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# Opportunities for Change in the First Two Years of College Mathematics

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**Abstract**
This is a survey of the developments in the first two years of undergraduate mathematics, beginning in the early 1950s and continuing up to the present. It documents the repeated efforts at making this instruction relevant to the partner disciplines, especially Biology, and describes the challenges for the future.

**Keywords** Calculus · Undergraduate mathematics

The series of articles in this issue of the *Bulletin of Mathematical Biology* was largely inspired by the provocative conference on *The Future of College Mathematics* (Ralston and Young 1983) and the subsequent series of articles that appeared in the *College Mathematics Journal* (Ralston 1984). Ralston sought to dethrone calculus from its role as the single unifying theme for the first two years of college mathematics. Reactions would serve to roil the mathematics community, precipitating the Calculus Reform Movement and echoing down to our current era.

To fully appreciate these events of the early 1980s, we must place them in their historical context, going back to the 1950s and the start of the national attempt to define the undergraduate curriculum in mathematics. We will sketch how Ralston's challenges reverberated through the succeeding decades. We will conclude by looking at the potential for another shake-up created by the attention that departments of mathematics are now paying to the needs of biologists.

## 1 In the beginning …

The concept of a department of mathematics in service to other disciplines had always existed in those institutions with a focus on engineering, but it only emerged broadly after the Second World War with the GI Bill, the expansion of public universities and the impetus in 1957 from Sputnik, with the Russians beating the USA into space.

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Most mathematics departments were small, offering courses in business mathematics, college algebra, trigonometry and analytic geometry—generally at a level that did not exceed what was taught in high school—plus calculus and perhaps advanced calculus (Tucker 2013). In 1953, dismayed by the inadequate state of mathematics instruction, the Mathematical Association of America (MAA) created a five-member standing Committee on the Undergraduate Program (CUP). Aware of the growing role of mathematics in support of other disciplines and the potential for the new "high-speed digital automata," CUP recommended a two-year sequence that began with a "universal course of mathematics" to be taken by all non-engineering college students in their first year.

This universal course would spend the first half of the year with functions, limits and elementary concepts of calculus. The second half would begin with sets and build up to simple algebraic systems and probability. In the second year, students in the social sciences would go on to study statistics. Science majors would get a full course in calculus. Engineers would spend their first year in "technical problem solving."

Universal Mathematics was pilot-tested at Tulane, where it proved to be a disaster. Instructors struggled to teach this material, and students found it far too difficult. Nevertheless, it did inspire John Kemeny and colleagues at Dartmouth to create *Modern Mathematical Methods and Models*, published by the MAA (Cogan et al. 1958). Intended as the second year of undergraduate mathematics for students in the biological and social sciences, it introduced matrices and vector algebras, basic ides of calculus, multivariable functions, optimization problems, probability, order relations, Markov chains and mathematical models. Its most enduring legacy was the recognition of linear algebra as a subject suitable for the first two years.

In 1958, the five-member CUP was disbanded and replaced by the Committee on the Undergraduate Program in Mathematics (CUPM), an oversight committee that would organize subcommittees to create recommendations across the breadth of undergraduate mathematics. CUPM was generously funded by the National Science Foundation (NSF). It sets up its headquarters at Berkeley and recruited an impressive list of prominent research mathematicians. Among the many reports and recommendations that emerged in the 1960s, the most influential was *A General Curriculum in Mathematics for Colleges* (CUPM 1965). It laid out a minimal program of 13 courses that could be taught in a department with as few as four faculty members.

In response to pressure from the physics community, calculus was moved to the first year. CUPM recognized that not all students who would need calculus would be ready for it when they entered college, so they also described Math 0, a course that condensed college algebra, trigonometry and all other precalculus topics into a single semester. For the first time, we see the emergence of what has become the standard curriculum for the first two years: two semesters of single variable calculus, one semester of multivariable calculus and differential equations, and one semester of linear algebra. Nine courses were now prescribed to be available for upper undergraduates. Statistics was among them, not accessible until students had completed a year of calculus plus courses in linear algebra and probability.

