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A Confidence-based Consensus Model for Multi-Attribute Group Decision Making: Exploring the Bounded Trust Propagation and Personalized Adjustment Willingness

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Abstract. Social trust network (STN) and minimum cost consensus (MCC) models have been widely used to address consensus issues in multi-attribute group decision-making (MAGDM) problems with limited resources. However, most researchers have overlooked the decision maker' (DMs)' confidence levels (CLs) and adjustment willingness implicit in their evaluations. To address these problems, this paper explores a confidence-based MCC model that considers DMs' adjustment willingness in the STN. The proposed model includes several modifications to the traditional trust propagation and consensus optimization models. Firstly, the improved method for measuring CLs of DMs and the confidence-based normalization approach are defined, respectively. Secondly, the bounded trust propagation operator is proposed, which considers the credibility of mediators to complete the STN. Thirdly, the identification rules based on the consensus index and CL are defined, and the MCC model with personalized cost functions and acceptable adjustment thresholds is built to automatically generate adjustment values for non-consensus DMs. Finally, a model to identify the non-cooperative behavior at the element level is established and the hybrid MCC model with persuasion strategies is provided. Finally, a case study is processed to verify the applicability of the proposed model, and comparison and sensitivity analysis are conducted to highlight its benefits.

Keywords: Confidence level, social trust network, bounded trust propagation, minimum cost consensus models, multi-attribute group decision making

1. Introduction

Multi-attribute group decision-making (MAGDM) method usually means that multiple decision makers (DMs) evaluate the alternatives under different attributes and select the optimal solution (Pang and Liang 2012). MAGDM, as opposed to individual multi-attribute decision-making, can combine the benefits of DMs in different fields to make more accurate decisions (Gupta et al. 2018). Based on this, MAGDM is widely used to deal with complex decision-making problems in corporate strategy (Krishankumar et al. 2021), internet venture capital (Gou and Xu 2021), and other fields. Especially, social trust network (STN), as a powerful tool for describing the relationships among DMs, is widely used to derive DM weights in MAGDM problems (Wu et al. 2019, Li et al. 2022, Liu et al. 2022a).

In real life, DMs may be unable to express their trust degree in other DMs due to a lack of familiarity, resulting in an incomplete STN (Gao et al. 2021). As such, the trust propagation used to estimate unknown trust relationships (TRs) has received a lot of attention lately. For example, Wu et al. (2021) proposed a t-norm trust propagation operator, Xu et al. (2021) developed a trust propagation method with uncertainty theory, Gao et al. (2021) defined the probabilistic linguistic trust relationship (PLTR) and the corresponding t-norm propagation operator. Su et al. (2022) further proposed the PLTR with CL in MAGDM. On this basis, some scholars pointed out that trust transmission is not infinite, and explored different ways to characterize the finiteness, such as relationship strength (Liu et al. 2019a), limited mediator paths (Tan et al. 2022), and relative importance of trust degree (Liu et al. 2022b). Current research on limited trust propagation focuses on objective factors and ignores the subjective factor of the uncertainty of the TR graph, which is vital information hidden in fuzzy graphs (Hao et al. 2015).

After determining the weights of DMs, the DMs' evaluations can be integrated into the collective evaluations. However, due to differences in knowledge backgrounds and positions among DMs, divergent evaluations commonly exist in the process of MAGDM (Wang and Wan 2020). Therefore, compared to directly integrating DMs' evaluations, the consensus reaching process (CRP) is easier to obtain accurate and reliable information which is accepted by all DMs (Tang et al. 2020). CRP mainly includes two stages: identification rules of non-consensus DMs and the feedback mechanism. For identification rules, most researchers designed them with the similarity measures among the DMs' evaluations (Chiclana et al. 2013) or with the collective evaluations (Gou et al. 2021). Some researchers adopted correlation measures (Wu and Liao 2019), clustering (Wu et al. 2019), satisfaction measures (Zhang et al. 2017), etc. Most identification rules are based on various measures between DMs, with little attention to the uncertainty of the DM's preference, which is the vital factor that influences DMs to adjust the evaluation.

For feedback mechanisms, there are two mainstream ways to provide modification suggestions for non-consensus DMs. One way is a linear combination of the original evaluations and the collective evaluations (Zhang et al. 2014). For example, Xing et al. (2023) proposed a feedback mechanism based on the STN and the bargaining game analysis. Cao et al. (2021) provided the feedback mechanism with the harmony degree. However, the linear combination way easily leads to the result that the collective evaluations are manipulated through feedback parameters. In contrast, the other way is adopting the modification model which seeks minimum deviation between the original and modified evaluations (Gao et al. 2021), which is more conducive to obtaining fair and reasonable results. For the second way, the minimum cost consensus (MCC) model was proposed in the context of limited resources (Ben-Arieh and Easton 2007). Many scholars focused on the MCC model, for example, Yu et al. (2023) developed an enhanced MCC model with flexible cost, García-Zamora et al. (2023) introduced the convex optimization theory into the MCC model, Liu et al. (2023a) proposed the MCC model for incomplete probabilistic linguistic preference relations (PLPR), where the unit cost is derived by the hesitancy index of PLPR. Meng et al. (2022) built the MCC model with the hybrid penalty mechanism for noncooperative behaviors. From above, it's worth noting that most studies hypothesize that the adjustment costs are known. Only Liu et al. (2023a) provided a hesitancy-based method to determine the adjustment costs. But the hesitancy-based method contains four parameters, which may cause errors in the results. Furthermore, most studies consider the adjustment willingness at the DM level, ignoring the differences in adjustment willingness for the same DM's evaluation of different alternatives under different attributes.

In complex and uncertain circumstances, DMs always display some personal psychological characteristics when they make decisions. Given this finding, some scholars investigated the influence of CL on the results of STN and MAGDM. Ding et al. (2019) studied how the CL affects the velocity of CRP in STN. Liu et al. (2019b) introduced the CL into the determination method of DM weights and the feedback mechanism based on linear combinations. Xie et al. (2022) proposed the probabilistic linguistic term set (PLTS) with the given confidence interval in CRP. You and Hou (2022) defined the CL as the entropy of PLTS and presented a minimum trust propagation operator. Li et al. (2022) derived the CL by the completeness of PLTS and the DM's hesitancy and preference among linguistic terms. Chen et al. (2022) defined the hesitancy degree of intuitionistic fuzzy sets as the CL and designed the feedback strategy to adjust the DMs' evaluation or weight penalty according to the confidence correlation degree of DMs. Liu et al. (2023b) developed personalized individual self-confidence in dynamic STN and implemented it into the interactive CRP, where the weights of DMs are derived by the trust aggregation method with the shortest path. For the optimization model, Zhang et al. (2019) and Liu et al. (2022b) established the minimum adjustment consensus (MAC) model with bounded confidence, successively. Research on CL has focused on two aspects: (1) studying the effect of CL on linear combination parameters in STN; (2) exploring the effect of CL on the adjustment range of DMs in the MCC model.

From the above review, it is clear that the impact of CL on the MCC model in STN has not been explored. Furthermore, the roles of CL and the trust degree of DMs in the MCC model for identifying noncooperative behavior and developing punishment strategies are ignored. As aforementioned, portraying DMs' attitudes and willingness in a more targeted and personalized manner in CRP facilitates obtaining the results that are supported by all DMs. Meanwhile, we can see that there is a wealth of research on PLTS in MAGDM problems. This is because that PLTS is a natural and powerful way of expressing information (Gao et al. 2021, Su et al. 2022, Wu and Liao 2019, Liu et al. 2023b, Xie et al. 2022, You and Hou 2022). Furthermore, the research on PLTS-based CL offers DMs a platform to communicate their uncertainty regarding evaluation through PLTS. Therefore, the motivations of this research are utilizing the PLTS-based CL and trust degree of DMs to bridge the theory of STN analysis and the MCC model and providing a new framework to solve the STN-based MAGDM problems. The main contributions and innovations are as follows:

1) To follow up on the role of the DM's CL in the MAGDM problems, an improved measurement of CL is defined by the completeness and entropy of PLTS, where the entropy is different from the one proposed by You and Hou (2022). On this basis, a confidence-based normalization method for incomplete PLTS is proposed,

2) To portray the efficiency of trust propagation flexibly and realistically in practical STN-based MAGDM problems, the confidence-based PLTR is defined and a bounded trust propagation operator is constructed, where the CL derived by the given PLTR means the certainty degree of a DM how he/she trusts another DM. In the process of trust transitivity, the CL is regarded as the credibility of mediators, where mediators refer to the trusted DMs.

3) To construct a more objective measurement for the unit adjustment cost, a cost function is defined as the CL of a DM under each alternative and attribute. Moreover, to consider the personalized adjustment willingness, an acceptable adjustment threshold captured by DMs' CL and trust degree in STN is inserted into the MCC model.

4) To identify the non-cooperative behavior of DMs in CRP fairly and accurately, an objective model is constructed at the element level. And the hybrid MCC model with the persuasion strategies is established, where the persuasion strategies are provided by DMs' CL and trust degree. The hybrid MCC model with more user-friendly strategies provides references for solving management problems such as major strategic decisions in which the attitudes and willingness of each DM cannot be ignored.

The remainder of this paper is organized as follows: Section 2 gives the definition and operations of PLTSs, and the related concepts of STN analysis. In Section 3, the confidence-based consensus method for MAGDM based on STN analysis is proposed. Section 4 represents a case study for the sustainable business model innovation and some discussions with the comparative analyses and simulation tests to verify the highlights of the proposed method. The conclusions are provided in Section 5.

2. Preliminaries

In this section, we briefly review the definition and operations of PLTS and the classical STN analysis.

2.1 PLTSs

Based on the additive linguistic term set $S = \left\{ s_\varphi \left| \varphi = 0, 1, 2, \cdots, \tau \right. \right\}$ (τ is a positive integer), PLTS was defined as (Pang et al. 2016):

$$= \left\{ s_l \left(p_l \right) \left| s_l \in S, p_l \geq 0, l = 1, 2, \cdots, \#L, \sum\nolimits_{l=1}^{\#L} p_l \leq 1 \right. \right\} \tag{1}$$

where $L^{(\varsigma)}\left(p^{(\varsigma)}\right)$ is a probabilistic linguistic element (PLE) that contains the linguistic term $L^{(\varsigma)}$ and its corresponding probability $p^{(\varsigma)}$, and $\#L\left(p\right)$ is the number of linguistic terms in $L\left(p\right)$. In general, a PLTS only needs to list the linguistic terms whose corresponding probability is greater than 0. $\left\{ p_l \left| l = 1, 2, \cdots, \#L \right. \right\}$ denotes the probability distribution corresponding to the linguistic terms in L. When $\sum_{l=1}^{\#L} p_l = 1$, the probability distribution of PLTS is complete; when $\sum_{l=1}^{\#L} p_l < 1$, $\hat{p} = 1 - \sum_{l=1}^{\#L} p_l$ is defined as the missing probability of PLTS. The missing probability can be redistributed using the normalization method. The normalized PLTS is denoted as:

$$\tilde{L} = \left\{ \tilde{s}_l \left(\tilde{p}_l \right) \left| \tilde{s}_l \in S, \tilde{p}_l \geq 0, l = 1, 2, \cdots, , \right. \right. \\ \left. \#\tilde{L} \sum_{l=1}^{\#L} \tilde{p}_l \leq 1 \right\} \tag{2}$$

To avoid the situation that the value of a linguistic term is out of range during the calculation, (Gou and Xu 2016) gave equivalent functions for the linguistic term set and PLTS as follows:

$$g : [0, \tau] \to [0, 1], g : [0, \tau] \to [0, 1], \\ g\left(s_\varphi\right) = \frac{\varphi}{\tau} = \gamma \tag{3}$$

$$g : [0, \tau] \to [0, 1] \ , g\left(\tilde{L}\right) = \\ \left\{ (l/\tau\ )\left(\tilde{p}_l\right) | l \in [0, \tau] \right\} = \tilde{L}_\gamma \tag{4}$$

$$g^{-1} : [0, 1] \to [0, \tau], g^{-1}(\gamma) = \\ \left\{ s_{\gamma \cdot \tau} \left| \gamma \in [0, 1] \right. \right\} \tag{5}$$

$$g^{-1} : [0, 1] \to [0, \tau], \ g^{-1}\left(\tilde{L}_\gamma\right) = \\ \left\{ \left(\tilde{s}_{\gamma \cdot \tau}\right)\left(\tilde{p}_l\right) \left| \gamma \in [0, 1] = \tilde{L} \right. \right\} \tag{6}$$

For improving the computability of PLTSs, the expectation function and deviation of a normalized PLTS were proposed as (Wu and Liao 2019):

$$E\left(\tilde{L}\right) = \sum_{l=1}^{\#\tilde{L}} \left(g\left(\tilde{s}_l\right) \times \tilde{p}_l \right) \tag{7}$$

$$\sigma\left(\tilde{L}\right) = \sqrt{\sum_{l=1}^{\#\tilde{L}} \left(\tilde{p}_l \left(g\left(\tilde{s}_l\right) - E\left(\tilde{L}\right) \right) \right)^2} \tag{8}$$

where l is the subscript of the linguistic term $\tilde{s}_l$.

2.2 STN Analysis

STN analysis is a set of norms and methods for exploring the structure and characteristics of the TRs among DMs (Gao et al. 2021). The concept of STN is represented as follows:

Definition 1 *Let $G(E, R, B)$ be a weighted graph where $E = \{e_k | k \in K\}$ represents the set of vertices, $R = \left\{r^{kh} | k \neq h \wedge k \ , k \in K\right\}$ denotes the set of edges, $T = \left\{T^{kh} | k \neq h \wedge k, k \in K\right\}$ is the set of weights with respect to edges. In the STN, the vertices e_k represent DMs, the edges r^{kh} represent trust relationships between DMs, and the weighted trust matrix T represents the strength of TRs.*

Based on Definition 2.1, there are three representations of STN (Gao et al. 2021): 1) graphic: a directed graph with vertices and edges showing the STN; 2) matrices, a matrix represents the strength of all TRs; 3) algebraic: the representation shows many combinations of relationships.

Furthermore, considering the transitivity of TR, the TR is classified into three types (Xu et al. 2021): 1) direct TR, where exists a directed edge between the vertices e_k and e_h; 2) indirect TR, where exists no directed edge, but it can be derived from the direct trust propagation paths that exist between them and mediators; 3) unrelated relationships, where exists no TR exist between the vertices e_k and e_h.

3. The Confidence-based Consensus Method for MAGDM Based on STN Analysis

In this section, a confidence-based consensus method for MAGDM is established to ensure that the final decision results are closer to the actual situation. The improved measurement of CL based on PLTS is proposed in Section 3.1, and a confidence-based normalization method for the incomplete PLTS is shown in Section 3.2. Then, the determination method of DMs' weights based on bounded trust propagation is explored in Section 3.3. Finally, a confidence-based CRP for MAGDM is developed in Section 3.4.

3.1 The Confidence-based Consensus Method for MAGDM based on STN Analysis

In MAGDM, the CL of a DM refers to the recognition degree of the evaluation given by the DM. The CL of DMs will significantly affect their choices and behaviors. For example, when a DM's CL is low, he/she will be more inclined to make decisions based on the choices of other DMs in the group. But, when his/her CL is high, it will be more difficult for him/her to accept modifying their initial evaluations during the CRP. Therefore, it is vital to consider the CL of DMs in the MAGDM.

