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Social Eavesdropping: A Game-Theoretic Analysis

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Abstract Here we extend the classic Hawk–Dove model of animal conflict to allow for continuous variation in fighting strengths. Whereas the winner of a fight is chosen at random in the discrete game, in our continuous game, the winner of any fight is the stronger individual, and costs are higher for more evenly matched opponents. We identify the evolutionary stable strength threshold beyond which an animal should be prepared to engage in aggressive behaviour and show that this threshold increases with variance in fighting strength when the costs of aggression are insensitive to the level of strength asymmetry, but decreases with variance when the costs are sensitive to the level of asymmetry. In contrast to the classic discrete game, population-wide aggressive behaviour occurs only when the costs of fighting are zero. It is now known that animals can eavesdrop on the outcome of contests between neighbours and modify their behaviour towards observed winners and losers. We therefore further extend our model to allow for social eavesdropping within networks comprising three individuals. Whereas earlier work showed that eavesdropping increases the frequency of mutually aggressive contests in the discrete game by enhancing the value of victory, here we show that aggression thresholds in the continuous game are always higher with eavesdropping than without it: for sufficiently weak animals, avoiding the costs of challenging an observed winner over-rides the potential benefit of winning, so that eavesdropping reduces the frequency of aggressive encounters. Thus, even though strength is not directly observable, information is extracted from the variation in fighting ability that the classic Hawk–Dove game ignores.

Keywords Animal conflict · Continuous Hawk–Dove game · Eavesdropping · Fighting · Game theory

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1. Introduction

Gathering information from interactions between other animals has been described as eavesdropping (McGregor, 1993, p. 241). The study of this phenomenon is still in its infancy: in particular, there has been very little theoretical analysis of its effect within species (Peake, 2005, p. 33). The principal contribution to this literature so far is that of Johnstone (2001), who extended the classic Hawk–Dove game (Maynard Smith, 1982) to allow bystanders to eavesdrop on contests between others, and to modify their behavior toward the contestants in response to the observed interaction. A surprising prediction from his analysis was that escalated contests should occur more frequently when eavesdropping is possible. But this prediction emerged from a model that ignores variation in strength or fighting ability, so that contestants are equally likely to win or lose a fight. Here we make some very different assumptions, namely, that the stronger of two animals always wins a fight if there is one, and that more evenly matched contests are more expensive (because it takes longer to discover who is really stronger). We also make some very different predictions, in particular, that escalated contests should occur less frequently when eavesdropping reveals information about relative strength.

The study of games on networks or populations of networks, in which interactions are fundamentally polyadic, is also in it infancy. This work relies heavily on computer simulation because analysis is difficult, the degree of difficulty increasing sharply with the size of the network. For that very reason, virtually the entire analytical literature on evolutionary game theory has assumed either dyadic interaction or a high degree of symmetry, or both, although there are some exceptions—e.g., a game-theoretic analysis of winner and loser effects by Mesterton-Gibbons (1999) that allows for asymmetric interaction within triads. Yet we cannot rely exclusively on computer simulation in the study of any behavioral phenomenon. Insights from analytical work, however idealized the underlying assumptions, are crucial in allowing us to explore the logic of verbal arguments rigorously. What this means in practice is that the study of triadic interactions has an important role to play because triads are both the simplest groups in which network phenomena can be studied and the groups beyond dyads in which analysis is most likely to be tractable, especially when allowing for intrinsic variation. In this paper, we present such an analysis of the phenomenon of social eavesdropping.

2. Hawk–Dove game with partial information

We begin by analyzing a Hawk–Dove game in which each contestant knows its own strength but not that of its opponent. Both strengths are drawn from the same distribution on $[0, 1]$. Stronger animals tend to escalate, weaker animals tend to defer. Let u be the aggression threshold for Player 1, the potential mutant (if its strength exceeds this value, then it escalates); let v be the corresponding threshold for Player 2, who represents the population. For each individual let fitness increase (beyond the basic level) by α for dominance (winning the contest) and by $b\alpha$ for non-subordination (neither opponent escalates in the contest), where $b < 1$, so that b is the value of shared resource access relative to the value of

exclusive access; in other words, b is an inverse measure of the advantage of dominance. It will be convenient to scale costs with respect to α. Accordingly, let $c(s_1, s_2)\,\alpha$ be the cost of a fight between a pair of animals whose strengths are s_1 and s_2. Let X be the strength of the u strategist, and let Y be that of the v strategist: thresholds are assumed heritable, whereas strengths are not. Then the associated (relative) payoff to the u strategist is

$$
F(u, v, X, Y) \;=\; \begin{cases}
0 & \text{if} \quad 0 < X < u, & v < Y < 1 \\
b\alpha & \text{if} \quad 0 < X < u, & 0 < Y < v \\
\alpha & \text{if} \quad u < X < 1, & 0 < Y < v \\
\{1 - c(X, Y)\}\alpha & \text{if} \quad u < X, & v < Y < X < 1 \\
-c(X, Y)\alpha & \text{if} \quad u < X < Y < 1, & Y > v
\end{cases} \tag{1}
$$

and the reward to a u strategist in a population of v strategists is

$$
f(u, v) \;=\; \mathrm{E}[F(u, v, X, Y)] \;=\; \int_0^1 \int_0^1 F(u, v, x, y) g(x) g(y) \, \mathrm{d}x \, \mathrm{d}y \tag{2}
$$

where E expected value and g the probability density function of the distribution from which X and Y are independently drawn. We calculate $f(u, v)$ in Appendix A.

A strategy v is an evolutionarily stable strategy or ESS in the sense of Maynard Smith (1982) if it is uniquely the best reply to itself, i.e., if $f(v, v) > f(u, v)$ for all $u \neq v$. In Appendix A, we show that the game defined by (2) has a unique ESS, which is the solution of the equation

$$
\int_v^1 c(v, y) g(y) \, \mathrm{d}y \;=\; (1 - b) G(v). \tag{3}
$$

Suppose, e.g., that strength is uniformly distributed between 0 and 1 and

$$
c(s_1, s_2) \;=\; c_0 \{1 - |s_1 - s_2|^k\} \tag{4}
$$

for $k > 0$, as illustrated in Fig. 1. Thus cost increases from a minimum of 0 when difference in strength is most extreme (either $s_1 = 1, s_2 = 0$ or $s_1 = 0, s_2 = 1$) to a maximum of c_0 when the difference is zero; k is a measure of the insensitivity of cost with respect to strength difference, in the sense that a small difference implies a large cost reduction when k is very low but virtually no cost reduction when k is very high. Given (4), then (3) reduces to

$$
1 - \frac{(1 - v)^k}{k + 1} \;=\; \frac{(1 - b)v}{c_0(1 - v)}. \tag{5}
$$

Let the solution of this equation, i.e., the ESS, be denoted by v^*. We obtain by inspection that $\lim_{k \to 0} v^* = 0$: if fighting is expensive only when contestants are
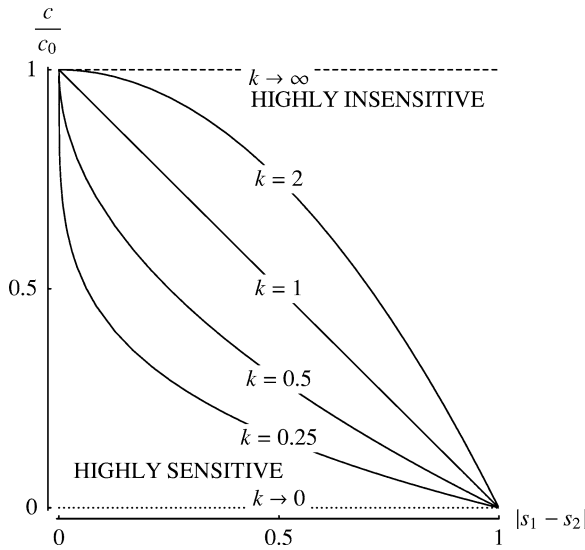
Fig. 1 Variation of cost c with strength difference $|s_1 - s_2|$ for various values of the insensitivity parameter k, according to (4). A small difference of strength implies a large cost reduction when k is very low but virtually no cost reduction when k is very high.

identical in strength, then they should always fight. Also by inspection, $\lim_{k \to \infty} v^* = c_0/(1 - b + c_0)$ as illustrated by the dashed curve in Fig. 2.