The 1960s were a heady time for mathematics departments. The number of majors soared from 11,400 in 1960 to 27,400 in 1970. Now spurred on by access to more challenging mathematics as undergraduates as well as the NSF Undergraduate Research

Participation Program (Astin 1969), the number of PhDs grew from 300 in 1960 to 1200 in 1970 (a quantity that far exceeded what the market could absorb; the production of PhDs in the mathematical sciences did not reach this level again until 2006). (NCES 2018)

## 2 The Reappraisal

Mathematics lost its sheen in the 1970s. By 1980, the number of majors had dropped to 11,000, now below the number produced in 1960. Part of this must be attributable to the appearance of computer science as an alternative, but the drop in mathematics majors was greater than the growth in computer science, and even the number of students studying Advanced Placement Calculus in high school declined in the early years of this decade. This may have been a reaction to the elitist nature of the mathematics program. Ralston's contribution was part of a larger effort in the early 1980s to re-evaluate the decisions of the 1960s.

Ralston was trained as a numerical analyst in the 1950s. In 1967, he founded the Computer Science Department at the University of Buffalo. He was acutely aware that the general curriculum in mathematics ill-served his students and was doing nothing to take advantage of the potential created by computers and handheld calculators. At the 1982 conference, Herb Wilf predicted that handheld calculators would soon be able to find solutions to differential equations, raising questions of which skills students actually need.

Ralston never proposed replacing calculus, but he did argue that the first two years should include a year's worth of discrete mathematics, adding graph theory, combinatorics, probability and statistics to the existing emphasis on linear algebra.

A year before the Ralston conference, CUPM had issued its own more moderate correctives. They asserted that "The mathematical sciences curriculum should be designed around the abilities and academic needs of the average mathematical sciences student," that classes should use interactive teaching drawing students to "discover new mathematics for themselves rather than present students with concisely sculptured theories," that applications should be used to illustrate and motivate the mathematics, and that the first courses "should be designed to appeal to as broad an audience as is academically reasonable." (Steen 1989, p. 5) In addition CUPM called for courses in computer science, applied probability, and statistics to be made available in the first two years and for students to have access to real-world modeling projects. This was also the first call from the MAA to require a statistics course of *all* mathematics majors.

Most mathematicians objected to anything that would decrease the emphasis on calculus. One of most measured objections came from Ron Douglas, then Chair of the Department of Mathematics at SUNY Stony Brook, later to become its Dean of Science and then Provost. While unwilling to relinquish the central position of calculus, he did acknowledge significant problems with how it was being taught. In 1986, he organized a conference at Tulane to examine calculus instruction. In the preamble to the conference proceedings, he wrote:

> The United States is currently experiencing a shortage of young people studying mathematics, science and engineering, and this shortage is expected to worsen. Calculus is the gateway and is fundamental to all such study. Hence, every student who does not complete calculus is lost to further study in science, mathematics or engineering. Moreover, many students who start calculus do not complete it successfully. The country cannot afford this now, if it ever could. Further, many of those who do finish the course, have taken a watered down, cookbook course in which all they learn are recipes, without even being taught what it is that they are cooking. Understandably, science and engineering faculties find it difficult to build on such a foundation, and they feel that they must teach their students elementary calculus as well as science or engineering. Finally, in past generations many students were sufficiently challenged and turned-on in their calculus course that they decided to become mathematicians. I don't believe that happens much today. To overcome these problems and to recapture that earlier excitement, I decided to see what could be done to improve calculus instruction. (Douglas 1986, p. iv)

More students than ever were now taking calculus. While the number of mathematics majors had recovered to only half their 1970 peak, the number of students taking calculus in 1985 was 50% higher than it had been in 1970 (Albers et al. 1987). Calculus was now being taught to a more inhomogeneous clientele, usually in large lecture halls. At the Joint Math Meetings that preceded the Tulane Conference, there was universal agreement with the complaints that Douglas raised in his preamble.

The papers written for the Tulane conference were filled with calls for more active learning, for the incorporation of technology, for the use of applications and open-ended problems, and for a trimmed-down syllabus that would allow for greater opportunity to engage with the key concepts. Realizing that the mathematical community was now facing a significant challenge, attention again turned to NSF to fund a major effort.

