Counting to 20:

Online Implementation of a Face-to-Face, Elementary Mathematics Methods Problem-Solving Activity

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Abstract

This study describes implementation of the same problem-solving activity in both online and face-to-face environments. The activity, done in the first class period or first module of a K-2 mathematics methods course, was initially used in a face-to-face class and then adapted later for use in an online class. While the task was originally designed for the face-to-face course, it is not the case that it is fully useful there and less useful in the online version. Rather, each context privileges different ideas about teaching and learning mathematics. Possibilities to address the challenges of each environment are considered with emphasis on task structure and the role of the instructor.

Keywords: elementary, mathematics, online, preservice teacher, problem-solving

I Can Count to 20 Before You Can!

Standards-based mathematics calls for teachers to engage students in the practices of a mathematical community while simultaneously promoting a strong conceptual understanding of and appreciation for mathematics content (NCTM, 2000; CCSSO & NGA, 2010). To this end, mathematics teacher educators strive to engage preservice teachers in problem-solving activities as a way to help them experience mathematics focused on content knowledge and the processes of doing mathematics. As online delivery becomes more prevalent, problem-solving activities usually taught in face-to-face elementary mathematics methods courses are necessarily moved into online learning environments. This paper examines the role that these different learning environments played in the implementation of a single problem-solving activity in an elementary mathematics methods course as it was redesigned for online delivery by a novice online instructor.

The "I Can Count to 20" activity is introduced the first day of the course. "I Can Count to 20" is a version of a NIM game in which partners take turns saying one or two numbers starting with one and counting consecutively to 20. For example, if player one says, "One, two," then player two could say "three" or "three, four." The game continues in this manner until one of the partners says 20. The person who says 20 wins the game. The challenge for preservice teachers is to figure out a strategy to win the game every time.

Several goals exist for this "first day" activity. First, the game sets the tone for the course. Many students come into the course anxious about mathematics and have only experienced traditional instruction of mathematics (Ball, 1988). The game engages students in a non-threatening way because it can be played multiple times with a partner and there are no high stakes consequences for losing. It is also a useful springboard for discussion about the teacher decision making that goes into an activity, the nature of mathematics, and problem-solving strategies. In addition, the game highlights the process standards (NCTM, 2000) such as communication and reasoning and proof. Finally, the instructor can do a quick, preliminary assessment of the preservice teachers' reasoning and problem-solving approaches as well as their dispositions towards mathematics.

Implementation in Different Environments

To understand the different ideas about teaching and learning mathematics that are afforded and constrained in the two contexts, the implementation of the activity in each learning environment must first be described. In the face-to-face environment, the game is played in a single class period on the first day of class. The instructor begins by reading the rules of the game and an example game is played between the instructor and a student. Then students in the class play with a partner as the instructor walks around and listens. After the game has been played just a few times, approximately two to five minutes, the instructor asks the class to see if they can work with their partner to figure out if there is a strategy to win every time. The class moves back and forth from partner work to whole group work several times as students share conjectures and then test them. Conjectures are listed on the board in the front of the room and are revised or abandoned as the work continues. Finally, the instructor pulls the whole group back together one last time when the majority of groups have a reasonable working strategy, even if it is tentative in their minds. The class discusses the strategies until a consensus is reached for how to win the game every time.

One goal of the course is to integrate the mathematics content and pedagogy so that preservice teachers are reasoning about a problem that is challenging for them mathematically but at the same time are thinking about mathematics pedagogy. Throughout the class, the instructor talks explicitly with students about the planned instructional decisions and those made in real time during the activity. For example, the instructor might explain why she chose to call on a particular student or why she asked a question in a certain way. While in-the-moment decision-making is so context specific that it is not easily "taught," in this way, preservice teachers begin to understand the kinds of considerations that inform improvisation (Sawyer, 2004) and thoughtful adaptation (Duffy, 2003) during teaching.

The online version of the class is comprised of modules which students complete asynchronously in the space of one week. Each module includes a group problem-solving task. The "I Can Count to 20" activity is the task for the first module. The class is broken into teams of four or five people. Each team has its own discussion board to work on the problem together. Rather than the instructor modeling an example game with a class member, students are given written directions and listen to an audio recording of two people playing the game. They are then told to work with their team to figure out the way to win. All discussion boards in the course require that students post on at least three different calendar days during the week and that their postings offer meaningful additions to the discussions. In other words, "I agree" is not enough. The instructor inputs questions and comments similar to those offered in class. One major difference in implementation is the isolation among the groups until the end of the activity. There is no interaction about the problem with the whole class until the second week when groups are asked to post their strategies and deliberations on a whole class discussion board.

