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A Novel Rolling and Fractional-ordered Grey System Model and Its Application for Predicting Industrial Electricity Consumption

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Abstract. Accurate and reasonable prediction of industrial electricity consumption is of great significance for promoting regional green transformation and optimizing the energy structure. However, the regional power system is complicated and uncertain, affected by multiple factors including climate, population and economy. This paper incorporates structure expansion, parameter optimization and rolling mechanism into a system forecasting framework, and designs a novel rolling and fractional-ordered grey system model to forecast the industrial electricity consumption, improving the accuracy of the traditional grey models. The optimal fractional order is obtained by using the particle swarm optimization algorithm, which enhances the model adaptability. Then, the proposed model is employed to forecast and analyze the changing trend of industrial electricity consumption in Fujian province. Experimental results show that industrial electricity consumption in Fujian will maintain an upward growth and it is expected to 186.312 billion kWh in 2026. Compared with other seven benchmark prediction models, the proposed grey system model performs best in terms of both simulation and prediction performance metrics, providing scientific reference for regional energy planning and electricity market operation.

Keywords: Electricity consumption, grey system theory, prediction model, fractional order

1. Introduction

1.1 Background

Electricity is a secure, high-quality, clean energy source, has permeated all sectors of national economy and stands as a pivotal energy pillar for China's economic development. Presently, China has attained the distinction of being the world's largest energy producer and consumer. As an efficient and convenient secondary energy, electricity plays a significant role in optimizing the energy structure, fostering industrial development and ensuring the stability of people's lives (Zhang et al. 2017). Numerous studies have elucidated a long-term, intimate interplay between electricity consumption and regional economic growth, mutually reinforcing one other (Xu et al. 2022). Particularly within the industrial sector, electricity consumption serves as the "barometer" and "weathervane" for the harmonious development of national economy.

China's economy is undergoing a transition from rapid growth to high-quality development, which requires further optimization of the industrial structure and the power market.

However, some provinces suffer from frequent power rationing and imbalances in electricity supply and demand due to regional disparities in China's development. Accurate prediction of industrial electricity consumption (IEC) is crucial for monitoring the energy market and facilitating industrial restructuring (Tang et al. 2019). This paper aims to build a scientific and reasonable IEC prediction model, which is also a hot academic topic in various fields.

In addition, we select the Fujian province as the case study to assess the effectiveness of prediction models. Fujian province was chosen as the case study for several reasons. First, Fujian province is one of the fastest-growing provinces in China, with an average annual GDP growth rate of 8.4% from 2010 to 2020 and a GDP of 5.31 trillion yuan in 2022, ranking seventh among all provinces. Second, Fujian province is also a pioneer in developing the digital economy and promoting industrial transformation and upgrading. Fujian province has a high proportion of industrial electricity consumption, accounting for 75.8% of the total electricity consumption in 2021, which reflects its strong industrial development. Third, Fujian province has experienced frequent power rationing and imbalances in electricity supply and demand since September 2021, which poses a great challenge for its economic and social development. During peak demand periods, inadequate electricity supply can result in blackouts or brownouts, causing disruptions in various industries such as manufacturing, healthcare, and information technology. These disruptions can lead to significant financial losses for businesses, reduced productivity, and even jeopardize public safety.

Consequently, accurate prediction and real-time monitoring of electricity consumption become indispensable for aligning electricity supply with industry demand. By developing robust forecasting models, policymakers, energy planners, and businesses can anticipate future electricity needs, optimize resource allocation, and ensure a stable and efficient energy market. This, in turn, fosters industrial growth, attracts investments, and enhances overall economic development. This paper will propose a novel grey system model for predicting and analyzing industrial electricity consumption in Fujian province, based on its current situation. The paper aims to offer some insights for the advancement of the intelligent power grid and the augmentation of sustainable energy structure in Fujian.

1.2 Literature Review

Numerous studies have demonstrated that industrial electricity consumption is affected by many complicated factors, encompassing regional economic development level, urban size, industrial activities and policy intervention (Yu et al. 2019). Consequently, the entire power system exhibits the characteristics of complexity and uncertainty, posing significant challenges in electricity consumption prediction. In recent years, scholars have conducted extensive investigations into forecasting approaches for electricity consumption. These methods can be categorized into four main groups, traditional statistical models, machine learning techniques, grey prediction models and hybrid approaches.

Statistical models offer specific advantages in analyzing the long-term development trends of the electricity consumption, and can effectively capture the potential time series changes based on extensive historical data. Some commonly used statistical models include Auto-Regressive Integrated Moving-Average (ARIMA) (Jamil 2020), exponential smoothing (Deng et al. 2021) and regression analysis models (Frondel et al. 2019, Cui et al. 2021, Tang et al. 2021). In general, these models are adept at identifying patterns and relationships within the data, enabling them to provide valuable insights into the long-term trends of electricity consumption. However, statistical models often require a substantial amount of observation data to ensure high accuracy and robustness in their predictions. When the power system is subject to numerous interference factors, the prediction accuracy may be compromised due to the increase in feature dimensions.

With the rapid development of big data in recent years, machine learning techniques have garnered a lot of interest from academics. The nonlinear relationship between electricity consumption data and its numerous influencing factors can be captured by various intelligent algorithms (He et al. 2019, Gul et al. 2021, Pappas et al. 2010, Albuquerque et al. 2022). Jiang et al. (2020) designed a novel composite electricity demand forecasting framework based support vector machine (SVM) to identify and measure any seasonal relationship that exists in electricity demand data. Laurinec and Lucka (2019) proposed a new interpretable approach for multiple data streams clustering used for the improvement of forecasting accuracy of electricity consumption. It is widely known that sufficient historical information is helpful for researchers to mine and explore the operation rules of a system. Therefore, statistical models and machine learning methods can get satisfactory results when modeling object is a system with numerous sample data or obvious distribution. However, they cannot perform well when facing limited historical data.

The grey prediction model, initially proposed by Professor Deng, represents a crucial research topic within the realm of grey system theory. Grey model demonstrates remarkable explanatory power when it comes to predicting uncertain systems with limited data and incomplete information. Consequently, it has fount extensive applications in various fields, including energy (Zhou et al. 2021), agriculture (Ou 2012), economy (Ding et al. 2020), environment (Zeng et al. 2021) and industrial engineering (Zhou et al. 2022a).

In order to leverage the strengths of different techniques, some scholars have developed hybrid predictive models that integrate the advantages of multiple approaches (Hu 2017, Wang and Jiang 2019). For instance, Liu et al. (2016) proposed a combined forecast framework with grey forecasting method and neural network back propagation model. The results showed that forecasting accuracy on energy consumption is greater than grey models separately. Xu et al. (2015) proposed a GM-autoregressive moving average model based on the Hodrick-Prescott (HP) filter method to forecast energy consumption.

From aforementioned studies, we found that grey prediction models demonstrated superior performance in modeling uncertain systems with limited data and incomplete information compared to alternative methods. The power energy system, known for its large and complex nature, possesses typical grey characteristics characterized by non-linear data changing patterns. Therefore, the construction of a grey model with enhanced prediction performance and the continuous improvement of grey system theory have become common scientific challenges. Existing studies have primarily focused on four main aspects, including model structure expansion, parameter optimization, incorporation of rolling mechanism and combined prediction models. The improvements for predicting energy system can be summarized in Table 1.

The refinement of these grey prediction models has been achieved through distinct avenues, encompassing enhancements in model structure, parameter solving, prediction mechanisms, and the amalgamation of advantages. These advancements have significantly elevated the prediction accuracy, thereby highlighting the adaptability of grey system models. Nevertheless, traditional grey univariate prediction models such as GM(1,1) and DGM(1,1), are increasingly challenged in their ability to address the complexities of engineering applications, such as nonlinear growth patterns and unbalanced information. Hence, scholars must prioritize the integration and unification of various improvement methodologies in their research endeavors. The electricity system is an intricate grey system, subject to the influences of diverse factors such as population structure, regional economic development stage, and electricity prices. It exhibits distinct 'grey' characteristics, rendering the grey system theory particularly suitable for electricity system analysis. However, the current research on electricity consumption prediction lacks a unified framework to enhance prediction performance. Most grey models have primarily focused on specific facets, such as parameter optimization, representing mere extensions of the traditional GM(1,1) approach. Yet, achieving heightened accuracy necessitates the integration of systematic concepts and global optimization into the construction of a well-founded grey system model. In reality, there exists no conflict between structure expansion, parameter optimization, and rolling mechanisms, as they encompass various aspects of grey modeling.