## 3 Calculus Reform

In 1988, NSF launched its Calculus Initiative. Over the next seven years, it would spend over $22 million to develop and promote new approaches to calculus instruction. A common theme of many of these efforts was the development of a curriculum that emphasized calculus as a tool for modeling dynamical systems.

At this time, I was on the faculty at the Pennsylvania State University. I remember talking with a Biology professor who complained that in an advanced course, with students who had all completed a full year of single variable calculus, he had been talking about exponential growth and decay. Seeing uncomprehending looks from his students, he asked whether they had learned about exponential growth and decay in calculus. All insisted it had never been mentioned.

I assured him that that was not true, that every first-semester calculus class discusses exponential functions and their role in modeling growth and decay. But I also think I understand why this knowledge was new to these students. In calculus then, as too often in calculus now, the focus had been on learning how to differentiate and integrate and solve a standard repertoire of problems. The use of exponential functions as models of growth and decay was nothing more than an aside. This application would not appear on a test and so could safely be ignored.

Eight major new calculus textbooks were funded by the calculus initiative. Half of them chose to begin the course with simple difference or differential equations, drawing students from the start with the idea of calculus as tool for modeling. Exponential functions arose early in the course in the context of modeling proportional growth or decay. All of the texts relied heavily on the use of computers or calculators, many incorporating computer algebra systems, with two of them specifically tied to *Mathematica*. Almost all were built around a student-centered approach to instruction, built around student exploration and other active learning approaches.

The reaction was swift and often vicious. One of the factors contributing to this reaction was the simultaneous release of *Curriculum and Evaluations Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM 1989). Many mathematicians believed that the real problem with calculus instruction was inadequate preparation in the high schools. The NCTM *Standards* were often interpreted as encouraging a weakening of this preparation. Calculus Reform was seen as furthering this lamentable trend, replacing the mastery of algebraic skills with reliance on black box computers. A Penn State colleague asserted that he had witnessed a calculus student, needing to find the average of 1 and 3, pull out his calculator, enter $1 + 3 \div 2 =$ and blithely write down 2.5.

While many faculty members embraced calculus reform and many departments experimented with it, there were significant problems in gaining broad acceptance even from faculty who were sympathetic. Moving an innovative approach beyond the initiators is always tricky, and there were a few monumental failures. Students, already stressed by the prospect of calculus as the most difficult course they were likely to encounter, had little patience when unfamiliar methods of instruction only made it more confusing.

Two events marked the end of the push for calculus reform. In 1995, NSF's calculus initiative was folded into other programs, losing its signature status. That same year, NSF ceased its support for *UME Trends* (UME = Undergraduate Mathematics Education), a journal sharing work on undergraduate mathematics education. It had been founded in 1988, at the start of the initiative. The intention was to continue, now supported by subscriptions. But the numbers were not there. It folded that year.

The Conference Board of the Mathematical Sciences (CBMS) tracked the use of computer assignments, writing assignments and group projects in Calculus I over the years 1990 to 2005. At the research universities, they peaked in 1995. In smaller colleges, their use continued to rise until 2000. But by 2005, their use had almost disappeared (Bressoud 2007). Only one of the eight calculus reform textbooks (Hughes-Hallett et al. 2019) is still in print, and it has drifted far toward the mainstream. It appeared that calculus reform was dead.

## 4 Continuing Initiatives

The period 1988 to 1995 was a time of high visibility and strong debate, but the reforms that were set in motion in the 1980s did not disappear. While the remaining reform text, Hughes-Hallett et al. have moved toward the mainstream, the mainstream itself has been diverted. One of the thrusts of calculus reform, the inclusion of graphical and numerical representations of functions in addition to the purely algebraic is now standard. Today the dominant textbooks all include opportunities for calculator or computer explorations, more realistic applications and invitations to exploration.

Calculus reform also gave a tremendous boost to the development of mathematics education research at the undergraduate level. In 1988, CBMS began publishing its series on *Issues in Mathematics Education.* In 1989, the joint *Committee on Research in Undergraduate Mathematics Education* (CRUME, perhaps unfortunately pronounced "crummy") was established. Over the following decade, it arranged for sessions on undergraduate mathematics education at the national meetings of the American Mathematical Society (AMS) and the MAA, began publishing *Research in Collegiate Mathematics Education* as a subseries of *Issues in Mathematics Education,* and organized its own conferences.