This study seeks to understand the ways these two different learning environments affected a single mathematics problem-solving activity as taught by an experienced face-to-face mathematics methods instructor attempting to transition to online teaching. Specifically, the study addresses the following research questions:

1) What are the affordances and constraints of the "I Can Count to 20" activity in the different learning environments?
2) What considerations do inexperienced online instructors face as they transition mathematics problem-solving activities across learning environments?

Methodology

The students encountered the game "I Can Count to 20" in a mathematics methods course that is the first of a two-course sequence in an undergraduate elementary teacher education program. The face-to-face class had 28 students and the online class had 18 students. The same instructor taught both courses. The instructor had five years experience teaching face-to-face methods courses, but this was the first online course taught.

Conjecture	Face-to-Face	Online
Odd/even numbers are important.		
It matters who goes first.		
You need to start thinking towards the end of the game		
Particular numbers are important (You will lose if you land on 15 and 16/win if you land on 17.)		
You must land on a particular string of numbers working backwards by 3 from 20 (17, 14, 11, 8, 5, 2.)		
Do the opposite of your opponent (If he says two numbers, you say one number and vice versa.)		
Control what they other person does (Ex. Get them to say multiples of three.)		
Say multiples of three.		
Mode strategy		

Table 1. Conjectures made about strategies for winning I can count to 20

Data Collected

Data collected from the face-to-face class included classroom observations and mathematics journals from students. Students keep these journals throughout the semester as a place to record their mathematical work, class notes, and responses to in-class writing prompts. Data collected from the online course were postings from the online discussion boards. The instructor kept a journal with entries written after each face-to-face class and two or three times per weekly module in the online course. The journal entries pertaining to the activity "I Can Count to 20" were also used. Research memos (Maxwell, 2005) were also written at each stage of the process and then mined for data as well.

Data Analysis

Because the goal of the activity was to have students work on both mathematics and pedagogy, these two categories served as the a priori groups for examining the data. Examples of data specific to the mathematics used included strategies employed by students while working on the activity such as "working backwards" or "trial and error." It also included conjectures made, that winning had something to do with odd and even numbers or multiples of three, for example. Data in the journals or observation notes that focused on instructional practice rather than mathematics were initially placed in an overarching "pedagogy" category. From there, constant comparative analysis (Miles & Huberman, 1994) was used to find emerging categories until saturation was reached (Glaser & Strauss, 1967). The categories were then used to compare the differences and similarities between the two environments.

Findings

The two different learning environments, face-to-face and online, afforded different strengths in terms of the mathematics and pedagogical strategies discussed by students. More constraints were expected in the online delivery for two reasons. First, the task was originally designed for face-to-face implementation. Secondly, the activity was in its first iteration of the online version and was designed by a mathematics teacher educator new to online delivery. While more constraints were in fact found in the online version, each context privileged different goals in the activity. The online context afforded deeper use of the practices of doing mathematics. The face-to-face context afforded better access to discussions of pedagogical issues during the activity, primarily because of the task structure and the role of the instructor.

Student Mathematics Outcomes

In terms of mathematics outcomes for students, the preservice teachers in both the face-to-face and online courses developed the same strategies common to the mathematics of the task. For example, both groups had students who conjectured that the solution had something to do with odd and even numbers, or who went first in the game, or the use of 17 as a target number (See Table 1).

Both groups recognized the need to look for patterns and the usefulness of working backwards a strategy. The online students, however, had more varied and thorough representations of their work, using tables, pictures, and detailed descriptions to explain their strategies. For example, the moves that each player should make to win were often explicitly recorded as in the following posting.

> When initially completing this activity, and after numerous attempts at strategizing, it appeared that the person who went second was more likely to win. However, after about 30 attempts using a variety of strategies and number patterns, I found that the likelihood decreased. I made sure to write each of the patterns down so that I had a means of comparing them. After hours of studying the numbers, the solution came to me: ensuring that YOU end with the number 17 in the next to last round/rotation, you are guaranteed a win!! For example, use the following sequence:

Player 1	Player 2
1, 2	3
4	5
6	7, 8
9, 10	11
12	13, 14
15	16, **17***
18, 19	20

> No matter how many times and scenarios I used, as long as I was the one ending in the number 17 prior to my last turn, I won each time!!!

