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Cost Allocation for Inventory Problem with Transportation Discount under Carbon Tax Policy

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Abstract. In this paper, we deal with the problem of cost allocation among multiple retailers in an inventory system with transportation quantity discount under the widely-used carbon tax regulation. We first develop an inventory model with transportation discount under the carbon tax policy, and determine the optimal order quantity per order such that the total cost is minimized in the case of individual and joint ordering. We show that the total cost for the group of retailers can be reduced by placing joint orders while the total carbon emissions may increase. Then, we provide a sufficient condition which indicates that when the costs and carbon emissions associated with each order initiated are relatively high, enterprises can achieve dual objectives (both carbon emission reduction and cost reduction) through joint ordering. To allocate the total cost among the retailers, we introduce an inventory game and show that this game is concave. Based on this, we propose a cost allocation rule, which belongs to the core of the game.

Keywords: Supply chain management, inventory systems, carbon tax, transportation discount, cooperative games

1. Introduction

In recent years, the study of cooperative behavior in supply chain management has attracted increasing awareness from scholars. Joint inventory management is a typical mode of horizontal cooperation in supply chain management, which refers to the situation where several agents facing individual inventory issues decide to cooperate by placing joint orders to reduce the associated costs.

Cooperative game theory has been demonstrated to be a powerful tool to solve the cooperation problem in inventory management (Fiestras-Janeiro et al. 2011a). The fundamental research on inventory cooperative games was introduced by Meca et al. (2004). Under the underlying economic order quantity (EOQ) model, they studied a class of inventory games that arise from situations in which a group of firms decide to cooperate by placing joint orders. And then a great deal of researchers extended Meca et al. (2004) 's work to different situations by considering reality factors such as delay in payments (Krichen et al. 2011, Li et al. 2014), product shortages (Guardiola et al. 2009), quantity discount on purchase (Krichen et al. 2011, Li et al. 2021), stochastic demand (Chen and Zhang 2009), carbon emissions (Feng et al. 2021, Zeng et al. 2022), private information (Zeng et al. 2024) and exemptable ordering costs (Fiestras-Janeiro et al. 2024).

In addition to the aforementioned factors, the cost of transportation is another important factor that scholars are concerned about. Buffa (1988) concluded through empirical analysis that cooperative transportation can effectively reduce the total transportation cost. In many studies in the field of inventory management, transportation costs are only considered by including them in the fixed cost of ordering. Recently, several works on inventory systems treat transportation cost as an independent component, and present different perspectives on the calculation of the transportation cost for cooperative retailers. For instance, Dror and Hartman (2007) and Fiestras-Janeiro et al. (2011b 2013) described the transportation costs of cooperative coalitions by considering different geographic locations of retailers. Saavedra-Nieves (2020) used a more general function to characterize the transportation cost per order from the supplier. However, the existing literature ignores an important factor that the quantity of transportation may affect the cost of transportation. Nollet and Beaulieu (2005) pointed out that by combining purchases and pooling transportation resources, multiple companies can combine smaller shipments to increase the quantity of shipments and thus reduce transportation cost. In many practical situations, quantity discounts depending on the number of unit loads delivered may occur due to economies of scale (Shinn et al. 1996). Transportation quantity discount is a common marketing strategy that has been extensively studied in supply chain network design (Tsao and Lu 2012), and multimodal freight transportation (K P and Panicker 2020). Indeed, the provision of quantity-based transportation discount gives strong incentives for retailers to cooperate due to the potential cost savings in transportation, which is worth investigating in inventory systems.

A lot of energy will be consumed in the process of transportation, storage and ordering of products, which inevitably leads to a lot of carbon emissions. At the same time, with the growing serious problem of climate change caused by greenhouse gas emissions such as carbon dioxide, the government has to implement various carbon emission regulatory policies (e.g., carbon tax, carbon cap, carbon cap-and-trade, carbon offset, etc.) on enterprises to limit carbon dioxide emissions. In this context, the inventory management strategies and operations of enterprises are not only aimed at enhancing economic benefits, but also need to consider environmental performance such as reducing carbon emissions for sustainable economic development. In the literature, a growing number of studies have incorporated carbon emission regulations into inventory management decisions. Some scholars have discussed how operational decisions are determined under carbon regulation policies. Benjaafar et al. (2012) are the first to introduce the carbon emissions into the field of supply chain management, and investigated how enterprises adjust operational decisions to reduce costs and mitigate carbon emissions. Chen et al. (2013) studied the optimal order quantity by considering the EOQ model under four different carbon constraint policies (including strict carbon cap, carbon tax, carbon cap-and-offset, and carbon cap-and-price), and showed that it is possible to reduce the carbon emission level by adjusting the order quantity. Dye and Yang

(2015) examined the impact of carbon emissions on the retailer's trade credit and replenishment strategies under carbon cap-and-trade and carbon offset policies. Khan et al. (2023) investigated the livestock farming firm's optimal prepayment installment, pricing and replenishment decisions for a growing item under cap-and-price, cap-and-trade and carbon tax regulations. San-José et al. (2024) identified the optimal inventory policy that maximizes the benefit per unit of time for deteriorating items in the context of carbon tax policy, in which carbon emissions originate from the transportation, storage and deterioration of products. Other scholars have found that financial schemes may have an impact on carbon emissions. For instance, Aljazzar et al. (2018) showed that delayed payments can improve both environmental and economic performance by considering a two-level coordinated supply chain. Shi et al. (2020 2023) studied the influence of different payments on the optimal ordering cycle of perishable products under carbon tax and cap-and-trade regulations, and discussed the most effective payment in curbing carbon emissions. More recently, it has been shown in several studies that cooperation between multiple firms may not only lead to cost savings but also reduce the level of carbon emissions. Feng et al. (2021) showed that under a carbon cap-and-trade policy, joint replenishment among multiple retailers can increase their total profits and reduce their total carbon emissions. Halat et al. (2021) studied multi-level supply chains considering different decision structures under carbon tax policies. They found that supply chain members could reduce their costs and carbon emissions through cooperation.

Energy consumption in transportation has been shown to be one of the major sources of carbon emissions (Li et al. 2019, Jia et al. 2021, Bai et al. 2023). However, existing studies of cooperative inventory systems with transportation processes have not explored the effect of carbon emissions. Our research is interested in exploring how joint ordering affects the total cost and carbon emissions of retailers when retailers face both transportation discount preferential policy and widely used carbon tax penalty policy[1], under what conditions cooperation can achieve the dual objectives of reducing cost and carbon emissions, and how the joint cost should be distributed among retailers to incentivize them to cooperate in ordering.

To do that, we consider a supply chain consisting of a single product, one supplier, one transporter, and multiple retailers. Retailers order products from the supplier to meet deterministic market demand and transport them through the transporter, and the economic activity of retailers is regulated by the government's carbon tax policy. In addition, the transporter offers retailers shipping discounts to incentivize them to increase the volume of single shipments. To save on costs, the retailers may cooperate by coordinating their orders as a larger one. We first develop an inventory model where each retailer receives a shipping discount from the transporter, and all retailers are penalized by the government's carbon tax policy for their carbon emissions. We determine the optimal order quantity per order for the retailer in the case of individual and joint ordering, respectively. By comparing before and after joint ordering, we show that joint ordering can reduce the total cost for the group of retailers while the total carbon emissions may increase. Then, we provide a sufficient condition which indicates that when the costs and carbon emissions associated with each order initiated are relatively high, enterprises can achieve dual objectives (both carbon emission reduction and cost reduction) through joint ordering. Then, to allocate the total cost from joint ordering among multiple retailers, we introduce an inventory game and show that this kind of game is concave. Based on this, we propose a cost allocation rule belongs to the core of the game.