In 1999, CRUME assisted in the formation of the Association for Research in Undergraduate Mathematics Education (ARUME), which later became the MAA Special Interest Group on RUME (RUME SIGMAA). RUME SIGMAA runs annual conferences drawing hundreds of participants, and it was instrumental in the foundation of the *International Journal of RUME*. All of these activities reflect a blossoming of the community of researchers in undergraduate mathematics education, establishing evidence for what works and laying solid foundations for improvements in teaching and learning.

Also, in the early 1990s, Brian Winkel founded *Problems, Resources, and Issues in Undergraduate Mathematics Education* (PRIMUS), a journal promoting the Scholarship of Teaching and Learning (SoTL). This journal has fostered the growth of SoTL within the community of mathematics faculty, encouraging carefully documented experimentation. Between the work of the RUME and SoTL communities, we now have a rich body of both formal and informal research on which to build.

One of the most important developments that appeared during the years of Calculus Reform was the MAA's Project NExT (New Experiences in Teaching). Each year, Project NExT admits 80 to 100 new or recent PhDs in the mathematical sciences and takes these NExT Fellows through a series of multi-day workshops that discuss all facets of being a new faculty member: publishing, getting grants, serving on committees, dealing with difficult colleagues and students. But the main emphasis is on teaching, introducing the fellows to techniques that promote active learning as well as equity and inclusion. Fellows stay in touch with each other through get-togethers at meetings and listserves where they share questions, frustrations and successes. Project NExT began in 1994. Most of the MAA leadership today has come through Project NExT, and these fellows, now numbering in the thousands, have become a powerful voice for continuing improvement in mathematics instruction.

## 5 The Early 21st Century

The CUPM recommendations from 1981 were reinforced and updated in 1991, adding the need for students to practice communication via both writing and speaking in their mathematics classes and encouraging the use of group projects. But the emphasis was almost entirely on serving prospective mathematics majors. As the ten-year anniversary of the 1991 report approached, CUPM decided that it needed to take a closer look at how departments of mathematics could better serve the "partner disciplines," those with mathematical prerequisites.

In preparation for the report that would eventually appear in 2004 (CUPM 2004), CUPM organized a series of discipline-focused three-day workshops. Each workshop brought together about a dozen faculty from that discipline, charged with creating a short description of the essential skills and understandings their majors needed to acquire from their classes in mathematics (Ganter and Barker 2004). The CUPM report focused on the following themes—which I slightly paraphrase—that had emerged from the workshops.

1. Understand the actual needs of the students who enroll in each class and monitor the effectiveness of the program in meeting those needs.
2. Develop what have since come to be known as the mathematical practices, especially mathematical thinking and communication skills.
3. Communicate the breadth and interconnections of the mathematical sciences.
4. Promote interdisciplinary cooperation.
5. Use computer technology as appropriate to support problem solving and to promote understanding.
6. Provide faculty support for curricular and instructional improvement.

The first decade of this century witnessed a great deal of work on different aspects of the undergraduate program, much of which was run under the auspices of the MAA. This included issues of placement, student assessment and program assessment. In 2007, the American Statistical Association (ASA) produced *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) (ASA 2016). Major initiatives emerged to promote inquiry-based learning, quantitative reasoning and alternative pathways for at-risk students. Problems of diversity, equity and inclusion gained more attention within the mathematical community. And the MAA launched its large-scale studies of calculus instruction in the USA (*Characteristics of Successful Programs in College Calculus* (CSPCC), NSF #0910240, 2005–2010, and *Progress through Calculus* (PtC), NSF #1430540, 2015–2020).