Although (5) can be solved analytically only for low integer values of k, e.g., $k = 1$ (see Appendix B), it is easily solved numerically. In the context of the classic Hawk–Dove game, it is usual to take $b = 1/2$. The corresponding ESS is shown in Figure 2 as a function of c_0 for various values of k. Unsurprisingly, the higher the cost, the higher the threshold for aggression; moreover, the greater the range of strength differences over which costs are significant, the higher the threshold for aggression.

A remark is in order before proceeding. Intuitively, individuals should have a lower aggression threshold (be more prepared to fight), the greater the value of a resource—relative to the cost of fighting. Because resource value (α) scales out of our analysis, a higher value to cost ratio corresponds to a lower value of c_0. Thus, intuition suggests that aggression thresholds should be lower when c_0 is lower—exactly what we see in Fig. 2.

Nevertheless, the uniform distribution is merely one of many possible distributions for fighting strength. We would like to know the effect of variance. In nature, distributions of fighting ability are typically fairly symmetric (see, e.g., McDonald, 1981, p. 135 et seq.), so an appropriate choice of distribution for theoretical purposes is one that is perfectly symmetric on [0, 1] with mean 1/2. Accordingly, we choose the symmetric Beta distribution defined by

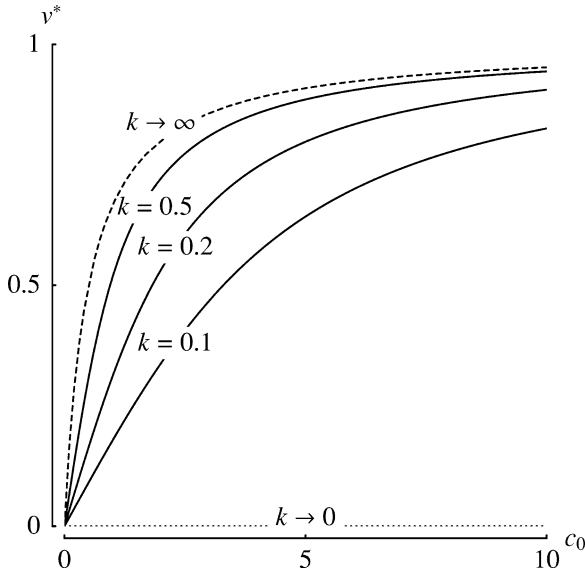$$g(\xi) = \frac{\Gamma(2a)}{\{\Gamma(a)\}^2} \xi^{a-1} (1 - \xi)^{a-1} \tag{6}$$

Fig. 2 The evolutionarily stable aggression threshold v^* as a function of maximum cost c_0 for $b = 1/2$ and various values of the insensitivity parameter k when strength is uniformly distributed between 0 and 1.

where Γ denotes the Euler gamma function, i.e., $\Gamma(\eta) = \int_0^\infty e^{-\xi}\xi^{\eta-1}\,d\xi$ (see, e.g., Kempthorne and Folks, 1971, p. 107). For $a = 1$ this distribution is uniform; for $a > 1$ it is unimodal, and its variance decreases with a according to

$$\sigma^2 = \frac{1}{4(1 + 2a)}. \tag{7}$$

We assume that $a \geq 1$, or $\sigma^2 \leq (1/12)$. The effect of variance on the ESS is illustrated by Fig. 3: for large k, the threshold increases with variance, for small k, it decreases. For sufficiently small k, the costs are appreciable only when contestants are equally matched, which is most likely when variance is least. Note that $v^* \to 1/2$ as $\sigma^2 \to 0$ for any k and that $v^* \to G^{-1}(c_0/\{1 - b + c_0\})$ as $k \to \infty$ for any σ^2, where $^{-1}$ denotes inverse; see Appendix C.

3. Eavesdropping model

For the sake of tractability, we consider only a population of animals above the basic ESS threshold for aggression, and we assume that this sub-population is subdivided into triads, the smallest groups in which investigation of the phenomenon of eavesdropping is possible. We assume that a sufficiently weak animal may defer to a prior winner it has observed, but that no individual defers to a loser or to an individual that it hasn't observed (consistent with our assumption that all animals are above the basic ESS threshold), and that sooner or later a fight breaks out
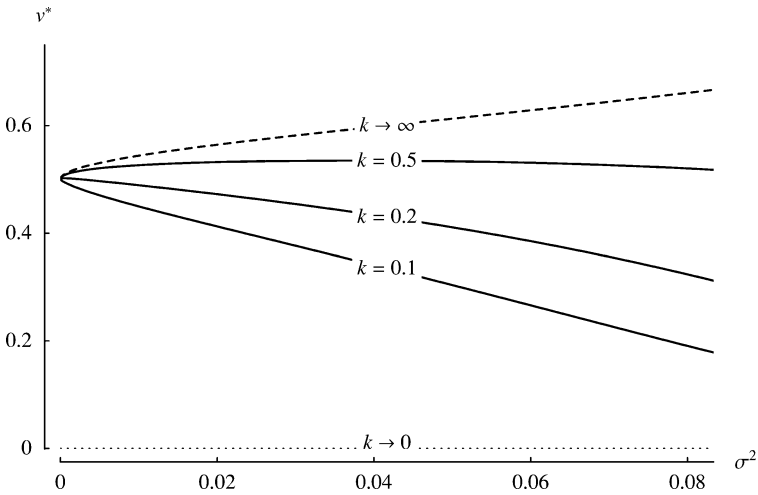
Fig. 3 The evolutionarily stable aggression threshold v^* as a function of variance σ^2 for $c_0 = 1$, $b = 1/2$ and various values of the insensitivity parameter k when strength has a symmetric Beta distribution on $[0, 1]$.

between two of the three animals; this fight is observed by the third animal, who subsequently picks a fight with the loser (its strength lies above the basic threshold, and the loser is not as strong as the winner). We assume that the direct benefit of beating a loser is the same as the direct benefit of beating a winner. Because the loser has not observed the first eavesdropper, there results a second actual fight (which is observed by the winner of the first). If the first loser wins its second fight, then the first winner picks a fight with the eavesdropper, but if the first loser loses again, then the first winner escalates only if it is sufficiently strong, and likewise for the eavesdropper. A strategy is now a two-dimensional vector, $u = (u_1, u_2)$ for the focal individual and $v = (v_1, v_2)$ for the population representative; the first component yields the aggression threshold against an observed winner for an animal that has won its first fight, and the second component is the aggression threshold against an observed winner for an animal that has lost its first fight. By assumption, all thresholds lie between v^* and 1, where v^* denotes the ESS for Section 2. Thus, an ESS under eavesdropping must have the form (V_1^*, V_2^*), where $v^* \leq V_1^* \leq 1$ and $v^* \leq V_2^* \leq 1$, and the question of interest is whether $V_1^* = v^* = V_2^*$ or whether at least one threshold exceeds the minimum. In the first case, eavesdropping does not reduce aggression; in the second case, it does. We are especially interested in whether $V_1^* > v^*$ because V_1^* is the less likely threshold to exceed its minimum.

Let the focal u strategist have strength X, and let the other two animals—both v strategists—have strengths Y and Z, all drawn from the same distribution on $[v^*, 1]$. If the density is h then

$$h(s) = \lambda g(s) \tag{8}$$

where $\lambda = 1/\int_{v^*}^1 g(s)\,\mathrm{d}s$ to ensure that $\int_{v^*}^1 h(s)\,\mathrm{d}s = 1$.