Based on the literature review in Introduction, You and Hou (2022) defined the CL as the entropy of PLTS, Li et al. (2022) and Zhong et al. (2022) successively proposed representation methods of CL based on the completeness of PLTS and the DM's hesitancy and preference among linguistic terms. However, the above three representation methods have the following aspects that can be improved: 1) The method proposed by Li et al. (2022) ignored the correlation between the DM's hesitancy and preference among linguistic terms, which may result in an underestimation of the results; 2) the method proposed by Zhong et al. (2022) used a subjectively designed function to fit the correlation between the two, which may cause the accuracy of results to be further in-depth inquiry; 3) the method proposed by You and Hou (2022) overlooked the completeness of PLTS, and the entropy of PLTS they used repeatedly computed the uncertainty of probabilities and linguistic terms.

Aiming at the above problems, the definition of DM's CL is given based on the completeness of PLTS and information entropy. Firstly, the concept of completeness for PLTS is given as follows:

Definition 2 *Let $\{e_1, \cdots, e_k, \cdots, e_K\}$ be the set of DMs, the PLTS given by DM e_k. be $L^k = \left\{ {s_l}^k \left({p_l}^k\right) \middle| {s_l}^k \in S, {p_l}^k \geq 0, l = 1, 2, \cdots, \#L, \sum_{l=1}^{\#L} {p_l}^k \leq 1 \right\}$, $\mathfrak{R}$ be the set of PLTSs given by all DMs, the map $EC_c^k : \mathfrak{R}^k \rightarrow [0,1]$ is called as the CL of DM based on the completeness of PLTS, where the EC_c^k can be calculated by:*

$$EC_c^k = \sum_{l=1}^{\#L} {p_l}^k \tag{9}$$

Then, we adopt the information entropy of the PLTS to simultaneously represent the DM's hesitancy and preference among linguistic terms, where the information entropy refers to the uncertainty of each possible linguistic term in a PLTS. Specifically, the more uniform the probability distribution of PLTS (the lower the DM's preference for a certain linguistic term), the greater the uncertainty. The greater the shown probability (the greater the DM's hesitancy), the greater the uncertainty. The detailed definition is as follows:

Definition 3 *Let $\{e_1, \cdots, e_k, \cdots, e_K\}$ be the set of DMs, the PLTS given by the DM $L^k = \left\{ {s_l}^k \left({p_l}^k\right) \middle| {s_l}^k \in S, {p_l}^k \geq 0, l = 1, 2, \cdots, \#L, \sum_{l=1}^{\#L} {p_l}^k \leq 1 \right\}$, $\mathfrak{R}$ be the set of PLTSs given by all DMs, the map $EC_u^k : \mathfrak{R}^k \rightarrow [0,1]$ is called as the CL of DM based on the uncertainty of PLTS, in which the EC_u^k can be calculated by*

$$EC_u^k = 1 - \frac{Ec_u^k - \min\limits_{L^k \in \mathfrak{R}} \left\{Ec_u^k\right\}}{\max\limits_{L^k \in \mathfrak{R}} \left\{Ec_u^k\right\} - \min\limits_{L^k \in \mathfrak{R}} \left\{Ec_u^k\right\}} \tag{10}$$

where $Ec_u^k = -\sum_{l=1}^{\#L}\left(\frac{p_l^k}{\sum_{l=1}^{\#L}p_l^k}\right)\ln\left(\frac{p_l^k}{\sum_{l=1}^{\#L}p_l^k}\right)$ *is the information entropy of PLTS. When* $Ec_u^k = 0$, $EC_u^k = 0$.

Remark 1 *The range of* p_l^k *in the equation* $Ec_u^k = -\sum_{l=1}^{\#L}\left(p_l^k/\sum_{l=1}^{\#L}p_l^k\right)\ln\left(p_l^k/\sum_{l=1}^{\#L}p_l^k\right)$ *is* $(0,1]$. *Particularly, when* $L^k = \varnothing$, *all* p_l^k *are equal to 0. At this time, the linguistic terms in the PLTS are completely determined, so we default that* $Ec_u^k = 0$.

Based on Definitions 2 and 3, the CL consists of two parts: completeness and uncertainty of PLTS. The higher the completeness and the lower the uncertainty of the PLTS given by a DM, the higher the CL of the DM, and vice versa. On this basis, we derive the concept for the CL of DM as:

Definition 4 *Let* $\{e_1,\cdots,e_k,\cdots,e_K\}$ *be the set of DMs, the PLTS given by the DM* e_k *be* $L^k = \left\{s_l{}^k\left(p_l{}^k\right)\left|s_l{}^k \in S, p_l{}^k \geq 0, l = 1,2,\cdots,\#L,\right.\right.$ $\left.\sum_{l=1}^{\#L}p_l{}^k \leq 1\right\}$, $\mathfrak{R}$ *be the set of PLTSs given by all DMs, the map* $EC_u^k : \mathfrak{R}^k \to [0,1]$ *is called as the CL of DM, in which* EC^k *can be calculated by*

$$EC^k = \beta\cdot EC_c^k + (1-\beta)\cdot EC_u^k,\quad \beta\in[0,1] \quad (11)$$

where $Ec_u^k = -\sum_{l=1}^{\#L}\left(p_l^k/\sum_{l=1}^{\#L}p_l^k\right)\ln\left(p_l^k/\sum_{l=1}^{\#L}p_l^k\right)$ *is the information entropy of PLTS. When* $Ec_u^k = 0$, $EC_u^k = 0$.

Property 1 $EC^k\left(L^k\right) = 1$, *if and only if* $L^k = \left\{s_\varphi(1)\right\}$; $EC^k\left(L^k\right) = 0$, *if and only if* $L^k = \varnothing$.

Proof. According to Definition 3.3, when $EC^k\left(L^k\right) = 1$, $EC_c^k = \sum_{l=1}^{\#L}p_l{}^k = 1$ and $EC_u^k = 1$. Then, when $EC_u^k = 1$, $Ec_u^k = 0$, which means all p_l^k in the PLTS are equal to 0 or one p_l^k is equal to 1 and the other p_l^k are equal to 0. Combined with $EC_c^k = \sum_{l=1}^{\#L}p_l{}^k = 1$, we can get that . Similarly, when $EC^k\left(L^k\right) = 0$, $EC_c^k = \sum_{l=1}^{\#L}p_l{}^k = 0$ and $EC_u^k = 0$, which means all p_l^k are equal to 0. Then, $L^k = \varnothing$. Proof completed. ■

3.2 The New Normalization Method for the Incomplete PLTS

Based on the improved measurement of CL proposed in Section 3.1, we further give a new normalization method for the incomplete PLTS. This method assigns the missing probabilities to the existing linguistic terms and unknown linguistic terms using the CL derived by the original PLTS as the assigned proportions.

Let $L^k = \left\{s_l{}^k\left(p_l{}^k\right)\left|s_l{}^k \in IS \in S, p_l{}^k \geq 0,\right.\right.$ $\left.l = 1,2,\cdots,\#L, \sum_{l=1}^{\#L}p_l{}^k \leq 1\right\}$ be an incomplete PLTS given by the DM e_k, $\hat{p}^k = 1-\sum_{l=1}^{\#L}p_l^k$ be the missing probability, $\tilde{L}^k = \left\{\tilde{s}_l^k\left(\tilde{p}_l^k\right)\left|\tilde{s}_l^k \in S, \tilde{p}_l^k \geq 0, l = 1,2,\cdots,\#\tilde{L}, \sum_{l=1}^{\#\tilde{L}}\right.\right.$ $\left.\tilde{p}_l^k = 1\right\}$ be the normalized PLTS, then the assigned formula is represented as:

$$\begin{cases} \tilde{p}_l^k = p_l^k + \left(\hat{p}^k\cdot EC^k\right)/\#IS\,, \\ \qquad \tilde{s}_l^k \in IS \\ \tilde{p}_l^k = \left(\hat{p}^k\cdot\left(1-EC^k\right)\right)/\left(\tau+1\right. \\ \qquad \left.-\#IS\right),\quad \tilde{s}_l^k \in S - IS \end{cases} \quad (12)$$

Remark 2 *When* $\hat{p}^k = 0$ *for a PLTS, that is, when its probability distribution is complete, its linguistic term length may not be consistent with other normalized PLTS. Hence, to reduce the computational difficulty between the PLTSs, it is necessary to add* $\tau+1-\#L$ *arbitrary linguistic terms and their corresponding probabilities (the probabilities are equal to 0) to the PLTS.*

3.3 The Determination Method of DMs' Weights based on Bounded Trust Propagation

In this section, we firstly propose a confidence-based PLTR by extending the concept of CL for DM proposed in Section 3.1 to the classical PLTR. Then, the comparison rules for any two confidence-based PLTRs are provided. Next, we extend the traditional t-norm trust propagation operator to a t-norm trust propagation operator considering the CL based on the PLTR. After that, the aggregation operator of confidence-based PLTRs is defined, and the determination method of DMs' weights is proposed based on the trust in-degree in STN and the CL of DMs in the decision matrix.

3.3.1 The Confidence-based PLTR

In the actual MAGDM, the TR in STN is closely related to the behavior and psychological factors of DMs. However, the definition of the classical PLTR overlooked the CL implicit in the trust evaluation given by the DM. The CL here represents the cognitive belief of the DM. When the trust evaluation given by the DM is more accurate, it means that the DM has higher confidence in accurately assessing how much he/she trusts other DMS. Based on this, the definition of confidence-based PLPR is represented as:

Definition 5 *Let $\{e_1, \cdots, e_k, \cdots, e_K\}$ be the set of DMs, the confidence-based PLPR between the DMs e_k and e_h is defined as:*

$$T^{kh} = \left\{ \left\{ s_l{}^{kh}(p_l{}^{kh}) \middle| s_l{}^{kh} \in S, p_l{}^{kh} \geq 0, \; l = 1, 2, \cdots, \#L, \sum_{l=1}^{\#L} p_l{}^{kh} \leq 1 \right\}, \left(EC^{kh} \middle| 0 \leq EC^{kh} \leq 1 \right) \right\} \tag{13}$$

where $s_l^{kh}\left(p_l^{kh}\right)$ is the probabilistic linguistic trust information (PLTI) given by the DM e_k, EC^{kh} is the CL derived by the PLTI based on Equation (11). The larger EC^{kh} is, the more explicit the DM e_k is about the TR with the DM e_h.

According to Equations (12) and (13), the normalized confidence-based PLPR between the DMs e_k and e_h can be got as:

$$\tilde{T}^{kh} = \left\{ \left\{ \tilde{s}_l{}^{kh}\left(\tilde{p}_l{}^{kh}\right) \middle| \tilde{s}_l{}^{kh} \in S, \tilde{p}_l{}^{kh} \geq 0, \; l = 1, 2, \cdots, \#\widetilde{T}, \sum_{l=1}^{\#L} \tilde{p}_l{}^{kh} = 1 \right\}, \left(EC^{kh} \middle| 0 \leq EC^{kh} \leq 1 \right) \right\} \tag{14}$$

For a normalized confidence-based PLPR, its expectation function and deviation are presented as:

$$E\left(\tilde{T}^{kh}\right) = \left(\sum_{l=1}^{\#\tilde{L}} \left(g\left(\tilde{s}_l^{kh}\right) \cdot \tilde{p}_l^{kh} \right) \right)^{1/EC^{kh}} \tag{15}$$

$$\sigma\left(\tilde{T}^{kh}\right) = \left(\sum_{l=1}^{\#\tilde{L}} \left(\left(g\left(\tilde{s}_l^{kh}\right) - E\left(\tilde{T}^{kh}\right) \right)^2 \cdot \tilde{p}_l^{kh} \right) \right)^{EC^{kh}/2} \tag{16}$$

where g is the equivalent function for the linguistic term set obtained by Equation (3).

Property 2 *For any normalized confidence-based PLPR, we have $0 \leq E\left(\tilde{T}^{kh}\right) \leq 1$.*

Proof. Since $0 \leq g\left(\tilde{s}_l^{kh}\right) \leq 1$, $0 \leq \tilde{p}_l^{kh} \leq 1$, and $\sum_{l=1}^{\#\tilde{L}} \tilde{p}_l^{kh} = 1$, then $0 \leq \sum_{l=1}^{\#\tilde{L}} \left(g\left(\tilde{s}_l^{kh}\right) \cdot \tilde{p}_l^{kh} \right) \leq 1$. Hence, when $\sum_{l=1}^{\#\tilde{L}}(g(\tilde{s}_l^{kh}) \cdot \tilde{p}_l^{kh}) = 0$, $E(\tilde{T}^{kh}) = 1$; when $0 < \sum_{l=1}^{\#\tilde{L}}(g(\tilde{s}_l^{kh}) \cdot \tilde{p}_l^{kh}) < 1$, since $\frac{1}{EC^{kh}} > 0$, $E(\tilde{T}^{kh})$ can be regarded as an exponential function with a range of $[0, 1)$; when $\sum_{l=1}^{\#\tilde{L}}(g\left(\tilde{s}_l^{kh}\right) \cdot \tilde{p}_l^{kh}) = 1$, $E(\tilde{T}^{kh}) = 1$. To sum up, $0 \leq E(\tilde{T}^{kh}) \leq 1$. ■

For any two normalized confidence-based PLPRs, $\tilde{T}^1 < \tilde{T}^2$, if and only if one of the following two conditions is satisfied:

1) $E\left(\tilde{T}^1\right) < E\left(\tilde{T}^2\right)$;

2) $E\left(\tilde{T}^1\right) = E\left(\tilde{T}^2\right)$ and $\sigma\left(\tilde{T}^1\right) > \sigma\left(\tilde{T}^2\right)$.