Amazingly, before 2010 we had had only local and anecdotal information on the percentage of students passing the first semester of calculus and continuing on to the second semester, who was taking it and why, and how it was being taught. Both the CSPCC and PtC projects surveyed mathematics departments across the country and then organized case study visits to the most interesting programs. Among the striking findings were the effectiveness of Calculus I in destroying student confidence in mathematical ability. This was particularly pronounced for women (Ellis et al. 2016). A summary of the CSPCC findings can be found in *Insights and Recommendation from the MAA National Study of College Calculus* (Bressoud et al. 2015). This study revealed that there is great concern about high failure rates and lack of persistence, especially at those universities serving large numbers of students. It discovered tremendous pressure to improve student outcomes, not just better grades but higher rates of persistence—especially for at-risk students—and better preparation for the downstream courses.

There is significant recognition of the need for pedagogical reformation. Even at research universities, usually the most conservative departments, the 2015 CBMS survey found that 62% reported having made major pedagogical changes over the period 2005–2015. A majority had at least one faculty member experimenting with active learning (64%), with flipped classes (61%) and with inquiry-based learning (56%) (Blair et al. 2018, Table SP.26). The PtC survey of 2015 revealed that over 40% of research universities consider the introduction of active learning practices to be "very important," although only 10% consider themselves to be "very successful" at it (Apkarian and Kirin 2017).

## 6 *Engage to Excel* and the Aftermath

The mathematical community was understandably shocked and dismayed when, in 2012, the President's Council of Advisors on Science and Technology (PCAST) published *Engage to Excel,* castigating mathematicians for "introductory mathematics courses [that] often leave students with the impression that all STEM fields are dull and unimaginative" (PCAST 2012, p. vi) and asserting that "Discipline-based education on effective undergraduate mathematics teaching also appears less developed when compared with other STEM fields." (PCAST 2012, p. 27)

The fact was that during the period 1995 to 2012 work on the undergraduate program in mathematics was flying below the radar of most departments of mathematics in research focused universities. The PCAST report served as a wake-up call to this community. Early in 2013, under the leadership of Phillip Griffiths of the Institute for Advanced Study, the Carnegie Corporation of New York convened a meeting of prominent research mathematicians, including Eric Friedlander of USC, Mark Green of UCLA, Brit Kirwan, Chancellor of the University of Maryland system and Uri Treisman, Director of the Charles A. Dana Center at the University of Texas-Austin, to begin the investigation of what should be taught, how it should be taught, and how to achieve improvement at scale.

From this initial meeting emerged Transforming Post-Secondary Education in Mathematics (TPSE Math). While addressing all of post-secondary mathematics, the leadership of TPSE Math was chosen to ensure credibility within the research community. This organization has established a network of members and runs frequent regional meetings built around its four priorities: lower-division pathways, upper-division pathways, graduate education and teaching strategies and practices. The last of these is informed by collaboration with Ithaka S + R, an organization that partners with leaders in higher education to provide research and evaluation.

Several other activities since the PCAST report are also worth mentioning. In 2012, the Association of Public and Land-grant Universities (APLU) launched its Mathematics and Teacher Education Partnership, working with the universities that produce most of the secondary mathematics teachers in the USA to improve their preparation. In 2016, the APLU began its NSF-sponsored program for Student Engagement in Mathematics through an Institutional Network of Active Learning (SEMINAL). It started by studying three large state universities that had introduced extensive use of active learning into their mathematics classes: University of Colorado-Boulder, University of Nebraska-Lincoln and San Diego State University. It then added twelve more universities seeking to effect comparable change. SEMINAL has since linked with the MAA's program on *Progress through Calculus*, which is studying university mathematics departments that are engaged in improving their programs.

In 2013, the National Research Council published *The Mathematical Sciences in 2025* (NRC 2013), describing for a general audience the role and importance of the mathematical sciences in today's world and calling for mathematics instruction that is attuned to these changing needs. Echoing much of Ralston's argument, this report questions the traditional calculus-focused curriculum. It suggests that "different pathways are needed for students who may go on to work in bioinformatics, ecology, medicine, computing, and so on. It is not enough to rearrange existing courses to create alternative curricula; *a redesigned offering of courses and majors is needed* [my emphasis]." (NRC 2013, p. S-9) The recent NRC report on *Data Science for Undergraduates* (NRC 2013) reinforces the point that if mathematics is to be engaged in the emerging field of data analytics, then the undergraduate curriculum will need to be restructured.