In this study, we will propose a novel grey forecasting modeling framework that integrates structure expansion, fractional-order, and rolling mechanism to enhance the accuracy of the grey model. By combining these different aspects of grey modeling, we aim to improve the accuracy and reliability of the prediction results. The developed model is called the Rolling and Fractional-ordered Grey Model, abbreviated as RFGM(1,r). '1' and 'r' denote the univariate prediction model and r-order accumulating generation operator, respectively.

This paper presents novelties in two aspects. First, in response to the limitations of the basic grey model, we have developed a novel grey modeling approach that incorporates three key elements: model structure expansion, parameter optimization, and rolling mechanism. By considering these components, our proposed approach offers an improved and more comprehensive solution for electricity consumption forecasting. Second,

Table 1 Summary of Research about Optimized Grey Prediction Models

Modelling Aspects	Model Name	Interpretation	Application Scope
Structure Expansion	FPGM(1,1,t_α) (Liu et al. 2020), UFNGBM (1,1) (Pu et al. 2021), FAGMO(1,1,k) (Wu et al. 2019), NGM(1,1,k) (Cui et al. 2013), DGM(1,1,α) (Javed and Cudjoe 2022), CCRGM(1,1) (Luo et al. 2020)	Nonlinear grey Bernoulli models and polynomial models	The grey system with nonlinear trend
	N_Verhulst (Zeng et al. 2020a), FD-Verhulst (Zhou et al. 2022), GMTGP (Zeng et al. 2020)	Structure improvement of traditional grey Verhulst model	The grey system with a saturated S-shape
	AWBO-DGGM(1,1) (Chen et al. 2021), SGM(1,1) (Wang et al. 2018), SIOGM (Zhou et al. 2022b)	Consider the seasonal fluctuations in the grey modeling process	The grey system with seasonal patterns
	GMC(1,N) (Wu et al. 2018), DBGM(1,N) (Zeng and Li 2018), GMP(1,1,N) (Luo and Wei 2017)	Consider some factors affecting electricity consumption in grey models	The grey system with multi-variables
Parameter Optimization	FGM(q,1) (Mao et al. 2016), WGM(1,1) (Wu et al. 2016), CFGOM(1,1) (Xie et al. 2020), SFOGM(1,1) Zhu et al. (2020)	Fractional grey prediction models and its extensions	Accumulating generation operator optimization
	IRGM(1,1) (Xu et al. 2017), GM(1,1)-$x(1)(n)$ (Dang et al. 2004), OICGM(1,1) (Xiong et al. 2014)	Grey prediction models based on improved initial value	Initial value condition optimization
	MDBGM(1,N) (Zeng et al. 2020b), DBGM(1,N) (Zeng and Li 2018), FOC_GM(1,N) (Huang et al. 2021)	Grey prediction models based on improved background value coefficient	Background-value optimization
Rolling Mechanism	GRPM(1,1) (Zhou et al. 2021), NOGM(1,1) (Ding et al. 2018)	Rolling mechanism in grey modeling	Grey system with new information priority
	Rolling-ALO-GM(1,1) (Zhao and Guo 2016), RGPMM(λ,1,1) (Sun et al. 2022)	Integration of rolling mechanism and other techniques	Hybrid models with new information priority

Combined Model	NNGM(1,1) (Hu 2017), GNF-IO model (Liu et al. 2016)	Integration of grey system model and machine learning method	Grey forecasting and big data technique
	NMGM-ARIMA (Wang and Jiang 2019), GM-ARMA (Xu et al. 2015)	Integration of grey system model and statistical method	Grey forecasting and statistical model

integrated contribution. One of the key contributions of our research is the integration of various techniques to enhance the prediction performance of the grey model for industrial electricity consumption, specifically for the context of Fujian province. By expanding the model structure, we capture the complex dynamics and non-linear patterns inherent in the electricity consumption data. Through parameter optimization, we determine the optimal fractional accumulating generation operator, ensuring the model adaptability. The incorporation of the rolling mechanism allows the model to dynamically incorporate new information as it becomes available.

The reminder of this paper is arranged as follows. A novel grey prediction model with fractional order and rolling mechanism is proposed in Section 2. Parameters estimation and recursive time response function of model are provided in this section. The modeling conditions and error checking method for the proposed model are discussed in Section 3. Section 4 employs the model in Fujian's industrial electricity consumption. Seven benchmark models are to test the superiority of the proposed approach. Further discussion and policy suggestions are presented in Section 5. Conclusions are summarized in Section 6.

2. Methodology

In this section, we present a novel grey system modeling approach, namely the rolling and fractional-ordered grey model, denoted as RFGM(1,r), which is designed for accurate forecasting of electricity consumption. The model incorporates a discrete time response function derived through recursive derivation, allowing for efficient capturing of the underlying dynamics. To optimize the performance of the model, we employ the particle swarm optimization algorithm to search for the optimal fractional-order parameter.

2.1 Basic Form of Grey System Model

Definition 1 *Assume that* $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)$ *is an original electricity consumption sequence, where* $x^{(0)}(k) \geq 0$ *for* $k = 1, 2, \cdots, n$. *D is an operator acting on* $X^{(0)}$, *and* $X^{(1)} = X^{(0)}D = \left(x^{(0)}(1)d, x^{(0)}(2)d, \cdots, x^{(0)}(n)d\right)$ *where*

$$x^{(0)}(k)d = \sum_{g=1}^{k} \frac{\Gamma\left(k+r-g\right)}{\Gamma\left(k-g+1\right)\Gamma\left(g\right)} x^{(0)}\left(g\right)$$

$$k = 1, 2, \cdots, n; r \in R^{+} \quad (1)$$

Then D is denoted as the r-order accumulating generation operator of the sequence $X^{(0)}$ *abbreviated as r-AGO. The new sequence obtained by r-AGO is called the fractional-order accumulating generation sequence, that is* $X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n)\right)$.

In particular, when $r = 1$, the above mentioned grey data process is the traditional first-order accumulating generation,that is $x^{(r)}(k) = \sum_{g=1}^{k} x^{(0)}(g)$, for $k = 1, 2, \cdots, k$.

In the grey modeling process, the introduce of r-AGO plays a crucial role in reducing the randomness and noise information of the raw data. Unlike the traditional 1-AGO that only considers the fixed first-order accumulation, r-AGO offers greater flexibility by allowing for fractional-order accumulations.

Based on r-AGO sequence $X^{(0)}$,the following new sequence $Z^{(r)}$ can be obtained, that is $Z^{(r)} = \left(z^{(r)}(2), z^{(r)}(3), \cdots, z^{(r)}(n)\right)$. where

$$z^{(r)}(k) = 0.5x\left[x^{(r)}(k) + x^{(r)}(k-1)\right] \tag{2}$$

Then $z^{(r)}$ is called the mean nearest neighbor sequence of $X^{(r)}$.

The process of returning to the original sequence from r-AGO sequence is called r-IAGO(r-order Inverse AGO), which is the inverse process of r-AGO. It satisfies the following requirements:

$$\begin{cases} \left(x^{(r)}(k)\right)^{(-r)} = \sum\limits_{g=0}^{k-1} \dfrac{\Gamma(r+1)}{\Gamma(g+1)\Gamma(r-g+1)} \cdot x^{(r)}(k-g) \\ x^{(r)}(1) = x^{(0)}(1) \end{cases}$$

Definition 2 *Assume that $X^{(r)}$, $Z^{(r)}$ are given by **Definition 1**, then*

$$x^{(r)}(k) - x^{(r)}(k-1) + r_1 z^{(1)}(k) = \frac{1}{2}(2k-1) r_2 + r_3 \tag{3}$$

is referred to as the basic form of Rolling and Fractional-ordered Grey Model with one variable and r-order accumulating generation operator, abbreviated as RFGM(1,r).

A notable distinction between the proposed model, RFGM(1,r), and the traditional GM(1,1) model is evident in Equation (3). It encompasses three undetermined parameters r_1, r_2, r_3. In addition, $0.5(2k-1)r_2 + r_3$ is the grey action quantity (or grey information coverage), which can represent all grey information in a system. It is linearly related to the time point k. Traditional GM(1,1) considers the grey action quantity as a constant r_2, thereby overlooking the dynamic nature of the system's evolution over time.