3.3.2 The Bounded Propagation Operator

In the actual STN-based MAGDM, it may be difficult for some DMs to directly provide the PLTR for other DMs since they are not familiar with each other. In this situation, the STN is incomplete. To obtain the unknown PLTR in the STN, the trust propagation operator is proposed to estimate the indirect TRs among DMs. The existing trust propagation methods focused on the relationship strength, path length, and propagation time, etc. However, the existing method have ignored the effect of TRs' uncertainty on trust transitivity. Inspired by this, we hypothesize that the efficiency of trust propagation is related to the uncertainty of TR graph, which is represented by the CL among DMs in STN. The CL indicates the credibility of mediators and the mediators refer to the trusted DMs. Specifically, the higher the credibility of mediators, the higher their propagation efficiency, and vice versa. On this basis, the bounded t-norm trust propagation operator for confidence-based PLTR is defined as:

Definition 6 *Let* $e_k \xrightarrow{1} e_{\theta(1)} \xrightarrow{2} e_{\theta(2)} \xrightarrow{3} \cdots \xrightarrow{z} e_{\theta(z)} \xrightarrow{z+1} e_h$ *be a path between the DMs* e_k *and* e_h*, where its length* $z+1$*, and* $\tilde{T}^{k,\theta(1)} = \left\{\left\{\tilde{s}_{l_1}{}^{k,\theta(1)}(\tilde{p}_{l_1}{}^{k,\theta(1)}) \middle| \tilde{s}_{l_1}{}^{k,\theta(1)} \in S, l_1 = 1,2,\cdots,\#\tilde{T}_1, \sum_{l_1=1}^{\#\tilde{T}} \tilde{p}_l{}^{k,\theta(1)} = 1\right\}, \left(EC^{k,\theta(1)} \middle| \Upsilon \leq EC^{k,\theta(1)} \leq 1\right)\right\}$, $\tilde{T}^{\theta(1),\theta(2)} = \left\{\left\{\tilde{s}_{l_2}{}^{\theta(1),\theta(2)}(\tilde{p}_{l_2}{}^{\theta(1),\theta(2)}) \middle| \tilde{s}_{l_2}{}^{\theta(1),\theta(2)} \in S, \tilde{p}_{l_2}{}^{\theta(1),\theta(2)} \geq 0, l_2 = 1,2,\cdots,\#\tilde{T}_2, \sum_{l_2=1}^{\#L_2} \tilde{p}_{l_2}{}^{\theta(1),\theta(2)} = 1\right\}, \left(EC^{\theta(1),\theta(2)} \middle| \Upsilon \leq EC^{\theta(1),\theta(2)} \leq 1\right)\right\}$, $\cdots$, $\tilde{T}^{\theta(z),h} = \left\{\left\{\tilde{s}^{\theta(z),h}_{l_{z+1}}(\tilde{p}^{\theta(z),h}_{l_{z+1}}) \middle| \tilde{s}^{\theta(z),h}_{l_{z+1}} \in S, \tilde{p}^{\theta(z),h}_{l_{z+1}} \geq 0, l_{z+1} = 1,2,\cdots,\#\tilde{T}_{z+1}, \sum_{l=1}^{\#L_{z+1}} \tilde{p}^{\theta(z),h}_{l_{z+1}} = 1\right\}, \left(EC^{\theta(z),h} \middle| \Upsilon \leq EC^{\theta(z),h} \leq 1\right)\right\}$*, then the bounded t-norm trust propagation operator for confidence-based PLTR can be got by*

$$\tilde{T}^{kh} = \Delta\left(\tilde{T}^{k,\theta(1)}, \tilde{T}^{\theta(1),\theta(2)}, \cdots, \tilde{T}^{\theta(z),h}\right) = \left\{\left\{g^{-1}\left(\bigcup_{\substack{\gamma^1_{l_1} \in g\left(\tilde{T}^{k,\theta(1)}\right), \\ \gamma^2_{l_2} \in g\left(\tilde{T}^{\theta(1),\theta(2)}\right) \\ \cdots \gamma^{z+1}_{l_{z+1}} \in g\left(\tilde{T}^{\theta(z),h}\right)}} \left\{\left(\gamma^1_{l_1} \times \gamma^2_{l_2} \times \cdots \times \gamma^{z+1}_{l_{z+1}}\right)\left(\tilde{p}^1_{l_1} \times \tilde{p}^2_{l_2} \times \cdots \tilde{p}^{z+1}_{l_{z+1}}\right)\right\}\right)\right\}, \left(EC^{k,\theta(1)} \times EC^{\theta(1),\theta(2)} \times \cdots \times EC^{\theta(z),h}\right)\right\},$$
$$l_1 = 1,2,\cdots,\#\tilde{T}^{k,\theta(1)}; \; l_2 = 1,2,\cdots,\#\tilde{T}^{\theta(1),\theta(2)}; \cdots; \; l_{z+1} = 1,2,\cdots,\#\tilde{T}^{\theta(z),h} \tag{17}$$

where Υ *is the confidence-based propagation threshold,* $\gamma^1_{l_1}, \gamma^2_{l_2}, \cdots, \gamma^{z+1}_{l_{z+1}}$ *is calculated by Equation* (3) *and the function* g^{-1} *is got by Equation* (5).

Remark 3 *The boundness of Definition 6 is embodied in the credibility of mediators, that is, the CL between the DM and the mediators. When* $\min\left(EC_{k,\theta(1)}, EC_{\theta(1),\theta(2)}, \cdots, EC_{\theta(z),h}\right) < \Upsilon$*, the credibility of at least one mediator is less than* Υ*, the propagation is interrupted. Here, the CL* $EC_{k,\theta}$ *refers to the certainty degree that DM* e_k *trusts the mediator* e^θ*. When the certainty degree is low, it is difficult for the DM* e_k *to trust the suggestions from the mediator* e^θ*, leading to the interruption of propagation. Therefore, only when* $\min\left(EC_{k,\theta(1)}, EC_{\theta(1),\theta(2)}, \cdots, EC_{\theta(z),h}\right) \geq \Upsilon$*, the effective indirect TR can be derived from the propagation path.*

Property 3 *For any operator* Δ*, we* *have* $\Delta\left(\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right) \leq \min\left\{\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right\}$.

Proof. According to Equations (5) and (17), we can obtain that

$$\Delta\left(\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right)=\Bigg\{\Bigg\{\left(s_{\tau\cdot\left(\gamma_1^1\times\gamma_1^2\times\cdots\times\gamma_1^{z+1}\right)}\right)\left(\tilde{p}_1{}^1\times\tilde{p}_1{}^2\times\cdots\times\tilde{p}_1{}^{z+1}\right),\left(s_{\tau\cdot\left(\gamma_1^1\times\gamma_2^2\times\cdots\times\gamma_2^{z+1}\right)}\right)$$
$$\left(\tilde{p}_1{}^1\times\tilde{p}_2{}^2\times\cdots\times\tilde{p}_2{}^{z+1}\right),\cdots,\left(s_{\tau\cdot\left(\gamma_1^1\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)}\right)\left(\tilde{p}_1{}^1\times\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right),$$
$$\left(s_{\tau\cdot\left(\gamma_2^1\times\gamma_1^2\times\cdots\times\gamma_1^{z+1}\right)}\right)\left(\tilde{p}_2{}^1\times\tilde{p}_1{}^2\times\cdots\times\tilde{p}_1{}^{z+1}\right),\left(s_{\tau\cdot\left(\gamma_2^1\times\gamma_2^2\times\cdots\times\gamma_2^{z+1}\right)}\right)\left(\tilde{p}_2{}^1\times\tilde{p}_2{}^2\times\cdots\times\tilde{p}_2{}^{z+1}\right),\cdots$$
$$\left(s_{\tau\cdot\left(\gamma_1^1\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)}\right)\left(\tilde{p}_2{}^1\times\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right),$$
$$\vdots$$
$$\left(s_{\tau\cdot\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\times\gamma_1^2\times\cdots\times\gamma_1^{z+1}\right)}\right)\left(\tilde{p}^1_{\#\tilde{T}^{k,\theta(1)}}\times\tilde{p}_1{}^2\times\cdots\times\tilde{p}_1{}^{z+1}\right),\left(s_{\tau\cdot\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\times\gamma_2^2\times\cdots\times\gamma_2^{z+1}\right)}\right)\left(\tilde{p}_2{}^1\times\tilde{p}_2{}^2\times\cdots\times\tilde{p}_2{}^{z+1}\right),$$
$$\cdots\left(s_{\tau\cdot\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)}\right)\left(\tilde{p}_2{}^1\times\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right),$$
$$\left(EC^{k,\theta(1)}\times EC^{\theta(1),\theta(2)}\times\cdots\times EC^{\theta(z),h}\right)\Bigg\}\Bigg\}$$

According to Equations (6) and (15), we can get that

$$E\left(\Delta\left(\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right)\right)=\Bigg(\left(\gamma_1^1\tilde{p}_1{}^1\times\gamma_1^2\tilde{p}_1{}^2\times\cdots\times\gamma_1^{z+1}\tilde{p}_1{}^{z+1}\right)+\left(\gamma_1^1\tilde{p}_1{}^1\times\gamma_2^2\tilde{p}_2{}^2\times\cdots\times\right.$$
$$\left.\gamma_2^{z+1}\tilde{p}_2{}^{z+1}\right)+\cdots+\left(\gamma_1^1\tilde{p}_1{}^1\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)+\left(\gamma_2^1\tilde{p}_2{}^1\times\gamma_1^2\tilde{p}_1{}^2\times\cdots\times\right.$$
$$\left.\gamma_1^{z+1}\tilde{p}_1{}^{z+1}\right)+\left(\gamma_2^1\tilde{p}_2{}^1\times\gamma_2^2\tilde{p}_2{}^2\times\cdots\times\gamma_2^{z+1}\tilde{p}_2{}^{z+1}\right)+\left(\gamma_1^1\tilde{p}_2{}^1\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)$$
$$+\cdots+\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\tilde{p}^1_{\#\tilde{T}^{k,\theta(1)}}\times\gamma_1^2\tilde{p}_1{}^2\times\cdots\times\gamma_1^{z+1}\tilde{p}_1{}^{z+1}\right)+\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\tilde{p}_2{}^1\times\gamma_2^2\tilde{p}_2{}^2\times\cdots\times\gamma_2^{z+1}\tilde{p}_2{}^{z+1}\right)+$$
$$\cdots+\left(\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\tilde{p}_2{}^1\times\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\times\cdots\times\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)\Bigg)^{1/EC^{k,\theta(1)}\times EC^{\theta(1),\theta(2)}\times\cdots\times EC^{\theta(z),h}}$$
$$=\left(\left(\gamma_1^1\tilde{p}_1{}^1+\gamma_2^1\tilde{p}_2{}^1+\cdots+\gamma^1_{\#\tilde{T}^{k,\theta(1)}}\tilde{p}_{\#\tilde{T}^{k,\theta(1)}}{}^1\right)\times\left(\gamma_1^2\tilde{p}_1{}^2+\gamma_2^2\tilde{p}_2^2+\cdots+\gamma^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\tilde{p}^2_{\#\tilde{T}^{\theta(1),\theta(2)}}\right)\times\cdots\right.$$
$$\left.\times\left(\gamma_1^{z+1}\tilde{p}_1{}^{z+1}+\gamma_2^{z+1}\tilde{p}_2^{z+1}+\cdots+\gamma^{z+1}_{\#\tilde{T}^{\theta(z),h}}\tilde{p}^{z+1}_{\#\tilde{T}^{\theta(z),h}}\right)\right)^{1/EC^{k,\theta(1)}\times EC^{\theta(1),\theta(2)}\times\cdots\times EC^{\theta(z),h}}$$
$$=\left(\left(\sum_{l_1=1}^{\#\tilde{T}^{k,\theta(1)}}\left(\gamma^1_{l_1}\tilde{p}_{l_1}{}^1\right)\right)\left(\sum_{l_2=1}^{\#\tilde{T}^{\theta(1),\theta(2)}}\left(\gamma^1_{l_2}\tilde{p}_{l_2}{}^1\right)\right)\times\cdots\right.$$
$$\left.\times\left(\sum_{l_{z+1}=1}^{\#\tilde{T}^{\theta(z),h}}\left(\gamma^1_{l_{z+1}}\tilde{p}_{l_{z+1}}{}^1\right)\right)\right)^{1/EC^{k,\theta(1)}\times EC^{\theta(1),\theta(2)}\times\cdots\times EC^{\theta(z),h}}$$

Let $\min\left\{\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right\}=\tilde{T}^{k,\theta(1)}$, then

$$E\left(\min\left\{\tilde{T}^{k,\theta(1)},\tilde{T}^{\theta(1),\theta(2)},\cdots,\tilde{T}^{\theta(z),h}\right\}\right)=E\left(\tilde{T}^{k,\theta(1)}\right)=\left(\sum_{l_1=1}^{\#\tilde{T}^{k,\theta(1)}}\left(\gamma^1_{l_1}\tilde{p}_{l_1}{}^1\right)\right)^{1/EC_{k,\theta(1)}};$$

Since $\Upsilon \leq EC \leq 1$, then $EC_{k,\theta(1)} \geq \left(EC_{k,\theta(1)} \times EC_{\theta(1),\theta(2)} \times \cdots \times EC_{\theta(z),h}\right)$, then $1/EC_{k,\theta(1)} \leq 1/\left(EC_{k,\theta(1)} \times EC_{\theta(1),\theta(2)} \times \cdots \times EC_{\theta(z),h}\right)$.

And since $0 \leq \gamma_l \leq 1$, $0 \leq \tilde{p}_l \leq 1$, and $\sum_{l=1}^{\#L} \tilde{p}_l = 1$, then $0 \leq \sum_{l_1=1}^{\#\tilde{T}^{k,\theta(1)}} (\gamma_{l_1}^1 \tilde{p}_{l_1}^1) \leq 1$, and thus, $\sum_{l_1=1}^{\#\tilde{T}^{k,\theta(1)}} (\gamma_{l_1}^1 \tilde{p}_{l_1}^1) \geq \left(\sum_{l_1=1}^{\#\tilde{T}^{k,\theta(1)}} \left(\gamma_{l_1}^1 \tilde{p}_{l_1}^1\right)\right)\left(\sum_{l_2=1}^{\#\tilde{T}^{\theta(1),\theta(2)}} (\gamma_{l_2}^2 \tilde{p}_{l_2}^2)\right) \times \cdots \times \left(\sum_{l_1=1}^{\#\tilde{T}^{\theta(z),h}} (\gamma_{l_{z+1}}^{z+1} \tilde{p}_{l_{z+1}}^{z+1})\right)$.

In summary, $E\left(\Delta\left(\tilde{T}^{k,\theta(1)}, \tilde{T}^{\theta(1),\theta(2)}, \cdots, \tilde{T}^{\theta(z),h}\right)\right) \leq E\left(\min\left\{\tilde{T}^{k,\theta(1)}, \tilde{T}^{\theta(1),\theta(2)}, \cdots, \tilde{T}^{\theta(z),h}\right\}\right)$; then $\Delta\left(\tilde{T}^{k,\theta(1)}, \tilde{T}^{\theta(1),\theta(2)}, \cdots, \tilde{T}^{\theta(z),h}\right) \leq \min\left\{\tilde{T}^{k,\theta(1)}, \tilde{T}^{\theta(1),\theta(2)}, \cdots, \tilde{T}^{\theta(z),h}\right\}$ holds true. ■

3.3.3 The Determination Method of DMs' Weights

Before determining the DMs' weights, we need to integrate the indirect TRs derived from multiple propagation paths. Inspired by the q-rung fuzzy weighted averaging operator defined in Gao et al. (2022), the confidence-based probabilistic linguistic trust ordered weighted averaging (PLT- OWA) operator is proposed as:

Definition 7 *Suppose that there are* Pr *propagation paths between the DMs e_k and e_h, and their confidence-based PLTRs are $\left\{\tilde{T}_1^{kh}, \tilde{T}_2^{kh}, \cdots, \tilde{T}_{\Pr}^{kh}\right\}$, then the aggregated confidence-based PLTR between the DMs e_k and e_h can be derived by the PLT-OWA operator:*

$$\begin{aligned}\tilde{T}^{kh} &= OWA\left(\tilde{T}_1^{kh}, \tilde{T}_2^{kh}, \cdots, \tilde{T}_{\Pr}^{kh}\right) \\ &= \oplus_{pr=1}^{\Pr} \psi_{pr} \tilde{T}_{pr}^{kh} \\ &= \left\{ \bigcup_{\gamma_{l_{pr}}^{pr} \in g\left(\tilde{T}_{pr}^{kh}\right)} \left\{ g^{-1}\left(1 - \prod_{pr=1}^{\Pr} \left(1 - \gamma_{l_{pr}}^{pr}\right)^{\Psi_{pr}}\right)\left(\prod_{pr=1}^{\Pr} \tilde{p}_{l_{pr}}^{pr}\right)\right\}, \frac{\sum_{pr=1}^{\Pr} \Psi_{pr} EC^{pr}}{\sum_{pr=1}^{\Pr} \Psi_{pr}} \right\}\end{aligned} \tag{18}$$

where $\tilde{T}_1^{kh}$ is the largest confidence-based PLTR in $\left\{\tilde{T}_1^{kh}, \tilde{T}_2^{kh}, \cdots, \tilde{T}_{\Pr}^{kh}\right\}$, $\Psi = (\Psi_1, \Psi_1, \cdots, \Psi_{\Pr})$ are the corresponding weights, $\Psi_{pr} \geq 0$ and $\sum_{pr=1}^{\Pr} \Psi_{pr} = 1$.