Finally, there is *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME), published jointly by the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) in 2015 and updated in 2019. It explains the central role of modeling and how it can be approached in preK-8, high school, and undergraduate mathematics.

## 7 The Future

In 1965 W.L. Duren, then chair of CUPM, looked ahead fifty years (Duren 1967). He was remarkably prescient in the two predictions he made. The first was that more and more students would arrive in college with more and more mathematics already behind them, many ready to start what was then regarded as graduate work. That has transpired. Roughly 800,000 students study calculus in high school, over 150,000 of them before their senior year. Large state universities such as the University of Illinois at Urbana-Champaign are struggling with how to determine placement for the hundreds of students who arrive having already studied several variable calculus.

Duren's other prediction was that rapid expansion in the number of students entering college would result in many more underprepared students. In the fall of 2015, over a quarter of a million students were taking non-credit pre-college level mathematics in our four-year undergraduate programs. Over half (57%) of the total mathematics enrollment that term was in classes normally taught in high school, that is to say CUPM's Math 0 or below (Blair et al. 2018). In the 1960s, CUPM had the luxury of proposing a curriculum for a relatively homogeneous population of students, overwhelmingly male, white and middle class. Duren was justifiably worried that mathematics faculty might not be able to accommodate the coming disparity in student preparation.

What Duren failed to grasp was how the computer would fundamentally affect the nature of the mathematical sciences and the preparation future students would need. Ralston saw this in part, but even he was naïve, believing that equipping students for the future lay in beefing up discrete mathematics in the first two years.

At Macalester College, we have taken our inspiration from the needs of the biology students. We now prescribe one semester of calculus and one semester of statistics for these majors. The first semester, replacing our traditional Calculus I, is a return to the ideas of so many of the Calculus Reform textbooks, building calculus through the study of models of dynamical systems. The second semester focuses on the construction of statistical models, replacing the commonly taught zoo of statistical tests with exploration of rich data sets, emphasizing regression, statistical inference, analysis of variance and multiple regressions. Since its formal adoption in 2004, Macalester's modeling approach has spread throughout all three semesters of calculus. Today it heavily influences our entire undergraduate program in mathematics. Most other disciplines find this fits their needs better than a standard calculus curriculum.

Macalester is a small liberal arts college with a great deal of freedom to experiment. Our curriculum would not be easy to export. But even large public and prominent research universities are recognizing the need to reform undergraduate instruction in mathematics. The MAA via *Progress through Calculus* and the APLU through SEMINAL are working with about two dozen large public and research universities to improve teaching and learning in the precalculus through single variable calculus sequence. A few, most notably Arizona State University and Oklahoma State University, have totally reconstructed their calculus instruction. The clearest lessons so far are that any reform must be based on a careful and honest assessment of the current strengths and weaknesses of the program, must be sensitive to local conditions and restraints, and must have support that extends from a group of enthusiastic faculty eager to work on new approaches to the chair and a cohort of senior faculty who can help ensure longevity of the effort, and from there to the dean and upper administration who can provide financial resources.

The challenge for the future is two-pronged. One direction is pedagogical, the need for more active engagement of students. Here I am optimistic. Deans and provosts no longer accept high failure rates in introductory mathematics classes, and we now have a solid body of evidence that active learning approaches make a real difference in student success rates (see e.g., Freeman et al. 2014). In 2016, the presidents of the professional societies in the mathematical sciences went on record to

> call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms (CBMS 2016).

The other direction is curricular. For too long, the emphasis in mathematics classes has been on what is easy to test, rather than the kind of knowledge that students actually need for subsequent work. In an earlier paper, I described the goal of our instruction to be classes that engage all students in the joy of mathematical exploration and the satisfaction of deep learning, not just the memorization of procedures but the ownership of them so that their principles can be applied flexibly in unfamiliar situations (Bressoud 2019).

In particular, it is essential that students understand the role of the mathematical sciences as a tool for modeling the world around us, whether this be through dynamical, discrete or statistical models. This is not just useful for biology majors. These understandings are needed by all students who wish to be able to use mathematics to comprehend and help to shape the world of the 21st century.

This will be the more difficult transformation for undergraduate mathematics education. An entire generation has fought for it with progress that has often been uncertain. But in the process, these pioneers have laid the foundations upon which their successors are now building.