Table 1 Payoff to a focal individual F whose first opponent is A, conditional on participation in the first of the three contests; B denotes the eavesdropper

Case,	Winners			Event	Payoff,
i	1st	2nd	3rd	$\Omega_i(u, v)$	$P_i(X, Y, Z)$
1	A	F	(**A**)	$Y > X > Z, Z < v_2$	$\{1 - c(X, Y)\}\alpha$
2	A	F	(**A**)	$Y > X > Z, Z > v_2$	$\{1 - c(X, Y) - c(X, Z)\}\alpha$
3	A	B	(**A**)	$Y > X, Z > X, Y > v_1, Z < v_1$	$-\{c(X, Y) + c(X, Z)\}\alpha$
4	A	B	(**A**)	$Y > Z > X, Y > v_1, Z > v_1$	$-\{c(X, Y) + c(X, Z)\}\alpha$
5	A	B	(**B**)	$Y > X, Z > X, Y < v_1, Z > v_1$	$-\{c(X, Y) + c(X, Z)\}\alpha$
6	A	B	(B)	$Z > Y > X, Y > v_1, Z > v_1$	$-\{c(X, Y) + c(X, Z)\}\alpha$
7	A	B	(**A/B**)	$Y > X, Z > X, Y < v_1, Z < v_1$	$-\{c(X, Y) + c(X, Z)\}\alpha$
8	F	(A)	F	$X > Y > Z, Z < v_2$	$\{2 - c(X, Y)\}\alpha$
9	F	(A)	F	$X > Y > Z, Z > v_2$	$\{2 - c(X, Y) - c(X, Z)\}\alpha$
10	F	(B)	F	$X > Y, Z > Y, X > u_1, Z < v_1$	$\{2 - c(X, Y)\}\alpha$
11	F	(B)	F	$X > Z > Y, X > u_1, Z > v_1$	$\{2 - c(X, Y) - c(X, Z)\}\alpha$
12	F	(B)	**B**	$X > Y, Z > Y, X < u_1, Z > v_1$	$\{1 - c(X, Y)\}\alpha$
13	F	(B)	B	$Z > X > Y, X > u_1, Z > v_1$	$\{1 - c(X, Y) - c(X, Z)\}\alpha$
14	F	(B)	**F/B**	$X > Y, Z > Y, X < u_1, Z < v_1$	$\{1 + b - c(X, Y)\}\alpha$

Note. Parentheses indicate a contest in which the focal individual is not involved. A bold letter indicates that the individual's opponent deferred.

We consider each order of interaction in turn. Let us first consider the payoff to a u strategist when it participates in the first of the triad's three dyadic interactions; this event occurs with probability 2/3 because the role of eavesdropper is randomly assigned, with probability 1/3 for each animal. Let the focal individual with strength X be denoted by F, let its opponent have strength Y and be denoted by A, and let the eavesdropper have strength Z and be denoted by B. We decompose the (conditional) sample space into constituent events as shown in Table 1. Thus, conditional on participation in the opening contest ($=$ fight), the reward to a u strategist in a population of v strategists is

$$f_O(u, v) = \sum_{i=1}^{14} \iiint_{\substack{(x,y,z) \\ \in \Omega_i(u,v)}} P_i(x, y, z)h(x)h(y)h(z)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z. \tag{9}$$

Let us next consider the payoff to a u strategist when it eavesdrops on the first of the triad's three dyadic interactions; this event occurs with probability $\frac{1}{3}$. We decompose the (conditional) sample space into constituent events as shown in Table 2. Thus, conditional on non-participation in the opening contest, the reward to a u strategist in a population of v strategists is

$$f_E(u, v) = \sum_{i=15}^{28} \iiint_{(x,y,z) \in \Omega_i(u,v)} P_i(x, y, z)h(x)h(y)h(z)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$$

$$= 2\sum_{i=15}^{21} \iiint_{\substack{(x,y,z) \\ \in \Omega_i(u,v)}} P_i(x, y, z)h(x)h(y)h(z)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \tag{10}$$

Table 2 Payoff to a focal individual F, conditional on being the eavesdropper during the first of the three contests

Case,	Winners			Event	Payoff,
i	1st	2nd	3rd	$\Omega_i(u, v)$	$P_i(X, Y, Z)$
15	(A)	B	**A**	$Y > Z > X, X < u_2$	$-c(X, Z)\alpha$
16	(A)	B	A	$Y > Z > X, X > u_2$	$-\{c(X, Z) + c(X, Y)\}\alpha$
17	(A)	F	**A**	$Y > Z, X > Z, X < u_1, Y > v_1$	$\{1 - c(X, Z)\}\alpha$
18	(A)	F	A	$Y > X > Z, X > u_1, Y > v_1$	$\{1 - c(X, Z) - c(X, Y)\}\alpha$
19	(A)	F	**F**	$Y > Z, X > Z, X > u_1, Y < v_1$	$\{2 - c(X, Z)\}\alpha$
20	(A)	F	F	$X > Y > Z, X > u_1, Y > v_1$	$\{2 - c(X, Z) - c(X, Y)\}\alpha$
21	(A)	F	**A/F**	$Y > Z, X > Z, X < u_1, Y < v_1$	$\{1 + b - c(X, Z)\}\alpha$
22	(B)	A	**B**	$Z > Y > X, X < u_2$	$-c(X, Y)\alpha$
23	(B)	A	B	$Z > Y > X, X > u_2$	$-\{c(X, Y) + c(X, Z)\}\alpha$
24	(B)	F	**B**	$Z > Y, X > Y, X < u_1, Z > v_1$	$\{1 - c(X, Y)\}\alpha$
25	(B)	F	B	$Z > X > Y, X > u_1, Z > v_1$	$\{1 - c(X, Y) - c(X, Z)\}\alpha$
26	(B)	F	**F**	$Z > Y, X > Y, X > u_1, Z < v_1$	$\{2 - c(X, Y)\}\alpha$
27	(B)	F	F	$X > Z > Y, X > u_1, Z > v_1$	$\{2 - c(X, Y) - c(X, Z)\}\alpha$
28	(B)	F	**A/F**	$Z > Y, X > Y, X < u_1, Z < v_1$	$\{1 + b - c(X, Y)\}\alpha$

Note. Parentheses indicate a contest in which the focal individual is not involved. A bold letter indicates that the individual's opponent deferred.

by symmetry. It now follows that the (unconditional) reward to a u strategist in a population of v strategists is

$$f(u, v) = \frac{2}{3} f_O(u, v) + \frac{1}{3} f_E(u, v) = f_1(u_1, v_1) + f_2(u_2) + f_3(v) \qquad (11)$$

where f_1 is defined in Appendix D by (D.1), f_2 is defined in Appendix D by (D.2) and $f_3(v)$ has no strategic effect because it is independent of u. Thus the reward function is separable (Mesterton-Gibbons, 2001, p. 229).