> *Bold text denotes red font in original post.

In contrast, written records for the face-to-face class, if they existed at all, usually consisted only of strings of numbers with no verbal explanations or indications of the relationship between the numbers and players who should say them. This finding makes sense because the online students did not talk with classmates, nor did they have the same non-verbal cues from their group members as those students in the face-to-face class. In addition, students in the face-to-face setting were playing the game with each other. In the online delivery, most students played with a person outside of class and reported back, although a few students made plans to play with each other using Skype.

When an online student's ideas were not clear, others asked for clarification using specific representations. After a student posed a theory about using multiples of three to win, another group member posted a message ent itled, "I AM A VISUAL LEARNER…PLEASE DIAGRAM THE PLAY BY PLAY OF THE MULTIPLES OF 3 [sic]…." While these same types of exchanges and requests for verification took place verbally in the face-to-face class, the effort to support conjectures was not as formal. Online students collected data about their tests of various conjectures so they could "report back" their findings to the group, recording the frequency of games won or lost with a particular strategy and presenting their tallies. Face-to-face students were more likely to explain that they "tried it a few times and the strategy didn't seem to make a difference." Consequently, the online version of the activity afforded development and use of multiple representations, explicitness in explanations, and preciseness about testing conjectures. These important mathematical foundations were not common among the face-to-face students.

Role of the Instructor

The role of the instructor was different in the online course in terms of the structure and pacing of the activity. In the face-to-face version, students moved back and forth from small group to whole group multiple times throughout the activity. The online students worked in small groups for the activity, but there was not a chance for groups to move fluidly back and forth to a whole group setting. This condition isolated both the variety of strategies considered by students as well as the instructor's comments about pedagogical moves. As noted above, the online group as a whole used the same strategies as the face-to-face group. The difference was that online students were not exposed to all of them. Rather they considered only the ideas raised in their own group. The whole group discussion board the next week became a reporting of solutions not a discussion of solving or of teaching points.

The isolation among groups also affected access to issues of pedagogy. In the face-to-face course, the instructor explicitly described pedagogical moves to the whole class so the preservice teachers began to think about teacher decision-making. For example, in the face-to-face class the instructor stopped the small group work after just a few minutes and asked the whole class, "Where did you start in your thinking about this problem?" Students then contributed some ideas that were written on the board as conjectures. Before the students started working in their small groups again to test the conjectures on the board, the instructor explained the reasoning behind asking the initial question in this particular way towards the beginning of the lesson. Asking the question before students have had much time to work eliminates the need for the person who answers to have the correct solution. Anyone can say where he or she started on a problem, so the discussion is easily opened in an atmosphere in which it is safe to risk putting forth an idea that might not be correct. In addition, students that had no idea where to start now had a list of ideas on the board that they could test. Similar explanations about the teacher decision-making continued throughout the face-to-face lesson.

Opportunities for pedagogical comments were less frequent in the online delivery as there was no think aloud of in-the-moment teacher decision-making by the instructor. When possible, however, these types of comments certainly were interjected by the instructor as in the following discussion board exchange, though they were isolated to the situations arising in particular groups rather than being accessible to whole class.

> **Student:** What I have found out for sure is that they [the student's 6th, 7th and 8th classes] are much better at this game than I. They win almost every time, but can't tell me how. They say it's easy, but then they don't know how to express it with logic. These are students I trust and I honestly believe them when they say they don't know how they are doing it. I have noticed that they really begin to think around the number 9. The look on their faces gets really intense by the number 14.
>
> **Instructor:** I wanted to quickly comment on students not being able to explain their thinking because this is not an uncommon occurrence. As you have so astutely noted they often get an intense look towards the end, usually around 14. Sometimes they will take a long pause at a particular number. I've found it helpful to stop them right at that point and ask them what they are thinking right then. They can even just think aloud. It may be jumbled, but the thoughts will come out.

In the future, the comments could easily be shared with other groups or the whole class later, but the explanation would not be in the context of a specific shared experience as it is when the comments are based on events within the face-to-face class.

