A Multi-Objective Multi-Period Model for Humanitarian Relief Logistics with Split Delivery and Multiple Uses of Vehicles

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Abstract. Disaster relief logistics is a significant element in the management of disaster relief operations. In this paper, the operational decisions of relief logistics are considered in the distribution of resources to the affected areas to include scheduling, routing, and allocation decisions. The proposed mathematical model simultaneously captures many aspects relevant to real life to face the challenging situation of disasters. Characteristics such as multiple uses of vehicles and split delivery allow for better use of vehicles as one of the primary resources of disaster response. A multi-period multi-criteria mixed-integer programming model is introduced to evaluate and address these features. The model utilizes a rolling horizon method that provides possibilities to adjust plans as more information becomes available. Three objectives of efficiency, effectiveness, and equity are jointly considered. The augmented epsilon constraint method is applied to solve the model, and a case study is presented to illustrate the potential applicability of our model. Computational results show that the model is capable of generating efficient solutions.

Keywords: Disaster management, relief logistics, vehicle routing problem, multi-trip, rolling horizon

1. Introduction

From 1998 to 2019, more than 4.4 billion people were affected by disasters, 1.3 million people died, and \$2 trillion of economic damage was reported only by natural disasters around the world. The 2004 Indian Ocean tsunami, the 2010 earthquake in Chile, and the 2011 earthquake in Japan have been the largest reported earthquakes since 2011 (Centre for Research on the Epidemiology of Disasters CRED 2019).

As a part of disaster management activities, logistics include "the process of planning, implementing, and controlling the efficient and effective flow of goods, material, and relevant information from source to destination to reduce the suffering of affected people" (Thomas and Kopczak 2005). Humanitarian logistics include several difficult optimization problems such as warehouse location, allocation, transportation, and vehicle routing. In this paper, we will focus on the operations of transportation and the distribution of aid commodities in humanitarian logistics. These operations involve the assignment of relief items, the selection of distribution path, and scheduling path.

Despite the similarity between this and commercial logistics problem, the postdisaster situation imposes challenging characteristics that make these two problems quite different. As argued by Balcik et al. (2008), these characteristics include the limited availability of resources, lack of knowledge of data, high stakes associated with delivering supplies, damaged transportation and communication infrastructure, temporary network structure that can only be set up after a disaster strikes, and non-profit objectives.

As already mentioned, a major challenge of disaster relief operations is the lack of access to accurate and reliable data and their evolution (Yi and Ozdamar 2007, Ozdamar et al. 2004). For example, the closure or repair of roads changes demands and travel times. As a result, solutions generated based on static models may significantly deviate from real situations and are, thus, impractical. Therefore, to guarantee efficient and effective response plans, a dynamic case must be considered in which information is revealed gradually over time, and projects are adjusted based on these changes.

In last-mile distribution operations, vehicles are usually small. However, in postdisaster situations, the volume of demands may be bigger than the vehicle's capacity in the affected areas. Therefore, an affected area can be served more than once by different vehicles. This feature is called split delivery in the relevant literature which was first introduced by Dror and Trudeau (1989). On the other hand, Split deliveries in last-mile relief distribution quickly serve demands using limited vehicles (Huang et al. 2012).

In the classic vehicle routing problem (VRP), only a single trip was performed by each vehicle. However, in many cases where a fast response is important, and the number of vehicles is limited, it may be feasible for each vehicle to make multiple trips. Not only does the efficacy of the source which is used increases, but also this attribute is the unique practical alternative for applications such as home delivery of perishable goods, the collection of livestock or humanitarian logistics where either vehicle routes are short with respect to the planning horizon or the vehicle capacity is small , and the total demand exceeds the capacity of the vehicle (Molina et al. 2018). This problem is known as the multitrip vehicle routing problem (MTVRP) which is an extension of the classical capacitated vehicle routing problem (CVRP) , and it was introduced by Fleischmann (1990). Cattaruzza et al. (2016) presented a survey on the MTVRP and stated that, despite the benefits which can be obtained by performing multiple trips, this

attribute had been neglected in the literature.

A major challenge of humanitarian relief operations is the presence of various stakeholders such as local governments, military, and NGOs. In this multifunctional environment, each of these actors might have different objectives and priorities, leading to conflicts and inefficiencies in practice (Ergun et al. 2010). Therefore, the best decisions are made when these goals are taken into account simultaneously. In order to do so, multi-objective models must be developed.