Because, from (11) and (D.2),

$$\frac{\partial f}{\partial u_2} = \frac{\partial f_2}{\partial u_2} = \alpha h(u_2) \int_{u_2}^{1} h(y) \left\{ c(u_2, y) \int_{u_2}^{y} h(z) \, dz \right\} dy \qquad (12)$$

is always positive, $u_2 = 1$ is always the best response after losing: an eavesdropper should never escalate after losing to the loser of the first contest because it cannot possibly win. Thus the ESS must have the form $v = (V^*, 1)$, where

$$v^* \leq V^* \leq 1. \qquad (13)$$

If $V^* = v^*$ then eavesdropping does not reduce aggression among prior winners; however, if $v^* < V^* \leq 1$, then eavesdropping raises their threshold. But

$$\frac{\partial f}{\partial u_1} \bigg|_{u_1 = v_1 = v^*} = 0, \quad \frac{\partial^2 f}{\partial u_1^2} \bigg|_{u_1 = v_1 = v^*} = \frac{4}{3} \alpha h(v^*)^2 \int_{v^*}^{1} c(v^*, y) h(y) \, dy > 0 \quad (14)$$
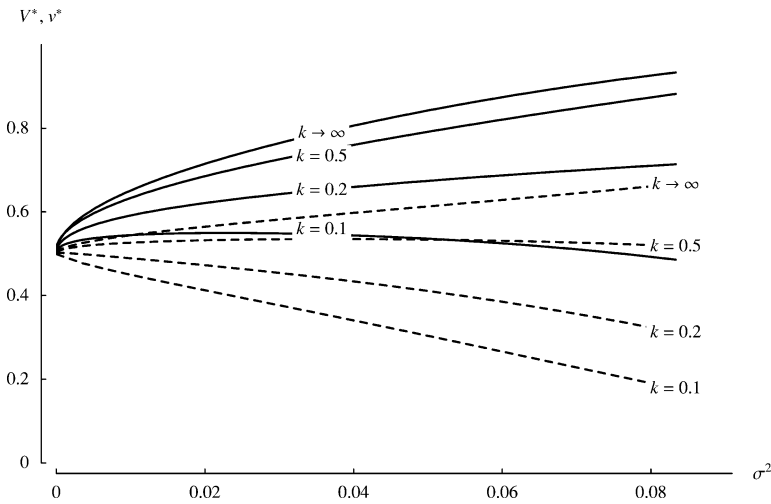
Fig. 4 The evolutionarily stable aggression threshold under eavesdropping (V^*, *solid curve*) as a function of variance σ^2 for $c_0 = 1$, $b = 1/2$ and various values of the insensitivity parameter k when strength has a symmetric Beta distribution on $[0, 1]$. In each case, the corresponding basic threshold (v^*, *dashed curve*) is also shown.

from Appendix D. We conclude that $V^* > v^*$, and hence that eavesdropping reduces aggression among prior winners as well as among prior losers. At the ESS

$$\left. \frac{\partial f}{\partial u_1} \right|_{u_1 = v_1 = V^*} = 0. \tag{15}$$

Assuming that $g(V^*) \neq 0$ (which holds in particular for the Beta distribution defined by (6)), it now follows from (8) and (D.8) that V^* is the solution of

$$\int_{V^*}^1 g(y) c(V^*, y) \, dy \int_{v^*}^{V^*} g(z) \, dz = (1-b) \int_{v^*}^{V^*} g(y) \int_{v^*}^y g(z) \, dz \, dy \tag{16}$$

(where g is the unconditional probability density function—in (8), λ scales out). For cost function (4), the ESS threshold is plotted against variance as a solid curve in Fig. 4 for $c_0 = 1$, $b = 1/2$ and various values of k, with the corresponding basic ESS from Section 2 shown as a dashed line. Note that $V^* \to (1/2)$ as $\sigma^2 \to 0$ for any k and that $v^* \to G^{-1}(c_0\{3(1-b)+2c_0\}/\{(1-b+c_0)(1-b+2c_0)\})$ as $k \to \infty$ for any σ^2; see Appendix C. Because observing an individual win is valuable only when there is information about its strength, if variance in strength is zero, then it does not serve to increase aggression thresholds.

One thing to bear in mind, however, is that whereas $b = 1/2$ is a fairly natural assumption when the population is not subdivided (as in Section 2), once groups have formed, it is possible that b may take a very different value. Accordingly, in Figure 5 the ESS threshold is plotted against variance as a solid curve for $c_0 = 1$, $k = 0.2$ and various values of b.
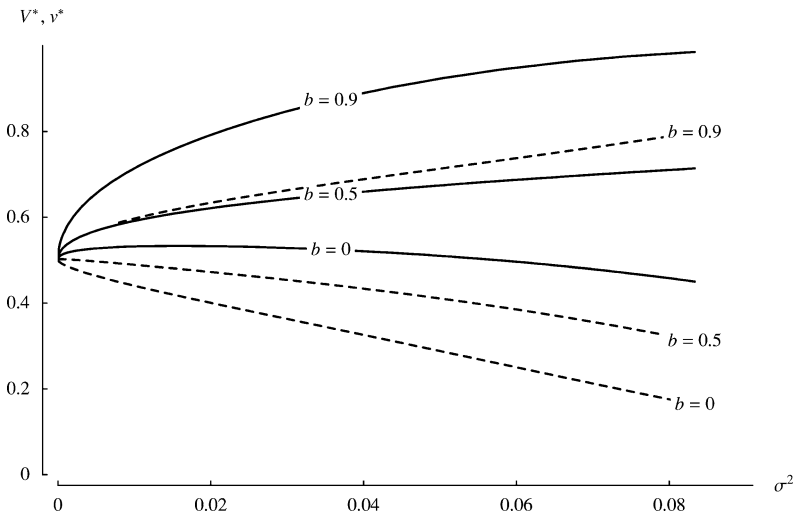
Fig. 5 The evolutionarily stable aggression threshold under eavesdropping (V^*, *solid curve*) as a function of variance σ^2 for $c_0 = 1$, $k = 0.2$ and various values of the (inverse) dominance advantage parameter b when strength has a symmetric Beta distribution on $[0, 1]$. In each case, the corresponding basic threshold (v^*, *dashed curve*) is also shown.

4. Discussion

Several recent analyses of the Hawk–Dove game with continuous variation in fighting ability predict that an individual should play Hawk (H, behave aggressively and prepare to escalate) if its fighting ability is above a certain critical threshold (Crowley, 2000; McNamara and Houston, 2005) and otherwise play Dove (D, display and prepare to retreat). In these studies, only four discrete payoff combinations were considered (HH, HD, DH, DD). Here, by contrast, we have allowed similarity of fighting strength to affect the cost of any mutually aggressive behaviour, rendering payoffs continuous rather than discrete. Our particular cost function can be justified on the basis of some form of sequential assessment of fighting skill (e.g., see Enquist and Leimar, 1983). In effect, one contestant should back down when it becomes evident that the other has greater fighting ability, which will take longer to realize when the individuals are more closely matched. Several empirical studies have lent support to the idea that differences between contestants are important in determining contest duration (e.g., see Moya-Laraño and Wise, 2000), although it is now recognized that alternative hypotheses, including one-sided assessment of fighting strengths, have not been ruled out (Taylor and Elwood, 2003). Indeed, contest duration may be affected by other factors such as cost thresholds (Morrell et al., 2005), and in some cases there may be no simple correlates (Kemp et al., 2006). Overall, however, we consider our cost function a pragmatic way of generating a continuous Hawk–Dove model, which is more biologically meaningful than simply interpolating between values in the discrete game (e.g., see Ahmed and Elgazzar, 2000) and more realistic than assuming fixed contest costs.

Our analysis of the basic continuous game without eavesdropping suggests that if fighting strength is variable among individuals, then the evolutionarily stable strength threshold beyond which an individual is prepared to fight should increase with the costs of fighting. In effect, individuals are less likely to adopt aggressive behaviour as the cost of fighting increases. Likewise we have found (Fig. 5, dashed curves) that the greater the value of the resource to individuals that mutually defer (the higher b), the higher the thresholds for fighting. These two results reflect an analogous property in the classic discrete game (although here the effect is mediated by different parameters): when the value V of the resource to the winner is less than the cost C to the loser ($V < C$) then a proportion V/C of the population should play Hawk (Maynard Smith, 1982).