The rest of this paper is organized as follows. Section 2 concentrates on some preliminaries on cooperative game theory. Section 3 introduces the inventory model with transportation discount under carbon tax policy. In Section 4, we present the inventory game with transportation discount under carbon tax policy and propose an allocation rule which belongs to the core. Section 5 provides conclusions.

2. Preliminaries on Cooperative Game Theory

A cooperative game with transferable utility (a cost TU-game) is a pair (N, C), where N is a finite set of players and $C : 2^N \longrightarrow \mathbb{R}$ is the characteristic function of the game with $C(\emptyset) = 0$. A nonempty subset S of N is called a coalition whose cardinality is denoted by s. For any $S \subseteq N$, $C(S)$ represents the minimum total cost that players in S need to pay when they cooperate.

A payoff is an n-dimensional vector $x \in \mathbb{R}^N$ that assigns a payoff x_i to each player $i \in N$. We denote the total amount allocated to coalition $S \subseteq N$ as $x(S)$, where $x(S) = \sum_{i\in S} x_i$. The core is one of the most fundamental concepts in TU-games, which is defined by

$$\text{Core}(N, C) = \left\{ x \in \mathbb{R}^N : \sum_{i\in N} x_i = C(N), \right.$$
$$\left. \sum_{i\in S} x_i \le C(S), \forall S \subseteq N \right\}$$

It is well known that the core of concave games is always nonempty. A cost TU-game (N, C) is called concave if and only if

$$C(S \cup \{i\}) - C(S) \ge C(T \cup \{i\}) - C(T)$$

for all $S, T \in 2^N$, $i \in N$ such that $S \subseteq T \subseteq N\backslash\{i\}$.

If the cost game (N, C) is concave, then no coalition has an incentive to split off from the grand coalition and form subcoalitions.

3. Inventory Model with Transportation Discount under Carbon Tax Policy

Suppose there is a supply chain consisting of one supplier, one transporter and n retailers in the market, and denote the set of n retailers by $N = \{1, 2, \cdots, n\}$. All retailers are located in the same region, and they order identical products from the supplier to meet deterministic market demand and transport the products through the transporter. To satisfy the demand of consumers, retailers usually order a certain amount of goods in advance and store them in their warehouses.

In the ordering process, four kinds of costs incurs, i.e., (i) the ordering fixed cost; (ii) the inventory holding cost; (iii) the transportation cost; (iv) the carbon tax. The ordering fixed cost a per order is only related to the number of orders rather than the order quantity. The inventory holding cost includes product maintenance, warehouse leasing and maintenance, equipment depreciation, lighting and other expenses. In this paper, we assume that the holding cost per unit product per unit time h is identical for all retailers. The transportation cost is associated with the quantity of shipments for a single order. In the model, we consider a situation where the transporter offers quantity discounts on the transportation to retailers. Under the quantity discount contract, retailers can enjoy a lower transportation cost when the transportation quantity exceeds a certain threshold. Specifically, let Q_0 be the threshold, when the quantity is greater than Q_0, the transporter offers a discount for the excess part.[2] In other words, when the transported quantity is not greater than Q_0, the price of the unit product is c_0, which is the starting price related to the transportation distance. We assume in our model that all retailers are located in the same region and therefore they have the same transportation starting price c_0. When the quantity is greater than Q_0, the price of the exceeding part is αc_0 per unit and the rest part is still c_0, where α is transportation discount rate satisfying $0 < \alpha < 1$. In addition, α is exogenous and thus the same for all retailers. Then, given that the quantity of retailer i needs to transport is q_i, the average price per unit of product that the retailer i has to pay can be expressed as

$$c(q_i) = \begin{cases} c_0, & \text{if } q_i \le Q_0 \\ \frac{c_0 Q_0 + \alpha c_0 (q_i - Q_0)}{q_i}, & \text{if } q_i > Q_0 \end{cases} \tag{1}$$

and the total transportation cost paid by retailer i for each order of the product is $q_i \cdot c(q_i)$.

Energy consumption during product ordering, storage and transportation results in a large amount of carbon emissions. For each retailer i, let $\hat{a}$, $\hat{h}$ and $\hat{e}$ denote the amount of carbon emissions associated with each order initiated, each unit held in inventory per unit time, and each unit transported, respectively. In addition, the retailers are subject to the regulation of the carbon tax policy. The carbon tax policy is essentially a financial penalty in which the government directly charges companies for their emissions without setting a cap. Following Shi et al. (2020) and Zeng et al. (2022), we assume that retailers are charged a fixed dollar amount β for every ton of emissions by the local government under the carbon tax regulation.

Next, we present the optimal order policies and the corresponding minimum costs per year for retailers in the case of individual ordering and joint ordering.

3.1 Optimal Order Policy for an Individual Retailer

When acting independently, retailer i's objective is to determine the optimal order quantity q_i^* to minimize his total cost. To determine q_i^*, we develop a total annual cost model for the retailer who orders individually. For any retailer $i \in N$, the total annual cost $C_i(q_i)$ involves the ordering fixed cost $a \cdot \frac{d_i}{q_i}$, inventory holding cost $\frac{hq_i}{2}$, transportation cost $c\left(q_i\right) d_i$ and carbon tax βX_i, which is given by

$$C_i(q_i) = a \cdot \frac{d_i}{q_i} + \frac{hq_i}{2} + c\left(q_i\right) d_i + \beta X_i \tag{2}$$

where β denotes the tax rate, i.e., the penalty of per unit carbon emitted, and X_i denotes the total carbon emission of retailer i, which is

$$X_i = \hat{a} \cdot \frac{d_i}{q_i} + \frac{\hat{h} q_i}{2} + \hat{e} d_i \tag{3}$$

Combining Eq. (1)-(3), we obtain the following total annual cost as a function of q_i,

$$C_i(q_i) = \begin{cases} C_{i1}(q_i), & \text{if } q_i \leq Q_0 \\ C_{i2}(q_i), & \text{if } q_i > Q_0 \end{cases} \tag{4}$$

where

$$\begin{aligned} C_{i1}(q_i) =& (a + \beta \hat{a}) \cdot \frac{d_i}{q_i} + \frac{(h + \beta \hat{h}) q_i}{2} \\ &+ (c_0 + \beta \hat{e}) d_i \end{aligned} \tag{5}$$

and

$$\begin{aligned} C_{i2}(q_i) =& [a + \beta \hat{a} + c_0 Q_0 (1 - \alpha)] \cdot \frac{d_i}{q_i} \\ &+ \frac{(h + \beta \hat{h}) q_i}{2} + (\alpha c_0 + \beta \hat{e}) d_i \end{aligned} \tag{6}$$

Subtract Eq. (6) from Eq. (5), we obtain

$$C_{i1}(q_i) - C_{i2}(q_i) = c_0 d_i (1 - \alpha)(1 - \frac{Q_0}{q_i}) \tag{7}$$

It is easy to check from Eq. (7) that $C_i(q_i)$ is continuous at $q_i = Q_0$, and if $q_i \leq Q_0$, then $C_{i1}(q_i) \leq C_{i2}(q_i)$, otherwise, $C_{i1}(q_i) > C_{i2}(q_i)$.