In contrast, other interactions were not isolated enough, changing the pacing for individual students. In the face-to-face class, for example, the instructor talks with partners or individuals about their work on the task. The exchange is private to the extent that it is between the individual and the instructor and those few students in close enough proximity to hear. In contrast, anything posted on a discussion board was seen by the group. By responding publically to a strategy proposed by a student on a discussion board, the instructor runs the risk of interrupting the math thinking of a different student who is taking a different approach or who is not yet ready for the next step/question. While the student who is not yet to the next step may be able to ignore the content of the message, it may still create a sense of urgency or "falling behind" in their minds if someone else has been asked a new or different question.

Finally, when a student quickly discerned how to win the game every time, they were eager to share their winning strategy. In the face-to-face course, the instructor chose to wait to call on that student until others had shared their thoughts and the class was closer to a solution. After the discussion, she then explained her choice to wait as a part of pedagogical decision making. In the online activity, two different times, a group member's first post was the solution. Subsequent postings from other group members served to simply verify the solution by testing it with other people, but the group members did not do any thinking about the task itself. Thus, the instructor had less control of pacing in the sense that choices about who shares what strategy and when they share it were no longer available. As a result, the explanation of the reasoning behind these teacher decisions was also not present.

While the students' mathematical representations were stronger in the online version of the "I Can Count to 20" activity, the amount of instructor's control over the pacing and the subsequent discussion of pedagogical decision-making decreased from the face-to-face activity to the online version.

Discussion

In the "I Can Count to 20" activity, each context afforded and constrained different aspects of teaching and learning mathematics. Online students were more detailed in their mathematical representations and more systematic in their testing of conjectures. Face-to-face students had more opportunities for discussion of pedagogical decision-making. Given the differences in student mathematics outcomes and the role of the instructor, the question raised by this study is how best to organize problem-solving activities in elementary methods courses to provide more affordances and remove some constraints in each learning environment. That is, how can face-to-face students be encouraged to represent their ideas more thoroughly and test their conjectures more formally? How can small teams of students working on a problem in an online format interact with the whole group in a fluid way that moves everyone's pedagogical thinking forward?

Two considerations in reconfiguring this activity are the structure and the pacing in terms of the instructor's interaction with students. For example, restructuring the task to add in a specific time in the face-to-face implementation for students to write about their thinking and then to compare notes might promote more detailed representations. Conversely, using a synchronous environment or limiting the amount of time to solve the given mathematics problem asynchronously could curtail the problem of students posting a complete solution in their first post.

Addressing the issue of pedagogical discussions is more complex. Researchers have successfully used case studies (McCrory, Putnam, & Jansen, 2008) and the Online Asynchronous Collaboration (OAS) model (Clay & Silverman, 2009) to foster online discussions of mathematics pedagogy. In the OAS model, students first respond privately to a mathematics task. The responses are then opened to a small group for comment. The instructor then leads a whole class discussion, choosing prompts that emerge from the small group discussion to synthesize and reflect upon ideas. They suggest three possible directions for these emergent prompts depending on the instructor's goals and the context of the discussion so far: 1) deepening mathematical understandings 2) focus on mathematics pedagogy, or 3) the process of doing mathematics: thinking like a mathematician.

These different structures offer real possibilities for increasing participation and collaboration in mathematics problem-solving and in subsequent pedagogical discussions for tasks such as "I Can Count to 20" that aim to concurrently convey ideas about both mathematics and teaching. However, one remaining issue is the common strategy in mathematics methods courses of having preservice teachers learn mathematics in the same way that they are being encouraged to teach it in their own classrooms. The considerations of the instructor are different in the two environments; there are not the same opportunities for discussions about what is informing the decision-making. When considering the representations used, one could make the argument that a higher level of discourse was present in the online discussions than the face-to-face class. But the explicit talk about how that discourse was being orchestrated was not present. Certainly the orchestration of discourse and other instructional strategies can be presented and discussed in an online environment, but not in the context of teachers who are currently experiencing the activity in the exact same delivery that they would use with their students.

Rich discussion about pedagogy, though critical, is not the same as capturing the messages that occur from the instructor's modeling of face-to-face mathematics teaching. More research is needed to understand what the value is for preservice teachers, if any, of learning in the same mode of delivery as they will be teaching. Secondly, in addition to the significant work already addressed by the literature on fostering discussions of mathematics and pedagogy in online environments, rethinking how online methods instructors might structure activities to make pedagogical moves explicit is essential, particularly those decisions that are improvisational (Sawyer, 2004) or thoughtfully adaptive (Duffy, 2003) and therefore usually situated within the particular context of the face-to-face classroom.

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