The value of Ψ_{pr} can be determined by the linguistic quantifier Q (Yager 1996):

$$\Psi_{pr} = Q\left(\frac{pr}{\Pr}\right) - Q\left(\frac{pr-1}{\Pr}\right) \tag{19}$$

where the monotone non-decreasing quantifier $Q = ir^{\delta}$, satisfies $\delta \geq 0$, $Q : [0,1] \rightarrow [0,1]$, $Q(0) = 0$, $Q(1) = 1$. In this paper, we set δ as 0.5.

Remark 4 *The PLTR-OWA operator differs from the traditional PL-OWA by utilizing the CL of PLTI to induce the reordering process. The CL of PLTI suggests its reliability. Therefore, it's crucial to consider both its value and CL level when determining its order. Specifically, the largest confidence-based PLTR must contain both high value and high CL level.*

Based on the bounded propagation operator and the aggregation operator, the incomplete STN matrix $IT = \left(\tilde{T}^{kh}\right)_{K\times K}$ can be transformed into the complete STN matrix $CT = \left(\tilde{T}^{kh}\right)_{K\times K}$. Then, we can obtain the in-degree trust (ID) and the integrated in-degree trust (CID) of DM e_k as:

$$ID^k = \frac{1}{K-1}\sum_{h=1,k\neq h}^{K} E\left(\tilde{T}^{hk}\right), \quad h = 1,2,\cdots,K \tag{20}$$

$$CID^k = \frac{ID^k}{\max\left(ID^k\right)}, \ k = 1,2,\cdots,K \tag{21}$$

And the out-degree trust (OD) and the integrated out-degree trust (COD) of the DM e_k:

$$OD^k = \frac{1}{K-1}\sum_{h=1,k\neq h}^{K} E\left(\tilde{T}^{kh}\right), \quad h = 1,2,\cdots,K \tag{22}$$

$$COD^k = \frac{OD^k}{\max\left(OD^k\right)}, \ k = 1,2,\cdots,K \tag{23}$$

where $E\left(\tilde{T}^{hk}\right)$ is the expectation function of $\tilde{T}^{hk}$.

When DMs have low CL in their evaluations, it indicates that the quality and accuracy of the evaluations are low. Hence, to obtain accurate and reliable group evaluations, it is necessary to consider the CL derived from DMs' evaluations when determining the DMs' weights. From this, we represent the DMs' weights based on the CL and CID of DMs as:

Definition 8 *Let $EC^{ij(k)}(i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ be the CL derived from the PLTS $L^{ij(k)}$ given by the DM e_k, then the weight of the DM e_k can be obtained by*

$$v_k = \frac{EC^k \cdot CID^k}{\sum_{k=1}^{K} EC^k \cdot CID^k} \tag{24}$$

where $EC^k = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} EC^{ij(k)}$.

Remark 5 *According to Definition 8, it is easy to find that even if a DM is highly trusted in the STN, his/her weight won't be very high if the quality of the evaluations they provided is low. Compared to the determination method that exclusively derives the weights from the CID in STN, the proposed method incorporating the objective evaluations given by DMs can more effectively prevent weight manipulation.*

3.4 The Confidence-based CRP for MAGDM

After determining the weights of experts, this section provides the integration method of group evaluations and calculates the group consensus index (CI) according to the similarity measures between individual evaluations and group evaluations. Then, we judge whether the DMs reach a consensus on the group evaluations. If the group does not reach a consensus, it is necessary to identify DMs with low CIs and supply their modification suggestions.

3.4.1 Consensus Measure

Firstly, we adopt the method proposed by Wu and Liao (2019) to collect $K \times m \times n$ PLTSs into $m \times n$ PLTSs. Let $A = \{A_1, \cdots A_i, \cdots, A_m\}$ be the set of alternatives, $C = \{C_1, \cdots, C_j, \cdots, C_n\}$ be set of attributes, $(w_1, \cdots, w_j, \cdots, w_n)$ be the weights of attributes, $E = \{e_1, \cdots, e_k, \cdots, e_K\}$ be the set of DMs, $(v_1, \cdots, v_k, \cdots, v_K)$ be the DMs' weights. The PLTS $L^{ij(k)} = \left\{s_l^{ij(k)}\left(p_l^{ij(k)}\right) \middle| s_l^{ij(k)} \in S, p_l^{ij(k)} \geq 0, l = 1,2,\cdots,\#L, \sum_{l=1}^{\#L} p_l^{ij(k)} \leq 1\right\}$ is the evaluation provided by the DM e_k with respect to the alternative A_i on the attribute C_j, and $\tilde{L}^{ij(k)} = \left\{\tilde{s}_l^{ij(k)}\left(\tilde{p}_l^{ij(k)}\right) \middle| \tilde{s}_l^{ij(k)} \in S, \tilde{p}_l^{ij(k)} \geq 0, l = 1,2,\cdots\#\tilde{L}, \sum_{l=1}^{\#\tilde{L}} \tilde{p}_l^{ij(k)} = 1\right\}$ is the normalized PLTS. Then, let the collective evaluation be $L^{ij(\Lambda)} = \left\{s_l^{ij(\Lambda)}\left(p_l^{ij(\Lambda)}\right) \middle| s_l^{ij(\Lambda)} \in S, p_l^{ij(\Lambda)} \geq 0, l = 1,2,\cdots,\#L, \sum_{l=1}^{\#L} p_l^{ij(\Lambda)} = 1\right\}$, $p_l^{ij(\Lambda)}$ can be calculated by

$$p_l^{ij(\Lambda)} = \sum_{k=1}^{K} v^k p_l^{ij'}, \ l = 1, 2, \cdots, \#L \tag{25}$$

where $p_l^{ij'} = \begin{cases} p_l^{ij'}, \text{if } s_l^{ij(\Lambda)} \in \tilde{L}^{ij(k)} \\ 0, \ \text{if } s_l^{ij(\Lambda)} \notin \tilde{L}^{ij(k)} \end{cases}$, $k = 1, 2, \cdots, K$.

Then, the similarity measure between the individual evaluations and collective evaluations is represented as Cao et al. (2021):

Definition 9 *Let* $\tilde{L}^k = \left(\tilde{L}^{ij(k)}\right)_{m\times n}$ *be the evaluation given by the DM* e_k*, the collective evaluation be* $L^{(\Lambda)} = \left(L^{ij(\Lambda)}\right)_{m\times n}$*, then the CIs in three levels is shown as:*

1) The CI of the DM e_k *on the element level:*

$$CI_{ij}^k = 1 - \left| E\left(L^{ij(\Lambda)}\right) - E\left(\tilde{L}^{ij(k)}\right) \right| \tag{26}$$

2) The CI of the DM e_k *on the alternative level:*

$$CI_i^k = \frac{1}{n} \sum_{j=1}^{n} CL_{ij}^k \tag{27}$$

3) The CI of the DM e_k *on the matrix level:*

$$CI^k = \frac{1}{m} \sum_{i=1}^{m} CL_i^k \tag{28}$$

Definition 10 *Let* CI_k *be the CI of the DM* e_k*, the group consensus index (GCI) is represented as:*

$$GCI = \frac{1}{K} \sum_{k=1}^{K} CI^k \tag{29}$$

Obviously, $GCI \in [0, 1]$. After setting the group consensus threshold ε, we can determine whether the group has reached an acceptable level of consensus. When $GCI \geq \varepsilon$, it indicates that the group consensus has been reached.

3.4.2 Feedback Mechanism

The traditional feedback mechanism directly identifies the non-consensus DMs and provides them with modification suggestions. However, if the non-consensus DMs have a high level of confidence in their evaluations, it is difficult to make them accept the modification suggestions, which finally causes the CRP fails. Hence, we establish identification rules which consider both the CL and CI of DM as follows:

Step 1 Identify the DMs with low CL and CI:

$$EXPCH = \left\{k \left| \left(CL_i^k < \varepsilon\right) \wedge \left(EC^k < \psi\right) \right.\right\} \tag{30}$$

Step 2 Identify the alternatives with low CL and CI for the non-consensus DMs:

$$ALT = \Big\{(k, i) \Big| (k \in EXPCH) \wedge \left(CL_i^k < \varepsilon\right) \wedge \left(EC^{i(k)} < \psi\right) \Big\} \tag{31}$$

Step 3 Identify the elements with low CL and CI for the non-consensus alternatives:

$$APS = \Big\{ (k, i, j) \Big| ((k, i) \in ALT) \wedge \left(CL_{ij}^k < \varepsilon\right) \wedge \left(EC^{ij(k)} < \psi\right) \Big\} \tag{32}$$

Remark 6 *The proposed identification rules aim to find out the DMs and the corresponding evaluations whose CI and CL are lower than the thresholds. There are two benefits of this design: one is to ensure that the group has a lower cost to persuade the non-consensus DMs to accept the modification suggestions; the other is to retain the evaluations with a high CL as much as possible, which will help ensure that the quality of group evaluations.*

After identifying the non-consensus DMs, we need to generate modification suggestions for them through the MCC model. Considering the impact of the DMs' CL on their behaviors, we introduce the DMs' CL into the cost function in the MCC model, the confidence-based unit adjustment cos function under each alternative and attribute for each DM is defined as:

Let $\tilde{L}^{ij(k)}$ be the original evaluation, EC_{ij}^{k} be the CL of the DM e^k for the evaluation $\tilde{L}^{ij(k)}$, $\bar{L}^{ij(k)}$ be the adjustment evaluation provided by the MCC model, the confidence-based unit adjustment cost function is obtained by

$$f_{(k,i,j)\in APS} = EC_{ij}^{k}\left|E\left(\bar{L}^{ij(k)}\right) - E\left(\tilde{L}^{ij(k)}\right)\right| \quad (33)$$

The total confidence-based adjustment cost function is represented by

$$f = \sum_{(k,i,j)\in APS} EC_{ij}^{k}\left|E\left(\bar{L}^{ij(k)}\right) - E\left(\tilde{L}^{ij(k)}\right)\right| \quad (34)$$

Remark 7 *The confidence-based unit adjustment cost function personalizes the definition of the DM's cost for modifying each evaluation, which is more in line with the psychological behavior of the DMs in actual situations. Due to the limitations of the professional field and experience, the cognitive levels of DMs for each alternative under different attributes may be inconsistent. That is, the CL of DMs for each evaluation may be different, then the unit adjustment cost may also be different. Specifically, if a DM lacks confidence in his/her evaluation, the cost to the group of persuading him/her to accept an adjustment is relatively low. The DM with a high CL, on the other hand, is more inclined to maintain the original evaluation, so the cost of persuasion is very high.*

Note, although that the group is willing to pay costs to persuade the non-consensus DMs to accept adjustments, the DMs' willingness to adjust can still not be ignored. In this section, we adopt the CL and COD of DMs to portray their willingness. The acceptable adjustment threshold is obtained by

$$uc_{ij}^{k} = \frac{\left(1 - EC_{ij}^{k}\right) + COD^{k}}{2} \quad (35)$$

Then, the adjustment evaluation needs to satisfy the following conditions:

$$\left|E\left(\bar{L}^{ij(k)}\right) - E\left(\tilde{L}^{ij(k)}\right)\right| \le uc_{ij}^{k}, \quad (i,j,k) \in APS \quad (36)$$

Remark 8 *Although both the cost function and the adjustment threshold involve the DM's CL, their meanings are different. The cost refers to the unit cost paid by the group to persuade the DM to adjust the evaluation, so only the DM's CL on the evaluation should be considered. The adjustment threshold refers to the maximum degree to which the DM is willing to adjust the evaluation. It depends not only on the CL, which is the internal cause that affects the DMs' adjustment willingness, but also on DM's trust in other DMs in the group. A DM with high CL can still increase his/her range of adjustment due to a high trust degree in other DMs, but the cost of persuading him/her to adjust the evaluation is still high.*

Based on the above thinking, we establish the following confidence-based MCC model:

Model 3.1

$$\min \sum_{(k,i,j)\in APS} EC_{ij}^{k}\left|E\left(\bar{L}^{ij(k)}\right)-E\left(\tilde{L}^{ij(k)}\right)\right|$$

$$\text{s.t.}\begin{cases} E\left(\bar{L}^{ij(k)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \bar{p}_{ij,\varphi}^{k}\right),\ (k,i,j)\in APS & (1)\\ E\left(\tilde{L}^{ij(h)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\tilde{s}_{\varphi}\right)\times \tilde{p}_{ij,\varphi}^{h}\right),\ (h,i,j)\notin APS & (2)\\ E\left(\bar{L}^{ij(\Lambda)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \left(\bar{p}_{ij,\varphi}^{\Lambda}\right)\right), k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} & (3)\\ \bar{p}_{ij,\varphi}^{\Lambda}=\sum_{k=1}^{K}v^{k}\left(\bar{p}_{ij,\varphi}^{k}+\tilde{p}_{ij,\varphi}^{k}\right),\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} & (4)\\ \sum_{\varphi=0}^{\tau}\bar{p}_{ij,\varphi}^{\Lambda}=1,\ i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} & (5)\\ \frac{1}{Kmn}\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(1-\left|E\left(\bar{L}^{ij(k)}\right)-E\left(\bar{L}^{ij(\Lambda)}\right)\right|\right)\geq\varepsilon, & \\ \qquad k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} & (6)\\ \left|E\left(\bar{L}^{ij(k)}\right)-E\left(\tilde{L}^{ij(k)}\right)\right|\leq uc_{ij}^{k},\ (k,i,j)\in APS & (7) \end{cases}$$

where $\tilde{L}^{ij(k)}$ is the original evaluation, $\bar{L}^{ij(k)}$ is the adjustment evaluation obtained from the model, $\bar{L}^{ij(\Lambda)}$ is the adjusted group evaluation. The objective function represents the overall cost to the group of persuading the DMs to adjust their evaluation from $\tilde{L}^{ij(k)}$ to $\bar{L}^{ij(k)}$, and it should be the smallest it can be. The constraints (1-3) indicate the expectations of $\tilde{L}^{ij(k)}$, $\bar{L}^{ij(k)}$, and $\bar{L}^{ij(\Lambda)}$. The constraints (4-5) provide a way to integrate probabilistic information for group evaluation $\bar{L}^{ij(\Lambda)}$.