For a more detailed comparison of these two models, we first note that only losers pay costs in the classic game. To modify it so that winners also pay costs, we must replace $(V - C)/2$ by $V/2 - C$ in the top-left hand corner of the matrix on page 12 of Maynard Smith (1982), the effect of which is to reduce the proportion of Hawks at the ESS when costs are high from V/C to $V/(2C)$. The classic game assumes that costs are constant, i.e., independent of strength, which is the infinitely insensitive limit of the model developed in Section 2. In this limit Maynard Smith's proportion $V/(2C)$ corresponds to $1/(2c_0)$ in our model, and on setting $b = 1/2$ and taking the limit as $k \to \infty$ in Section 2, we find that the proportion of Hawks is $1/(1 + 2c_0)$, which is always lower, as also predicted by McNamara and Houston (2005). Note that this result holds (in the infinitely insensitive limit) for arbitrary variance: intuitively, aggression is lower because information is extracted from the variation in fighting ability, which the classic Hawk–Dove game ignores. Effectively, in the discrete Hawk–Dove game, a weaker individual always has an even chance of winning a fight, whereas in our continuous game a weaker individual is bound to lose; therefore, an animal should not escalate unless its strength is high enough for it to have a sufficient chance of being the stronger, and the overall effect at the population level is to raise the aggression threshold at the ESS. We also note that unconditional Hawk is the classic ESS when costs are low. In our model, by contrast, the equivalent of pure Hawk (zero aggression threshold) is the ESS only if costs are zero ($k = 0$, see Appendix C): for low cost, the threshold is merely low. Again, information is extracted from the variation in fighting ability, which the classic Hawk–Dove game ignores.

An important parameter in our analysis is k, which measures the insensitivity of the relationship between cost and strength difference at low asymmetry in fighting strengths. We have shown that at high sensitivity in this relationship (low k), aggression thresholds decrease with variance in strengths among individuals; but at low sensitivity (high k), thresholds increase with variance. With low k, the costs of fighting are only appreciable in size if the difference in fighting ability between contesting individuals is low, so that individuals adopt higher thresholds when the variance is low. By contrast, at high k, maximal costs would apply to fighters under a far wider range of differences in strengths. As the variance in fighting strength increases, costly mutually aggressive interactions are reduced through the adoption of more conservative fighting thresholds. Little empirical work has been conducted to evaluate the role of among-individual variation in fighting strengths in mediating the frequency of fighting behaviour, but our analysis suggests that the relationship is not, in general, a simple one.

Our key result centres on what happens to the evolutionary stable thresholds when one allows for eavesdropping. The possibility that bystanders are capable of altering their behaviour in a strategic way when interacting with observed winners or losers has been increasingly recognized (see McGregor and Peake, 2000; Peake, 2005). One might expect that there would be a greater reluctance to fight with a previously observed victor, especially if there were significant costs of fighting, and this is precisely what our model predicts. At an extreme, we have shown (rather than assumed) that an individual should never escalate against an individual who was victorious over the individual that in turn defeated it. Thus, in a population at the ESS, animals behave as though they are using transitive inference, even though we have not assumed that they have this capability—we have merely assumed a propensity to attack losers in prefence to winners and an ability to react differently in the roles of winner and loser. It has recently been demonstrated under controlled conditions that pinyon jays use transitive reasoning (Paz-y Miño C et al., 2004), and it has been shown theoretically that transitive-inference strategies are superior to several alternatives (Nakamaru and Sasaki, 2003); however, these authors did not propose a mechanism through which a transitive-inference strategy could emerge ab initio. Our analysis suggests that social eavesdropping is a possible candidate.

More generally, we have shown that the evolutionary stable aggression threshold against an animal that has been observed to win its first fight (V^* in Section 3) is always higher at positive variance than the aggression threshold against an individual that has lost its first fight (v^* by assumption). This prediction already has a degree of empirical support. For instance, Oliveira et al. (1998) showed that male Siamese fighting fish (*Betta splendens*) monitor the outcomes of contests between neighbours and found that males are more willing to fight individuals that they have seen losing compared to individuals they have seen winning, relative to an intermediate response to previously unseen individuals. Similarly, Earley and Dugatkin (2002) investigated the dynamics of aggressive interactions in the green swordtail fish (*Xiphophorus helleri*) and found that eavesdropping reduced the bystander's propensity to initiate aggression, escalate, and win against an earlier observed winner.

In an insightful paper, Johnstone (2001) incorporated an eavesdropping strategy into the classic Hawk–Dove model. His eavesdroppers by definition played Dove against an observed winner in the previous round but Hawk against an observed loser. Somewhat surprisingly, Johnstone predicted escalated conflicts to occur more frequently in his eavesdropping model than in the classic Hawk–Dove game because the benefits of winning in any given round could cascade through rounds, with eavesdroppers always deferring to observed winners. As Johnstone (2001, p. 9180) noted, his formulation could usefully be extended to allow for individual variation in fighting ability. Here we have done just that, yet we have arrived at a rather different conclusion: when strength varies, eavesdropping reduces aggression. In particular, in the limit of infinite insensitivity corresponding to the classic discrete Hawk–Dove game, eavesdropping further reduces the proportion of Hawks from $1/(1 + 2c_0)$ to $1/(1 + 2c_0)(1 + 4c_0)$; see Appendix C. In our continuous game, avoiding the costs of challenging an observed winner overrides the potential benefit of winning for animals that are sufficiently weak, so

that eavesdropping decreases the frequency of aggressive encounters. Only at zero variance in strength among individuals will eavesdropping have no effect on aggression thresholds. Clearly, observing an individual win is valuable only when it provides information about its strength, which happens only when individuals differ and cues about this difference are available to the bystander.

Note that our model differs in several important respects from that of Johnstone (2001). For example, as a consequence of continuous variation in strength, we have been able to allow the winner of a contest to be the stronger of two contestants, rather than being decided at random. Moreover, in Johnstone's model, the response to eavesdropping at the ESS is a fixed behaviour exhibited by a certain proportion of the population, whereas in our model, it is a conditional strategy that all individuals adopt. McElreath (2003) similarly explored the effects of reputation-based strategies and concluded that reputation would tend to generate less fighting at equilibrium in his model than if the reputation effect were absent. All of these mathematical models have one thing in common, however; they serve to demonstrate that the wider social environment in which interactions take place can strongly influence the nature of the contest behaviours that are exhibited.

All models are idealizations, and ours are no exception. In particular, our payoff structure is not the most general in several respects. We assume throughout that fights are more costly for more evenly matched opponents and that costs are the same for both winner and loser, but there are many other plausible assumptions about cost structure that we could have made instead. We justify our special assumptions by our need for simplicity: the most important thing is to use the same cost structure in Section 3 as in Section 2. Similarly, we have assumed throughout that the stronger of two contestants always wins a fight. But judicious approximation is the essence of modelling: with an analytical model, it is almost always necessary—not only for tractability, but also for clarity of insight—to exclude effects that are small in a real population. Thus, at the very least, we expect our model to capture the essence of social eavesdropping in populations where the stronger animal has a high probability of winning. In Appendix A, we make winning probabilistic for the basic Hawk–Dove game by introducing a parameter r that measures the reliability of strength difference as a predictor of fight outcome. The analysis there shows that decreasing r reduces aggression thresholds, to the extent that a threshold of 0 (unconditional aggression) must be an ESS for sufficiently small r for sufficiently small costs. Unfortunately, it is only in the limit as $r \to \infty$ that we are able to extend our basic Hawk–Dove game to a tractable analytical model for social eavesdropping. Nevertheless, it is reasonable to conjecture that reducing r would have the effect of weakening the extent to which eavesdropping raises aggression thresholds.