To find the optimal order quantity per order q_i^* for retailer i, taking the first-order derivatives of $C_{i1}(q_i)$ and $C_{i2}(q_i)$ with respect to q_i, we obtain

$$\frac{dC_{i1}(q_i)}{dq_i} = -(a + \beta \hat{a}) \cdot \frac{d_i}{q_i^2} + \frac{(h + \beta \hat{h})}{2} \tag{8}$$

and

$$\begin{aligned} \frac{dC_{i2}(q_i)}{dq_i} =& - [a + \beta \hat{a} + c_0 Q_0 (1 - \alpha)] \cdot \frac{d_i}{q_i^2} \\ &+ \frac{(h + \beta \hat{h})}{2} \end{aligned} \tag{9}$$

Taking the second-order derivatives of Eq. (8) and Eq. (9) with respect to q_i, we get

$$\frac{d^2 C_{i1}(q_i)}{dq_i^2} = (a + \beta \hat{a}) \cdot \frac{2d_i}{q_i^3} > 0$$

and

$$\frac{d^2 C_{i2}(q_i)}{dq_i^2} = [a + \beta \hat{a} + c_0 Q_0 (1 - \alpha)] \cdot \frac{2d_i}{q_i^3} > 0$$

Setting Eq. (8) and Eq. (9) to zero, we obtain the optimal solution q_{i1}^* of $C_{i1}(q_i)$, which is

$$q_{i1}^* = \sqrt{\frac{2(a + \beta \hat{a}) d_i}{h + \beta \hat{h}}}$$

and the optimal solution q_{i2}^* of $C_{i2}(q_i)$, which is

$$q_{i2}^* = \sqrt{\frac{2[a + \beta \hat{a} + c_0 Q_0 (1 - \alpha)] d_i}{h + \beta \hat{h}}}$$

It can be easily verified that

$$q_{i1}^* < q_{i2}^*$$

In the following, we determine the optimal order quantity q_i^* that minimizes the total annual cost $C_i(q_i)$ in Eq. (4), and we obtain the corresponding minimum annual cost and annual carbon emissions for retailer i.

Theorem 1 *For each retailer i that orders independently, there exists a unique discriminant Δ^* such that the following statements hold, where*

$$\Delta^* = \frac{2(h + \beta \hat{h}) \left(\sqrt{a + \beta \hat{a} + c_0 Q_0 (1 - \alpha)} - \sqrt{a + \beta \hat{a}} \right)^2}{[(1 - \alpha) c_0]^2}$$

1) *if $d_i \leq \Delta^*$, the optimal order quantity per order for retailer i is*

$$q_i^* = \sqrt{\frac{2(a + \beta \hat{a}) d_i}{h + \beta \hat{h}}}$$

the corresponding minimum annual cost is

$$C_i(q_i^*) = \sqrt{2(a + \beta \hat{a})(h + \beta \hat{h}) d_i} + (c_0 + \beta \hat{e}) d_i$$

and the amount of carbon emissions generated by retailer i is

$$X_i(q_i^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}} + \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i$$

2) *if $d_i > \Delta^*$, the optimal order quantity per order for retailer i is*

$$q_i^* = \sqrt{\frac{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{h+\beta\hat{h}}}$$

the corresponding minimum annual cost is

$$C_i(q_i^*) = \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_i} + (\alpha c_0+\beta\hat{e})d_i$$

and the amount of carbon emissions generated by retailer i is

$$X_i(q_i^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} + \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i$$

Proof. To find the global minimum solution of the function $C_i(q_i)$ in Eq. (4), we discuss the following three cases, i.e., (i) $q_{i1}^* < q_{i2}^* \le Q_0$; (ii) $Q_0 < q_{i1}^* < q_{i2}^*$; (iii) $q_{i1}^* \le Q_0 < q_{i2}^*$.

Case (i) $q_{i1}^* < q_{i2}^* \le Q_0$.

By Eq. (7), it holds that

$$C_{i1}(q_{i2}^*) \le C_{i2}(q_{i2}^*)$$

Since q_{i1}^* is the minimum point of $C_{i1}(q_i)$, we obtain

$$C_{i1}(q_{i1}^*) < C_{i1}(q_{i2}^*)$$

Therefore,

$$\min C_i(q_i) = C_{i1}(q_{i1}^*)$$

That is, the optimal order quantity per order for retailer i is q_{i1}^*, the corresponding minimum annual cost is

$$C_{i1}(q_{i1}^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_i} + (c_0+\beta\hat{e})d_i \tag{10}$$

and the amount of carbon emissions generated by retailer i is

$$X_{i1}(q_{i1}^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}} + \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i \tag{11}$$

Case (ii) $Q_0 < q_{i1}^* < q_{i2}^*$.

By Eq. (7), it holds that

$$C_{i2}(q_{i1}^*) < C_{i1}(q_{i1}^*)$$

Since q_{i2}^* is the minimum point of $C_{i2}(q_i)$, we obtain

$$C_{i2}(q_{i2}^*) < C_{i2}(q_{i1}^*)$$

Therefore,

$$\min C_i(q_i) = C_{i2}(q_{i2}^*)$$

That is, in Case (ii), the optimal order quantity per order for retailer i is q_{i2}^*, the corresponding minimum annual cost is

$$C_{i2}(q_{i2}^*) = \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_i} + (\alpha c_0+\beta\hat{e})d_i \tag{12}$$

and the amount of carbon emissions generated by retailer i is

$$X_{i2}(q_{i2}^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} + \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i \tag{13}$$

Case (iii) $q_{i1}^* \le Q_0 < q_{i2}^*$.

In this case, we need to compare $C_{i1}(q_{i1}^*)$ and $C_{i2}(q_{i2}^*)$. To this end, let $F = C_{i2}(q_{i2}^*) - C_{i1}(q_{i1}^*)$, and we obtain that

$$F = \sqrt{2(h+\beta\hat{h})d_i}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)} - \sqrt{a+\beta\hat{a}}\right) - (1-\alpha)c_0d_i \tag{14}$$

Taking the first-order and second-order derivatives of F with respect to d_i, we get

$$\frac{dF}{dd_i} = \sqrt{\frac{h+\beta\hat{h}}{2}}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)} - \sqrt{a+\beta\hat{a}}\right)d_i^{-\frac{1}{2}} - (1-\alpha)c_0$$

and

$$\frac{d^2F}{dd_i^2} = -\frac{\sqrt{2(h+\beta\hat{h})}}{4}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)} - \sqrt{a+\beta\hat{a}}\right)d_i^{-\frac{3}{2}} < 0$$

Thus, $\frac{dF}{dd_i}$ is strictly monotonically decreasing in d_i. Note that the condition of $q_{i1}^* \le Q_0 < q_{i2}^*$ is equivalent to $\Delta_1 < d_i \le \Delta_2$, where $\Delta_1 = \frac{(h+\beta\hat{h})Q_0^2}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}$ and $\Delta_2 = \frac{(h+\beta\hat{h})Q_0^2}{2(a+\beta\hat{a})}$. It can be easily verified that

$$\frac{dF}{dd_i}|_{d_i=\Delta_1} = \frac{1}{Q_0}\left(a+\beta\hat{a} - \sqrt{(a+\beta\hat{a})[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}\right) < 0$$

Hence, $\frac{dF}{dd_i} < 0$ always holds. In other words, F is also decreasing in $d_i \in (\Delta_1, \Delta_2]$. Furthermore, it is easy to check that

$$F(\Delta_1) = \frac{\Delta_1}{Q_0}\left(2(a+\beta\hat{a}) + c_0Q_0(1-\alpha) - 2\sqrt{(a+\beta\hat{a})[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}\right) > 0 \tag{15}$$

and

$$F(\Delta_2) = -\frac{\Delta_2}{Q_0}\left(2(a+\beta\hat{a}) + c_0Q_0(1-\alpha) - 2\sqrt{(a+\beta\hat{a})[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}\right) < 0 \tag{16}$$

where the inequalities (15), (16) follow from the fact that

$$\begin{aligned}&[2(a+\beta\hat{a}) + c_0Q_0(1-\alpha)]^2 - 4(a+\beta\hat{a})[a+\beta\hat{a}+c_0Q_0(1-\alpha)]\\ =&[(a+\beta\hat{a}) + a+\beta\hat{a}+c_0Q_0(1-\alpha)]^2 - 4(a+\beta\hat{a})[a+\beta\hat{a}+c_0Q_0(1-\alpha)]\\ =&[c_0Q_0(1-\alpha)]^2\\ >&0\end{aligned}$$