2.2 Parameters Estimation

Theorem 1 *Assume that $X^{(r)}, Z^{(r)}$ are given in Definition 1,if $\hat{p} = (r_1, r_2, r_3)^T$ is the estimated parameter matrix of RFGM(1,r), and*

$$M = \begin{bmatrix} -0.5 \cdot \left[x^{(r)}(2) + x^{(r)}(1)\right] & \frac{3}{2} & 1 \\ -0.5 \cdot \left[x^{(r)}(3) + x^{(r)}(2)\right] & \frac{5}{2} & 1 \\ \vdots & \vdots & \vdots \\ -0.5 \cdot \left[x^{(r)}(n) + x^{(r)}(n-1)\right] & \frac{2n-1}{2} & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} x^{(r)}(2) - x^{(r)}(1) \\ x^{(r)}(3) - x^{(r)}(2) \\ \vdots \\ x^{(r)}(n) - x^{(r)}(n-1) \end{bmatrix}$$

then it satisfies

$$\hat{p} = \left(r_1, r_2, r_3\right)^T = \left(M^T M\right)^{-1} M^T N \tag{4}$$

Proof is provided in Appendix.

2.3 The Recursive Time Response Function

Theorem 2 *Assume that$X^{(r)}, Z^{(r)}$are given in Definition 2, and $\hat{p} = (r_1, r_2, r_3)^r$ is the parameter vector of the RFGM(1,r) model, we set the initial* *value of the model as $\hat{x}^{(0)}(1) = x^{(0)}(1)$, then the recursive time response function of RFGM(1,r) can be deduced as*

$$\hat{x}^{(r)}(t) = \alpha^{t-1}x^{(0)}(1) + \sum_{g=0}^{t-2}\left[(t-g)\beta + \gamma\right]\alpha^{g} \tag{5}$$

In the above Equation, $\alpha = \frac{1-0.5r_1}{1+0.5r_1}, \beta = \frac{r_2}{1+0.5r_1}, \gamma = \frac{r_3-0.5r_2}{1+0.5r_1}$. The detailed proof is provided in Appendix.

2.4 Determination of the Optimal Fractional Order

In the recursive time response function of RFGM(1,r), the accumulating generation order r is predetermined. The traditional grey model assumes a fixed order of $r = 1$, which will limit its effectiveness in capturing the dynamics of the system. To address this question, the optimal order of the RFGM(1,r) model is determined by minimizing the error, as described in reference (Zhou et al. 2022b).The optimization condition involves solving the following optimization problem to obtain the optimal fractional order parameters.

$$\min f(r) = \sum_{k=2}^{n}\left[\frac{x^{(r)}(k) - \hat{x}^{(r)}(k)}{x^{(r)}(k)}\right]^2 \tag{6}$$

$$\text{s.t.}\begin{cases}\hat{x}^{(r)}(k) = \alpha^{t-1}x^{(0)}(1) - \sum_{g=0}^{t-2}\left[(t-g)\beta+\gamma\right]\alpha \\ \alpha = \frac{1-0.5r_1}{1+0.5r_1}, \beta = \frac{r_2}{1+0.5r_1}, \gamma = \frac{r_3-0.5r_2}{1+0.5r_1} \\ (r_1, r_2, r_3)^{\top} = (M^{\top}M)^{-1}M^{\top}N \\ r \in R^{+}\end{cases}$$

Equation (6) represents a complex nonlinear problem that is challenging to solve using classical ordinary least square methods. To address this, an intelligent optimization algorithm, particle swarm optimization (PSO) is employed to search for the optimal fractional order of RFGM(1,r). PSO is a group iterative method inspired by the collective behavior of a flock of birds flying in space.. Each particle can be seen as a flying bird constantly searching for the best foraging location. And each particle has two essential attributes, position and velocity. PSO is widely used in engineering applications owing to its structural interpretability, and ease of implementation (Zhou et al. 2022b, Zhu et al. 2020). By leveraging PSO, the optimal fractional order for RFGM(1,r) can be determined effectively.

The procedures of searching the optimal fractional order by PSO are as follows.

Step 1: Initialize the random populations as well as the position, velocity of each particle.

Step 2: Calculate the fitness value of each particle. The fitness function is obtained based on Equation (6).

Step 3: Find the local extreme $fbest_i$ and global extreme $gbest_i$ of each particle i. Particularly,the $fbest_i$ represents the best objective function value achieved by the particle i within the entire swarm at a particular iteration. It serves as a measure of the best solution found by any individual particle in the search process. The $gbest_i$ stands for global best and represents the best position and corresponding objective function value achieved by the i particle in the entire swarm throughout the entire optimization process.

Step 4: Update the velocity and position of the particle by using the following formula:

$$v_i = w \times v_i + c_1 \times rand_1\left(fbest_i - x_i\right) + c_2 \times rand_2\left(gbest_i - x_i\right) \tag{7}$$

$$x_i = v_i + x_i \tag{8}$$

Where c_1 and c_2 the individual learning factor and the social learning factor, respectively and usually are set to 2. rand and $rand_2$ are two random number between 0 and 1.

Step 5: End loop until maximum iterations or converged fitness value.

3. Modeling Conditions and Evaluation Criterion of RFGM(1,r)

3.1 Modeling Conditions of RFGM(1,r)

Definition 3 *Assume that* $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)$ *is an original non-negative sequence, where* $x^{(0)}(k) \geq 0$ *for* $k = 1, 2, \cdots, n$*, then*

$$\rho(k) = \frac{x^{(0)}(k)}{\sum_{i=1}^{k-1} x^{(0)}(i)}, \; k = 2, 3, \cdots, n \tag{9}$$

is referred to as the smoothness ratio of $X^{(0)}$.

The smoothness ratio is a measure of how consistent or steady the data changes are within the series. In general, when the data exhibits more stable and regular patterns of variation, the smoothness ratio tends to be smaller.

Definition 4 *Suppose that* $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)$ *is given in Definition 3, if* $X^{(0)}$ *satisfies the following three principles, then* $X^{(0)}$ *is a quasi-smooth sequence and can be used for grey modeling.*

1) $\sigma(k) = \frac{\rho(k)}{\rho(k-1)} < 1, \; k = 3, 4, \cdots, n$

2) $\rho(k) \in [0, \delta], k = 3, 4, \cdots, n$

3) $\delta < 0.8$

Quasi-smoothness condition serves as a crucial criterion for evaluating the suitability of a sequence for constructing a grey prediction model. If the modeling sequence fails to satisfy the quasi-smoothness test, it suggests that the sequence exhibits a high level of uncertainty and instability, which is unfavorable for establishing a reliable grey model.

3.2 Performance Evaluation

Suppose that $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)$ is the modeling sequence, where $x^{(0)}(k) \geq 0$ for $k = 1, 2, \cdots, n$. $\hat{X}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n)\right)$ is the simulation sequence of $X^{(0)}$, then the error sequence of $X^{(0)}$ is

$e^{(0)} = \left(e^{(0)}(1), e^{(0)}(2), \cdots, e^{(0)}(n)\right)$ where

$$e^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), k = 1, 2, \cdots, n \tag{10}$$

The absolute percentage simulation error of $X^{(0)}$ is

$\Delta = (\Delta(1), \Delta(2), \cdots, \Delta(n))$ where

$$\Delta(k) = \left|\frac{e^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, \quad k = 1, 2, \cdots, n \tag{11}$$

The mean absolute percentage simulation error (MAPE) of $X^{(0)}$ is

$$\Delta = \frac{1}{n}\sum_{k=1}^{n} \Delta(k), \quad k = 1, 2, \cdots, n \tag{12}$$

In general, a smaller mean absolute percentage error indicates a better fitting result of the model. It represents the average deviation between the predicted values and the actual values, with a value closer to 0 indicating a more accurate prediction. Furthermore, two other important evaluation metrics are the root mean square error (RMSE) and the model fitness R^2.

The RMSE quantifies the overall difference between the predicted values and the actual values, taking into account both the magnitude and direction of the errors. A lower RMSE value suggests a better model performance in terms of capturing the variability in the data. The model fitness R^2, also known as the coefficient of determination, assesses the proportion of the total variation in the dependent variable

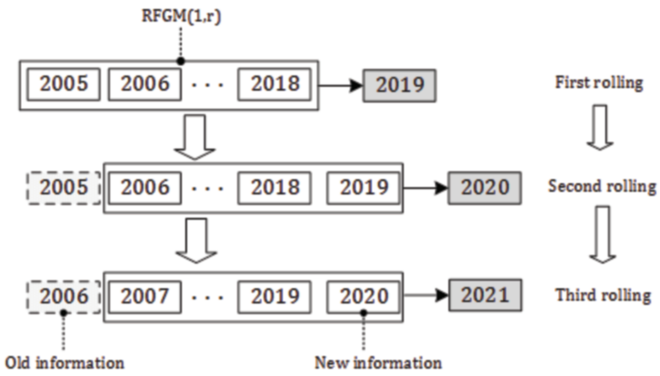

Figure 1 Forecasting Procedures of RFGM(1,r)

that can be explained by the independent variables in the model. A higher R^2 value indicates a stronger relationship between the predicted values and the actual values, indicating a better simulation effect of the model.

