 - \delta Z_6^T + {}_6 + \delta S_6^T$$

$$\Theta_{3_{44}}^i = -R_2 + \delta X_4^T + \delta X_4$$
$$\Theta_{3_{63}}^i = -\delta X_3^T + \delta Y_3^T$$
$$\Theta_{3_{64}}^i = \delta X_6^T - \delta X_4^T + {}_4^T$$
$$\Theta_{3_{65}}^i = -\delta X_5^T + \delta Y_5^T - {}_6^T$$
$$\Theta_{3_{66}}^i = -\delta X_6 - \delta X_6^T + {}_6^T + \delta Y_6$$
$$\phi = \delta^2 R_1 + \delta^2 R_2 + \delta^2 R_3 + \tau_n^2 R_4$$
$$\Upsilon_{11}^i = \delta N \, \Upsilon_{12}^i = \Upsilon_{21}^i = \delta S \, \Upsilon_{22}^i = \Upsilon_{31}^i = \delta X \, \Upsilon_{32}^i = \delta Y$$

$$\Theta_3^i = \begin{bmatrix} 0 & * & * & * & * & * & * & * & * \\ 0 & -R_1 & * & * & * & * & * & * & * \\ 0 & R_1 & -R_1 - R_2 & * & * & * & * & * & * \\ \delta X_1^T & \delta X_2^T & \delta X_3^T + R_2 & \Theta_{3_{44}}^i & * & * & * & * & * \\ -\delta Y_1^T & -\delta Y_2^T & -\delta Y_3^T & {}_5^T - \delta Y_4^T & -\delta Y_5^T - \delta Y_5 & * & * & * & * \\ -\delta X_1^T + \delta Y_1^T & -\delta X_2^T + {}_2^T & \Theta_{3_{63}}^i & \Theta_{3_{64}}^i & \Theta_{3_{65}}^i & \Theta_{3_{66}}^i & * & * & * \\ 0 & 0 & 0 & \delta X_7^T & -\delta X_7^T & -{}_7^T + \delta Y_7^T & 0 & * & * \\ 0 & 0 & 0 & \delta X_8^T & -\delta Y_8^T & -{}_8^T + \delta Y_8^T & 0 & 0 & * \\ 0 & 0 & 0 & \delta X_9^T & -\delta Y_9^T & -{}_9^T + \delta Y_9^T & 0 & 0 & 0 \end{bmatrix} < 0,$$

then, switched interval network (5) is global robust asymptotic stable, the switching rule is chosen as $\sigma(x(t)) = i$ whenever $x(t) \in \bar{\Omega}_i$.

Proof Consider the following Lyapunov–Krasovskii functional

$$V(t,x_t) = V_1(t,x_t) + V_2(t,x_t) + V_3(t,x_t) + V_4(t,x_t) + V_5(t,x_t) \tag{9}$$

where

$$V_1(t,x_t) = x^T(t)Px(t) + 2\sum_{i=1}^{n}\gamma_{1,i}\int_0^{x_i(t)} f_i(s)ds$$

$$V_2(t,x_t) = \int_{t-\tau(t)}^{t} [x^T(s)T_1x(s) + f^T(x(s))T_2f(x(s))]ds$$

$$V_3(t,x_t) = \int_{t-\tau_n}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau_1}^{t} x^T(s)Q_2x(s)ds + \int_{t-\tau_2}^{t} x^T(s)Q_3x(s)ds + \int_{t-\tau_N}^{t} x^T(s)Q_4x(s)ds$$

$$V_4(t,x_t) = \delta\int_{t-\tau_1}^{t-\tau_n}\int_s^t \dot{x}^T(\theta)R_1\dot{x}(\theta)d\theta ds + \delta\int_{t-\tau_2}^{t-\tau_1}\int_s^t \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta ds + \delta\int_{t-\tau_N}^{t-\tau_2}\int_s^t \dot{x}^T(\theta)R_3\dot{x}(\theta)d\theta ds$$

$$V_5(t,x_t) = \tau_n\int_{t-\tau_n}^{t}\int_s^t \dot{x}^T(\theta)R_4\dot{x}(\theta)d\theta ds$$

Calculating the time derivative of $V(t, x_t)$ along the trajectory of (6), it can follow that

$$\begin{aligned}\dot{V}_1(t,x_t) &= 2x^T(t)P\dot{x}(t) + 2\sum_{i=1}^{n}\gamma_{1,i}f_i(x_i(t))\dot{x}_i(t)\\
&= 2x^T(t)P[-A_0^ix(t) + B_{10}^if(x(t)) + B_{20}^if(x(t-\tau(t)) + E^i\Delta^i(t)]\\
&\quad + 2f^T(x(t))\gamma_1[-A_0^ix(t) + B_{10}^if(x(t)) + B_{20}^if(x(t-\tau(t)) + E^i\Delta^i(t)]\\
&= -2x^T(t)(A_0^i)^TPx(t) + 2f^T(x(t))(B_{10}^i)^TPx(t) + 2f^T(x(t-\tau(t))(B_{20}^i)^TPx(t)\\
&\quad + 2(\Delta^i(t))^T(E^i)^TPx(t) - 2f^T(x(t))\gamma_1A_0^ix(t) + 2f^T(x(t))\gamma_1B_{10}^if(x(t))\\
&\quad 2f^T(x(t))\gamma_1B_{20}^if(x(t-\tau(t))) + 2f^T(x(t))\gamma_1E^i\Delta^i(t)\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_2(t,x_t) &= x^T(t)T_1x(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))T_1x(t-\tau(t)) + f^T(x(t))T_2f(x(t)) - (1-\dot{\tau}(t))f^T(x(t-\tau(t)))\\
&\quad T_2f(x(t-\tau(t))) \le x^T(t)T_1x(t) + f^T(x(t))T_2f(x(t)) - (1-\mu)x^T(t-\tau(t))T_1x(t-\tau(t))\\
&\quad - (1-\mu)f^T(x(t-\tau(t)))T_2f(x(t-\tau(t)))\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_3(t,x_t) &= x^T(t)(Q_1+Q_2+Q_3+Q_4)x(t) - x^T(t-\tau_n)Q_1x(t-\tau_n) - x^T(t-\tau_1)Q_2\\
&\quad x(t-\tau_1) - x^T(t-\tau_2)Q_3x(t-\tau_2) - x^T(t-\tau_N)Q_4x(t-\tau_N)\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_4(t,x_t) &= \delta^2\dot{x}^T(t)R_1\dot{x}(t) - \delta\int_{t-\tau_1}^{t-\tau_n}\dot{x}^T(s)R_1\dot{x}(s)ds + \delta^2\dot{x}^T(t)R_2\dot{x}(t) - \delta\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds\\
&\quad + \delta^2\dot{x}^T(t)R_3\dot{x}(t) - \delta\int_{t-\tau_M}^{t-\tau_2}\dot{x}^T(s)R_3\dot{x}(s)ds\end{aligned} \tag{13}$$