Therefore, there must exist a unique point Δ^* that satisfies $F(\Delta^*) = 0$, where

$$\Delta^* = \frac{2(h+\beta\hat{h})\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)} - \sqrt{a+\beta\hat{a}}\right)^2}{[(1-\alpha)c_0]^2}$$

From the above discussion, we obtain the conclusion that if $\Delta_1 < d_i \le \Delta^*$, then $C_{i1}(q_{i1}^*) \le C_{i2}(q_{i2}^*)$, the optimal order quantity per order for retailer i is q_{i1}^*, the corresponding minimum annual cost is $C_{i1}(q_{i1}^*)$ as shown in Eq. (10), and the amount of carbon emissions generated by retailer i is $X_{i1}(q_{i1}^*)$ as shown in Eq. (11); if $\Delta^* < d_i \le \Delta_2$, then $C_{i1}(q_{i1}^*) > C_{i2}(q_{i2}^*)$, the optimal order quantity per order for retailer i is q_{i2}^*, the corresponding minimum annual cost is $C_{i2}(q_{i2}^*)$ as shown in Eq. (12), and the amount of carbon emissions generated by retailer i is $X_{i2}(q_{i2}^*)$ as shown in Eq. (13).

Notice that the condition of Case (i) is equivalent to $d_i \le \Delta_1$, and the condition of Case (ii) is equivalent to $d_i > \Delta_2$. Combining Case (i)-(iii), we obtain that

1) if $d_i \le \Delta^*$, then the optimal order quantity per order for retailer i is q_{i1}^*, the corresponding minimum annual cost is $C_{i1}(q_{i1}^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_i} + (c_0 + \beta\hat{e})d_i$, and the amount of carbon emissions generated by retailer i is $X_{i1}(q_{i1}^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}} + \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i$;

2) if $d_i > \Delta^*$, then the optimal order quantity per order for retailer i is q_{i2}^*, the corresponding minimum annual cost is $C_{i2}(q_{i2}^*) = \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_i} + (\alpha c_0 + \beta\hat{e})d_i$, and the amount of carbon emissions generated by retailer i is $X_{i2}(q_{i2}^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} + \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{2(h+\beta\hat{h})}} + \hat{e}d_i$.

■

It can be seen from the expression of the discriminant Δ^* that the smaller the threshold Q_0, the smaller the discriminant Δ^*. This indicates that the retailer is more likely to choose q_{i2}^* as the optimal order quantity per order to benefit from the transportation discount when the discount offered by the transporter is more available. However, an increase in carbon tax rate β will increase the discriminant Δ^*. In other words, when the carbon tax penalty imposed by the government is more severe, retailers are more willing to forfeit discounts and choose q_{i1}^* as the optimal order quantity per order to avoid higher carbon tax penalties.

3.2 Optimal Order Policy for a Coalition under Joint Ordering

When the set of retailers $S \subseteq N$ decide to order jointly, they will determine the common ordering cycle length and the total order quantity q_S at each period to minimize their total cost. For convenience, we use d_S to denote the total demand of coalition S, where $d_S = \sum_{i\in S} d_i$.

The goal of coalition S is to minimize their total annual cost, which also involves the following four parts:

1) The ordering fixed cost of placing joint orders which equals to $a \cdot \frac{d_S}{q_S}$;
2) The inventory holding cost which equals to $\frac{hq_S}{2}$;
3) The transportation cost which equals to $c(q_S)d_S$;
4) The carbon tax which equals to βX_S, where $X_S = \hat{a} \cdot \frac{d_S}{q_S} + \frac{\hat{h}q_S}{2} + \hat{e}d_S$ is the total annual carbon emissions of the coalition S.

Hence, the total annual cost for coalition S is given by

$$C_S(q_S) = \begin{cases} C_{S1}(q_S), & \text{if } q_S \le Q_0 \\ C_{S2}(q_S), & \text{if } q_S > Q_0 \end{cases} \tag{17}$$

where

$$C_{S1}(q_S) = (a+\beta\hat{a}) \cdot \frac{d_S}{q_S} + \frac{(h+\beta\hat{h})q_S}{2} + (c_0+\beta\hat{e})d_S$$

and

$$C_{S2}(q_S) = [a+\beta\hat{a}+c_0Q_0(1-\alpha)] \cdot \frac{d_S}{q_S} + \frac{(h+\beta\hat{h})q_S}{2} + (\alpha c_0+\beta\hat{e})d_S$$

Next, we determine the optimal order quantity q_S^* that minimizes the total annual cost $C_S(q_S)$ in Eq. (17) and the corresponding minimum annual cost and annual carbon emissions for coalition S.

Theorem 2 *Given the set of retailers $S \subseteq N$ that order jointly,*

1) *if $d_S \le \Delta^*$, the optimal order quantity per order for coalition S is*

$$q_S^* = \sqrt{\frac{2(a+\beta\hat{a})d_S}{h+\beta\hat{h}}}$$

the corresponding minimum annual cost is

$$C_S(q_S^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S} + (c_0+\beta\hat{e})d_S$$

and the amount of carbon emissions generated by coalition S is

$$X_S(q_S^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2(a+\beta\hat{a})}} + \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_S}{2(h+\beta\hat{h})}} + \hat{e}d_S$$

2) *if $d_S > \Delta^*$, the optimal order quantity per order for coalition S is*

$$q_S^* = \sqrt{\frac{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}{h+\beta\hat{h}}}$$

the corresponding minimum annual cost is

$$C_S(q_S^*) = \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_S} + (\alpha c_0+\beta\hat{e})d_S$$

and the amount of carbon emissions generated by coalition S is

$$X_S(q_S^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} + \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}{2(h+\beta\hat{h})}} + \hat{e}d_S$$

The proof of Theorem 2 is similar to the proof of Theorem 1, and hence we omit it here.

The next theorem indicates that joint ordering reduces the total annual cost for retailers.

Theorem 3 *Joint ordering between multiple retailers results in a reduction in the total annual cost, that is, $C_S(q_S^*) \le \sum_{i\in S} C_i(q_i^*)$.*

Proof. We first divide S into S_1 and S_2 according to whether retailers in the coalition S can enjoy transportation discounts when ordering individually. Specifically, we define

$$S_1 = \{i | d_i \le \Delta^*, i \in S\}$$

and

$$S_2 = \{i | d_i > \Delta^*, i \in S\}$$

where $S = S_1 \cup S_2$, and $S_1 \cap S_2 = \emptyset$.

According to Theorem 1, for all $i \in S_1$, we have

$$C_i(q_i^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_i} + (c_0+\beta\hat{e})d_i \tag{18}$$

and for all $i \in S_2$, we obtain

$$C_i(q_i^*) = \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_i} + (\alpha c_0+\beta\hat{e})d_i \tag{19}$$

The proof proceeds by considering the following two cases, i.e., (i)$S_2 = \emptyset$ and (ii)$S_2 \ne \emptyset$.

Case (i) $S_2 = \emptyset$.

For all $i \in S$, we obtain $d_i \le \Delta^*$. Then, we divided this situation into two cases.

Subcase 1. $d_S \le \Delta^*$.