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x^{(0)}(k) - \hat{x}^{(0)}(k)\right)^2} \quad (13)$$

$$R^2 = 1 - \frac{\sum_{k=1}^{n}\left[x^{(0)}(k) - \hat{x}^{(0)}(k)\right]^2}{\sum_{k=1}^{n}\left[x^{(0)}(k) - \overline{x}^{(0)}\right]^2} \quad (14)$$

$$\overline{x}^{(0)} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k) \quad (15)$$

If the above performance evaluation metrics meet the requirements of system modeling, the$(n+p)$ th point data $\{p = 1, 2, \cdots\}$ can be calculated based on the recursive time response function of RFGM(1,r).

3.3 Rolling Mechanism

In traditional grey models, the prediction of subsequent data at various time points from $n+1$ to $n+t$ is performed directly based on effective fitting techniques. However, this approach overlooks the evolving uncertainty of grey information within a system. To address this limitation and fully consider the influence of new information on the modeling process, a rolling modeling mechanism based on the principle of information metabolism is introduced. The following procedures outline the implementation of the rolling mechanism.

Step 1: Set the modeling size of RFGM(1,r) to n, and get the first predicted data $\hat{x}^{(0)}(n+1)$. The prediction error of the$(n+1)\,th$ point data is

$$\Delta_{F1} = \left|\frac{x^{(0)}(n+1) - \hat{x}^{(0)}(n+1)}{x^{(0)}(n+1)}\right| \times 100\% \quad (16)$$

Step 2: Add the new modeling data $x^{(0)}(n+1)$, and remove the oldest information $x^{(0)}(1)$to form a new modeling sequence, that is $Y_1 = \left(x^{(0)}(2), x^{(0)}(2), \cdots, x^{(0)}(n), x^{(0)}(n+1)\right)$. The new round of RFGM(1,r) with the same modelling size is employed to predict the next data $\hat{x}^{(0)}(n+2)$. Similarly, the prediction error of the $(n+2)$th point data is

$$\Delta_{F2} = \left|\frac{x^{(0)}(n+2) - \hat{x}^{(0)}(n+2)}{x^{(0)}(n+2)}\right| \times 100\% \quad (17)$$

Step 3: Repeat the Step 1 and Step 2 until we finish forecasting the required future steps. In this study, the training data consists of industrial electricity consumption records from 2005 to 2018. To enhance the prediction accuracy, the rolling mechanism is incorporated into the subsequent prediction process. This mechanism, as depicted in Figure 1, enables the model to dynamically update and adapt to new

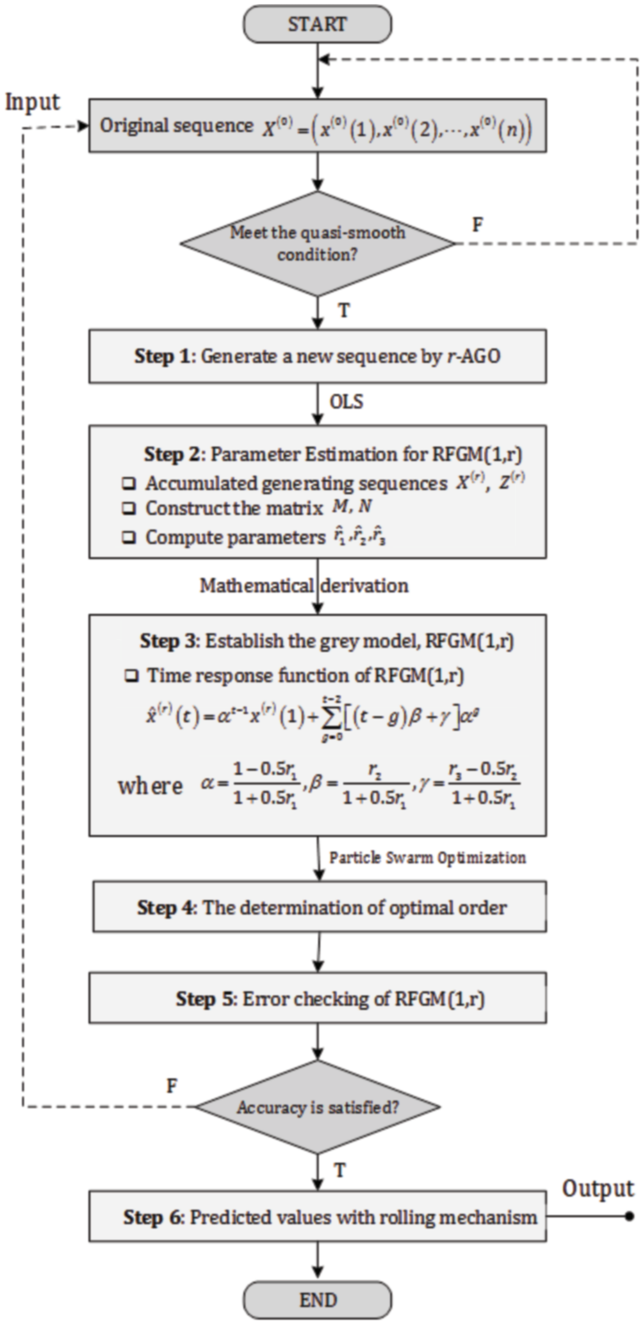

Figure 2 The Modeling Flowchart of RFGM(1,r)

information as it becomes available, thereby improving the forecasting performance.

Based on the above contents including parameters estimation, model derivation, fractional-order determination and model evaluation criterion of RFGM(1,r), the modeling flowchart is shown in Figure 2.

4. Forecast of Fujian's Industrial Electricity Consumption

4.1 Data and Sources

The smooth operation of the electricity system in Fujian province plays a vital role in ensuring sustainable development within the economic zone on the west coast of the Taiwan Strait. Accurate forecast and assessment of industrial electricity consumption are essential prerequisites for the long-term development of industries in Fujian Province. The original electricity consumption data used in this research were collected from Fujian Provincial Bureau of Statistics (https://tjj.fujian.gov.cn/) and Municipal Bureau of Statistics. Table 2 presents Fujian's industrial electricity consumption data from 2005 to 2020. In this study, original data was divided into two parts: training data spanning from 2005 to 2018, and testing data covering the years 2019 to 2020.

According to Figure 3, it is evident that Fujian's electricity consumption has experienced a noticeable upward trend. Among the different sectors contributing to Fujian's electricity consumption, the secondary industry holds the largest share, accounting for 84.9% of the total. It indicates that the secondary industry remains the primary power consumer in Fujian. On the other hand, the primary industry has the smallest proportion of electricity consumption, representing only 1.34% of the total. Since the beginning of the 21st century, the heavy industry and material industry have been the key development targets, driving economic growth and increasing the demand for industrial electricity in Fujian.

4.2 Modeling Condition Checking

To verify the applicability of the proposed model for predicting Fujian's industrial electricity consumption, we examine the quasi-smooth condition of the original sequence. The quasi-smooth checking results for the modeling sequence are demonstrated in Figure 4.

It can be seen from Figure 4 that all quasi-smooth results satisfy $\rho(k)\in[0, 0.8]$. $\max(\sigma(k)) = \sigma(14) = 0.997$, and $\sigma(k) < 1$ for $k = 3, 4, \cdots, 14$. Therefore, the original electricity consumption sequence $X^{(0)}$ can be used to establish the RFGM(1,r) model.

4.3 Simulation and Prediction of Fujian's Industrial Electricity Consumption

In this section, the proposed model RFGM(1,r) along with other seven benchmark prediction models, GM(1,1), DGM(1,1), ARIMA, Pearl model, OICGM(1,1), FGM(1,1) and GM(1,1)-$x(1)(n)$ are employed to simulate and forecast Fujian's industrial electricity consumption. For the purpose of model establishment, the data from 2005 to 2018 are specifically utilized. The remaining data, covering the years 2019 to 2020, are then employed for model testing and evaluation.