By applying Lemma 2.1, we have

$$\begin{aligned}\dot{V}_5(t,x_t) &= \tau_n^2\dot{x}^T(t)R_4\dot{x}(t) - \tau_n\int_{t-\tau_n}^{t}\dot{x}^T(s)R_4\dot{x}(s)ds\\
&\le \tau_n^2\dot{x}^T(t)R_4\dot{x}(t) + \begin{bmatrix} x(t) \\ x(t-\tau_n)\end{bmatrix}^T\begin{bmatrix} -R_4 & R_4 \\ R_4 & -R_4\end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-\tau_n)\end{bmatrix}\end{aligned} \tag{14}$$

Based on (10)–(14), we can get

$$\begin{aligned}\dot{V}(t,x_t) &\le x^T(t)[(A_0^i)^TP - PA_0^i + T_1 + Q_1 + Q_2 + Q_3 + Q_4]x(t) + f^T(x(t))[\gamma_1B_{10}^i + B_{10}^i\gamma_1\\
&\quad + T_2]f(x(t)) + 2(\Delta^i(t))^T(E^i)^TPx(t) + 2f^T(x(t-\tau(t)))(B_{20}^i)^TPx(t)\\
&\quad + 2f^T(x(t))\gamma_1B_{20}^if(x(t-\tau(t))) - x^T(t-\tau_n)Q_1 x(t-\tau_n) + 2f^T(x(t))\gamma_1E^i\Delta^i(t)\\
&\quad + 2f^T(x(t))[(B_{10}^i)^TP - \gamma_1A_0^i]x(t) - (1-\mu)x^T(t-\tau(t))T_1x(t-\tau(t)) - x^T(t-\tau_1)Q_2\\
&\quad \times x(t-\tau_1) - (1-\mu)f^T(x(t-\tau(t))) \times T_2f(x(t-\tau(t))) - x^T(t-\tau_2)Q_3x(t-\tau_2)\end{aligned}$$

$$+\dot{x}^T(t)\phi\dot{x}(t) - x^T(t-\tau_N)Q_4x(t-\tau_N)$$
$$\times + \begin{bmatrix} x(t) \\ x(t-\tau_n) \end{bmatrix}^T \begin{bmatrix} -R_4 & R_4 \\ R_4 & -R_4 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_n) \end{bmatrix}$$
$$-\delta \int_{t-\tau_1}^{t-\tau_n} \dot{x}^T(s)R_1\dot{x}(s)ds - \delta \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds$$
$$\times -\delta \int_{t-\tau_M}^{t-\tau_2} \dot{x}^T(s)R_3\dot{x}(s)ds \quad (15)$$

By the assumption $(\mathcal{H}_1)$, one has

$$[f_i(x_i(t)) - \check{l}_i x_i(t)][f_i(x_i(t)) - \hat{l}_i x_i(t)] \le 0 \quad (16)$$

$$[f_i(x_i(t-\tau(t))) - \check{l}_i x_i(t-\tau(t))][f_i(x_i(t-\tau(t))) - \hat{l}_i x_i(t - \tau(t))] \le 0 \quad (17)$$

It follows from (16) and (17) that

$$2\sum_{i=1}^{n} \gamma_{2,i}[f_i(x_i(t)) - \check{l}_i x_i(t)][f_i(x_i(t)) - \hat{l}_i x_i(t)]$$
$$= \sum_{i=1}^{n} \gamma_{2,i} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} 2\check{l}_i\hat{l}_i e_i e_i^T & * \\ -(\check{l}_i + \hat{l}_i)e_i^T e_i & 2e_ie_i^T \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}$$
$$= \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} 2\gamma_2 L_1 & * \\ -\gamma_2 L_2 & 2\gamma_2 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \le 0 \quad (18)$$

$$2\sum_{i=1}^{n} \gamma_{3,i}[f_i(x_i(t-\tau(t))) - \check{l}_i x_i(t-\tau(t))][f_i(x_i(t-\tau(t))) - \hat{l}_i x_i(t-\tau(t))]$$
$$= \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} 2\gamma_3 L_1 & * \\ -\gamma_3 L_2 & 2\gamma_3 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \le 0 \quad (19)$$

where e_i denotes the unit column vector with a "$1''$" on its ith row and zeros elsewhere.