According to Theorem 1 and Theorem 2, for all $i \in S$ we have

$$C_i(q_i^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_i} + (c_0+\beta\hat{e})d_i \tag{20}$$

and

$$C_S(q_S^*) = \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S} + (c_0+\beta\hat{e})d_S \tag{21}$$

By Eq. (20) and Eq. (21), it is easy to obtain that $C_S(q_S^*) \le \sum_{i\in S} C_i(q_i^*)$.

Subcase 2. $d_S > \Delta^*$.

According to Theorem 2, we obtain

$$\begin{aligned} C_S(q_S^*) &= \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_S} + (\alpha c_0+\beta\hat{e})d_S \\ &\le \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S} + (c_0+\beta\hat{e})d_S \\ &\le \sum_{i\in S}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_i} + (c_0+\beta\hat{e})d_S \\ &= \sum_{i\in S} C_i(q_i^*) \end{aligned}$$

Case (ii) $S_2 \neq \emptyset$.

Since $S_2 \neq \emptyset$, there exists at least one retailer i such that $d_i > \Delta^*$, and thus $d_S > \Delta^*$. According to Theorem 2, we obtain

$$C_S(q_S^*) = \sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_S} + (\alpha c_0 + \beta\hat{e})d_S$$

and

$$C_{S_2}(q_{S_2}^*) = \sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_{S_2}} + (\alpha c_0 + \beta\hat{e})d_{S_2} \tag{22}$$

We discuss this situation in the following two cases.

Subcase 1. $d_{S_1} \leq \Delta^*$.

According to Theorem 2, we obtain

$$C_{S_1}(q_{S_1}^*) = \sqrt{2(a + \beta\hat{a})(h + \beta\hat{h})d_{S_1}} + (c_0 + \beta\hat{e})d_{S_1} \tag{23}$$

It holds that

$$\begin{aligned}
&C_S(q_S^*) - C_{S_2}(q_{S_2}^*) - C_{S_1}(q_{S_1}^*) \\
=&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})}\left(\sqrt{d_S} - \sqrt{d_{S_2}}\right) \\
&- \sqrt{2(a + \beta\hat{a})(h + \beta\hat{h})d_{S_1}} - (1-\alpha)c_0 d_{S_1} \\
\leq&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})}\left(\sqrt{d_{S_1} + \Delta^*} \right. \\
&\left. -\sqrt{\Delta^*}\right) - \sqrt{2(a + \beta\hat{a})(h + \beta\hat{h})}\left(\sqrt{d_{S_1} + \Delta^*} \right. \\
&\left. -\sqrt{\Delta^*}\right) - (1-\alpha)c_0 d_{S_1} \\
=&\sqrt{2(h + \beta\hat{h})(d_{S_1} + \Delta^*)}\left(\sqrt{a + \beta\hat{a} + c_0 Q_0(1-\alpha)} \right. \\
&\left. -\sqrt{a + \beta\hat{a}}\right) \\
&- \sqrt{2(h + \beta\hat{h})\Delta^*}\left(\sqrt{a + \beta\hat{a} + c_0 Q_0(1-\alpha)} \right. \\
&\left. -\sqrt{a + \beta\hat{a}}\right) + (1-\alpha)c_0\Delta^* - (1-\alpha)c_0(d_{S_1} + \Delta^*) \\
=&\sqrt{2(h + \beta\hat{h})(d_{S_1} + \Delta^*)}\left(\sqrt{a + \beta\hat{a} + c_0 Q_0(1-\alpha)} \right. \\
&\left. -\sqrt{a + \beta\hat{a}}\right) - (1-\alpha)c_0(d_{S_1} + \Delta^*) \\
&- (1-\alpha)c_0(d_{S_1} + \Delta^*) \\
<&0
\end{aligned} \tag{24}$$

The last equality holds because F in Eq. (14) satisfies $F(\Delta^*) = 0$, and the last inequality holds since F is decreasing in d_i.

By Eq. (18) and Eq. (23), we have

$$C_{S_1}(q_{S_1}^*) \leq \sum_{i \in S_1} C_i(q_i^*) \tag{25}$$

By Eq. (19) and Eq. (22), we have

$$C_{S_2}(q_{S_2}^*) \leq \sum_{i \in S_2} C_i(q_i^*) \tag{26}$$

By Eq. (24), Eq. (25) and Eq. (26), it can be easily verified that

$$\begin{aligned}
C_S(q_S^*) &\leq C_{S_1}(q_{S_1}^*) + C_{S_2}(q_{S_2}^*) \\
&\leq \sum_{i \in S_1} C_i(q_i^*) + \sum_{i \in S_2} C_i(q_i^*) \\
&= \sum_{i \in S} C_i(q_i^*)
\end{aligned}$$

Subcase 2. $d_{S_1} > \Delta^*$.

According to Theorem 2, we obtain

$$\begin{aligned}
&C_{S_1}(q_{S_1}^*) \\
=&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_{S_1}} \\
&+ (\alpha c_0 + \beta\hat{e})d_{S_1}
\end{aligned}$$

It holds that

$$\begin{aligned}
&C_S(q_S^*) \\
=&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_S} \\
&+ (\alpha c_0 + \beta\hat{e})d_S \\
\leq&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_{S_1}} \\
&+ \sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})d_{S_2}} \\
&+ (\alpha c_0 + \beta\hat{e})d_S \\
=&C_{S_1}(q_{S_1}^*) + C_{S_2}(q_{S_2}^*)
\end{aligned} \tag{27}$$

By Eq. (25), Eq. (26) and Eq. (27), we obtain

$$\begin{aligned}
C_S(q_S^*) &\leq C_{S_1}(q_{S_1}^*) + C_{S_2}(q_{S_2}^*) \\
&\leq \sum_{i \in S_1} C_i(q_i^*) + \sum_{i \in S_2} C_i(q_i^*) \\
&= \sum_{i \in S} C_i(q_i^*)
\end{aligned}$$

To sum up, $C_S(q_S^*) \leq \sum_{i\in S} C_i(q_i^*)$ holds. ■

In the following, we are interested in investigating whether joint ordering reduces the total amount of carbon emissions generated by retailers. Unfortunately, the answer is negative, as shown by the example below.

Example 1 *Consider a supply chain consisting of one supplier, one transporter, and two retailers $N = \{1, 2\}$ who are located in the same region. The two retailers order the same product from the supplier and transport it through the transporter. The transporter provides a half-price discount for products transported in excess of 1,000 units. In addition, retailers are required to pay carbon taxes to the government for the carbon emissions generated during replenishment activities. The parameters for this example are listed in Table 1.*

Table 1 The Parameter Settings for Example 1

a	$\hat{a}$	h	$\hat{h}$	c_0	Q_0	α	β	$\hat{e}$	d_1	d_2
10	1	1	0.5	2	1000	0.5	2	1	3200	3500

By Theorem 1, it is easy to calculate the discriminant $\Delta^* = 3214.4$, the amount of carbon emissions generated by retailer 1 is $X_1(q_1^*) = 3265.3$, and the amount of carbon emissions generated by retailer 2 is $X_2(q_2^*) = 3972.4$. By Theorem 2, we calculate the amount of carbon emissions generated by coalition N is 7353.6. Hence, $X_N(q_N^*) = 7353.6 > X_1(q_1^*) + X_2(q_2^*) = 7237.7$. This shows that joint ordering between retailers 1 and 2 increases the total amount of carbon emissions.

From this example, we observe that joint ordering between multiple retailers does not necessarily decrease the total amount of carbon emissions, but it still reduces the total cost. However, enterprises with a focus on low-carbon practices also aim for cooperation to reduce carbon emissions, thus achieving the effect of killing two birds with one stone. The following theorem provides a sufficient condition under which joint ordering reduces carbon emissions.