The experimental results of different prediction models are shown in Table 3. Moreover, the accessory model parameters of RFGM(1,r) are $(r_1, r_3, r_3)^\intercal = (-0.0039, 170.9155, 431.2757)^\intercal$. The optimal fractional order is $r = 1.3011$. Figure 5 illustrates the iterative process of determining the optimal fractional order using the particle swarm optimization (PSO) algorithm. The specific form of the recursive time response function can be expressed as follows:

$$\hat{x}^{(1.3011)}(k) = 480.74 \times 0.9781^{k-1} + \sum_{g=0}^{k-3}[228.7801 \times (k-g) + 397.6616] \times 0.9781^{g}, \quad k = 2, 3, \cdots, n$$

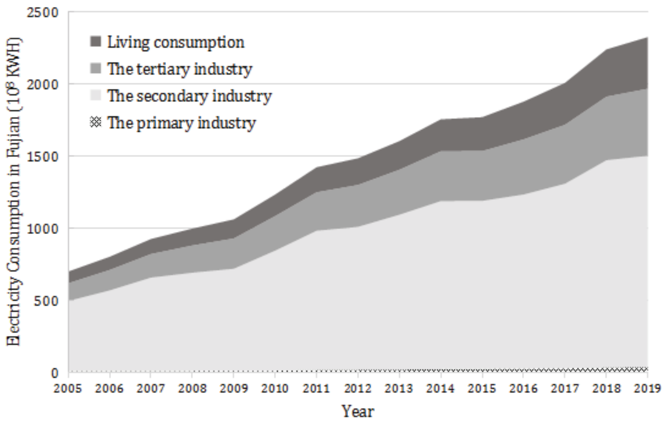

Figure 3 Industrial Distribution of Fujian's Electricity Consumption

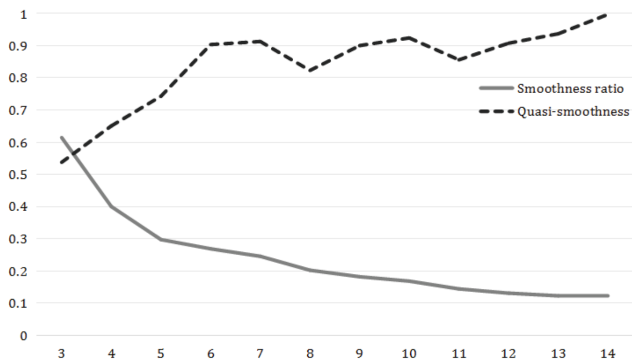

Figure 4 The Quasi-smoothness Checking Results of Modeling Sequence

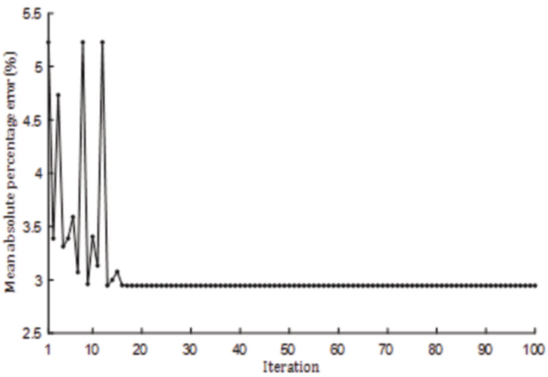

Figure 5 The Searching Process of Optimal Fractional Order by PSO Algorithm

Table 2 Industrial Electricity Consumption of Fujian (Unit: 10^8 kWh)

Year	2005	2006	2007	2008	2009	2010	2011	2012
IEC	480.74	549.51	633.05	665.01	691.69	810.82	939.21	962.64
Year	2013	2014	2015	2016	2017	2018	2019	2020
IEC	1041.68	1137.62	1137.61	1181.31	1251.62	1401.13	1424.52	1444.01

The iterative solution process of the particle swarm optimization algorithm, as depicted in Figure 5, reveals that convergence is achieved for the mean absolute percentage error after the 20th iteration, with an optimal fractional-order parameter yielding a value of 2.779%. Table 3 demonstrates the superior performance of the proposed model in this study compared to other benchmark models, as it exhibits the lowest simulation and prediction error. The simulated values closely align with the actual values, indicating the model's accuracy. The simulation curves of various prediction models are shown in Figure 6.

4.4 Performance Comparison

To validate the superiority of the proposed model RFGM(1,r), the errors results of the aforementioned prediction models are calculated and analyzed. Figure 7 presents a columnar diagram depicting the mean absolute percentage error for eight prediction models. This visualization allows for a comparative analysis of the performance of each model.

Figure 7 provides a visual representation of the prediction performance of the different models. It is evident that the RFGM(1,r) model achieves the overall best performance. The highest absolute percentage error observed for the RFGM(1,r) model is 8.453%, while the GM(1,1) model exhibits a higher error rate of 10.533%, and the ARIMA model shows a significantly higher error rate of 23.479%. Although the Pearl model demonstrates similar simulation performance to RFGM(1,r), it deviates from the actual changing trend of Fujian's industrial electricity consumption as the forecasting curve gradually becomes horizontal. In contrast, the RFGM(1,r) model captures the underlying trend accurately, leading to more reliable and robust predictions.

From the basic form of traditional grey model, we find that the final time response function of GM(1,1) is $\hat{x}^{(0)}(k)=(1-e^{a})\left(x^{(0)}(1)-b\cdot a^{-1}\right)\cdot e^{-a(k-1)}$, which is a typical exponential function. However, the dynamic trend of Fujian's IEC is affected by several external factors, so it is not suitable for forecasting Fujian's IEC with non-exponential trend.

Table 4 provides a comprehensive evaluation of the performance of different models, including GM(1,1), ARIMA model, OICGM(1,1), Pearl model, FGM(1,1), GM(1,1)-$x(1)(n)$ and the proposed RFGM(1,r).

The results in Table 4 highlight the superiority of the RFGM(1,r) model compared to the other benchmark models. The RFGM(1,r) model achieves an R^2 value of 0.990, which is closest to the critical value of 1, indicating a higher level of fitness to the data. Additionally, the RMSE value of RFGM(1,r) is smaller than that of the competing models, further confirming its superior predictive performance. Based on the mean absolute percentage error, the top five prediction models are identified as RFGM(1,r), FGM(1,1), OICGM(1,1), ARIMA, and GM(1,1)-$x(1)(n)$. These models present lower prediction errors, indicating their effectiveness in forecasting Fujian's industrial electricity consumption.

In summary, the proposed RFGM(1,r) demonstrates higher accuracy compared to the FGM(1,1), OICGM(1,1), ARIMA model, Pearl model, GM(1,1)-$x(1)(n)$, GM(1,1) and DGM(1,1), indicating that the novel grey

Table 3 Simulated and Predicted Results of Diverse Models

Year	Actual value	RFGM(1,r)	GM(1,1)	ARIMA	OICGM(1,1)	DGM(1,1)	Pearl	FGM(1,1)	GM-x(1)(n)
Training data									
2005	480.74	480.74	480.74	593.61	563.19	480.74	453.53	480.74	480.74
2006	549.51	546.08	607.39	559.52	604.10	608.11	522.6	549.51	606.39
2007	633.05	606.14	651.51	607.93	647.98	652.24	596.42	624.44	650.43
2008	665.01	677.61	698.84	684.51	695.04	699.57	673.77	692.07	697.68
2009	691.69	750.16	749.60	725.31	745.53	750.34	753.16	756.46	748.35
2010	810.82	821.85	804.04	754.78	799.68	804.80	832.91	819.73	802.71
2011	939.21	892.22	862.45	856.17	857.77	863.20	911.35	883.08	861.02
2012	962.64	961.18	925.09	979.21	920.07	925.84	986.87	947.33	923.56
2013	1041.68	1028.82	992.29	1022.36	986.90	993.03	1058.14	1013.05	990.64
2014	1137.62	1095.21	1064.36	1094.29	1058.59	1065.10	1124.12	1080.65	1062.60
2015	1137.61	1160.47	1141.67	1185.48	1135.48	1142.40	1184.13	1150.52	1139.78
2016	1181.31	1224.69	1224.60	1203.52	1217.96	1225.30	1237.83	1222.96	1222.57
2017	1251.62	1287.98	1313.55	1247.76	1306.42	1314.22	1285.2	1298.26	1311.38
2018	1401.13	1350.40	1408.96	1307.29	1401.32	1409.60	1326.45	1376.69	1406.63
Training error									
	2.779%	4.323%	5.235%	5.436%	4.345%	3.886%	2.953%	4.281%	
Testing data									
2019	1424.52	1412.05	1511.31	1438.99	1503.11	1511.90	1361.98	1458.49	1508.80
2020	1444.01	1473.77	1621.08	1495.43	1612.29	1621.62	1392.29	1543.93	1618.40
Testing error									
	1.468%	9.177%	2.288%	3.986%	8.585%	9.217%	4.652%	8.997%	
Comprehensive error									
	2.615%	4.930%	4.867%	3.899%	5.829%	4.954%	3.166%	4.870%	