The goal of constraint (6) is to guarantee that the deviation between the group evaluation $\bar{L}^{ij(\Lambda)}$ and the individual adjustment evaluation $\bar{L}^{ij(k)}$ can satisfy the consensus threshold. The constraint (7) stipulates that the deviation between the adjustment evaluation $\bar{L}^{ij(k)}$ and the original evaluation $\tilde{L}^{ij(k)}$ must not exceed the range of DMs' willingness. The range uc_{ij}^{k} is represented by the certainty of evaluation (CL) and the DMs' trust in other DMs (COD) in the STN.

Since Model 3.1 is a nonlinear programming model, it is necessary to convert it to a linear programming model through mathematical transformation, which is shown as follows:

Model 3.2

$$\min \sum_{(k,i,j)\in APS} EC_{ij}^{k}\cdot a^{ij(k)}$$

$$\text{s.t.}\begin{cases} E\left(\bar{L}^{ij(k)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \bar{p}_{ij,\varphi}^{k}\right),\ (k,i,j)\in APS\\ E\left(\tilde{L}^{ij(k)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\tilde{s}_{\varphi}\right)\times \tilde{p}_{ij,\varphi}^{k}\right),\ (k,i,j)\notin APS\\ E\left(\bar{L}^{ij(\Lambda)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \left(\bar{p}_{ij,\varphi}^{\Lambda}\right)\right),\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ \bar{p}_{ij,\varphi}^{\Lambda}=\sum_{k=1}^{K}v^{k}\left(\bar{p}_{ij,\varphi}^{k}+\tilde{p}_{ij,\varphi}^{k}\right),\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ md^{ij(k)}=E\left(\bar{L}^{ij(k)}\right)-E\left(\tilde{L}^{ij(k)}\right),\ (k,i,j)\in APS\\ md^{ij(k)}\leq a^{ij(k)},\ (k,i,j)\in APS\\ -md^{ij(k)}\leq a^{ij(k)},\ (k,i,j)\in APS\\ x^{ij(k)}=E\left(\bar{L}^{ij(k)}\right)-E\left(\bar{L}^{ij(\Lambda)}\right),\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ x^{ij(k)}\leq y^{ij(k)},\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ -x^{ij(k)}\leq y^{ij(k)},\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ \frac{1}{Kmn}\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(1-y^{ij(k)}\right)\geq\varepsilon,\ k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\}\\ a^{ij(k)}\leq uc_{ij}^{k},\ (k,i,j)\in APS\\ \sum_{\varphi=0}^{\tau}\bar{p}_{ij,\varphi}^{\Lambda}=1,\ i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \end{cases}$$

where $md^{ij(k)} = E\left(\bar{L}^{ij(k)}\right) - E\left(\tilde{L}^{ij(k)}\right)$, $a^{ij(k)} = \left|md^{ij(k)}\right|$, $x^{ij(k)} = E\left(\bar{L}^{ij(k)}\right) - E\left(\bar{L}^{ij(\Lambda)}\right)$ and $y^{ij(k)} = \left|x^{ij(k)}\right|$.

In the case of setting thresholds ε and ψ, Model 3.2 may not have a feasible solution, which means some DMs refuse to accept the adjustment suggestion generated by the Model 3.2. Hence, we represent the following:

Property 4 *Let uc_{ij}^{k*} be the optimal solution of the Model 3.3, then if all uc_{ij}^{k} satisfy the conditions $uc_{ij}^{k} \geq uc_{ij}^{k*}$, then the Model 3.2 exists in at least one optimal solution; if any uc_{ij}^{k} do not satisfy the conditions $uc_{ij}^{k} \geq uc_{ij}^{k*}$, then there is no solution to the Model 3.2, which means that it cannot generate the adjustment suggestions that make all DMs to accept by the Model 3.2.*

Model 3.3

$$\min \sum_{(k,i,j)\in APS} uc_{ij}^{k}$$

$$\text{s.t.} \begin{cases} E\left(\bar{L}^{ij(k)}\right) = \sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \bar{p}_{ij,\varphi}^{k}\right), (k,i,j)\in APS \\ E\left(\tilde{L}^{ij(k)}\right) = \sum_{\varphi=0}^{\tau}\left(g\left(\tilde{s}_{\varphi}\right)\times \tilde{p}_{ij,\varphi}^{k}\right), (k,i,j)\notin APS \\ E\left(\bar{L}^{ij(\Lambda)}\right) = \sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times \left(\bar{p}_{ij,\varphi}^{\Lambda}\right)\right), k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ \bar{p}_{ij,\varphi}^{\Lambda} = \sum_{k=1}^{K} v^{k}\left(\bar{p}_{ij,\varphi}^{k}+\tilde{p}_{ij,\varphi}^{k}\right), k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ md^{ij(k)} = E\left(\bar{L}^{ij(k)}\right) - E\left(\tilde{L}^{ij(k)}\right), (k,i,j)\in APS \\ md^{ij(k)} \leq a^{ij(k)}, (k,i,j)\in APS \\ -md^{ij(k)} \leq a^{ij(k)}, (k,i,j)\in APS \\ x^{ij(k)} = E\left(\bar{L}^{ij(k)}\right) - E\left(\bar{L}^{ij(\Lambda)}\right), k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ x^{ij(k)} \leq y^{ij(k)}, k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ -x^{ij(k)} \leq y^{ij(k)}, k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ \frac{1}{Kmn}\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(1-y^{ij(k)}\right) \geq \varepsilon, k\in\{1,2,\cdots,K\}; i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \\ a^{ij(k)} \leq uc_{ij}^{k}, (k,i,j)\in APS \\ \sum_{\varphi=0}^{\tau}\bar{p}_{ij,\varphi}^{\Lambda} = 1, i\in\{1,2,\cdots,m\}; j\in\{1,2,\cdots,n\} \end{cases}$$

When we cannot reach a consensus through the Model 3.2, it is necessary to identify the non-cooperative DMs who refuse to accept the suggestions generated by the Model 3.2. Then, the group can utilize the impact of the STN on DMs' preferences to persuade them to accept the new adjustment strategy. Firstly, the DMs and the corresponding evaluations who refuse adjustments are identified by

$$RPS = \left\{\left(k',i,j\right)\left|uc_{ij}^{k'} < uc_{ij}^{k*}\right.\right\} \tag{37}$$

Then, the new adjustment strategy is represented as:

$$\overline{L}_{ij}^{k'} = uc_{ij}^{k'} L^{ij(\Lambda)} + \left(1-uc_{ij}^{k'}\right)\tilde{L}_{ij}^{k'} \tag{38}$$

Remark 9 *The traditional adjustment strategies in the STN-based MAGDM usually use the evaluation of the trusted DMs to generate suggestions, which is easy to ignore the evaluation of DMs on the edge of the STN. Therefore, in order to avoid the collective evaluation being manipulated by DMs at the center of the STN, we use the current group evaluation to generate adjustment suggestions.*

Then, we construct the following hybrid

model to provide the adjustment suggestions for the other elements in the set APS:

Model 3.4

$$\min \sum_{(k,i,j)\in APS/RPS} EC_{ij}^{k}\left|E\left(\bar{L}^{ij(k)}\right)-E\left(\tilde{L}^{ij(k)}\right)\right|$$

$$\text{s.t.}\begin{cases}
E\left(\bar{L}^{ij(k)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times\bar{p}_{ij,\varphi}^{k}\right),(k,i,j)\in APS/RPS & (1)\\
E\left(\bar{L}^{ij\left(k'\right)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times\bar{p}_{ij,\varphi}^{k'}\right),(k,i,j)\in RPS & (2)\\
E\left(\bar{L}^{ij(h)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times\tilde{p}_{ij,\varphi}^{h}\right),(h,i,j)\notin APS & (3)\\
E\left(\bar{L}^{ij(\Lambda)}\right)=\sum_{\varphi=0}^{\tau}\left(g\left(\bar{s}_{\varphi}\right)\times\left(\bar{p}_{ij,\varphi}^{\Lambda}\right)\right), & \\
\qquad k\in\{1,2,\cdots,K\};i\in\{1,2,\cdots,m\};j\in\{1,2,\cdots,n\} & (4)\\
\bar{p}_{ij,\varphi}^{\Lambda}=\sum_{k=1}^{K}v^{k}\left(\bar{p}_{ij,\varphi}^{k}+\bar{p}_{ij,\varphi}^{k'}+\tilde{p}_{ij,\varphi}^{k}\right), & \\
\qquad k\in\{1,2,\cdots,K\};i\in\{1,2,\cdots,m\};j\in\{1,2,\cdots,n\} & (5)\\
\sum_{\varphi=0}^{\tau}\bar{p}_{ij,\varphi}^{\Lambda}=1,i\in\{1,2,\cdots,m\};j\in\{1,2,\cdots,n\} & (6)\\
\bar{p}_{ij,\varphi}^{k'}=\frac{\left(uc_{ij}^{k'}+OD^{k'}\right)}{2}p_{ij,\varphi}^{\Lambda}+\left(1-\frac{\left(uc_{ij}^{k'}+OD^{k'}\right)}{2}\right)\tilde{p}_{ij,\varphi}^{k'},\ (k,i,j)\in RPS & (7)\\
\frac{1}{Kmn}\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(1-\left|E\left(\bar{L}^{ij(k)}\right)-E\left(\bar{L}^{ij(\Lambda)}\right)\right|\right)\geq\varepsilon, & \\
\qquad k\in\{1,2,\cdots,K\};i\in\{1,2,\cdots,m\};j\in\{1,2,\cdots,n\} & (8)\\
\left|E\left(\bar{L}^{ij(k)}\right)-E\left(\tilde{L}^{ij(k)}\right)\right|\leq uc_{ij}^{k},(k,i,j)\in APS & (9)
\end{cases}$$

where $\bar{L}^{ij(k')}$ is the adjustment evaluation for non-cooperative DMs. The constraints (4-5) indicate the integration of group evaluation. The constraint (6) provides the strategy to adjust the evaluation of non-cooperative DMs. The remaining variables, objective function, and constraints are the same as in the Model 3.1.

3.4.3 Selection Process

After the group reached a consensus and the final group decision matrix $L^{(\Lambda)}=\left(L^{ij(\Lambda)}\right)_{m\times n}$ was obtained, the score of the alternatives A_i can by calculated by $CE_i=\sum_{j=1}^{n}w_jE\left(L^{ij(\Lambda)}\right)$., where we set $w_j=\frac{1}{n}$. Then, we can obtain the rankings of alternatives and select the optimal one by comparing the CE_i.

3.5 Procedure of the Proposed Method

Based on the STN analysis with the bounded trust propagation proposed in Section 3.3 and the confidence-based MCC model in MAGDM proposed in Section 3.4, we summarize the procedure of the proposed method in Figure 1, and present the detailed steps as follows:

Step 1 Invite DMs to give the PLTS matrix ($L^k=\left(L^{ij(k)}\right)_{m\times n}$), the incomplete PLTR matrix ($IT=\left(T^{kh}\right)_{K\times K}$), and determine the values of the parameter and three thresholds β, Υ, ε, and ψ. Then, we use Equation (11) to obtain the CL of the PLTS ($EC^{ij(k)}$) and the CL of the PLTR (EC^{kh});

Step 2 Utilize Equation (12) to acquire the normalized PLTS ($\tilde{L}^k=\left(\tilde{L}^{ij(k)}\right)_{m\times n}$) and the PLTR ($I\tilde{T}=\left(\tilde{T}^{kh}\right)_{K\times K}$);

Step 3 Apply the bounded trust propagation operator in Equation (17) and the PLT-OWA operator in Equation (18) to calculate the missing value in the incomplete PLTR matrix for obtaining the complete PLTR matrix

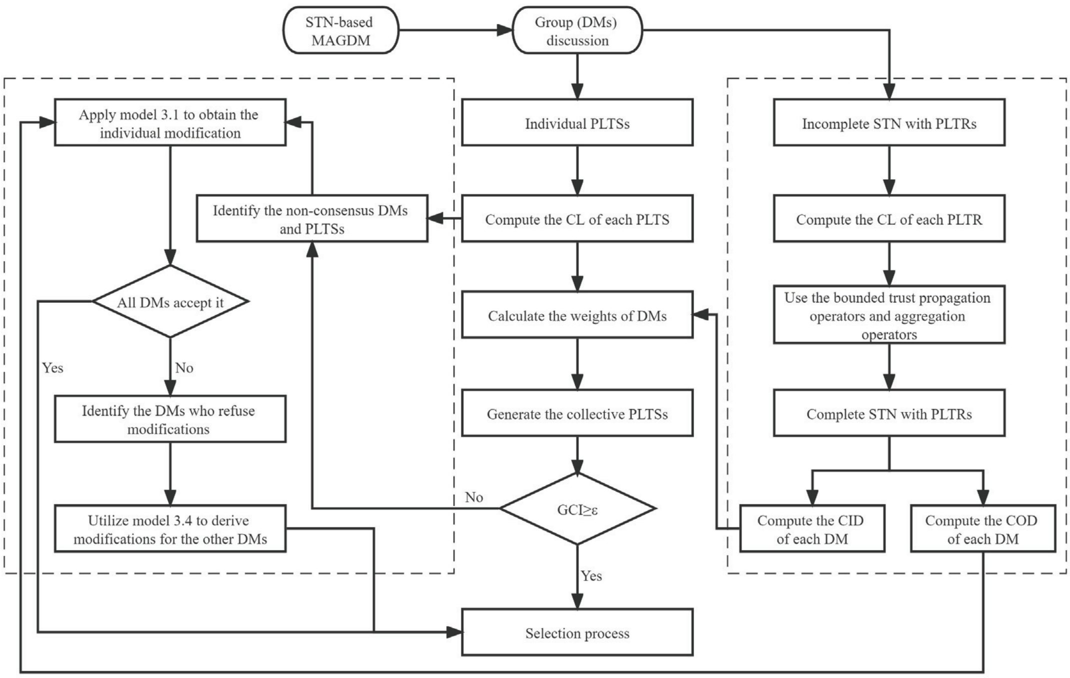

Figure 1 Confidence-based Consensus Framework for MAGDM in STN

($C\tilde{T} = (\tilde{T}^{kh})_{K\times K}$);

Step 4 Use Equation (24) to calculate the weights of DMs $(v_1, \cdots, v_k, \cdots, v_K)$, and apply Equation (25) to derive the collective matrix $(L^{\Lambda} = (L^{ij(\Lambda)})_{m\times n})$;

Step 5 Utilize Equations (26)-(29) to calculate the CIs of all DMs and the GCI, if $GCI \geq \varepsilon$, then the group has reached a consensus, go to Step 9; Otherwise, proceed to Step 6;

Step 6 Utilize Equations (30)-(32) to identify the non-consensus DMs and the corresponding elements set APS, and then apply Model 3.1 to seek the modification suggestions, if the Model 3.1 has a feasible solution, go to Step 9; Otherwise, proceed to Step 7;

Step 7 Apply the Model 3.3 to obtain uc_{ij}^{k*}, if the Model 3.3 has a feasible solution, then we use Equation (33) to identify the DMs who refuse to accept the modification suggestions, and utilize Equation (34) to calculate the adjustment values of the DMs; Otherwise, return to Step 1;

Step 8 Utilize the Model 3.4 to calculate the adjustment values of the other DMs in the set APS;

Step 9 Use Equation (25) to obtain the final collective matrix and the ranking of all alternatives.