By substituting (7) and (18), (19) into (15), it yields

$$\dot{V}(t,x_t) \le x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)\Pi^i\eta(t)$$
$$-\delta \int_{t-\tau_1}^{t-\tau_n} \dot{x}^T(s)R_1\dot{x}(s)ds - \delta \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds$$
$$-\delta \int_{t-\tau_M}^{t-\tau_2} \dot{x}^T(s)R_3\dot{x}(s)ds \quad (20)$$

where

$$\eta^T(t) = [x^T(t)\, x^T(t-\tau_n)\, x^T(t-\tau_1)\, x^T(t-\tau_2) \times x^T(t-\tau_N)\, x^T(t-\tau(t))\, f^T(x(t))\, f^T(x(t-\tau(t)))\, (\Delta^i(t))^T]$$

In the following, we will consider three cases: that is $t \in \mathcal{S}_1$, $t \in \mathcal{S}_2$, $t \in \mathcal{S}_3$.

Case 1: when $t \in \mathcal{S}_1$, i.e. $\tau(t) \in [\tau_n, \tau_1]$.

By using Lemma 2.1, we have

$$-\delta \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \times \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} \quad (21)$$

$$-\delta \int_{t-\tau_N}^{t-\tau_2} \dot{x}^T(s)R_3\dot{x}(s)ds \le \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \times \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix} \quad (22)$$

Combing (20)–(22), and applying Newton-Leibniz formula and adding the free weighting matrices N and M, it can be obtained

$$\dot{V}(t,x_t) \le x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)\Pi^i\eta(t)$$
$$-\delta \int_{t-\tau_1}^{t-\tau_n} \dot{x}^T(s)R_1\dot{x}(s)ds$$
$$+\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}$$
$$+\begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix}$$
$$+2\delta\eta^T(t)N\left[x(t-\tau_n) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_n} \dot{x}(s)ds\right]$$
$$+2\delta\eta^T(t)M\left[x(t-\tau(t)) - x(t-\tau_1) - \int_{t-\tau_1}^{t-\tau(t)} \dot{x}(s)ds\right] \quad (23)$$

It is easy to deduce the following inequality:

$$-2\delta\eta^T(t)N \int_{t-\tau(t)}^{t-\tau_n} \dot{x}(s)ds = \delta \int_{t-\tau(t)}^{t-\tau_n} 2\eta^T(t)(-N)\dot{x}(s)ds$$
$$\le (\tau(t) - \tau_n)\delta\eta^T(t)NR_1^{-1}N^T\eta(t) + \delta \int_{t-\tau(t)}^{t-\tau_n} \dot{x}^T(s)R_1\dot{x}(s)ds \quad (24)$$

$$-2\delta\eta^T(t)M \int_{t-\tau_1}^{t-\tau(t)} \dot{x}(s)ds \le (\tau_1 - \tau(t))\delta\eta^T(t)MR_1^{-1}M^T\eta(t)$$
$$+\delta \int_{t-\tau_1}^{t-\tau(t)} \dot{x}^T(s)R_1\dot{x}(s)ds \quad (25)$$

By substituting (24)–(25) into (23), it follows that

$$\begin{aligned}\dot{V}(x(t)) \leq\ & x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)[\Pi^i + \Theta_1^i \\ & + (\tau(t) - \tau_n)\delta N R_1^{-1} N^T \\ & + (\tau_1 - \tau(t))\delta M R_1^{-1} M^T]\eta(t)\end{aligned} \tag{26}$$

when $m = n = 1$, using Schur complement, (8) is equivalent to

$$\Pi^i + \Theta_1^i + \delta^2 N R_1^{-1} N^T < 0 \tag{27}$$

Similarly, when $m = 1$ and $n = 2$, (8) is equivalent to

$$\Pi^i + \Theta_1^i + \delta^2 M R_1^{-1} M^T < 0 \tag{28}$$

From (27) and (28), by using Lemma 2.2, we can obtain

$$\begin{aligned}\Pi^i + \Theta_1^i + (\tau(t) - \tau_n)\delta N R_1^{-1} N^T + (\tau_1 \\ - \tau(t))\delta M R_1^{-1} M^T < 0\end{aligned} \tag{29}$$

Therefore, we finally obtain from (26) and (29) that

$$\begin{aligned}\dot{V}(x(t)) < x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t), \\ \forall i = 1, 2, \ldots, N,\ t \in \mathcal{S}_1\end{aligned} \tag{30}$$

Case 2: when $t \in \mathcal{S}_2$, i.e. $\tau(t) \in (\tau_1, \tau_2]$.
Similar to case 1, we have

$$-\delta \int_{t-\tau_1}^{t-\tau_n} \dot{x}^T(s)R_1\dot{x}(s)ds \leq \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix} \tag{31}$$

$$-\delta \int_{t-\tau_N}^{t-\tau_2} \dot{x}^T(s)R_3\dot{x}(s)ds \leq \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix} \tag{32}$$

Combing (20), (31), (32), and applying Newton-Leibniz formula and adding the free weighting matrices S and Z, it can be obtained

$$\begin{aligned}\dot{V}(t, x_t) \leq\ & x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)\Pi^i\eta(t) \\ & - \delta \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds \\ & + \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_2 & -R_1 \end{bmatrix} \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix} \\ & + \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_N) \end{bmatrix} \\ & + 2\delta\eta^T(t)S\left[x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1} \dot{x}(s)ds\right] \\ & + 2\delta\eta^T(t)Z\left[x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)} \dot{x}(s)ds\right]\end{aligned} \tag{33}$$