Finally, for analytical tractability, our social network structure in the case of eavesdropping (Section 3) comprises only collections of three individuals. Although many social networks will be limited in size and three individuals suffice to capture the effect of eavesdropping, it is possible that larger networks could produce qualitatively different results. Ongoing simulation work is seeking to extend our insights to larger networks.
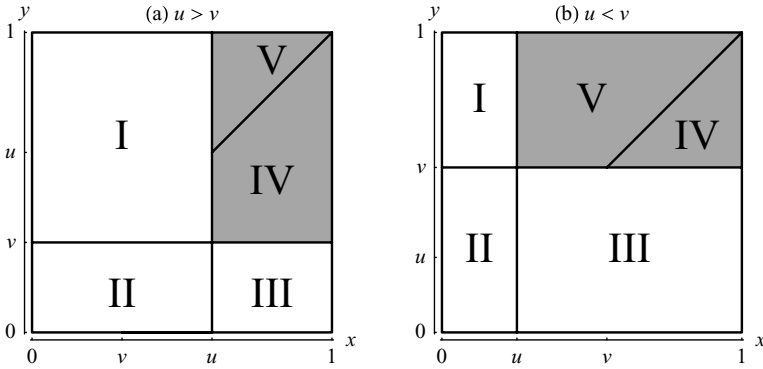
Fig. A.1 The sample space of pairs of strengths.

Appendix A: Calculation of the Hawk–Dove reward function and ESS

We assume that $g(s) \neq 0$ for all $s \in (0, 1)$, which holds in particular for (6). Then the distribution function G, defined by $G(\eta) = \int_0^\eta g(\xi) \, d\xi$, is strictly increasing. From (1) to (2), to calculate the reward, we must add together the integrals of $b\alpha g(x)g(y)$, $\alpha g(x)g(y)$, $\alpha\{1 - c(x, y)\}g(x)g(y)$, and $-\alpha c(x, y)g(x)g(y)$ over regions II, III, IV and V, respectively, of Fig. A.1. The integrals over regions II and III always yield $b\alpha G(u)G(v)$ and $\alpha G(v)\{1 - G(u)\}$, respectively. The shapes of regions IV and V depend on which of u or v is larger, as indicated in Fig. A.1. In either case, however, $-\alpha c(x, y)g(x)g(y)$ must be integrated over the whole of the shaded rectangle, yielding

$$f_{\mathrm{SR}}(u, v) \;=\; -\alpha \int_u^1 g(x) \int_v^1 c(x, y)g(y) \, dy \, dx. \tag{A.1}$$

For $u > v$, we must add the integral of $\alpha g(x)g(y)$ over trapezoidal region IV, which yields $\alpha\{\int_u^1 g(x)G(x) \, dx - G(v)\{1 - G(u)\}\}$, so that the last part of the expression cancels out the integral over region III. For $u < v$, we must add the integral of $\alpha g(x)g(y)$ over triangular region IV, which yields the expression for $u > v$ with u replaced by v. Hence, after simplification, we obtain

$$f(u, v) \;=\; \alpha \left\{ bG(u)G(v) + \int_u^1 g(x)G(x) \, dx \right\} + f_{\mathrm{SR}}(u, v) \tag{A.2a}$$

for $u \geq v$ and

$$f(u, v) \;=\; \alpha \left\{ G(v)\{(b - 1)G(u) + G(v)\} + \int_v^1 g(x)G(x) \, dx \right\} + f_{\mathrm{SR}}(u, v) \tag{A.2b}$$

for $u \leq v$, with f_{SR} defined by (A.1). It is readily verified that the function f defined by (A.2) is continuously differentiable where $u = v$ (in other words, that either (A.2a) or (A.2b) yields the correct expression for $\partial f / \partial u|_{u=v} = 0$); and because G is an increasing function, the two expressions for $\partial f / \partial u$ that (A.2) implies can be conveniently combined as

$$\frac{\partial f}{\partial u} = \alpha g(u) \left\{ bG(v) - \max\{G(u), G(v)\} + \int_v^1 c(u, y)g(y)\,\mathrm{d}y \right\}. \qquad (A.3)$$

Because (A.3) is both positive in the neighborhood of $u = 0$ for $v = 0$ and negative in the neighborhood of $u = 1$ for $v = 1$ (with $b < 1$), neither 0 nor 1 is ever an ESS (for positive c); however, there must be an interior ESS at $u = v$ where $\partial f / \partial u|_{u=v} = 0$, i.e., where v is defined by (3).

It is possible to relax the assumption that fights are always won by the stronger contestant in the Hawk–Dove game of Section 2. For example, we could instead assume that the probability that a focal individual wins a fight is

$$p(\Delta s) = \frac{\Gamma(2r)}{\Gamma(r)^2} B\left(\frac{1}{2} + \frac{1}{2}\Delta s, r, r\right) \qquad (A.4)$$

where Δs is the strength difference between it and its opponent; here $\Delta s \in [-1, 1]$ and B denotes the incomplete beta function, i.e., $B(\eta, p_1, p_2) = \int_0^\eta \xi^{p_1-1}(1-\xi)^{p_2-1}\,\mathrm{d}\xi$. The function p defined by (A.4) is sigmoidal (Fig. A.2a); $p \to 1/2$ as $r \to 0$, and p approaches a step function as $r \to \infty$. Thus, the parameter r is a measure of the reliability of strength difference to predict fight outcome, and our analysis corresponds to the limit as $r \to \infty$. With the above assumption, the fourth and fifth lines of (1) would combine as $F(u, v, X, Y) = \{p(X - Y) - c(X, Y)\}\alpha$ if

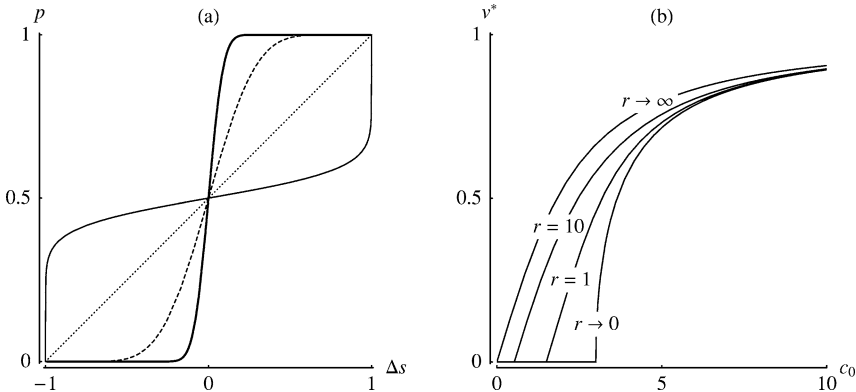**Fig. A.2** (a) p defined by (A.4) for $r = 0.1$ (*thin solid curve*), $r = 1$ (*dotted*), $r = 10$ (*dashed*) and $r = 100$ (*thick solid curve*). (b) The evolutionarily stable aggression threshold v^* as a function of maximum cost c_0 for $b = 1/2$, $k = 0.2$ and various values of the reliability parameter r when strength is uniformly distributed between 0 and 1. The *top-most curve* is identical to the second curve from the bottom in Fig. 2.

$u < X < 1$, $v < Y < 1$ and integrating over regions II, III, and the shaded triangle in Fig. A.1 would yield

$$f(u, v) = \alpha \left\{ \{1 - (1 - b)G(u)\}G(v) + \int_u^1 g(x) \int_v^1 \{p(x - y) \right.$$

$$\left. -c(x, y)\}g(y)\, dy\, dx \right\} \tag{A.5}$$

and

$$\frac{\partial f}{\partial u} = \alpha g(u) \left\{ (b - 1)G(v) - \int_v^1 \{p(u - y) - c(u, y)\}g(y)\, dy \right\} \tag{A.6}$$

in place of (A.2) respectively. This expression is always negative in the neighborhood of $u = 1$ for $v = 1$ (because $b < 1$), so that 1 is never an ESS; however, (A.3) is positive in the neighborhood of $u = 0$ for $v = 0$, making 0 an ESS, if

$$\int_0^1 p(-y)g(y)\, dy > \int_0^1 c(y)g(y)\, dy. \tag{A.7}$$

Otherwise, there is an interior ESS at $u = v$ where $\partial f / \partial u|_{u=v} = 0$ or

$$(b - 1)G(v) + \int_v^1 c(v - y)g(y)\, dy = \int_v^1 p(v - y)g(y)\, dy \tag{A.8}$$

which reduces to (3) in the limit as $r \to \infty$. The lower the value of r, the lower the threshold given by (A.8); moreover, it is clear that (A.7) must eventually be satisfied for sufficiently low r for sufficiently low c_0, as illustrated by Fig. A.2b.