Theorem 4 *Joint ordering between multiple retailers leads to a reduction in the total amount of carbon emissions if*

$$\sqrt{1 + \frac{c_0 Q_0(1-\alpha)}{a + \beta\hat{a}}} \leq \frac{\sum_{i\in S}\sqrt{d_i}}{\sqrt{d_S}}, s > 1 \tag{28}$$

Proof. We divide S into $S_1 = \{i|d_i \leq \Delta^*, i \in S\}$ and $S_2 = \{i|d_i > \Delta^*, i \in S\}$, where $S = S_1 \cup S_2$, and $S_1 \cap S_2 = \emptyset$. According to Theorem 1 and Theorem 2, we obtain

$$\begin{aligned}\sum_{i\in S_1} X_i(q_i^*) = \sum_{i\in S_1} \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}} \\ + \sum_{i\in S_1} \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}} + \hat{e}d_{S_1}\end{aligned} \tag{29}$$

and

$$\begin{aligned}\sum_{i\in S_2} X_i(q_i^*) = \sum_{i\in S_2} \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} \\ + \sum_{i\in S_2} \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{2(h+\beta\hat{h})}} \\ + \hat{e}d_{S_2}\end{aligned} \tag{30}$$

The proof proceeds by considering the following two cases, i.e., (i)$d_S \leq \Delta^*$ and (ii)$d_S > \Delta^*$.

Case (i) $d_S \leq \Delta^*$.

If $d_S \leq \Delta^*$, then for any $i \in S$ satisfies $d_i \leq \Delta^*$. By Theorem 2, we have

$$X_S(q_S^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2(a+\beta\hat{a})}} + \hat{h}\sqrt{\frac{(a+\beta\hat{a})d_S}{2(h+\beta\hat{h})}} + \hat{e}d_S$$

Hence, we have

$$\begin{aligned}
&\sum_{i\in S} X_i(q_i^*)\\
&=\sum_{i\in S}\hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}}+\sum_{i\in S}\hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}}+\hat{e}d_S\\
&\geq\hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2(a+\beta\hat{a})}}+\hat{h}\sqrt{\frac{(a+\beta\hat{a})d_S}{2(h+\beta\hat{h})}}+\hat{e}d_S\\
&=X_S(q_S^*)
\end{aligned}$$

Case (ii) $d_S > \Delta^*$.

According to Theorem 2, we have

$$X_S(q_S^*) = \hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}} + \hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}{2(h+\beta\hat{h})}}+\hat{e}d_S \tag{31}$$

By Eq. (29), (30) and (31), we obtain

$$\begin{aligned}
&X_S(q_S^*)-\sum_{i\in S_1}X_i(q_i^*)-\sum_{i\in S_2}X_i(q_i^*)\\
&=\hat{a}\sqrt{\frac{(h+\beta\hat{h})d_S}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}}\\
&+\hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}{2(h+\beta\hat{h})}}+\hat{e}d_S\\
&-\Big(\sum_{i\in S_1}\hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2(a+\beta\hat{a})}}+\sum_{i\in S_1}\hat{h}\sqrt{\frac{(a+\beta\hat{a})d_i}{2(h+\beta\hat{h})}}+\hat{e}d_{S_1}\Big)\\
&-\Big(\sum_{i\in S_2}\hat{a}\sqrt{\frac{(h+\beta\hat{h})d_i}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}}\\
&+\sum_{i\in S_2}\hat{h}\sqrt{\frac{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_i}{2(h+\beta\hat{h})}}+\hat{e}d_{S_2}\Big)\\
&=\hat{a}\sqrt{\frac{(h+\beta\hat{h})}{2}}\Big(\frac{\sqrt{d_S}-\sum_{i\in S_2}\sqrt{d_i}}{\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}}\\
&-\frac{\sum_{i\in S_1}\sqrt{d_i}}{\sqrt{(a+\beta\hat{a})}}\Big)\\
&+\frac{\hat{h}}{\sqrt{2(h+\beta\hat{h})}}\Big(\sqrt{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}\\
&-\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}(\sum_{i\in S_2}\sqrt{d_i})\\
&-\sqrt{a+\beta\hat{a}}(\sum_{i\in S_1}\sqrt{d_i})\Big)\\
&\leq\hat{a}\sqrt{\frac{(h+\beta\hat{h})}{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)]}}\Big(\sqrt{d_S}-\sum_{i\in S}\sqrt{d_i}\Big)\\
&+\frac{\hat{h}}{\sqrt{2(h+\beta\hat{h})}}\Big(\sqrt{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}\\
&-\sqrt{a+\beta\hat{a}}(\sum_{i\in S}\sqrt{d_i})\Big)
\end{aligned}$$

If the following inequality holds

$$\sqrt{[a+\beta\hat{a}+c_0Q_0(1-\alpha)]d_S}\leq\sqrt{a+\beta\hat{a}}(\sum_{i\in S}\sqrt{d_i})$$

i.e., $\sqrt{1+\frac{c_0Q_0(1-\alpha)}{a+\beta\hat{a}}}\leq\frac{\sum_{i\in S}\sqrt{d_i}}{\sqrt{d_S}}$, then $X_S(q_S^*)-\sum_{i\in S_1}X_i(q_i^*)-\sum_{i\in S_2}X_i(q_i^*)\leq 0$. ■

Indeed, Eq. (28) provides a sufficient condition for joint ordering to reduce the total carbon emissions. It can be seen that the condition in Eq. (28) always holds when the transporter does not offer the transport discount ($\alpha = 1$). In other words, the presence of discounts potentially increases total carbon emissions for retailers. Another finding is that when the costs and carbon emissions associated with each initiated order are relatively high, enterprises can more easily achieve the dual goals of reducing both costs and carbon emissions through cooperation. Moreover, the presence of transportation discounts makes this condition even more stringent.

From our previous analysis, we know that it is beneficial for retailers to cooperate by placing joint orders. As a result, a new problem naturally arises: how to share the total cost among retailers? We discuss this problem within the framework of cooperative game theory in the next section.

4. The Inventory Game with Transportation Discount under Carbon Tax Policy

In this section, we introduce a new set of cost TU-games that correspond to the inventory problem with transportation discount under carbon tax policy. We denote the inventory game corresponding to the situation described above by a pair (N, C), where $N = \{1, 2, \cdots, n\}$ is the finite set of all retailers and $C : 2^N \longrightarrow \mathbb{R}$ is the characteristic function of the game with $C(\emptyset) = 0$. For every coalition $S \subseteq N$, $C(S)$ is interpreted as the minimal cost per year of co-operating retailers in S. For any $S \subseteq N$, it holds that

$$C(S) = \begin{cases} \sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S} + (c_0+\beta\hat{e})d_S, \\ \text{if } d_S \leq \Delta^* \\ \sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_S} \\ +(\alpha c_0+\beta\hat{e})d_S, \ \text{ if } d_S > \Delta^* \end{cases} \tag{32}$$

Concavity is one of the most important properties of cooperative games. If the inventory game with transportation discount under carbon tax policy is concave, then this game has a nonempty core. We prove this result in the following proposition.

Proposition 1 *The inventory game with transportation discount under carbon tax policy (N, C) is a concave game.*

The proof of Proposition 1 is given in Appendix A.

Proposition 1 shows that the corresponding game has a nonempty core. Constructing an allocation in the core is one of the main concerns of cooperative game theory. In the following, we propose an allocation rule to share the total annual cost in the grand coalition and prove that the rule belongs to the core.