Then, according to a similar method in Case 1, we have

$$\begin{aligned}\dot{V}(x(t)) \leq\ & x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)[\Pi^i + \Theta_2^i \\ & + (\tau(t) - \tau_1)\delta S R_2^{-1} S^T + (\tau_2 - \tau(t))\delta Z R_2^{-1} Z^T]\eta(t)\end{aligned} \tag{34}$$

when $m = 2$, $n = 1$, using Schur complement, (8) is equivalent to

$$\Pi^i + \Theta_2^i + \delta^2 S R_2^{-1} S^T < 0 \tag{35}$$

Similarly, when $m = 2$ and $n = 2$, (8) is equivalent to

$$\Pi^i + \Theta_2^i + \delta^2 Z R_2^{-1} Z^T < 0 \tag{36}$$

From (35) and (36), by using Lemma 2.2, it yields

$$\Pi^i + \Theta_2^i + (\tau(t) - \tau_1)\delta S R_2^{-1} S^T + (\tau_2 - \tau(t))\delta Z R_2^{-1} Z^T < 0 \tag{37}$$

Therefore, we finally obtain from (34) and (37) that

$$\begin{aligned}\dot{V}(x(t)) < x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t),\ \forall i = 1, 2, \ldots, N, \\ t \in \mathcal{S}_2\end{aligned} \tag{38}$$

Case 3: when $t \in \mathcal{S}_3$, i.e. $\tau(t) \in (\tau_2, \tau_N]$.
From the above (21) and (31), we can get

$$\begin{aligned}\dot{V}(t, x_t) \leq\ & x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) + \eta^T(t)\Pi^i\eta(t) \\ & - \delta \int_{t-\tau_N}^{t-\tau_2} \dot{x}^T(s)R_3\dot{x}(s)ds \\ & + \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t-\tau_n) \\ x(t-\tau_1) \end{bmatrix} \\ & + \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} \\ & + 2\delta\eta^T(t)X\left[x(t-\tau_2) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_2} \dot{x}(s)ds\right] \\ & + 2\delta\eta^T(t)Y\left[x(t-\tau(t)) - x(t-\tau_N) - \int_{t-\tau_N}^{t-\tau(t)} \dot{x}(s)ds\right]\end{aligned} \tag{39}$$

Similar to the analysis methods in case 1 and case 2, it can be obtained:

$$\begin{aligned}\dot{V}(x(t)) < x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t), \quad \forall i = 1, 2, \ldots, N, \\ t \in \mathcal{S}_3\end{aligned} \tag{40}$$

From the above discussions, for all $t > 0$, (8) with $m = 1, 2, 3$, $n = 1$ and 2, we can get the following equality:

$$\dot{V}(x(t)) < x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t), \quad \forall i = 1, 2, \ldots, N, \\ t > 0 \tag{41}$$

By the condition (i) and Proposition 2.1, the system of matrices $G_i(A_0^i, Q_1, Q_2, Q_3)$ is strictly complete. Then we have

$$\bigcup_{i=1}^{N} \bar{\Omega}_i = R^n \backslash \{0\}, \quad \bar{\Omega}_i \bigcap \bar{\Omega}_j = \phi, \quad i \neq j \tag{42}$$

Hence, for any $x(t) \in R^n$, there exists $i \in \{1, 2, \ldots, N\}$ such that $x(t) \in \bar{\Omega}_i$. By choosing switching rule as $\sigma(x) = i$ whenever $\sigma(x) \in \bar{\Omega}_i$, from (41), it can derive

$$\dot{V}(x(t)) < x^T(t)G_i(A_0^i, Q_1, Q_2, Q_3)x(t) < 0, \quad t > 0 \tag{43}$$

According to Definition 2.1, the switched interval network (5) is global robust asymptotic stable. The proof is completed. □

Next, we will consider the situation when the time-varying delay $\tau(t)$ becomes the fast delay, by structuring the different Lyapunov–Krasovskii functional, it is easy to obtain the following corollary:

Corollary 3.1 *Under the assumption* $(\mathcal{H}_1)$ *and* $(\mathcal{H}_3)$, *if there exist matrices* $P > 0$, $Q_j > 0$, $R_j > 0$ $(j = 1, 2, 3, 4)$, *and diagonal matrices* $\gamma_k = diag\{\gamma_{k,1}, \gamma_{k,2}, \ldots, \gamma_{k,n}\} > 0 (k = 1, 2, 3)$, *and matrices* N_l, M_l, Z_l, $S_l\,(l = 1, 2, \ldots, 9)$ *with appropriate dimensions such that for all m and n, the following LMIs hold*:

(i) $\exists \xi^i \geq 0, i = 1, 2, \ldots, N, \sum_{i=1}^{N} \xi^i > 0 :$
$\sum_{i=1}^{N} \xi^i G_i(A_0^i, Q_1, Q_2, Q_3) < 0.$

(ii)

$$\begin{bmatrix} \Pi^i + \Theta_m^i & * \\ \Upsilon_{mn}^i & -R_m^i \end{bmatrix} < 0, \quad m = 1, 2, 3, n = 1, 2 \tag{44}$$

where

$$\Pi^i = \begin{bmatrix}
\Pi_{11}^i & * & * & * & * & * & * & * & * \\
R_4 & -Q_1 - R_4 & * & * & * & * & * & * & * \\
0 & 0 & -Q_2 & * & * & * & * & * & * \\
0 & 0 & 0 & -Q_3 & * & * & * & * & * \\
0 & 0 & 0 & 0 & -Q_4 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & \Pi_{66}^i & * & * & * \\
\Pi_{71}^i & 0 & 0 & 0 & 0 & 0 & \Pi_{77}^i & * & * \\
\Pi_{81}^i & 0 & 0 & 0 & 0 & \gamma_3 L_2 & \Pi_{87}^i & \Pi_{88}^i & * \\
(E^i)^T P - (E^i)^T \phi A_0^i & 0 & 0 & 0 & 0 & 0 & \Pi_{97}^i & \Pi_{98}^i & \Pi_{99}^i
\end{bmatrix} < 0,$$