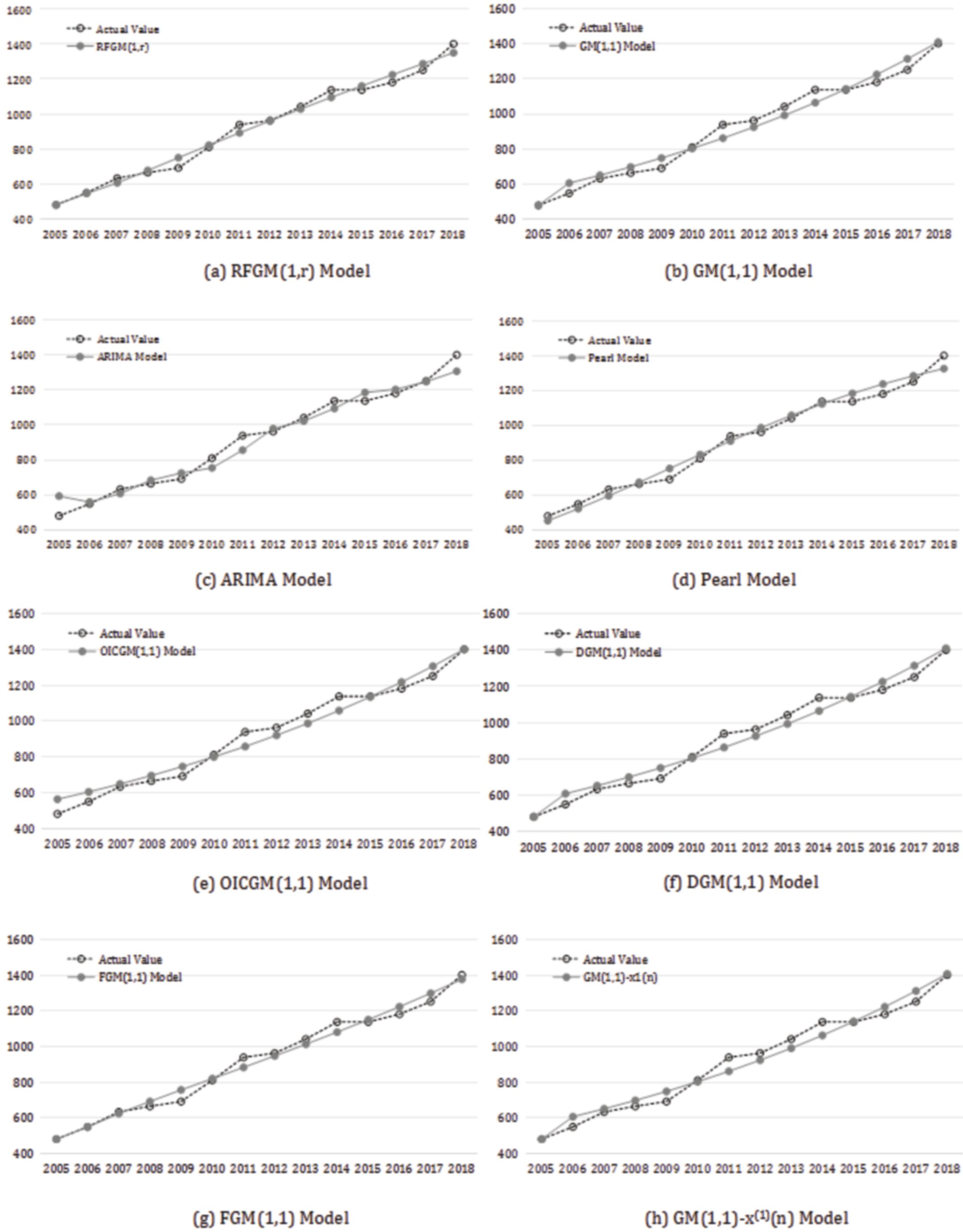

Figure 6 Simulated Curves of Different Prediction Models

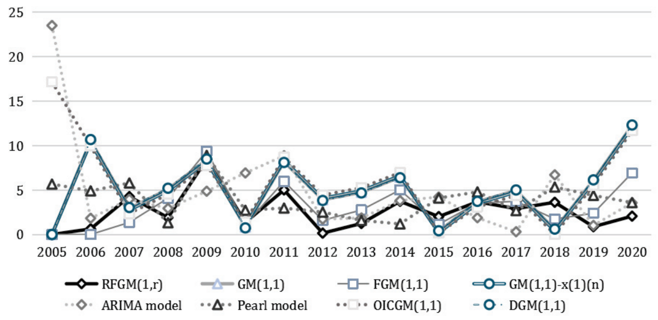

Figure 7 The Absolute Percentage Error of Different Prediction Models

Table 4 Performance Evaluation Metrics of Diverse models

Model	R^2	$RMSE$	$\Delta_S/\%$	$\Delta_F/\%$	$\Delta/\%$	Ranking
RFGM(1,1)	0.990	31.528	2.779%	1.441%	2.612%	1
GM(1,1)	0.955	65.200	4.32%	9.18%	4.93%	6
ARIMA	0.972	51.236	5.24%	2.29%	4.87%	4
OICGM(1,1)	0.982	41.545	5.44%	3.99%	3.90%	3
DGM(1,1)	0.954	66.362	4.35%	8.59%	5.83%	8
Pearl model	0.955	65.364	3.89%	9.22%	4.95%	7
FGM(1,1)	0.981	42.089	2.95%	4.65%	3.17%	2
GM(1,1)-$x(1)(n)$	0.956	64.507	4.28%	9.00%	4.87%	5

model, incorporating structure expansion, optimal fractional-order and rolling mechanism, contributes to the improvement of prediction accuracy and enhances the overall performance of electricity consumption forecasting.

4.5 Forecast of Fujian's Industrial Electricity Consumption with Rolling Mechanism

In view of the outstanding performance and validity demonstrated by the RFGM(1,r) model, we proceed to apply it with rolling mechanism to forecast Fujian's industrial electricity consumption for the period from 2021 to 2026. The rolling prediction modeling approach involves maintaining a consistent system capacity of 14 in each round of forecasting. The testing data in the last forecasting year is 2020. The detailed procedures are shown as follows.

Step 1: Add the new information in 2020,$x^{(0)}(16) = 1444.01$ to the original modeling sequence $X^{(0)}$ of RFGM(1,r), and remove the first point (that is the oldest information) to keep the modeling steps constant. Then, the modeling sequence with 14 years is updated as $Y_1^{(0)} = (633.05, 655.01, \cdots, 1424.52, 1444.01)$.

Step 2: Rebuild the RFGM(1,r) model based on $Y_1^{(0)}$, and we can obtain the newly predicted value $\hat{x}^{(0)}(17) = 1494.01$ for the next fore¦casting year.

Step 3: Similarly, add the next predicted value $\hat{x}^{(0)}(17)$ to the sequence $Y_1^{(0)}$ and remove the first point data $x^{(0)}(2)$.Then, the new round of RFGM(1,r) is constructed based on the updated system sequence with same length to complete the next prediction process.

Step 4: Repeat the process until the last forecasting year is 2026. To capture the medium and long-term changing trend of Fujian's industrial electricity consumption, we utilize the RFGM(1,r) model proposed in this study to conduct forecasting. The detailed results are presented in Table 5.

The forecast results in Table 5 provide valuable insights into the future trends of industrial electricity consumption in Fujian province. It is evident that the IEC is projected to continue growing steadily in the coming years, with a consistent and moderate growth rate. It is estimated that the industrial electricity consumption will reach 186.312 billion kWh in 2026. The annual growth rate is expected to at around 4.34%. Furthermore, an interesting observation is that the fractional-order parameter varies for different years, ranging from 0 to 2. Notably, in the prediction of RFGM(1,r), the initial data in the system is continuously updated as new data are added for prediction. Thus, the predicted data from 2021 to 2026 are obtained through the construction of six round of RFGM(1,r) models. The minimum fractional order of 0.1815 suggests that inherent growth patterns in the original system can be excavated without heavy reliance on the accumulating generation operator. This finding highlights the effectiveness of the proposed RFGM(1,r) model in capturing and forecasting the growth trends of industrial electricity consumption in Fujian province.

From the forecasting trend depicted in Figure 8, the eight prediction models utilized in this study can be categorized into four types in terms of their projected industrial power consumption trends in Fujian Province. Firstly, there are models that indicate a gradually flattening trend, with the Pearl model being the representative example. The underlying assumption of this model is that the system has reached saturation, and electricity consumption will plateau based on the historical data. Secondly, the results suggest a sharp upward trend in industrial electricity consumption, represented by GM(1,1), DGM(1,1) and OICGM(1,1) model. These models project an increasing growth rate that is accelerating over time. The forecasted values from these three models exhibit a high degree of consistency, with an expected consumption level of around 240 billion kWh in 2026. The third type includes the FGM(1,1) model, which also demonstrates a rising trend in electricity consumption, albeit at a slower growth rate compared to the GM(1,1) model. Lastly, the RFGM(1,r) and ARIMA models exhibit similar prediction results, suggesting a stable growth trend in industrial electricity consumption over the next six years. However, the growth rate projected by these models is slower compared to the FGM(1,1) model.