The proposed method sufficiently considers the psychological factors existing in the practical STN-based MAGDM and the fairness in CRP, which can make the results high quality and obtain the support of all DMs.

4. Numerical Example and Discussion

In this section, we apply the proposed methods to a sustainable business model selection case where the evaluations are given by the DMs with PLTSs for verifying its reliability. Moreover, some comparative analyses and discussions for the parameters are made to further show the advantages of the proposed methods.

4.1 Numerical Example

In the current economic situation and market environment, a small manufacturing enterprise is facing low-cost competition, shortage of funds, and technological lag. To get rid of the current dilemma, the enterprise makes the sustainable business model innovation to break the old framework for enhancing its sustainable development capabilities under economic uncertainty. By consulting experts in related fields and referring to the successful cases of other enterprises, the enterprise initially drafted the following five innovation modes: Industry alliance mode (A_1), Media marketing mode (A_2), Precise service mode (A_3), Product innovation mode (A_4), Resource management mode (A_5).

To select the best alternative, the CEO (as the coordinator) and the experts establish three indicators to evaluate the five m odes, which are "Profitability (C_1)", "Feasibility (C_2)", and "Risk (C_3)". At the same time, they set the thresholds as $\beta = 0.4$, $\Upsilon = 0.5$, $\varepsilon = 0.85$, and $\psi = 0.71$. After determining the index system, the CEO of the enterprise invites five DMs on the board of directors to provide evaluations in the form of PLTSs for the five modes under the three indicators, the detailed evaluations are shown in Tables 1-5.

Additionally, the five DMs are invited to give their trust degree to each other. However, since the choice of a sustainable business innovation mode is a crucial but highly uncertain strategic decision, DMs can only provide an incomplete STN as follows:

$$IT = \begin{bmatrix} - & \{s_4(1)\} & - & \left\{\begin{matrix} s_0(0.3), s_1(0.28), \\ s_2(0.1), s_3(0.32) \end{matrix}\right\} & \{s_3(1)\} \\ \{s_2(1)\} & - & \{s_0(1)\} & \{s_1(0.4), s_2(0.6)\} & \{s_0(0.2)\} \\ \{s_0(0.56)\} & - & - & - & - \\ \left\{\begin{matrix} s_1(0.03), \\ s_3(0.54), \\ s_4(0.43) \end{matrix}\right\} & \left\{\begin{matrix} s_0(0.36), s_1(0.2), \\ s_2(0.1), s_3(0.26) \end{matrix}\right\} & \{s_0(0.6), s_2(0.4)\} & - & \{s_3(0.44)\} \\ - & - & - & \{s_2(0.77)\} & - \end{bmatrix}$$

where the dotted lines in Figure 2 represent the indirect TRs among DMs.

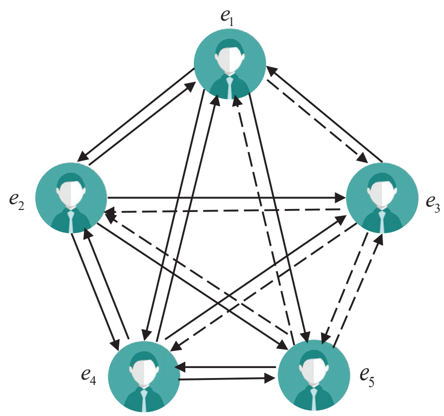

Figure 2 The incomplete STN

Step 1. Use Equation (11) to calculate EC_L based on the PLTSs and EC_T based on the PLTRs as follows:

$$EC_L^1 = \begin{bmatrix} 0.7643 & 0.7476 & 0.4529 \\ 1 & 0.8480 & 0.5790 \\ 1 & 0.5673 & 0.8289 \\ 0.5600 & 0.4820 & 0.6725 \\ 0.7466 & 1 & 0.4887 \end{bmatrix}$$

$$EC_L^2 = \begin{bmatrix} 0.5725 & 0.5738 & 0.5105 \\ 0.6093 & 0.6108 & 0.7753 \\ 1 & 0.5142 & 0.5600 \\ 0.5357 & 0.5920 & 0.5071 \\ 1 & 0.5578 & 0.5125 \end{bmatrix}$$

$$EC_L^3 = \begin{bmatrix} 0.6247 & 0.7504 & 0.7979 \\ 0.7441 & 0.5743 & 0.6197 \\ 1 & 0.6026 & 1 \\ 0.5392 & 0.7536 & 0.4609 \\ 0.6063 & 0.8501 & 0.7681 \end{bmatrix}$$

$$EC_L^4 = \begin{bmatrix} 0.5663 & 0.6355 & 0.7434 \\ 0.5600 & 1 & 0.5876 \\ 0.6825 & 0.4475 & 1 \\ 0.6760 & 0.7234 & 0.4650 \\ 0.5790 & 0.6035 & 0.6677 \end{bmatrix}$$

$$EC_L^5 = \begin{bmatrix} 0.7434 & 0.4243 & 0.5203 \\ 0.6455 & 0.6983 & 1 \\ 0.5600 & 0.4458 & 0.7029 \\ 0.5254 & 0.6075 & 0.5788 \\ 0.6072 & 0.9520 & 0.8057 \end{bmatrix}$$

$$EC_T = \begin{bmatrix} - & 1 & - & 0.5125 & 1 \\ 1 & - & 1 & 0.7496 & 0.56 \\ 0.7016 & - & - & - & - \\ 0.6972 & 0.4918 & 0.7491 & - & 0.6403 \\ - & - & - & 0.8321 & - \end{bmatrix}$$

Step 2 Utilize Equation (12) to obtain the normalized PLTSs and PLTRs (shown in Tables 6-10 and the matrix $\widetilde{IT}$);

$$I\tilde{T} = \begin{bmatrix} - & \{s_4(1)\} & - & \left\{\begin{matrix} s_0(0.3), s_1(0.28), \\ s_2(0.1), s_3(0.32) \end{matrix}\right\} & \{s_3(1)\} \\ \{s_2(1)\} & - & \{s_0(1)\} & \{s_1(0.4), s_2(0.6)\} & \left\{\begin{matrix} s_0(0.64), s_1(0.09), \\ s_2(0.09), s_3(0.09), \\ s_4(0.09) \end{matrix}\right\} \\ \left\{\begin{matrix} s_0(0.87), \\ s_1(0.04), \\ s_2(0.03), \\ s_3(0.03), \\ s_4(0.03) \end{matrix}\right\} & - & - & - & - \\ \left\{\begin{matrix} s_1(0.03), \\ s_3(0.54), \\ s_4(0.43) \end{matrix}\right\} & \left\{\begin{matrix} s_0(0.37), s_1(0.21), \\ s_2(0.11), s_3(0.27), \\ s_4(0.04) \end{matrix}\right\} & \{s_0(0.6), s_2(0.4)\} & - & \left\{\begin{matrix} s_0(0.05), s_1(0.05), \\ s_2(0.05), s_3(0.8), \\ s_4(0.05) \end{matrix}\right\} \\ - & - & - & \left\{\begin{matrix} s_0(0.01), s_1(0.01), \\ s_2(0.96), s_3(0.01), \\ s_4(0.01) \end{matrix}\right\} & - \end{bmatrix}$$

Table 1 The Bounded Trust Propagation Paths for Missing Values

Missing value	The bounded trust propagation paths
T^{13}	$e_1 \to e_2 \to e_3$; $e_1 \to e_4 \to e_3$;$e_1 \to e_2 \to e_4 \to e_3$;$e_1 \to e_5 \to e_4 \to e_3$;$e_1 \to e_2 \to e_5 \to e_4 \to e_3$
T^{32}	$e_3 \to e_1 \to e_2$
T^{34}	$e_3 \to e_1 \to e_4$; $e_3 \to e_1 \to e_2 \to e_4$; $e_3 \to e_1 \to e_5 \to e_4$;$e_3 \to e_1 \to e_2 \to e_5 \to e_4$
T^{35}	$e_3 \to e_1 \to e_5$; $e_3 \to e_1 \to e_2 \to e_5$; $e_3 \to e_1 \to e_4 \to e_5$; $e_3 \to e_1 \to e_2 \to e_4 \to e_5$
T^{51}	$e_5 \to e_4 \to e_1$; $e_5 \to e_4 \to e_3 \to e_1$
T^{52}	$e_5 \to e_4 \to e_1 \to e_2$
T^{53}	$e_5 \to e_4 \to e_3$; $e_5 \to e_4 \to e_1 \to e_2 \to e_3$

Step 3. Apply Equations (17)-(18) to calculate the missing value in $\widetilde{IT}$, the trust propagation paths are shown in Table 11.

$$T^{13} = \left\{ \left\{ \begin{array}{l} s_0(0.25), s_{0.08}(0.26), s_{0.16}(0.05), s_{0.2}(0.14), s_{0.24}(0.06), \\ s_{0.28}(0.08), s_{0.32}(0.04), s_{0.36}(0.04), s_{0.44}(0.04), s_{0.48}(0.01), \\ s_{0.52}(0.01), s_{0.56}(0.01), s_{0.68}(0.01) \end{array} \right\} 0.5003 \right\};$$

$$T^{32} = \{\{s_0(0.87), s_1(0.04), s_2(0.03), s_3(0.03), s_4(0.03)\}\ 0.7016\};$$

$$T^{34} = \left\{ \left\{ \begin{array}{l} s_0(0.68), s_{0.04}(0.02), s_{0.08}(0.01), s_{0.12}(0.05), s_{0.16}(0.02), s_{0.2}(0.03), \\ s_{0.24}(0.03), s_{0.28}(0.01), s_{0.36}(0.02), s_{0.4}(0.03), s_{0.48}(0.01), s_{0.52}(0.02), \\ s_{0.6}(0.03), s_{0.8}(0.01), s_{0.84}(0.03) \end{array} \right\} 0.5018 \right\};$$

$$T^{35} = \left\{ \left\{ \begin{array}{l} s_0(0.7), s_{0.04}(0.03), s_{0.08}(0.05), s_{0.12}(0.04), s_{0.16}(0.01), \\ s_{0.2}(0.01), s_{0.24}(0.01), s_{0.28}(0.02), s_{0.4}(0.03), s_{0.52}(0.01), \\ s_{0.84}(0.03), s_{1.36}(0.03), s_2(0.03) \end{array} \right\} 0.5165 \right\};$$

$$T^{51} = \left\{ \left\{ \begin{array}{l} s_{0.24}(0.01), s_{0.56}(0.02), s_{0.64}(0.01), s_{1.12}(0.01), s_{1.2}(0.02), \\ s_{1.32}(0.29), s_{1.36}(0.22), s_{1.6}(0.02), s_{1.68}(0.02), s_{1.76}(0.18) \end{array} \right\} 0.5802 \right\};$$

$$T^{52} = \{\{s_0(0.01), s_{0.52}(0.03), s_{0.76}(0.01), s_{1.52}(0.52), s_2(0.41), s_{2.24}(0.01), s_3(0.01)\}\ 0.5802\};$$

$$T^{53} = \{\{s_0(0.59), s_{0.36}(0.01), s_{0.72}(0.38), s_{1.12}(0.01), s_{1.56}(0.01)\}\ 0.6107\};$$

Step 4 Use Equation (24) to get the weights of DMs $v = (0.3041, 0.2469, 0.0268, 0.1646, 0.2576)$, and utilize Equation (25) to derive the group evaluations (shown in Table 12);

Step 5 The CIs of five DMs are calculated as $CI = (0.8317, 0.8304, 0.8509, 0.8346, 0.8484)$, and the GCI is equal to 0.8392. Since $GCI < 0.85$, go to Step 6;

Step 6 Utilize Equations (30)-(32) to identify the non-consensus DMs $EXPCH = \{2,4,5\}$, and the corresponding elements set $APS = \{(2,2,2),(2,3,3),(2,5,2),(4,5,2),(5,3,1),(5,3,3)\}$, and then apply the model 3.1 to seek the modification suggestions. Since there is no feasible solution in the Model 3.1, proceed to Step 7;

Step 7 Apply the Model 3.3 to obtain that $uc_{22}^{2*} = 0$, $uc_{33}^{2*} = 0.7266$, $uc_{52}^{2*} = 0.273$, $uc_{52}^{4*} = 0.1044$,$uc_{31}^{5*} = 0.4155$, $uc_{33}^{5*} = 0.8649$, and compare them with $uc_{22}^{2} = 0.4228$, $uc_{33}^{2} = 0.4482$, $uc_{52}^{2} = 0.4493$, $uc_{52}^{4} = 0.6144$, $uc_{31}^{5} = 0.4449$, $uc_{33}^{5} = 0.3735$ to obtain the elements that rejected the adjustment $RPS = \{(2,3,3),(5,3,3)\}$. Then, we utilize Equation (34) to calculate the adjustment values as: $\overline{L}_{33}^{2} = \{s_0(0.53), s_1(0.06), s_2(0.07), s_3(0.09), s_4(0.25)\}$, $\overline{L}_{33}^{2} = \{s_0(0.53), s_1(0.06), s_2(0.07), s_3(0.09),$

$s_4(0.25)\}$;

Step 8 Utilize the Model 3.4 to calculate the adjustment values in the set APS/RPSas $\overline{L}_{22}^{2} = \{s_0(0.67), s_4(0.33)\}$, $\overline{L}_{52}^{2} = \{s_0(1)\}$, $\overline{L}_{52}^{4} = \{s_0(0.77), s_0(0.23)\}$, $\overline{L}_{31}^{5} = \{s_0(0.15), s_3(0.85)\}$;

Step 9 Use Equation (25) to obtain the final group evaluations (shown in Table 13). It is easy to find that only four PLTSs have been adjusted located in the coordinates $\{(2,2),(3,1),(3,3),(5,2)\}$, which means that the original information is preserved to the greatest extent. Moreover, the revised CIs for the five DMs are $CI^{'} = (0.8462, 0.8725, 0.8547, 0.8358, 0.8914)$ and $GCL^{'} = 0.8601$. The total adjustment cost for CRP is 0.6168 and the final ranking of the five modes is $A_3 > A_1 > A_4 > A_2 > A_5$.

4.2 Comparative Analysis and Discussion

In this section, sensitivity analyses and comparative analyses are performed to demonstrate the viability of the proposed method. The discussion on the effect of different values of ε and ψ on the results of the Model 3.1 is generated.

4.2.1 Sensitivity Analyses

Since the proposed method contains three thresholds (Υ, ε and ψ), it is necessary to explore their effects on the results in the numerical example. Here we design two simulation tests as follows:

1) The analysis based on the propagation threshold Υ

Let the value of Υ vary from 0 to 1 with the step 0.0001, and use the proposed method to obtain the weights of the five DMs, the results are shown in Figure 3.