where

$$\Pi_{11}^i = (A_0^i)_0^T i + Q_4 - R_4 - \gamma_2 L_1 - L_1 \gamma_2 + (F_A^i)^T F_A^i$$
$$\Pi_{66}^i = -\gamma_3 L_1 - L_1 \gamma_3$$
$$\Pi_{71}^i = L_2 \gamma_2 + (B_{10}^i)^T P - \gamma_1 A_0^i - (B_{10}^i)_0^T i + (F_A^i)^T F_1^i$$
$$\Pi_{77}^i = \gamma_1 B_{10}^i + (B_{10}^i)^T \gamma_1 - \gamma_2 - \gamma_2^T + (B_{10}^i)_{10}^T i + (F_1^i)^T F_1^i$$
$$\Pi_{81}^i = (B_{20}^i)^T P - (B_{20}^i)_0^T i + (F_A^i)^T F_2^i$$
$$\Pi_{87}^i = (B_{20}^i)^T \gamma_1 + (B_{20}^i)_{10}^T i + (F_1^i)^T F_2^i$$
$$\Pi_{88}^i = -\gamma_3 - \gamma_3^T + (B_{20}^i)_{20}^T i + (F_2^i)^T F_2^i$$
$$\Pi_{97}^i = (E^i)^T \gamma_1 + (E^i)^T \phi^T (B_{10}^i)^T$$
$$\Pi_{98}^i = (E^i)^T \phi^T (B_{20}^i)^T$$
$$\Pi_{99}^i = (E^i)^T \phi E^i - I$$

$$\Theta_1^i = \begin{bmatrix}
0 & * & * & * & * & * & * & * & * \\
\delta N_1^T & \delta N_2^T + \delta N_2^T & * & * & * & * & * & * & * \\
-\delta M_1^T & \delta N_3^T - \delta M_2^T & \Theta_{1_{33}}^i & * & * & * & * & * & * \\
0 & \delta N_4^T & R_2 - \delta M_4^T & -R_2 - R_3 & * & * & * & * & * \\
0 & \delta N_5^T & -\delta M_5^T & R_3 & -R_3 & * & * & * & * \\
\delta M_1^T - \delta N_1^T & \Theta_{1_{62}}^i & \Theta_{1_{63}}^i & -\delta N_4^T + \delta M_4^T & \Theta_{1_{65}}^i & \Theta_{1_{66}}^i & * & * & * \\
0 & \delta N_7^T & -\delta M_7^T & 0 & 0 & -{}_7^T + \delta M_7^T & 0 & * & * \\
0 & \delta N_8^T & -\delta M_8^T & 0 & 0 & -{}_8^T + \delta M_8^T & 0 & 0 & * \\
0 & \delta N_9^T & -\delta M_9^T & 0 & 0 & -{}_9^T + \delta M_9^T & 0 & 0 & 0
\end{bmatrix} < 0,$$

$$\Theta^i_{1_{33}} = -R_2 - \delta M_3^T - \delta M_3$$
$$\Theta^i_{1_{62}} = \delta N_6^T - \delta N_2^T + {}_2^T$$
$$\Theta^i_{1_{63}} = -\delta N_3^T + \delta M_3^T - {}_6^T$$
$$\Theta^i_{1_{65}} = -\delta N_5^T + \delta M_5^T$$

then, switched interval network (5) *is global robust asymptotic stable, the switching rule is chosen as* $\sigma(x(t)) = i$ *whenever* $x(t) \in \bar{\Omega}_i$.

$$\Theta^i_2 = \begin{bmatrix} 0 & * & * & * & * & * & * & * & * \\ 0 & -R_1 & * & * & * & * & * & * & * \\ \delta Z_1^T & R_1 + \delta Z_2^T & \Theta^i_{2_{33}} & * & * & * & * & * & * \\ -\delta S_1^T & -\delta S_2^T & \delta Z_4^T - {}_3^T & \Theta^i_{2_{44}} & * & * & * & * & * \\ 0 & 0 & \delta Z_5^T & R_3 - \delta S_5^T & -R_3 & * & * & * & * \\ -\delta Z_1^T + \delta S_1^T & -\delta Z_2^T + {}_2^T & \Theta^i_{2_{63}} & \Theta^i_{2_{64}} & \Theta^i_{2_{65}} & \Theta^i_{2_{66}} & * & * & * \\ 0 & 0 & \delta Z_7^T & -\delta S_7^T & 0 & -{}_7^T + \delta S_7^T & 0 & * & * \\ 0 & 0 & \delta Z_8^T & -\delta S_8^T & 0 & -{}_8^T + \delta S_8^T & 0 & 0 & * \\ 0 & 0 & \delta Z_9^T & -\delta S_9^T & 0 & -{}_9^T + \delta S_9^T & 0 & 0 & 0 \end{bmatrix} < 0,$$

$$\Theta^i_{2_{33}} = -R_1 + \delta Z_3 + \delta Z_3^T$$
$$\Theta^i_{2_{44}} = -R_3 - \delta S_4^T - \delta S_4$$
$$\Theta^i_{2_{63}} = \delta Z_6^T - \delta Z_3^T + {}_3^T$$
$$\Theta^i_{2_{64}} = -\delta Z_4^T + \delta S_4^T - {}_6^T$$
$$\Theta^i_{2_{65}} = -\delta Z_5 + \delta S_5$$
$$\Theta^i_{2_{66}} = -\delta Z_6 - \delta Z_6^T + {}_6 + \delta S_6^T$$