   Unfortunately, extending our Hawk–Dove game in Section 2 as above would not lead to a tractable analytical model for social eavesdropping in Section 3, and so we have analyzed this phenomenon only in the limit as $r \to \infty$.

Appendix B: Uniform distribution with linear costs

When $a = 1$ in (6) and $k = 1$ in (4), the ESS can be found analytically. In this case, (5) reduces to $c_0(1 - v^2) = 2(1 - b)v$ with solution

$$v = v^* = \frac{c_0}{1 - b + \sqrt{(1 - b)^2 + c_0^2}} \tag{B.1}$$

and (16) reduces to $c_0(1 - V^{*2}) = (1 - b)(V^* - v^*)$ with solution

$$V^* = \frac{2\{c_0 + (1 - b)v^*\}}{1 - b + \sqrt{(1 - b)^2 + 4(1 - b)c_0 v^* + 4c_0^2}}. \tag{B.2}$$

Appendix C: On the limit of the ESS as $k \to \infty$, $k \to 0$ or $\sigma^2 \to 0$

Substituting from (4) into (3) and taking the limit as $k \to \infty$ yields

$$(1 - b)G(v) = \int_v^1 c_0 g(y)\,dy = c_0 \int_v^1 g(y)\,dy = c_0\{1 - G(v)\} \tag{C.1}$$

or

$$G(v) = \frac{c_0}{1 - b + c_0} \tag{C.2}$$

for arbitrary G. So the ESS becomes $v^* = G^{-1}(c_0/\{1 - b + c_0\})$, as illustrated by the dashed curves in Figs. 2–3. In this limit of infinite insensitivity, the probability of aggression at the ESS is $\text{Prob}(Y > v) = 1 - G(v) = \{1 - b\}/\{1 - b + c_0\}$ $(= 1/\{1 + 2c_0\}$ when $b = 1/2$, as in the classic Hawk–Dove game).

From (A.3) and (4) in the limit as $k \to 0$ we obtain $\partial f/\partial u\big|_{u=v} = \alpha(b - 1)g(v)G(v)$, which is negative for $0 < v \le 1$; hence 0 is the only possible ESS, and we confirm directly from (2) that $f(0, 0) - f(u, 0) = \int_0^u g(x)G(x)\,dx$ is positive for all $u \ne 0$, so that 0 is a best reply to itself.

As $\sigma^2 \to 0$, $s_1 \to s_2$ in (4). Hence substituting from (4) into (3) and taking the limit as $\sigma^2 \to 0$ again yields (C.1)–(C.2); but now, in addition, we have $G(v) \to G_{\min}(v)$ where $G_{\min}$ is the multifunction defined by $G_{\min}(s) = 0$ if $0 \le s < 1/2$, $G_{\min}(s) \in [0, 1]$ if $s = 1/2$ and $G_{\min}(s) = 1$ if $1/2 < s \le 1$. Thus, $v^* \to G_{\min}^{-1}(c_0/\{1 - b + c_0\}) = 1/2$ as $\sigma^2 \to 0$.

Similarly, substituting from (4) into (16), taking the limit as $k \to \infty$ and simplifying greatly, we obtain $c_0\{1 - G(V^*)\} = (1/2)b\{G(V^*) - G(v^*)\}$ or

$$G(V^*) = \frac{c_0\{3(1 - b) + 2c_0\}}{(1 - b + c_0)(1 - b + 2c_0)} \tag{C.3}$$

for arbitrary G. So the ESS becomes $v^* = G^{-1}(c_0\{3(1 - b) + 2c_0\}/\{(1 - b + c_0)(1 - b + 2c_0)\})$, as illustrated by the uppermost solid curve in Fig. 4. In this limit of infinite insensitivity, the probability of aggression at the ESS is $\text{Prob}(Y > V^*) = 1 - G(V^*) = (1 - b)^2/\{(1 - b + c_0)(1 - b + 2c_0)\}$ $(= 1/\{(1 + 2c_0)(1 + 4c_0)\}$ when $b = 1/2$, as in the classic Hawk–Dove game—although there is no longer any special reason why b should equal $1/2$).

As $\sigma^2 \to 0$, $s_1 \to s_2$ in (4). Hence substituting from (4) into (16) and taking the limit as $\sigma^2 \to 0$ again yields (C.3); but now, in addition, we have $G(V^*) \to G_{\min}(V^*)$ where $G_{\min}$ is defined above. Hence $V^* \to G_{\min}^{-1}(c_0\{3(1 - b) + 2c_0\}/\{(1 - b + c_0)(1 - b + 2c_0)\}) = 1/2$ as $\sigma^2 \to 0$.

Appendix D: Calculation of the eavesdropping reward function and ESS

$$
f_1(u_1, v_1) = \tfrac{2}{3} \sum_{i=10}^{14} \iiint_{\substack{(x,y,z) \\ \in \Omega_i(u,v)}} P_i(x, y, z)h(x)h(y)h(z)\, dx\, dy\, dz
$$

$$
+ \tfrac{2}{3} \sum_{i=17}^{21} \iiint_{\substack{(x,y,z) \\ \in \Omega_i(u,v)}} P_i(x, y, z)h(x)h(y)h(z)\, dx\, dy\, dz
$$

$$
= \tfrac{2}{3}\alpha \Bigg\{ \int_{u_1}^1 h(x) \int_{v^*}^{v_1} h(z) \int_{v^*}^{\min(x,z)} \{2 - c(x, y)\}h(y)\, dy\, dz\, dx
$$

$$
+ \int_{\max(u_1,v_1)}^1 h(x) \int_{v_1}^x h(z) \int_{v^*}^z \{2 - c(x, y) - c(x, z)\}h(y)\, dy\, dz\, dx
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{v_1}^1 h(z) \int_{v^*}^{\min(x,z)} \{1 - c(x, y)\}h(y)\, dy\, dz\, dx \qquad \text{(D.1)}
$$

$$
+ \int_{\max(u_1,v_1)}^1 h(z) \int_{u_1}^z h(x) \int_{v^*}^x \{1 - c(x, y) - c(x, z)\}h(y)\, dy\, dx\, dz
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{v^*}^{v_1} h(z) \int_{v^*}^{\min(x,z)} \{1 + b - c(x, y)\}h(y)\, dy\, dz\, dx
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{v_1}^1 h(y) \int_{v^*}^{\min(x,y)} \{1 - c(x, z)\}h(z)\, dz\, dy\, dx
$$

$$
+ \int_{\max(u_1,v_1)}^1 h(y) \int_{u_1}^y h(x) \int_{v^*}^x \{1 - c(x, z) - c(x, y)\}h(z)\, dz\, dx\, dy
$$

$$
+ \int_{u_1}^1 h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{\min(x,y)} \{2 - c(x, z)\}h(z)\, dz\, dy\, dx
$$

$$
+ \int_{\max(u_1,v_1)}^1 h(x) \int_{v_1}^x h(y) \int_{v^*}^y \{2 - c(x, z) - c(x, y)\}h(z)\, dz\, dy\, dx
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{\min(x,y)} \{1 + b - c(x, z)\}h(z)\, dz\, dy\, dx \Bigg\}
$$

and

$$
f_2(u_2) = -\tfrac{2}{3}\alpha \int_{v^*}^1 \int_x^1 \int_x^y c(x, z)h(x)h(y)h(z)\, dz\, dy\, dx
$$

$$
- \tfrac{2}{3}\alpha \int_{u_2}^1 \int_x^1 \int_x^y c(x, y)h(x)h(y)h(z)\, dz\, dy\, dx \qquad \text{(D.2)}
$$

from Tables 1 and 2. The first five terms of (D.1) are merely a rearrangement of the last five, and doubling the sum of the last five terms yields

$$
\begin{aligned}
f_1(u_1, v_1) \;=\; \tfrac{4}{3}\alpha\Bigg\{ & \int_{v^*}^{u_1} h(x) \int_{v_1}^{1} h(y) \int_{v^*}^{\min(x,y)} \{1 - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{\max(u_1, v_1)}^{1} h(y) \int_{u_1}^{y} h(x) \int_{v^*}^{x} \{1 - c(x, z) - c(x, y)\}h(z)\,dz\,dx\,dy \\
& + \int_{u_1}^{1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{\min(x,y)} \{2 - c(x, z)\}h(z)\,dz\,dy\,dx \qquad \text{(D.3)} \\
& + \int_{\max(u_1, v_1)}^{1} h(x) \int_{v_1}^{x} h(y) \int_{v^*}^{y} \{2 - c(x, z) - c(x, y)\}h(z)\,dz\,dy\,dx \\
& + \int_{v^*}^{u_1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{\min(x,y)} \{1 + b - c(x, z)\}h(z)\,dz\,dy\,dx \Bigg\}.
\end{aligned}
$$