In Figure 3, we divide the value of Υ into 8 intervals according to the nodes where the weights of DMs change, which is conducive to concisely and clearly showing the influence of the value of Υ on the weights. It can be seen that as Υ changes, the weight of each DM changes. Especially, when the value of Υ increases from the interval $[0, 0.4917]$ to $[0.4918, 0.5129]$, the weights of the DMs e_4 and e_5 decrease significantly, and the weight of DM e_2 increases significantly; when the value of Υ increases from the interval $[0.6403, 0.6972]$ to $[0.6973, 0.7016]$, the weights of the DMs e_1 and e_2 decrease significantly, and the weights of the DMs e_4 and e_5 increase significantly. Combined with the CL matrix based on the TRs EC_T, we can obtain the following findings. When the value of Υ is greater than 0.4917, it is difficult for the DM e_4 to trust the advice from the mediator e_2, so the propagation paths containing $e_4 \to e_2$ are interrupted; When the value of Υ is greater than 0.6972, the propagation paths containing $e_1 \to e_4$, $e_2 \to e_5$, $e_4 \to e_2$, and $e_4 \to e_5$ are all interrupted. These findings demonstrate the effectiveness of bounded trust propagation.

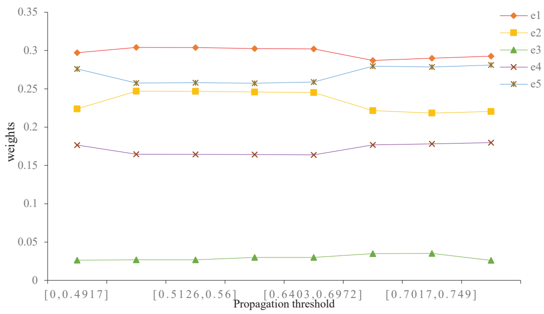

Figure 3 The Effect of Υ on DMs' Weights

2) The analysis based on the consensus threshold ε and the CL threshold ψ

Let the value of ε change in the interval $[0.8393, 1]$ and the value of ψ change in the in-

Table 2 The PLTSs Given by the DM e_1

	C_1	C_2	C_3
A_1	$\{s_1(0.33), s_3(0.67)\}$	$\{s_0(0.59), s_1(0.41)\}$	$\{s_0(0.15), s_2(0.24), s_3(0.17), s_4(0.26)\}$
A_2	$\{s_3(1)\}$	$\{s_1(0.89), s_2(0.08), s_3(0.03)\}$	$\{s_0(0.05), s_1(0.42), s_3(0.4), s_4(0.06)\}$
A_3	$\{s_2(1)\}$	$\{s_0(0.42), s_1(0.29), s_2(0.12)\}$	$\{s_3(0.17), s_4(0.83)\}$
A_4	$\{s_0(0.2)\}$	$\{s_0(0.14), s_2(0.05), s_3(0.34), s_4(0.24)\}$	$\{s_2(0.58), s_4(0.18)\}$
A_5	$\{s_0(0.42), s_4(0.58)\}$	$\{s_0(1)\}$	$\{s_0(0.1), s_1(0.42), s_2(0.31), s_3(0.1), s_4(0.07)\}$

Table 3 The PLTSs Given by the DM e_2

	C_1	C_2	C_3
A_1	$\{s_1(0.33), s_2(0.23), s_3(0.42), s_4(0.02)\}$	$\{s_2(0.39), s_4(0.15)\}$	$\{s_0(0.1), s_1(0.35), s_3(0.29), s_4(0.26)\}$
A_2	$\{s_0(0.43), s_3(0.19), s_4(0.38)\}$	$\{s_0(0.18), s_2(0.03), s_3(0.2), s_4(0.59)\}$	$\{s_2(0.29), s_4(0.71)\}$
A_3	$\{s_4(1)\}$	$\{s_0(0.11), s_1(0.26), s_2(0.4), s_4(0.23)\}$	$\{s_0(0.2)\}$
A_4	$\{s_1(0.35), s_2(0.37), s_3(0.07), s_4(0.21)\}$	$\{s_0(0.43), s_1(0.03), s_2(0.13), s_3(0.41)\}$	$\{s_0(0.21), s_1(0.02), s_2(0.37), s_4(0.17)\}$
A_5	$\{s_0(1)\}$	$\{s_0(0.11), s_1(0.42), s_2(0.25)\}$	$\{s_0(0.31), s_2(0.09), s_3(0.31), s_4(0.29)\}$

Table 4 The PLTSs Given by the DM e_3

	C_1	C_2	C_3
A_1	$\{s_2(0.48), s_3(0.15), s_4(0.37\}$	$\{s_0(0.61), s_1(0.39)\}$	$\{s_2(0.77), s_4(0.23)\}$
A_2	$\{s_0(0.44), s_1(0.56)\}$	$\{s_0(0.34), s_1(0.04), s_2(0.37)\}$	$\{s_1(0.51), s_2(0.29), s_3(0.2)\}$
A_3	$\{s_0(1)\}$	$\{s_0(0.37), s_4(0.32)\}$	$\{s_2(1)\}$
A_4	$\{s_0(0.37), s_1(0.18), s_4(0.24)\}$	$\{s_1(0.63), s_3(0.37)\}$	$\{s_0(0.18), s_1(0.21), s_2(0.25), s_3(0.25)\}$
A_5	$\{s_0(0.37), s_1(0.43), s_4(0.18)\}$	$\{s_0(0.8)\}$	$\{s_1(0.72), s_4(0.24)\}$

Table 5 The PLTSs Given by the DM e_4

	C_1	C_2	C_3
A_1	$\{s_0(0.24), s_2(0.47), s_3(0.01), s_4(0.14)\}$	$\{s_1(0.27), s_3(0.48)\}$	$\{s_0(0.55), s_3(0.45)\}$
A_2	$\{s_0(0.2)\}$	$\{s_2(1)\}$	$\{s_0(0.33), s_1(0.09), s_4(0.43)\}$
A_3	$\{s_0(0.36), s_3(0.51)\}$	$\{s_0(0.15), s_1(0.2), s_2(0.11), s_3(0.29)\}$	$\{s_4(1)\}$
A_4	$\{s_0(0.41), s_1(0.01), s_4(0.49)\}$	$\{s_0(0.66), s_1(0.03), s_3(0.31)\}$	$\{s_1(0.26), s_2(0.2), s_3(0.07), s_4(0.26)\}$
A_5	$\{s_0(0.39), s_1(0.2)\}$	$\{s_0(0.37), s_2(0.07), s_4(0.42)\}$	$\{s_0(0.33), s_1(0.04), s_4(0.56)\}$

Table 6 The PLTSs Given by the DM e_5

	C_1	C_2	C_3
A_1	$\{s_1(0.45), s_4(0.55)\}$	$\{s_0(0.11), s_1(0.28), s_2(0.13), s_3(0.29), s_4(0.19)\}$	$\{s_0(0.22), s_1(0.09), s_3(0.35), s_4(0.34)\}$
A_2	$\{s_0(0.3), s_1(0.55), s_4(0.05)\}$	$\{s_0(0.01), s_1(0.56)\}$	$\{s_3(1)\}$
A_3	$\{s_0(0.2)\}$	$\{s_1(0.19), s_2(0.15), s_3(0.29), s_4(0.13)\}$	$\{s_0(0.56)\}$
A_4	$\{s_0(0.27), s_1(0.25), s_4(0.08), s_4(0.4)\}$	$\{s_1(0.2), s_2(0.44), s_4(0.36)\}$	$\{s_0(0.42), s_2(0.06), s_3(0.01), s_4(0.34)\}$
A_5	$\{s_0(0.42), s_1(0.2), s_4(0.38)\}$	$\{s_0(0.97), s_4(0.03)\}$	$\{s_1(0.78), s_4(0.22)\}$

Table 7 The Normalized PLTSs Given by the DM e_1

	C_1	C_2	C_3
A_1	$\{s_1(0.33), s_3(0.67)\}$	$\{s_0(0.59), s_1(0.41)\}$	$\{s_0(0.17), s_1(0.1), s_2(0.26), s_3(0.19), s_4(0.28)\}$
A_2	$\{s_3(1)\}$	$\{s_1(0.89), s_2(0.08), s_3(0.03)\}$	$\{s_0(0.06), s_1(0.43), s_2(0.03), s_3(0.41), s_4(0.07)\}$
A_3	$\{s_2(1)\}$	$\{s_0(0.45), s_1(0.32), s_2(0.15), s_3(0.04), s_4(0.04)\}$	$\{s_3(0.17), s_4(0.83)\}$
A_4	$\{s_0(0.64), s_1(0.09), s_2(0.09), s_3(0.09), s_4(0.09)\}$	$\{s_0(0.17), s_1(0.12), s_2(0.08), s_3(0.36), s_4(0.27)\}$	$\{s_0(0.03), s_1(0.03), s_2(0.66), s_3(0.02), s_4(0.26)\}$
A_5	$\{s_0(0.42), s_4(0.58)\}$	$\{s_0(1)\}$	$\{s_0(0.1), s_1(0.42), s_2(0.31), s_3(0.1), s_4(0.07)\}$

Table 8 The Normalized PLTSs Given by the DM e_2

	C_1	C_2	C_3
A_1	$\{s_1(0.33), s_2(0.23), s_3(0.42), s_4(0.02)\}$	$\{s_0(0.07), s_1(0.07), s_2(0.52), s_3(0.06), s_4(0.28)\}$	$\{s_0(0.1), s_1(0.35), s_3(0.29), s_4(0.26)\}$
A_2	$\{s_0(0.43), s_3(0.19), s_4(0.38)\}$	$\{s_0(0.18), s_2(0.03), s_3(0.2), s_4(0.59)\}$	$\{s_2(0.29), s_4(0.71)\}$
A_3	$\{s_4(1)\}$	$\{s_0(0.11), s_1(0.26), s_2(0.4), s_4(0.23)\}$	$\{s_0(0.64), s_1(0.09), s_2(0.09), s_3(0.09), s_4(0.09)\}$
A_4	$\{s_1(0.35), s_2(0.37), s_3(0.07), s_4(0.21)\}$	$\{s_0(0.43), s_1(0.03), s_2(0.13), s_3(0.41)\}$	$\{s_0(0.24), s_1(0.05), s_2(0.4), s_3(0.11), s_4(0.2)\}$
A_5	$\{s_0(1)\}$	$\{s_0(0.14), s_1(0.45), s_2(0.28), s_3(0.03), s_4(0.1)\}$	$\{s_0(0.31), s_2(0.09), s_3(0.31), s_4(0.29)\}$

Table 9 The Normalized PLTSs Given by the DM e_3

	C_1	C_2	C_3
A_1	$\{s_2(0.48), s_3(0.15), s_4(0.37)\}$	$\{s_0(0.61), s_1(0.39)\}$	$\{s_2(0.77), s_4(0.23)\}$
A_2	$\{s_0(0.44), s_1(0.56)\}$	$\{s_0(0.39), s_1(0.09), s_2(0.42), s_3(0.05), s_4(0.05)\}$	$\{s_1(0.51), s_2(0.29), s_3(0.2)\}$
A_3	$\{s_0(1)\}$	$\{s_0(0.47), s_1(0.04), s_2(0.04), s_3(0.04), s_4(0.41)\}$	$\{s_2(1)\}$
A_4	$\{s_0(0.41), s_1(0.22), s_2(0.04), s_3(0.05), s_4(0.28)\}$	$\{s_1(0.63), s_3(0.37)\}$	$\{s_0(0.19), s_1(0.23), s_2(0.26), s_3(0.26), s_4(0.06)\}$
A_5	$\{s_0(0.37), s_1(0.43), s_4(0.18)\}$	$\{s_0(0.96), s_1(0.01), s_2(0.01), s_3(0.01), s_4(0.01)\}$	$\{s_1(0.74), s_4(0.26)\}$

Table 10 The Normalized PLTSs Given by the DM e_4

	C_1	C_2	C_3
A_1	$\{s_0(0.26), s_1(0.06), s_2(0.49), s_3(0.03), s_4(0.16)\}$	$\{s_0(0.03), s_1(0.35), s_2(0.03), s_3(0.56), s_4(0.03)\}$	$\{s_0(0.55), s_3(0.45)\}$
A_2	$\{s_0(0.64), s_1(0.09), s_2(0.09), s_3(0.09), s_4(0.09)\}$	$\{s_2(1)\}$	$\{s_0(0.36), s_1(0.12), s_2(0.03), s_3(0.03), s_4(0.46)\}$
A_3	$\{s_0(0.4), s_1(0.02), s_2(0.01), s_3(0.56), s_4(0.01)\}$	$\{s_0(0.18), s_1(0.23), s_2(0.14), s_3(0.31), s_4(0.14)\}$	$\{s_4(1)\}$
A_4	$\{s_0(0.43), s_1(0.03), s_2(0.02), s_3(0.01), s_4(0.51)\}$	$\{s_0(0.66), s_1(0.03), s_3(0.31)\}$	$\{s_0(0.11), s_1(0.28), s_2(0.22), s_3(0.1), s_4(0.29)\}$
A_5	$\{s_0(0.1), s_1(0.31), s_2(0.06), s_3(0.06), s_4(0.06)\}$	$\{s_0(0.4), s_1(0.03), s_2(0.1), s_3(0.02), s_4(0.45)\}$	$\{s_0(0.34), s_1(0.06), s_2(0.01), s_3(0.01), s_4(0.58)\}$

Table 11 The Normalized PLTSs Given by the DM e_5

	C_1	C_2	C_3
A_1	$\{s_1(0.45), s_4(0.55)\}$	$\{s_0(0.11), s_1(0.28), s_2(0.13), s_3(0.29), s_4(0.19)\}$	$\{s_0(0.22), s_1(0.09), s_3(0.35), s_4(0.34)\}$
A_2	$\{s_0(0.32), s_1(0.57), s_2(0.02), s_3(0.02), s_4(0.07)\}$	$\{s_0(0.16), s_1(0.72), s_2(0.04), s_3(0.04), s_4(0.04)\}$	$\{s_3(1)\}$
A_3	$\{s_0(0.64), s_1(0.09), s_2(0.09), s_3(0.09), s_4(0.09)\}$	$\{s_0(0.13), s_1(0.22), s_2(0.18), s_3(0.31), s_4(0.16)\}$	$\{s_0(0.88), s_1(0.03), s_2(0.03), s_3(0.03), s_4(0.03)\}$
A_4	$\{s_0(0.27), s_1(0.25), s_4(0.08), s_4(0.4)\}$	$\{s_1(0.2), s_2(0.44), s_4(0.36)\}$	$\{s_0(0.44), s_1(0.07), s_2(0.09), s_3(0.03), s_4(0.37)\}$
A_5	$\{s_0(0.42), s_1(0.2), s_4(0.38)\}$	$\{s_0(0.97), s_4(0.03)\}$	$\{s_1(0.78), s_4(0.22)\}$

Table 12 The Original Group Evaluations

	C_1	C_2	C_3
A_1	$\{s_0(0.04), s_1(0.31), s_2(0.15), s_3(0.32), s_4(0.18)\}$	$\{s_0(0.25), s_1(0.28), s_2(0.17), s_3(0.18), s_4(0.12)\}$	$\{s_0(0.23), s_1(0.14), s_2(0.1), s_3(0.29), s_4(0.24)\}$
A_2	$\{s_0(0.31), s_1(0.18), s_2(0.02), s_3(0.37), s_4(0.12)\}$	$\{s_0(0.09), s_1(0.46), s_2(0.22), s_3(0.07), s_4(0.16)\}$	$\{s_0(0.08), s_1(0.17), s_2(0.09), s_3(0.39), s_4(0.27)\}$
A_3	$\{s_0(0.26), s_1(0.03), s_2(0.33), s_3(0.11), s_4(0.27)\}$	$\{s_0(0.24), s_1(0.26), s_2(0.21), s_3(0.15), s_4(0.14)\}$	$\{s_0(0.38), s_1(0.03), s_2(0.06), s_3(0.08), s_4(0.45)\}$
A_4	$\{s_0(0.35), s_1(0.19), s_2(0.14), s_3(0.05), s_4(0.27)\}$	$\{s_0(0.27), s_1(0.12), s_2(0.17), s_3(0.27), s_4(0.17)\}$	$\{s_0(0.2), s_1(0.09), s_2(0.37), s_3(0.07), s_4(0.27)\}$
A_5	$\{s_0(0.58), s_1(0.11), s_2(0.01), s_3(0.01), s_4(0.29)\}$	$\{s_0(0.68), s_1(0.12), s_2(0.09), s_3(0.01), s_4(0.1)\}$	$\{s_0(0.16), s_1(0.36), s_2(0.12), s_3(0.11), s_4(0.25)\}$