Proof By choosing the following Lyapunov–Krasovskii functional:

$$V(t, x_t) = V_1(t, x_t) + V_2(t, x_t) + V_3(t, x_t) + V_4(t, x_t) + V_5(t, x_t)$$

where

$$\Theta^i_3 = \begin{bmatrix} 0 & * & * & * & * & * & * & * & * \\ 0 & -R_1 & * & * & * & * & * & * & * \\ 0 & R_1 & -R_1 - R_2 & * & * & * & * & * & * \\ \delta X_1^T & \delta X_2^T & \delta X_3^T + R_2 & \Theta^i_{3_{44}} & * & * & * & * & * \\ -\delta Y_1^T & -\delta Y_2^T & -\delta Y_3^T & {}_5^T - \delta Y_4^T & -\delta Y_5^T - \delta Y_5 & * & * & * & * \\ -\delta X_1^T + \delta Y_1^T & -\delta X_2^T + {}_2^T & \Theta^i_{3_{63}} & \Theta^i_{3_{64}} & \Theta^i_{3_{65}} & \Theta^i_{3_{66}} & * & * & * \\ 0 & 0 & 0 & \delta X_7^T & -\delta X_7^T & -{}_7^T + \delta Y_7^T & 0 & * & * \\ 0 & 0 & 0 & \delta X_8^T & -\delta Y_8^T & -{}_8^T + \delta Y_8^T & 0 & 0 & * \\ 0 & 0 & 0 & \delta X_9^T & -\delta Y_9^T & -{}_9^T + \delta Y_9^T & 0 & 0 & 0 \end{bmatrix} < 0,$$

$$\Theta^i_{3_{44}} = -R_2 + \delta X_4^T + \delta X_4$$
$$\Theta^i_{3_{63}} = -\delta X_3^T + \delta Y_3^T$$
$$\Theta^i_{3_{64}} = \delta X_6^T - \delta X_4^T + {}_4^T$$
$$\Theta^i_{3_{65}} = -\delta X_5^T + \delta Y_5^T - {}_6^T$$
$$\Theta^i_{3_{66}} = -\delta X_6 - \delta X_6^T + {}_6^T + \delta Y_6$$
$$\phi = \delta^2 R_1 + \delta^2 R_2 + \delta^2 R_3 + \tau_n^2 R_4$$
$$\Upsilon^i_{11} = \delta N\, \Upsilon^i_{12} = \Upsilon^i_{21} = \delta S\, \Upsilon^i_{22} = \Upsilon^i_{31} = \delta X\, \Upsilon^i_{32} = \delta Y$$

$$V_1(t,x_t) = x^T(t)Px(t) + 2\sum_{i=1}^{n}\gamma_{1,i}\int_0^{x_i(t)} f_i(s)ds$$

$$V_2(t,x_t) = \int_{t-\tau_n}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau_1}^{t} x^T(s)Q_2x(s)ds + \int_{t-\tau_2}^{t} x^T(s)Q_3x(s)ds + \int_{t-\tau_N}^{t} x^T(s)Q_4x(s)ds$$

$$V_3(t,x_t) = \delta\int_{t-\tau_1}^{t-\tau_n}\int_s^t \dot{x}^T(\theta)R_1\dot{x}(\theta)d\theta ds + \delta\int_{t-\tau_2}^{t-\tau_1}\int_s^t \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta ds + \delta\int_{t-\tau_N}^{t-\tau_2}\int_s^t \dot{x}^T(\theta)R_3\dot{x}(\theta)d\theta ds$$

$$V_4(t,x_t) = \tau_n\int_{t-\tau_n}^{t}\int_s^t \dot{x}^T(\theta)R_4\dot{x}(\theta)d\theta ds$$

the derivation process of Corollary 3.1 is similar to Theorem 3.1.

Remark 3 In (Zhang et al. 2009), author investigate the global asymptotic stability of a class of recurrent neural networks with interval time-varying delays via delay-decomposing approach, the variation interval of the time delay is divided into two subintervals with equal length by introducing its central point, several new stability criteria are derived in terms of LMIs. However, in this paper, we divide the interval time delay into three subintervals, as we all know, when the number of the divided subintervals increases, the corresponding criteria can be improved in results, hence, the proposed criteria expand and improve the results in the existing literatures. Moreover, when $N=1$ and without regard to robustness in (5), the model in our paper is degenerated as the nonlinear functional differential equation (1) in (Zhang et al. 2009), so model studied in (Zhang et al. 2009; Shen and Cao 2011; Liu and Cao 2011; Phat and Trinh 2010) can be seen a special case of the model (5).

An illustrative example

In this section, an illustrative example will be given to check the validity and effectiveness of the proposed stability criterion obtained in Theorem 3.1.

Example Consider the the following second-order switched interval networks with interval time-varying delay described by

$$\begin{cases} \dot{x_i}(t) = -a_{i_\sigma}x_i(t) + \sum_{j=1}^{2} b_{ij_\sigma}^{(1)}f_j(x_{ij}(t)) + \sum_{j=1}^{2} b_{ij_\sigma}^{(2)}f_j(x_{ij}(t-\tau(t))) \\ \qquad a_{i_\sigma}\in[\underline{a}_{i_\sigma},\overline{a}_{i_\sigma}], \quad b_{ij_\sigma}^{(k)}\in[\underline{b}_{ij_\sigma}^{(k)},\overline{b}_{ij_\sigma}^{(k)}], \quad k=1,2, \end{cases} \tag{45}$$

where $\sigma(x(t)): R^n \to \{1,2\}$, and $\check{l}_1 = 0.1$, $\check{l}_2 = 0.2$, $\hat{l}_1 = 0.3$, $\hat{l}_2 = 0.6$, $\tau_n = 0.5$, $\tau_N = 2$, $\mu = \delta = 0.5$, The networks system parameters are defined as