In summary, the industrial electricity consumption in Fujian province is expected to exhibit a growth trend according to the predictions of various models, although the growth rates may differ. Among the models considered, the proposed RFGM(1,r), which has demonstrated the highest comprehensive accuracy, is employed to predict Fujian's industrial electricity consumption from 2021 to 2026. The results indicate a steady increase from 144.41 billion kWh in 2020 to 186.312 billion kWh in 2026. The findings hold great significance for regional energy planning and future policy-making endeavors.

Table 5 Prediction of Fujian's Electricity Consumption of Industrial Sector from 2021 to 2026 (Unit: 10^8 kWh)

Year	2021	2022	2023	2024	2025	2026
Fractional order	0.1815	1.8102	1.9219	1.8703	1.9367	0.9570
IEC	1494.01	1583.52	1664.16	1714.62	1792.01	1863.12
Growth rate	3.46%	5.99%	5.09%	3.03%	4.51%	3.97%

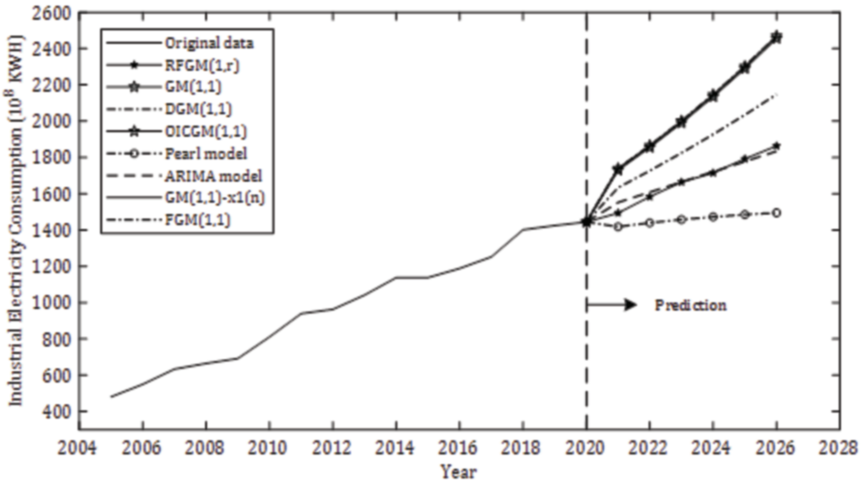

Figure 8 The Changing Trend of Industrial Electricity Consumption by Using Different Models

5. Discussion and Policy Suggestions

The prediction trend of Fujian's industrial electricity consumption indicates a continuous growth with no signs of slowing down. Therefore, electricity substitution in Fujian needs to be further strengthened to meet the growing demand for industrial sectors. To further improve the existing electricity energy structure, the following several policy suggestions can be considered.

Firstly, it is important to actively enhance the intelligent level of Fujian's power grid. By implementing regional smart grid technologies, such as smart substations and automatic distribution devices in urban core areas, the power information collection and supply reliability can be significantly improved. This will optimize energy resource allocation, enhance the operational efficiency, and elevate the maintenance standards of the power grid.

Secondly, advocating tax reduction and exemption for electricity consumption of lower-consuming industrial enterprises can be an effective measure. Such government initiatives will reduce the electricity costs for these industries, promoting their competitiveness. Additionally, offering support and subsidies to industrial enterprises that adopt clean energy power generation technologies, such as wind, solar, and tidal energy, will incentivize the development and exploration of clean energy sources.

Lastly, the steady promotion of electricity energy substitution (EES) can play a crucial role in improving the energy structure of Fujian province. EES involves the substitution of traditional energy sources with electricity generated from renewable energy and ultralow emission coal-fired power units. This approach not only expands power consumption but also contributes to a higher proportion of clean energy usage and reduces air pollutant emissions.

By implementing these policy suggestions, Fujian can meet the growing demand for industrial electricity consumption while simultaneously improving the energy structure, promoting clean energy adoption, and reducing environmental impacts.

6. Conclusions

Scientific and accurate forecasting of industrial electricity consumption serves as a crucial guideline for optimizing energy structures and fostering regional green development. However, The complexity of the energy market and the uncertainty of the external environment pose a great challenge to electricity consumption forecasting.

In this paper, a rolling and fractional-ordered grey system modeling approach RFGM(1,r) is designed for accurately forecasting Fujian's industrial electricity consumption. The model incorporates structural expansion, parameter optimization, and a rolling mechanism, providing advantages over traditional grey models. By extending the grey action quantity and optimizing the fractional order parameter using a particle swarm optimization algorithm, the proposed model enhances the accuracy and flexibility of the forecasting process. The effectiveness of the RFGM(1,r) model is verified by applying it to forecast industrial electricity consumption in Fujian from 2005 to 2020. Three main evaluation metrics APE, R2 and the root mean square error are introduced to test the comprehensive performance compared with other seven benchmark prediction models. The experimental results show that the proposed grey model outperforms seven benchmark models in terms of simulation and prediction. In view of the excellent performance that the accuracy is 97.388%, which demonstrates the superiority of the proposed grey model in capturing the changing trends of Fujian's industrial electricity consumption. Some reasonable policy suggestions are provided, which will be helpful for energy planning and regional green development in Fujian province.

However, it is important to acknowledge the limitations of the modeling process. Factors such as the background value coefficient and external interventions like policy changes and unforeseen events can influence the accuracy of the forecasts. Future research should explore the development of self-adaptive grey system models with multiple parameter optimization and consider the incorporation of inevitable intervention factors in the prediction framework.

Appendix A The Proof of Theorem 1

Theorem 1 *Assume that* $X^{(r)}, Z^{(r)}$ *are given in Definition* 1*,if* $\hat{p} = (r_1, r_2, r_3)^T$ *is the estimated parameter matrix of RFGM(1,r), and*

$$M = \begin{bmatrix} -0.5 \cdot \left[x^{(r)}(2) + x^{(r)}(1)\right] & \frac{3}{2} & 1 \\ -0.5 \cdot \left[x^{(r)}(3) + x^{(r)}(2)\right] & \frac{5}{2} & 1 \\ \vdots & \vdots & \vdots \\ -0.5 \cdot \left[x^{(r)}(n) + x^{(r)}(n-1)\right] & \frac{2n-1}{2} & 1 \end{bmatrix},$$

$$N=\begin{bmatrix} x^{(r)}(2)-x^{(r)}(1) \\ x^{(r)}(3)-x^{(r)}(2) \\ \vdots \\ x^{(r)}(n)-x^{(r)}(n-1) \end{bmatrix} \tag{A1}$$

then it satisfies

$$\hat{p}=(r_1,r_2,r_3)^T=\left(M^TM\right)^{-1}M^TN \tag{A2}$$

Proof. We substitute $X^{(r)}$ and $Z^{(r)}$ into the proposed model, RFGM(1,r) defined in Definition 2, and can obtain:

$$\begin{cases} x^{(r)}(2)-x^{(r)}(1)=-r_1z^{(r)}(2)+\frac{3}{2}r_2+r_3 \\ x^{(r)}(3)-x^{(r)}(2)=-r_1z^{(r)}(3)+\frac{5}{2}r_2+r_3 \\ \vdots \\ x^{(r)}(n)-x^{(r)}(n-1)=-r_1z^{(r)}(n)+\frac{2n-1}{2}r_2+r_3 \end{cases} \tag{A3}$$

The Equation (2) can be expressed as

$$N=M\hat{p} \tag{A4}$$

$x^{(r-1)}(k)$ can be approximately estimated as $-r_1z^{(r)}(k)+\frac{2k-1}{2}r_2+r_3$, then the residual sequence is

$$\varepsilon=\left(N-M\hat{p}\right) \tag{A5}$$

In order to get the minimum simulation error, let

$$S=\min_{r_1,r_2,r_3}\left(\varepsilon^T\varepsilon\right)=\min\sum_{k=2}^{n}\left[x^{(r)}(k)-x^{(r)}(k-1)+ r_1z^{(r)}(k)-\frac{2k-1}{2}r_2-r_3\right]^2 \tag{A6}$$