## References

Albers DJ, Anderson RD, Loftsgaarden DO (1987) Undergraduate programs in the mathematical and computer sciences: the 1985–1986 survey. MAA, Washington, DC

American Statistical Association (ASA) (2016) Guidelines for assessment and instruction in statistics education college report 2016. ASA, Alexandria

Apkarian N, Kirin D (2017) Progress through calculus: census survey technical report. Retrieved January 20, 2020 from https://www.maa.org/sites/default/files/PtC%20Technical%20Report_Final.pdf

Astin AW (1969) A preliminary evaluation of the undergraduate research participation program of the national science foundation. J Educ Res 62(5):217–221

Blair R, Kirkman EE, Maxwell JW (2018) Statistical abstract of undergraduate programs in the mathematical sciences in the United States. AMS, Providence

Bressoud D (2007) Reform fatigue. Launchings, June 2007. Retrieved January 20, 2020 from https://www.macalester.edu/~bressoud/launchings/2007/launchings_06_07.html

Bressoud D (2019) Calculus reform: what is different this time?, Retrieved January 20, 2020 from https://mathvalues.squarespace.com/masterblog/launchings201906-z45y4-fhkpj

Bressoud D, Mesa V, Rasmussen C (eds) (2015) Insights and recommendations from the MAA national study of college calculus. MAA Press, Washington, DC

Cogan EJ, Kemeny JG, Norman RZ, Snell JL (1958) Modern. Mathematical Association of America, Buffalo

Committee on the Undergraduate Program in Mathematics (CUPM) (1965) A general curriculum in mathematics for colleges. CUPM, Berkeley

Committee on the Undergraduate Program in Mathematics (CUPM) (2004) CUPM curriculum guide 2004. MAA, Washington, DC

Conference Board of the Mathematical Sciences (CBMS) (2016) Active learning in post-secondary mathematics education. Retrieved January 20, 2020 from https://www.cbmsweb.org/2016/07/active-learning-in-post-secondary-mathematics-education/

Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM) (2019) Guidelines for assessment and instruction in mathematical modeling education, 2nd edn. SIAM, Philadelphia

Douglas R (1986) Toward a lean and lively calculus. MAA notes #6. MAA, Washington, DC

Duren WL Jr (1967) CUPM the history of an idea. Am Math Mon 74(1):23–37

Ellis J, Fosdick BK, Rasmussen C (2016) Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: lack of mathematical confidence a potential culprit. PLoS ONE 5(6):23–67. https://doi.org/10.1371/journal.pone.0157447

Freeman S, Eddy S, McDonough M, Smith M, Okoroafor N, Jordt H, Wenderoth M (2014) Active learning increases student performance in science, engineering, and mathematics. Proc Natl Acad Sci U. S. A. 111(23):8410–8415

Ganter S, Barker W (eds) (2004) Curriculum foundations project: voices of the partner disciplines. MAA, Washington, DC

Hughes-Hallett D, Gleason AM, McCallum WG (2019) Calculus: single and multi-variable. Wiley, Hoboken

National Center for Education Statistics (NCES) (2018) Table 325.65. Retrieved January 20, 2020 from https://nces.ed.gov/programs/digest/d18/tables/dt18_325.65.asp

National Council of Teachers of Mathematics (NCTM) (1989) Curriculum and evaluations standards for school mathematics. NCTM, Reston

National Research Council (NRC) (2013) The mathematical sciences in 2025. The National Academies Press, Washington, DC. https://doi.org/10.17226/15269

President's Council of Advisors on Science and Technology (PCAST) (2012) Engage to excel: producing one million additional college graduates with degrees in science, technology, engineering, and mathematics. The White House, Washington, DC

Ralston A (1984) Will discrete mathematics surpass calculus in importance? Coll Math J 15(5):371–373

Ralston A, Young GS (eds) (1983) The future of college mathematics. Springer, New York

Steen LA (ed) (1989) Reshaping college mathematics. MAA notes #13. MAA, Washington, DC

Tucker A (2013) The history of the undergraduate program in mathematics in the United States. Am Math Mon 120(8):689–705