Based on the proportion of each retailer's demand in the total demand of coalition N, we propose a cost allocation rule φ which defined as

$$\varphi_i(C) = \frac{d_i}{d_N}C(N)$$
$$= \begin{cases} \frac{d_i}{\sqrt{d_N}}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})} + (c_0+\beta\hat{e})d_i, \\ \text{if } d_N \leq \Delta^* \\ \frac{d_i}{\sqrt{d_N}}\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})} \\ +(\alpha c_0+\beta\hat{e})d_i, \ \text{ if } d_N > \Delta^* \end{cases}$$

Theorem 5 *The cost allocation rule φ lies in the core of the inventory game with transportation discount under carbon tax policy (N, C).*

Proof. The proof simply involves checking whether $\sum_{i\in S}\varphi_i(C) \leq C(S)$ holds for all $S \subseteq N$.

If $d_N \leq \Delta^*$, then $d_S \leq \Delta^*$ holds for any $S \subseteq N$. It is easily verified that

$$\begin{aligned} &\sum_{i\in S}\varphi_i(C) \\ =&\frac{d_S}{\sqrt{d_N}}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})} + (c_0+\beta\hat{e})d_S \\ \leq&\frac{d_S}{\sqrt{d_S}}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})} + (c_0+\beta\hat{e})d_S \\ =&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S} + (c_0+\beta\hat{e})d_S \\ =&C(S) \end{aligned}$$

If $d_N > \Delta^*$, then there exists the following two cases, i.e., (i)$d_S > \Delta^*$; (ii)$d_S \leq \Delta^*$.

Case (i) $d_S > \Delta^*$.

In this case, we obtain

$$\begin{aligned}
&\sum_{i\in S}\varphi_i(C)\\
=&\frac{d_S}{\sqrt{d_N}}\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\\
&+(\alpha c_0+\beta\hat{e})d_S\\
\leq&\frac{d_S}{\sqrt{d_S}}\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\\
&+(\alpha c_0+\beta\hat{e})d_S\\
=&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_S}\\
&+(\alpha c_0+\beta\hat{e})d_S\\
=&C(S)
\end{aligned}$$

Case (ii) $d_S \leq \Delta^*$.

In this case, it holds that

$$\begin{aligned}
&\sum_{i\in S}\varphi_i(C)\\
=&\frac{d_S}{\sqrt{d_N}}\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\\
&+(\alpha c_0+\beta\hat{e})d_S\\
\leq&\frac{d_S}{\sqrt{d_N}}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}+(c_0+\beta\hat{e})d_S\\
\leq&\frac{d_S}{\sqrt{d_S}}\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}+(c_0+\beta\hat{e})d_S\\
=&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S}+(c_0+\beta\hat{e})d_S\\
=&C(S)
\end{aligned}$$

Therefore, for all $S \subseteq N$, it holds that $\sum_{i\in S}\varphi_i \leq C(S)$, which completes the proof. ■

Next, we use an example to show how the allocation rule shares the total annual cost when multiple retailers cooperate.

Example 2 *Consider a supply chain consisting of one supplier, one transporter, and three retailers $N = \{1,2,3\}$ who are located in the same region. The three retailers order the same product from the supplier and transport it through the transporter. The transporter provides a half-price discount for products transported in excess of 1,000 units. In addition, retailers are required to pay carbon tax to the government for the carbon emissions generated during replenishment activities. The parameter settings are given in Table 2.*

Table 2 The Parameter Settings for Example 2

a	$\hat{a}$	h	$\hat{h}$	c_0	Q_0	α	β	$\hat{e}$	d_1	d_2	d_3
1000	100	1	0.5	2	1000	0.5	2	1	1000	2000	3000

By Theorem 1, it is easy to calculate the discriminant $\Delta^* = 601.5$. Then, we determine d_S, i.e., the total demand for coalition S, and the results are shown in Table 3.

Table 3 The d_S

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	N
d_S	1000	2000	3000	3000	4000	5000	6000

For all $S \subseteq N$ with $s > 1$, it can be checked that the condition in Eq. (28) holds. Thus, cooperation among any two or more players in the grand coalition N leads to lower costs and lower carbon emissions. By Eq. (32), we calculate $C(S)$ for all $S \subseteq N, S \neq \emptyset$. We describe the inventory game with transportation discount under carbon tax policy (N, C) in Table 4.

Table 4 Characteristic Values in Example 2

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	N
$C(S)$	5966.5	10195.2	14138.1	14138.1	17933.0	21633.2	25266.4

It is easy to check that $\varphi = (4211.1, 8422.1, 12633.2)$. For all $i \in N$, it holds that $\varphi_i < C(\{i\})$, and $\varphi_1 + \varphi_2 = 12633.2 < C(\{1,2\}) = 14138.1$, $\varphi_1 + \varphi_3 = 16844.3 < C(\{1,3\}) = 17933.0$, $\varphi_2 + \varphi_3 = 21055.3 < C(2,3) = 21633.2$, $\varphi_1 + \varphi_2 + \varphi_3 = 25266.4 = C(N)$. Thus, φ belongs to the core of the game (N, C).

5. Conclusion

In this paper, we mainly study the cost allocation problem of inventory system with transportation discount under carbon tax policy. We first model the inventory system with transportation discount under carbon tax policy, and then identify the optimal order quantity for the individual ordering retailer (see Theorem 1) and the joint ordering coalition (see Theorem 2), respectively. The results show that placing joint orders can reduce the total cost for retailers (see Theorem 3) but may increase total carbon emissions (see Example 1). The sufficient condition provided in Theorem 4 indicates that when the costs and carbon emissions associated with each order initiated are relatively high, enterprises can achieve dual objectives (both carbon emission reduction and cost reduction) through joint ordering. Then, we introduce an inventory game with transportation discount under carbon tax policy and verify that this game is concave (see Proposition 1). Finally, we propose a cost allocation rule based on the proportion of each retailer's demand in the total demand of the coalition, and show that this rule lies in the core of the game (see Theorem 5).

We acknowledge that the model presented in this paper is only a simplified model of a complex real-world problem. For future research, this work can be extended in several directions. First, this study considers the effect of the quantity of products transported on the cost of transportation. In addition to the quantity transported, the distance transported also affects the cost of transportation (Fiestras-Janeiro et al. 2011b 2013). Researchers could further consider the comprehensive impact of the quantity and distance of the transported products on the transportation cost. Second, the carbon tax price is an exogenous given parameter considered in this paper as in many studies (Shi et al. 2020, Zeng et al. 2022). However, carbon tax rates are not constant in real economic markets (Heutel 2012, Annicchiarico and Di Dio 2015, Chan 2020). It is interesting to explore the pricing problem of the tax charged by the government on each unit of carbon emissions. Finally, this paper considers a special incremental quantity discount scheme with one threshold. Although this particular discount structure is often observed in real-world settings, the general incremental quantity discount scheme with wide application in supply chain management is of more practical interest (Taleizadeh et al. 2015, Tamjidzad et al. 2017). A challenging question for future research is to extend our research to the general incremental quantity discount scheme [3].

Appendix A

Proof of Proposition 1. We only need to show for all $i \in N$ and all $S \subseteq T \subseteq N \backslash \{i\}$, it holds that

$$C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T)$$

The proof proceeds by considering the following three cases, i.e., (i) $d_{S\cup\{i\}} > d_S > \Delta^*$; (ii) $d_{S\cup\{i\}} > \Delta^* \geq d_S$; (iii) $\Delta^* \geq d_{S\cup\{i\}} > d_S$.

Case (i) $d_{S\cup\{i\}} > d_S > \Delta^*$.