$$\underline{A}_1 = \begin{pmatrix} 17.99 & 0 \\ 0 & 14.99 \end{pmatrix}, \quad \overline{A}_1 = \begin{pmatrix} 18.01 & 0 \\ 0 & 15.01 \end{pmatrix},$$

$$\underline{B}_{11} = \begin{pmatrix} -0.17 & 0.1 \\ 0.13 & -0.14 \end{pmatrix},$$

$$\overline{B}_{11} = \begin{pmatrix} -0.15 & 0.12 \\ 0.15 & -0.12 \end{pmatrix}, \underline{B}_{21} = \begin{pmatrix} -0.47 & 0.13 \\ 0.11 & -0.54 \end{pmatrix},$$

$$\overline{B}_{21} = \begin{pmatrix} -0.45 & 0.15 \\ 0.13 & -0.52 \end{pmatrix},$$

$$\underline{A}_2 = \begin{pmatrix} 15.99 & 0 \\ 0 & 16.99 \end{pmatrix}, \quad \overline{A}_2 = \begin{pmatrix} 16.01 & 0 \\ 0 & 17.01 \end{pmatrix},$$

$$\underline{B}_{12} = \begin{pmatrix} -0.208 & 0 \\ 0 & -0.208 \end{pmatrix},$$

$$\overline{B}_{12} = \begin{pmatrix} -0.188 & 0.02 \\ 0.02 & -0.188 \end{pmatrix}, \quad \underline{B}_{22} = \begin{pmatrix} -0.12 & 0.14 \\ 0.05 & -0.12 \end{pmatrix},$$

$$\overline{B}_{22} = \begin{pmatrix} -0.09 & 0.16 \\ 0.07 & -0.09 \end{pmatrix},$$

Solving the LMI in condition (ii) by using appropriate LMI solver in the Matlab, the feasible positive definite matrices P, Q_1, Q_2, Q_3, and diagonal matrices could be as

$$P = \begin{pmatrix} 1.6853 & 0.0095 \\ 0.0095 & 1.6431 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} 17.6516 & -0.0006 \\ -0.0006 & 17.6480 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 17.6708 & -0.0000 \\ -0.0000 & 17.6655 \end{pmatrix}, Q_3 = \begin{pmatrix} 17.6835 & 0.0000 \\ 0.0000 & 17.6809 \end{pmatrix},$$

Let $\xi_1 = 0.1, \xi_2 = 0.9$, it can be shown that

$$G_1(A_0^1, Q_1, Q_2, Q_3) = \begin{pmatrix} -7.6637 & -0.3142 \\ -0.3142 & 3.7002 \end{pmatrix},$$

$$G_2(A_0^2, Q_1, Q_2, Q_3) = \begin{pmatrix} -0.9226 & -0.3142 \\ -0.3142 & -2.8724 \end{pmatrix}$$

Moreover, the sum

$$\xi_1G_1(A_0^1, Q_1, Q_2, Q_3) + \xi_2G_2(A_0^2, Q_1, Q_2, Q_3) = \begin{pmatrix} -1.5967 & -0.3142 \\ -0.3142 & -2.2152 \end{pmatrix} < 0$$

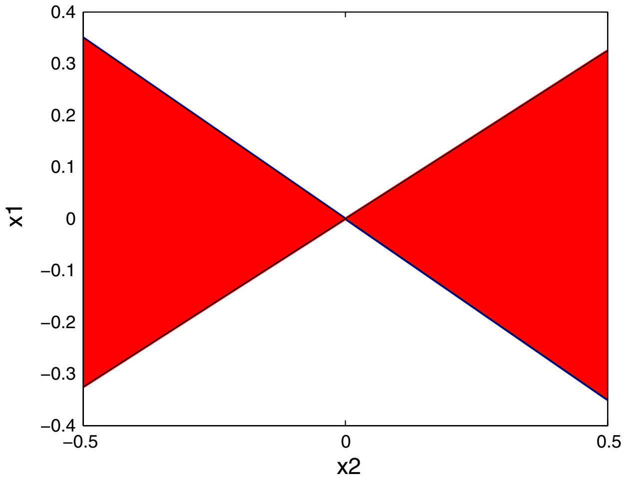

Fig. 1 Regions $\bar{\Omega}_1$

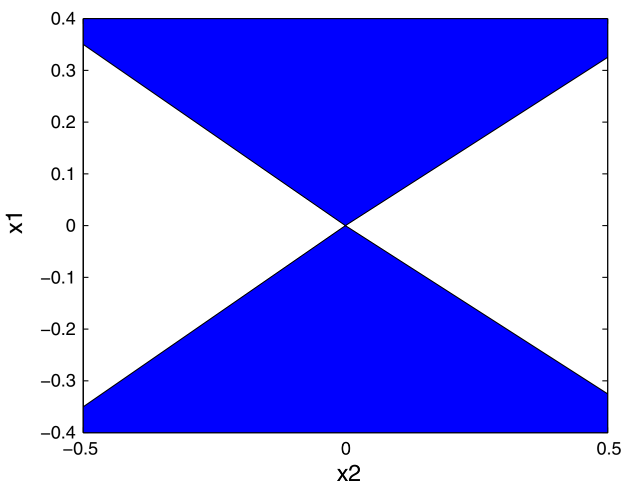

Fig. 2 Regions $\bar{\Omega}_2$

The sets Ω_1 and Ω_2 are given as

$$\Omega_1 = \{(x_1, x_2) \in R^2 : -7.6637x_1^2 - 0.6284x_1x_2 + 3.7002x_2^2 < 0\},$$
$$\Omega_2 = \{(x_1, x_2) \in R^2 : 0.9226x_1^2 + 0.6284x_1x_2 + 2.8724x_2^2 > 0\}.$$

then, the switching regions (Figs. 1, 2) are defined as

$$\bar{\Omega_1} = \{(x_1, x_2) \in R^2 : -7.6637x_1^2 - 0.6284x_1x_2 + 3.7002x_2^2 \leq 0\},$$
$$\bar{\Omega_2} = \{(x_1, x_2) \in R^2 : -7.6637x_1^2 - 0.6284x_1x_2 + 3.7002x_2^2 \geq 0\}.$$

The switching rule $\sigma(x(t))$ can be given by

$$\sigma(t) = \begin{cases} 1, & if\ x(t) \in \bar{\Omega_1}, \\ 2, & if\ x(t) \in \bar{\Omega_2}. \end{cases}$$

By Theorem 3.1, this switched interval network (45) is global robust asymptotic stable.