So for $u_1 > v_1 \geq v^*$ we have

$$
\begin{aligned}
f_1(u_1, v_1) \;=\; \tfrac{4}{3}\alpha\Bigg\{ & \int_{v^*}^{v_1} h(x) \int_{v_1}^{1} h(y) \int_{v^*}^{x} \{1 - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{v_1}^{u_1} h(x) \int_{v_1}^{x} h(y) \int_{v^*}^{y} \{1 - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{v_1}^{u_1} h(x) \int_{x}^{1} h(y) \int_{v^*}^{x} \{1 - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{u_1}^{1} h(y) \int_{u_1}^{y} h(x) \int_{v^*}^{x} \{1 - c(x, z) - c(x, y)\}h(z)\,dz\,dx\,dy \\
& + \int_{u_1}^{1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{y} \{2 - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{u_1}^{1} h(x) \int_{v_1}^{x} h(y) \int_{v^*}^{y} \{2 - c(x, z) - c(x, y)\}h(z)\,dz\,dy\,dx \\
& + \int_{v^*}^{v_1} h(x) \int_{v^*}^{x} h(y) \int_{v^*}^{y} \{1 + b - c(x, z)\}h(z)\,dz\,dy\,dx \qquad \text{(D.4)} \\
& + \int_{v^*}^{v_1} h(x) \int_{x}^{v_1} h(y) \int_{v^*}^{x} \{1 + b - c(x, z)\}h(z)\,dz\,dy\,dx \\
& + \int_{v_1}^{u_1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{y} \{1 + b - c(x, z)\}h(z)\,dz\,dy\,dx \Bigg\}
\end{aligned}
$$

implying that

$$
\frac{\partial f}{\partial u_1} = \frac{\partial f_1}{\partial u_1} = \tfrac{4}{3}\alpha h(u_1) \Bigg\{ \int_{v_1}^{u_1} h(y) \int_{v^*}^{y} \{1 - c(u_1, z)\} h(z) \, dz \, dy
$$

$$
+ \int_{u_1}^{1} h(y) \int_{v^*}^{u_1} \{1 - c(u_1, z)\} h(z) \, dz \, dy
$$

$$
- \int_{u_1}^{1} h(y) \int_{v^*}^{u_1} \{1 - c(u_1, z) - c(u_1, y)\} h(z) \, dz \, dy \qquad \text{(D.5)}
$$

$$
- \int_{v^*}^{v_1} h(y) \int_{v^*}^{y} \{2 - c(u_1, z)\} h(z) \, dz \, dy
$$

$$
- \int_{v_1}^{u_1} h(y) \int_{v^*}^{y} \{2 - c(u_1, z) - c(u_1, y)\} h(z) \, dz \, dy
$$

$$
+ \int_{v^*}^{v_1} h(y) \int_{v^*}^{y} \{1 + b - c(u_1, z)\} h(z) \, dz \, dy \Bigg\}
$$

and for $u_1 < v_1$ we have

$$
f_1(u_1, v_1) = \tfrac{4}{3}\alpha \Bigg\{ \int_{v^*}^{u_1} h(x) \int_{v_1}^{1} h(y) \int_{v^*}^{x} \{1 - c(x, z)\} h(z) \, dz \, dy \, dx
$$

$$
+ \int_{v_1}^{1} h(y) \int_{u_1}^{y} h(x) \int_{v^*}^{x} \{1 - c(x, z) - c(x, y)\} h(z) \, dz \, dx \, dy
$$

$$
+ \int_{u_1}^{v_1} h(x) \int_{v^*}^{x} h(y) \int_{v^*}^{y} \{2 - c(x, z)\} h(z) \, dz \, dy \, dx
$$

$$
+ \int_{u_1}^{v_1} h(x) \int_{x}^{v_1} h(y) \int_{v^*}^{x} \{2 - c(x, z)\} h(z) \, dz \, dy \, dx \qquad \text{(D.6)}
$$

$$
+ \int_{v_1}^{1} h(x) \int_{v^*}^{v_1} h(y) \int_{v^*}^{y} \{2 - c(x, z)\} h(z) \, dz \, dy \, dx
$$

$$
+ \int_{v_1}^{1} h(x) \int_{v_1}^{x} h(y) \int_{v^*}^{y} \{2 - c(x, z) - c(x, y)\} h(z) \, dz \, dy \, dx
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{v^*}^{x} h(y) \int_{v^*}^{y} \{1 + b - c(x, z)\} h(z) \, dz \, dy \, dx
$$

$$
+ \int_{v^*}^{u_1} h(x) \int_{x}^{v_1} h(y) \int_{v^*}^{x} \{1 + b - c(x, z)\} h(z) \, dz \, dy \, dx \Bigg\}
$$

implying that

$$
\begin{aligned}
\frac{\partial f}{\partial u_1} = \frac{\partial f_1}{\partial u_1} = {} & \tfrac{4}{3}\alpha h(u_1)\Bigg\{ \int_{v_1}^{1} h(y) \int_{v^*}^{u_1} \{1 - c(u_1, z)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& - \int_{u_1}^{1} h(y) \int_{v^*}^{u_1} \{1 - c(u_1, z) - c(u_1, y)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& - \int_{v^*}^{u_1} h(y) \int_{v^*}^{y} \{2 - c(u_1, z)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& - \int_{u_1}^{v_1} h(y) \int_{v^*}^{u_1} \{2 - c(u_1, z)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& + \int_{v^*}^{u_1} h(y) \int_{v^*}^{y} \{1 + b - c(u_1, z)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& + \int_{u_1}^{v_1} h(y) \int_{v^*}^{u_1} \{1 + b - c(u_1, z)\}h(z)\,\mathrm{d}z\,\mathrm{d}y \Bigg\}.
\end{aligned}
\tag{D.7}
$$

So taking the limit as $u_1 \to v_1$ (in (D.6) for $v_1 = v^*$, in (D.7) for $v_1 = 1$, and in either expression for $v^* < v_1 < 1$), we have

$$
\begin{aligned}
\frac{\partial f}{\partial u_1}\bigg|_{u_1 = v_1} = {} & \tfrac{4}{3}\alpha h(u_1)\Bigg\{ \int_{v_1}^{1} h(y) \int_{v^*}^{v_1} c(v_1, y)h(z)\,\mathrm{d}z\,\mathrm{d}y \\
& - (1 - b)\int_{v^*}^{v_1} h(y) \int_{v^*}^{y} h(z)\,\mathrm{d}z\,\mathrm{d}y \Bigg\}.
\end{aligned}
\tag{D.8}
$$

Setting $v_1 = v^*$ in (D.8) now immediately yields the first result of (14). To obtain the second result of (14), we differentiate (D.5) with respect to u_1 and then take the limit as $u_1 \to v_1 = v^*$.

Acknowledgements

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