According to the ordinary least square method, the parameters r_1,r_2,r_3 of RFGM(1,r) will satisfy

$$\begin{cases} \frac{\partial S}{\partial r_1}=-2\sum_{k=2}^{n}\left[x^{(r)}(k)-x^{(r)}(k-1)+r_1z^{(r)}(k)-\frac{2k-1}{2}r_2-r_3\right]\cdot\left[-z^{(r)}(k)\right]=0 \\ \frac{\partial S}{\partial r_2}=2\sum_{k=2}^{n}\left[x^{(r)}(k)-x^{(r)}(k-1)+r_1z^{(r)}(k)-\frac{2k-1}{2}r_2-r_3\right]\cdot\frac{2k-1}{2}=0 \\ \frac{\partial S}{\partial r_3}=2\sum_{k=2}^{n}\left[x^{(r)}(k)-x^{(r)}(k-1)+r_1z^{(r)}(k)-\frac{2k-1}{2}r_2-r_3\right]=0 \end{cases} \tag{A7}$$

To simplify it, we can get

$$M^{\top}\varepsilon=0\Leftrightarrow M^{\top}\left(N-M\hat{p}\right)=M^{\top}N-M^{\top}M\hat{p}=0 \tag{A8}$$

Then $\hat{p}=(M^{\top}M)^{-1}M^{\top}N$. ■

Appendix B The Proof of Theorem 2

Theorem 2 *Assume that $X^{(r)},Z^{(r)}$ are given in Definition 2, and $\hat{p}=(r_1,r_2,r_3)^T$ is the parameter vector of the RFGM(1,r) model, we set the initial value of the model as $\hat{x}^{(0)}(1)=x^{(0)}(1)$, then the recursive time response function of RFGM(1,r) can be deduced as*

$$\hat{x}^{(r)}(t)=\alpha^{t-1}x^{(0)}(1)+\sum_{g=0}^{t-2}\left[\left(t-g\right)\beta+\gamma\right]\alpha^{g},\quad t=2,3,\cdots,n \tag{B1}$$

In the above Equation, $\alpha=\frac{1-0.5r_1}{1+0.5r_1},\beta=\frac{r_2}{1+0.5r_1},\gamma=\frac{r_3-0.5r_2}{1+0.5r_1}$.

Proof. According to Definition 1 and Definition 2, we can get

$$\begin{cases} x^{(0)}(k) = \left(x^{(r)}(k)\right)^{(-r)} \\ z^{(r)}(k) = 0.5 \cdot \left(x^{(r)}(k) + x^{(1)}(k-1)\right) \\ x^{(r)}(k) - x^{(r)}(k-1) + r_1 z^{(1)}(k) \\ = \frac{1}{2}(2k-1)r_2 + r_3 \end{cases}$$

$$\Rightarrow (1+0.5r_1)\, x^{(r)}(k) = 0.5 \cdot (2k-1)\, r_2 + r_3 + (1-0.5r_1)\, x^{(r)}(k-1) \quad \text{(B2)}$$

Rearranging it, then

$$x^{(r)}(k) = \frac{1-0.5r_1}{1+0.5r_1} \cdot x^{(r)}(k-1) + \frac{r_2}{1+0.5r_1} \cdot k + \frac{r_3-0.5r_2}{1+0.5r_1} \quad \text{(B3)}$$

Let $\hat{x}^{(0)}(k)$ be the simulation value of $x^{(0)}(k)$, $\hat{x}^{(r)}(k)$ be the simulation value of $\chi^{(r)}(k)$, then when $k = 2$,

$$\hat{x}^{(r)}(2) = \frac{1-0.5r_1}{1+0.5r_1} \cdot \hat{x}^{(r)}(1) + \frac{r_2}{1+0.5r_1} \cdot 2 + \frac{r_3-0.5r_2}{1+0.5r_1} \quad \text{(B4)}$$

when $k = 3$,

$$\hat{x}^{(r)}(3) = \frac{1-0.5r_1}{1+0.5r_1} \cdot \hat{x}^{(r)}(2) + \frac{r_2}{1+0.5r_1} \cdot 3 + \frac{r_3-0.5r_2}{1+0.5r_1} \quad \text{(B5)}$$

By substituting Equation (B4) into Equation (B5), we can get

$$\begin{aligned} \hat{x}^{(r)}(3) = &\left(\frac{1-0.5r_1}{1+0.5r_1}\right)^2 \cdot \hat{x}^{(r)}(1) \\ &+ \left(\frac{1-0.5r_1}{1+0.5r_1}\right) \cdot \left(\frac{r_2}{1+0.5r_1} \cdot 2 + \frac{r_3-0.5r_2}{1+0.5r_1}\right) \\ &+ \frac{r_2}{1+0.5r_1} \cdot 3 + \frac{r_3-0.5r_2}{1+0.5r_1} \end{aligned} \quad \text{(B6)}$$

Similarly, when $k = 4$,

$$\begin{aligned} \hat{x}^{(r)}(4) = &\left(\frac{1-0.5r_1}{1+0.5r_1}\right)^3 \cdot \hat{x}^{(r)}(1) \\ &+ \left(\frac{1-0.5r_1}{1+0.5r_1}\right)^2 \cdot \left(\frac{r_2}{1+0.5r_1} \cdot 2 + \frac{r_3-0.5r_2}{1+0.5r_1}\right) \\ &+ \left(\frac{1-0.5r_1}{1+0.5r_1}\right) \cdot \left(\frac{r_2}{1+0.5r_1} \cdot 3 \right. \\ &\left. + \frac{r_3-0.5r_2}{1+0.5r_1}\right) + \frac{r_2}{1+0.5r_1} \cdot 4 + \frac{r_3-0.5r_2}{1+0.5r_1} \end{aligned} \quad \text{(B7)}$$

When $k = t$,

$$\hat{x}^{(r)}(t) \quad \text{(B8)}$$

$$\begin{aligned} = &\left(\frac{1-0.5r_1}{1+0.5r_1}\right)^{t-1} \cdot \hat{x}^{(r)}(1) \\ &+ \left(\frac{1-0.5r_1}{1+0.5r_1}\right)^{t-2} \cdot \left(\frac{r_2}{1+0.5r_1} \cdot 2 + \frac{r_3-0.5r_2}{1+0.5r_1}\right) \\ &+ \cdots \left(\frac{1-0.5r_1}{1+0.5r_1}\right) \cdot \left(\frac{r_2}{1+0.5r_1} \cdot (t-1) + \frac{r_3-0.5r_2}{1+0.5r_1}\right) \\ &+ \frac{r_2}{1+0.5r_1} \cdot t + \frac{r_3-0.5r_2}{1+0.5r_1} \end{aligned} \quad \text{(B9)}$$

To arrange the form of $\hat{x}^{(r)}(t)$, we can get

$$\begin{aligned} \hat{x}^{(r)}(t) = &\left(\frac{1-0.5r_1}{1+0.5r_1}\right)^{t-1} \cdot \hat{x}^{(r)}(1) \\ &+ \sum_{g=0}^{t-2}\left[\frac{r_2}{1+0.5r_1}(t-g) + \frac{r_3-0.5r_2}{1+0.5r_1}\right] \cdot \left(\frac{1-0.5r_1}{1+0.5r_1}\right)^g, \end{aligned}$$

$$t = 2, 3, \cdots, n \quad \text{(B10)}$$

when $\alpha = \dfrac{1-0.5r_1}{1+0.5r_1}$, $\beta = \dfrac{r_2}{1+0.5r_1}$, $\gamma = \dfrac{r_3-0.5r_2}{1+0.5r_1}$.

In the grey modelling process, the first input $x^{(0)}(1)$ determines the starting point of a system and is usually used as the initial condition. Then it satisfies that $\hat{x}^{(r)}(1) = \hat{x}^{(0)}(1) = x^{(0)}(1)$,then Equation (17) can be simplified as

$$\hat{x}^{(r)}(t) = \alpha^{t-1} \cdot \hat{x}^{(0)}(1) + \sum_{g=0}^{t-2}\left[\beta(t-g) + \gamma\right] \cdot \left(\alpha\right)^g \quad \text{(B11)}$$

Through the inverse calculation of r-AGO given in Definition 1, we can get

$$\hat{x}^{(0)}(k) = \sum_{g=0}^{k-1} \frac{\Gamma(r+1)}{\Gamma(g+1)\Gamma(r-g+1)} \cdot \hat{x}^{(r)}(k-g) \quad \text{(B12)}$$

Equation (B12) is called the recursive time response function of RFGM(1,r). When $k = 2, 3, \cdots, n$, $\hat{x}^{(0)}(k)$ is called the fitted value of RFGM(1,r). And when $k = n+1, n+2, \cdots$, $\hat{x}^{(0)}(k)$ is called the predicted value. ■

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Data Availability

The datasets generated during and analysed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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