In this case, $d_{T\cup\{i\}} > d_T > \Delta^*$. According to Eq. (32), we obtain

$$\begin{aligned} &C(S \cup \{i\}) - C(S) \\ =&\sqrt{2[a + \beta\hat{a} + c_0 Q_0(1-\alpha)](h + \beta\hat{h})}(\sqrt{d_{S\cup\{i\}}} - \sqrt{d_S}) \\ &+ (\alpha c_0 + \beta\hat{e})d_i \end{aligned} \tag{A.1}$$

and

$$\begin{aligned}&C(T\cup\{i\})-C(T)\\=&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}(\sqrt{d_{T\cup\{i\}}}-\sqrt{d_T})\\&+(\alpha c_0+\beta\hat{e})d_i\end{aligned}\tag{A.2}$$

Since $\sqrt{x}$ is a concave function, it holds that $C(S\cup\{i\})-C(S)\geq C(T\cup\{i\})-C(T)$.

Case (ii) $d_{S\cup\{i\}}>\Delta^*\geq d_S$.

According to Eq. (32), we obtain

$$\begin{aligned}&C(S\cup\{i\})-C(S)\\=&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_{S\cup\{i\}}}\\&-\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_S}+c_0(\alpha d_{S\cup\{i\}}-d_S)\\&+\beta\hat{e}d_i\\\geq&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S})\\&+(\alpha c_0+\beta\hat{e})d_i\end{aligned}\tag{A.3}$$

Next, we divide this situation into the following two cases.

Subcase 1. $d_{T\cup\{i\}}>d_T>\Delta^*$

By Eq. (A.1) to (A.3), we obtain

$$\begin{aligned}&C(S\cup\{i\})-C(S)\\\geq&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S})\\&+(\alpha c_0+\beta\hat{e})d_i\\\geq&C(T\cup\{i\})-C(T)\end{aligned}$$

Subcase 2. $d_{T\cup\{i\}}>\Delta^*\geq d_T$

According to Eq. (32), we obtain

$$\begin{aligned}&C(T\cup\{i\})-C(T)\\=&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_{T\cup\{i\}}}\\&-\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_T}+c_0(\alpha d_{T\cup\{i\}}-d_T)\\&+\beta\hat{e}d_i\end{aligned}\tag{A.4}$$

By Eq. (A.3) and Eq. (A.4), we get

$$\begin{aligned}&C(S\cup\{i\})-C(S)-[C(T\cup\{i\})-C(T)]\\=&-\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\Big(\sqrt{d_{T\cup\{i\}}}\\&-\sqrt{d_{S\cup\{i\}}}\Big)+\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}(\sqrt{d_T}-\sqrt{d_S})\\&+(1-\alpha)c_0d_{T\setminus S}\\\geq&-\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}(\sqrt{d_{T\setminus S}+\Delta^*}\\&-\sqrt{\Delta^*})+\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\Big(\sqrt{d_{T\setminus S}+\Delta^*}\\&-\sqrt{\Delta^*}\Big)+(1-\alpha)c_0d_{T\setminus S}\\=&-\sqrt{2(h+\beta\hat{h})(d_{T\setminus S}+\Delta^*)}\Big(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\\&-\sqrt{a+\beta\hat{a}}\Big)\\&+\sqrt{2(h+\beta\hat{h})\Delta^*}\Big(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\\&-\sqrt{a+\beta\hat{a}}\Big)-(1-\alpha)c_0\Delta^*+(1-\alpha)c_0(d_{T\setminus S}+\Delta^*)\\=&-\sqrt{2(h+\beta\hat{h})(d_{T\setminus S}+\Delta^*)}\Big(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\\&-\sqrt{a+\beta\hat{a}}\Big)+(1-\alpha)c_0(d_{T\setminus S}+\Delta^*)\\\geq&0\end{aligned}$$

The last equality holds because F in Eq. (14) satisfies $F(\Delta^*)=0$, and the last inequality holds since $-F$ is increasing in d_i.

Case (iii) $\Delta^*\geq d_{S\cup\{i\}}>d_S$.

According to Eq. (32), we obtain

$$\begin{aligned}&C(S\cup\{i\})-C(S)\\=&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S})\\&+(c_0+\beta\hat{e})d_i\end{aligned}\tag{A.5}$$

We further discuss this situation by analyzing the following three cases.

Subcase 1. $d_{T\cup\{i\}}>d_T>\Delta^*$

By subtracting Eq. (A.5) to Eq. (A.2), we obtain

$$
\begin{aligned}
&C(S\cup\{i\})-C(S)-\left[C(T\cup\{i\})-C(T)\right]\\
=&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S}\right)\\
&-\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\left(\sqrt{d_{T\cup\{i\}}}\right.\\
&\left.-\sqrt{d_T}\right)+(1-\alpha)c_0d_i\\
\geq&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_i+\Delta^*}-\sqrt{\Delta^*}\right)\\
&-\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})}\left(\sqrt{d_i+\Delta^*}\right.\\
&\left.-\sqrt{\Delta^*}\right)+(1-\alpha)c_0d_i\\
=&-\sqrt{2(h+\beta\hat{h})(d_i+\Delta^*)}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\right.\\
&\left.-\sqrt{a+\beta\hat{a}}\right)\\
&+\sqrt{2(h+\beta\hat{h})\Delta^*}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\right.\\
&\left.-\sqrt{a+\beta\hat{a}}\right)-(1-\alpha)c_0\Delta^*+(1-\alpha)c_0(d_i+\Delta^*)\\
=&-\sqrt{2(h+\beta\hat{h})(d_i+\Delta^*)}\left(\sqrt{a+\beta\hat{a}+c_0Q_0(1-\alpha)}\right.\\
&\left.-\sqrt{a+\beta\hat{a}}\right)+(1-\alpha)c_0(d_i+\Delta^*)\\
\geq&0
\end{aligned}
$$

Subcase 2. $d_{T\cup\{i\}}>\Delta^*\geq d_T$

According to Eq. (32), we obtain

$$
\begin{aligned}
&C(T\cup\{i\})-C(T)\\
=&\sqrt{2[a+\beta\hat{a}+c_0Q_0(1-\alpha)](h+\beta\hat{h})d_{T\cup\{i\}}}\\
&+(\alpha c_0+\beta\hat{e})d_{T\cup\{i\}}\\
&-\left(\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_T}+(c_0+\beta\hat{e})d_T\right)\\
\leq&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_{T\cup\{i\}}}+(c_0+\beta\hat{e})d_{T\cup\{i\}}\\
&-\left(\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})d_T}+(c_0+\beta\hat{e})d_T\right)\\
=&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_{T\cup\{i\}}}-\sqrt{d_T}\right)\\
&+(c_0+\beta\hat{e})d_i\\
\leq&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S}\right)\\
&+(c_0+\beta\hat{e})d_i\\
=&C(S\cup\{i\})-C(S)
\end{aligned}
$$

Subcase 3. $\Delta^*\geq d_{T\cup\{i\}}>d_T$

According to Eq. (32), we obtain

$$
\begin{aligned}
&C(T\cup\{i\})-C(T)\\
=&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_{T\cup\{i\}}}-\sqrt{d_T}\right)\\
&+(c_0+\beta\hat{e})d_i\\
\leq&\sqrt{2(a+\beta\hat{a})(h+\beta\hat{h})}\left(\sqrt{d_{S\cup\{i\}}}-\sqrt{d_S}\right)\\
&+(c_0+\beta\hat{e})d_i\\
=&C(S\cup\{i\})-C(S)
\end{aligned}
$$

Therefore, the inventory game with transportation discount under carbon tax policy (N, C) is a concave game. ■

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Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Conflict of interest

The authors declare no conflict of interest.

Endnotes

[1] Among various carbon emission regulations, the carbon tax policy is the most widely used policy, which has been implemented in many countries (e.g., Norway, Sweden, etc.) (Stavins 2019).

[2] This type of discount is used in the context of inventory problems when considering purchase cost (Li et al. 2021).

[3] We thank one of the reviewers for suggesting the general incremental quantity discount scheme to us.

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