Table 13 The Final Group Evaluations

	C_1	C_2	C_3
A_1	$\{s_0(0.04), s_1(0.31), s_2(0.15), s_3(0.32), s_4(0.18)\}$	$\{s_0(0.25), s_1(0.28), s_2(0.17), s_3(0.18), s_4(0.12)\}$	$\{s_0(0.23), s_1(0.14), s_2(0.1), s_3(0.29), s_4(0.24)\}$
A_2	$\{s_0(0.31), s_1(0.18), s_2(0.02), s_3(0.37), s_4(0.12)\}$	$\{s_0(0.22), s_1(0.46), s_2(0.21), s_3(0.02), s_4(0.09)\}$	$\{s_0(0.08), s_1(0.17), s_2(0.09), s_3(0.39), s_4(0.27)\}$
A_3	$\{s_0(0.13), s_2(0.31), s_3(0.31), s_4(0.25)\}$	$\{s_0(0.24), s_1(0.26), s_2(0.21), s_3(0.15), s_4(0.14)\}$	$\{s_0(0.31), s_1(0.02), s_2(0.06), s_3(0.08), s_4(0.53)\}$
A_4	$\{s_0(0.35), s_1(0.19), s_2(0.14), s_3(0.05), s_4(0.27)\}$	$\{s_0(0.27), s_1(0.12), s_2(0.17), s_3(0.27), s_4(0.17)\}$	$\{s_0(0.2), s_1(0.09), s_2(0.37), s_3(0.07), s_4(0.27)\}$
A_5	$\{s_0(0.58), s_1(0.11), s_2(0.01), s_3(0.01), s_4(0.29)\}$	$\{s_0(0.95), s_4(0.05)\}$	$\{s_0(0.16), s_1(0.36), s_2(0.12), s_3(0.11), s_4(0.25)\}$

terval $[0.5, 1]$, where the steps are both 0.0001, the setting ranges of ε and ψ for the Models 3.1 and 3.4 are explored and the final results are as follows:

As shown in Table 14, the range of ε is $[0.8393, 0.8520]$, when $\varepsilon \in [0.83930.8485]$, we can obtain all modification suggestions for non-consensus DMs through the Model 3.1; when $\varepsilon \in [0.84840.8520]$, we need to apply the Model 3.3 to identify the DMs who refuse modification and use the Model 3.4 to generate modification suggestions for the other DMs; when $\varepsilon \in [0.85211]$, the group fails to reach a consensus, we need to re-invite DMs to give their evaluations. Combined with the identification rules in Subsection 3.4, it can be seen that the smaller the value of ψ, the better it can ensure that the evaluations with a higher CL can be retained. Based on Table 14, as the value of ε increases from 0.8393 to 0.8520, the minimum value of ψ increases from 0.6902 to 0.7030, which means that more evaluations are identified in the adjusted range.

4.2.2 Comparative Analyses

1) The comparison for the measurements of CL

To observe the difference between the proposed improvement measurement of CL (No. 1) with the method proposed by You and Hou (2022) (No. 2), the method proposed by Li et al. (2022) (No. 3) and the method proposed by Zhong et al. (2022) (No. 4), we design a simulation test.

We set the linguistic term set as $S = \{s_0, s_1, s_2, s_3, s_4\}$ and the length of PLTS is $\#L \leq 5$. Considering the method 3 contains two parameters $\alpha = 0.4$, $\beta_1 = 0.3$, we set the parameter in our method as $\beta_2 = \alpha = 0.4$ since the entropy can represent the hesitancy and preference of DMs. Then, we randomly generate 1000 PLTSs through MATLAB software and use the above four methods to calculate their CLs, the results are shown as:

From Figure 4, the value of CL obtained by the method 1(green line) is generally higher than the value of CL obtained by the method 3 (red line), which confirms that the method 3 has an underestimation of the CLs. Moreover, it is easy to find that the values of CL obtained by the methods 2 (blue line) and 4 (yellow line) fluctuate more than the method 1, which means the lack of completeness or the subjective design for the correlation function between the hesitancy and preference can lead to underestimation or overestimation of the results. The comparative analysis shows that the proposed method can acquire more stable results.

2) The comparison for the determination method of the DMs' weights

To further illustrate the effectiveness of the weight determination method based on bounded trust propagation, we compare the method proposed in Subsection 3.3 (No. 1)

Table 14 The Setting Ranges of ε and ψ

Consensus threshold ε	CL threshold ψ	Model selection
$[0.8393, 0.8397]$	$[0.6902, 0.7752]$, $[0.8289, 1]$	Model 3.1
$[0.8398, 0.8405]$	$[0.6651, 0.7752]$, $[0.8289, 1)$	Model 3.1
$[0.8406, 0.8465]$	$[0.6651, 0.7434]$	Model 3.1
$[0.8466, 0.8485]$	$[0.6902, 0.6913]$	Model 3.1
$[0.8484, 0.8509]$	$[0.7030, 0.7434]$	Model 3.4
$[0.8410, 0.8520]$	$[0.7030, 0.7128]$	Model 3.4

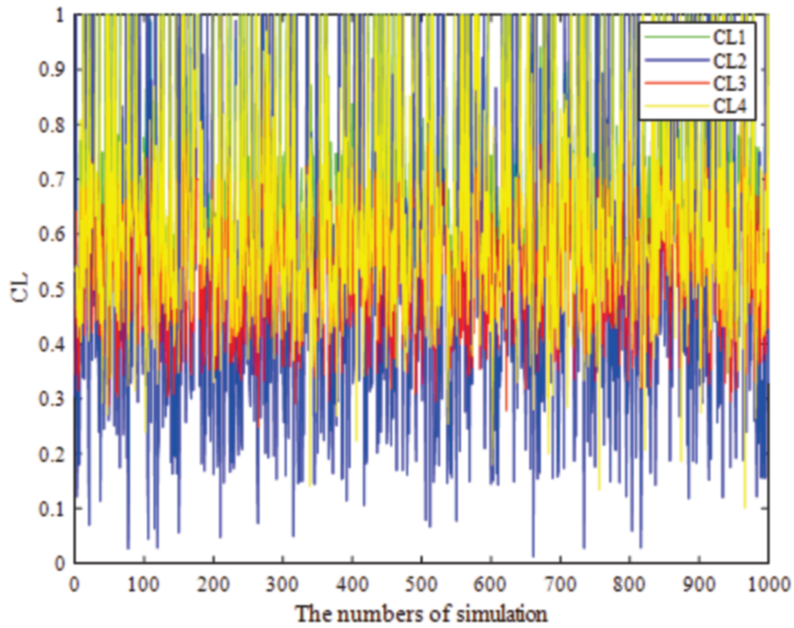

Figure 4 The Comparative Result for the Measurements of CL

with the method of removing the CL of DMs in Equation (24) (No. 2) and the method proposed by Gao et al. (2021) (No. 3), the results are shown in Figure 5.

From Figure 5, it can be seen that the weights obtained by the three methods are certainly different. In detail, the weights of the DMs e_1, e_2, e_4 and e_5 acquired by the method 3 are quite different from the weights obtained by the methods 1 and 2. And there is much difference between the weights of the DMs e_1 and e_2 get by the method 1 and their weights obtained by the method 2. This shows that it is necessary to consider CL and bounded trust propagation when determining the weights of DMs.

3) The comparison for CRP

To explore the advantages of the confidence-based CRP for MAGDM proposed in Subsection 3.4, we compare it with the traditional MCC model and the linear combination method proposed by You and Hou (2022), the results are shown in Table 15.

Compared with the traditional MCC model and the linear combination method proposed by You and Hou (2022), the adjustment amount

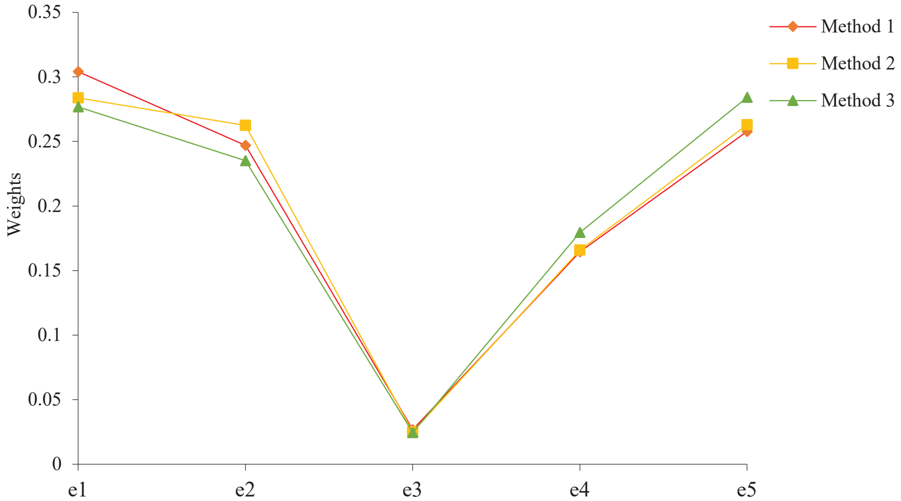

Figure 5 The Comparative Result for the Weight Determination Method

Table 15 Comparison for Different Consensus Method

Method	Iterations	Adjustment number	Adjustment value	GCI	Ranking of alternatives
Method of You and Hou [33]	1	15	4.6017	0.8542	$A_1 > A_2 > A_3 > A_4 > A_5$
Traditional MCC model	1	20	3.186	0.8808	$A_3 > A_1 > A_2 > A_4 > A_5$
Proposed method	1	6	1.2137	0.8601	$A_3 > A_1 > A_4 > A_2 > A_5$

of evaluations and overall adjustment value obtained by the proposed method is the smallest, but the final GCI is between the two methods. At the same time, it can be seen that the ranking results obtained by the linear combination method are quite different from those based on the MCC model. Moreover, the difference between the ranking of the alternatives obtained by the traditional MCC model and the proposed method is mainly reflected in the ranking of the alternatives A_2 and A_4. It demonstrates that the proposed method can achieve a better level of consensus on the basis of retaining the original evaluations as much as possible and respecting the adjustment willingness of DMs. In other words, the decision-making results obtained through the proposed method can better reflect the needs and concerns of all DMs.

4.2.3 Simulation Tests

Through the above sensitivity analysis, it can be seen that whether the Model 3.1 has a feasible solution largely depends on the values of ε and ψ. Hence, we design simulation tests to explore the ranges of ε and ψ, which can serve as references for the coordinator who sets these two parameters in the actual decision-making process. Specifically, we set the number of alternatives as $m = 5$, the number of attributes as $n = 5$, the number of DMs as $K = 5$. Then, we use MATLAB to randomly generate 1000 decision matrices and let the value of ε change in the interval $[0.8, 1]$ and the value of ψ change in the interval $[0.5, 1]$, where the steps are both 0.01. After that, we can obtain the average adjustment cost for the 1000 matrices where we set the adjustment cost as 0 when Model 3.1 has no solution. The results are shown in Figure 6.

From Figure6, we can see when the value

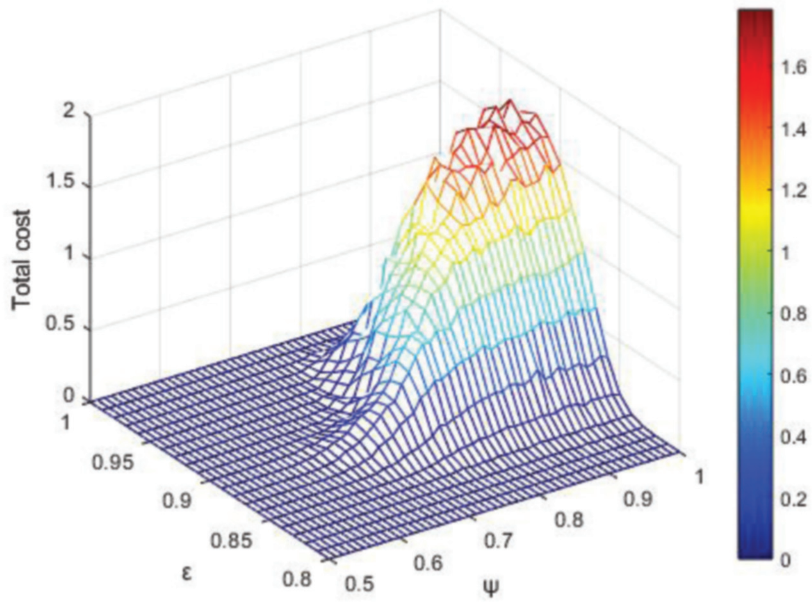

Figure 6 The Effect of ε and ψ on Adjustment Cost in the Model 3.1

of ψ is greater than 0.65, the Model 3.1 has a feasible solution. Moreover, when the values of ε and ψ increase, the average adjustment cost increases. When $\varepsilon \in [0.8, 0.9]$ and $\psi \in [0.65, 0.75]$, the average adjustment cost is low. Furthermore, the larger the value of ψ, the more evaluations of DMs are modified. And the larger the value of ε, the higher the consensus level of group. Hence, the coordinator needs to make a compromise between a high consensus level and retaining the original evaluation in the actual decision- making process, which both are critical to gain support from all DMs.

5. Conclusions

Psychological factors of DMs play a vital role in the actual decision-making process. To obtain accurate and reliable group evaluations supported by all DMs, we utilize the CL and trust degree of DMs to bridge the theory of STN analysis and the MCC model. It provides a new framework to solve the STN-based MAGDM problems. In this framework, the bounded trust propagation is proposed to describe the psychology of DMs when they indirectly trust others. Then, the determination method of DMs' weights is proposed based on the CID in STN and the CL of DMs in the decision matrix. In the CRP, we adopt the CL to fit the unit adjustment cost at the element level and combine it with the COD in STN to represent the acceptable adjustment range for DMs. Thus, the proposed confidence-based CRP can consider the psychology of all DMs while ensuring the pursuit of the minimum cost. Meanwhile, it offers an approach that is impartial for determining DMs' noncooperative behavior.

However, this paper still has some limitations that should be addressed in future research. Firstly, only the CL and trust degree are considered when characterizing the psychology of DMs, and future research can further explore conflicts of interest among DMs in CRP. Next, the bounded trust propagation only considers the interactions between two DMs, the future research can focus on more complex interactions among more than two DMs.

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Data Availability

All data generated or analysed during this study are included in this published article (and its supplementary information files).

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