Conclusion

In this paper, we have proposed a new scheme of switched interval networks with interval time-varying delay and general activation functions. By introducing the delay fractioning approach, the variation interval of the time delay is divided into three subintervals, by checking the variation of the Lyapunov functional for the case when the value of the time delay is in every subinterval, the switching rule which depends on the state of the network is designed and some new delay-dependent robust stability criteria are derived in terms of LMIs. An illustrative example has been also provided to demonstrate the validity of the proposed robust asymptotic stability criteria for switched interval networks.

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References

Balasubramaniam P, Vembarasan V, Rakkiyappan R (2011) Leakage delays in T–S fuzzy cellular neural networks. Neural Process Lett 33:111–136

Boyd S, Ghaoui LE, Feron E, Balakrishnan V (1994) Linear matrix inequalities in system and control theory. SIAM, Philadelphia

Brown TX (1989) Neural networks for switching. IEEE Commun Mag 27(11):72–81

Han QL, Yue D (2007) Absolute stability of Lur'e systems with time-varying delay. Control Theory Appl IET 1(3):854–859

He W, Cao J (2008) Robust stability of genetic regulatory networks with distributed delay. Cogn Neurodyn 2(4):355–361

Hu J, Wang Z (2011) A delay fractioning approach to robust sliding mode control for discrete-time stochastic systems with randomly occurring non-linearities. IMA J Math Control Inf 28:345–363

Huang H, Qu Y, Li H (2005) Robust stability analysis of switched Hopfield neural networks with time-varying delay under uncertainty. Phys Lett A 345(4–6):345–354

Li P, Cao J (2007) Global stability in switched recurrent neural networks with time-varying delay via nonlinear measure. Nonlinear Dyn 49(1–2):295–305

Lian J, Zhang K (2011) Exponential stability for switched Cohen–Grossberg neural networks with average dwell time. Nonlinear Dyn 63:331–343

Liberzon D (2003) Switching in systems and control. Springer, Berlin

Liu X, Cao J (2011) Local synchronization of one-to-one coupled neural networks with discontinuous activations. Cogn Neurodyn 5(1):13–20

Liu H, Chen G (2007) Delay-dependent stability for neural networks with time-varying delay. Chaos Solitons Fractals 33(1):171–177

Liu L, Han Z, Li W (2009) Global stability analysis of interval neural networks with discrete and distributed delays of neutral-type. Expert Syst Appl Part 2 36(3):7328–7331

Niamsup P, Phat VN (2010) A novel exponential stability condition of hybrid neural networks with time-varying delay. Vietnam J Math 38(3):341–351

Phat VN, Trinh H (2010) Exponential stabilization of neural networks with various activation functions and mixed time-varying delays. IEEE Trans Neural Netw 21(7):1180–1184

Ratchagit K, Phat VN (2011) Stability and stabilization of switched linear discrete-time systems with interval time-varying delay. Nonlinear Anal Hybrid Syst 5:605–612

Shen J, Cao J (2011) Finite-time synchronization of coupled neural networks via discontinuous controllers. Cogn Neurodyn 5(4):373–385

Thanha N, Phat V (2013) Decentralized stability for switched nonlinear large-scale systems with interval time-varying delays in interconnections. Nonlinear Anal Hybrid Syst 11:22–36

Uhlig F (1979) A recurring theorem about pairs of quadratic forms and extensions. Linear Algebra Appl 25:219–237

Wang Y, Wang Z, Liang J (2008) Delay fractioning approach to global synchronization of delayed complex networks with stochastic disturbances. Phys Lett A 372(39):6066–6073

Wang Y, Yang C, Zuo Z (2012) On exponential stability analysis for neural networks with time-varying delays and general activation functions. Commun Nonlinear Sci Numer Simulat 17:1447–1459

Xu H, Wu H, Li N (2012) Switched exponential state estimation and robust stability for interval neural networks with discrete and distributed time delays. Abstr Appl Anal 20 ID:103542. doi:10.1155/2012/103542

Ye H, Michel AN, Hou L (1998) Stability theory for hybrid dynamical systems. IEEE Trans Autom Control 43(4):461–474

Yue D (2004) Robust stabilization of uncertain systems with unknown input delay. Automatica 40:331–336

Zeng H, He Y, Wu M, Zhang C (2011) Complete delay-decomposing approach to asymptotic stability for neural networks with time-varying delays. IEEE Trans Neural Netw 22(5):806–811

Zhang W, Yu L (2009) Stability analysis for discrete-time switched time-delay systems. Automatica 45(10):2265–2271

Zhang Y, Yue D, Tian E (2009) New stability criteria of neural networks with interval time-varying delay: a piecewise delay method. Appl Math Comput 208:249–259

Zhang H, Liu Z, Huang GB, Wang Z (2010) Novel weighting-delay-based stability criteria for recurrent neural networks with time-varying delay. IEEE Trans Neural Netw 21(1):91–106