The need to address these features motivated us to study a multi-criteria optimization model for an extension of the vehicle scheduling and routing problem in humanitarian lastmile distribution. This problem is known as the multi-objective multi-period multi-trip vehicle scheduling and routing problem and incorporates decisions related to last-mile distribution such as transport quantities, delivery quantities, scheduling, and delivery routes. The offered model takes into account various assumptions in which a set of vehicles depart from a multi-depot, and each vehicle may make many trips. It does not restrict the number of times a customer can be visited and, thus, split deliveries are allowed. As more information becomes available, decisions are updated over a multi-period horizon. We consider three objectives of response time, meeting demand, and fair distribution of resources.

The rest of the article is organized as follows: section 2 provides a review of the relevant literature. The general problem description statement and model are given in section 3. section 4 introduces the proposed solution approach to the problem. In section 5, we present a case study for potential earthquakes in Tehran, and numerical results are presented in section 6. Finally, we provide conclusions and recommendations for future studies in section 7.

2. Literature Review

This paper will not review all decision models for humanitarian relief logistics, and we will only focus on last-mile distribution, the distribution of supplies from the storage point to final beneficiaries, based on detailed path planning and vehicle routing in a humanitarian supply chain. Several in-depth literature reviews exist covering the VRP in the postdisaster response phase (DelaTorre et al. 2012, Pillac et al. 2013, Ozdamar and Ertem 2015).

Reviewing the literature in this area reveals that most of the pertinent studies assume each vehicle performs a single trip (Haghani and Oh 1996, Barbaroso and Arda 2004, Ozdamar et al. 2004, Yi and Kumar 2007, Balcik et al. 2008, Campbell et al. 2008, Ozdamar and Yi 2008, Shen et al. 2009, Najafi et al. 2014, Talebian and Salari 2015, Wang et al. 2015, Al and Murray 2017). Whilst the models in which vehicles are permitted to have plenty of trips, are more realistic than a single trip model in disaster situations (Smilowitz and Dolinskaya 2011). Few works have studied the MTVRP. For example, (Ozdamar et al. 2002) proposed a deterministic single period model as a two-stage programming framework to involve helicopter routing, pilot allocation, and transportation decisions in disaster relief operations, and developed an interactive heuristic method. The model was solved using very small instances with relief networks of up to 10 nodes and 3 helicopters. Assignment costs were minimized during the top level, and makespan was considered as the base level related goal. More recently, MartinezSalazar et al. (2015) and Rivera et al. (2016) have presented MTCVRP that minimizes the sum of arrival times. MartinezSalazar et al. (2015) have developed two mixed- integer formulations based on a multi-level network. Instances with up to 30 nodes are solved optimally. They applied a GRASP metaheuristic algorithm for the solution of large-size test problems. Two flow-based and set partitioningbased models have been developed by Rivera et al. Rivera et al. (2016) for MTCVRP. An exact algorithm is introduced that reformulates the problem as a shortest path problem for larger cases. The results of the comparison between the two models with the exact method represent the good performance of the proposed method. In the latter two, the authors considered only a single vehicle and depot for performing delivery, and a multi-period planning situation is not taken into consideration.

The presence of various stakeholders with conflict criteria in humanitarian relief operations makes the problem a multi-criteria problem. Gutjahr and Nolz (2016) reviewed multicriteria optimization models and classified optimization criteria in humanitarian aid in seven groups: cost, response time, travel distance, coverage, reliability, security, and equity. A few publications meet the attribute of multiobjective as the problem. Lin et al. (2011) formulated a multi-item, multi-vehicle, and multi-period logistics model for the distribution of prioritized relief goods. They allowed split delivery strategies. The objective was to combine total travel time, unsatisfied demand, and equity in service among nodes. Equity was calculated by measuring the deviation in service level across nodes. The scalarization approach was utilized to solve the multi-criteria model. Huang et al. (2012) presented a vehicle routing model to deliver a single type of supply from a depot for one single period. In their proposed model, each vehicle had to perform a single trip. The authors considered three objectives, i.e., time, demand satisfaction, and equity. They also showed how the different objective functions influenced the overall behaviour of their proposed model. Using the scalarization approach, the authors converted the multi-objective problem into a single-objective problem. Ferrer et al. (2018) introduced a model based on deterministic multi-criteria transportation that attempted at last-mile distribution of humanitarian aids, which constituted some objectives as time, cost, equity, priority, security, and reliability. Regarding security, all vehicles must traverse an arc at the same time for forming convoys. They introduced a compromise programming model for solving the multi-criteria optimization problem.

Given the literature reviewed above, it can be concluded that some features that make the problem closer to the real emergency situation have not received much attention in the literature. In most previous works, for instance, each vehicle performed a single trip, and the number of vehicles was supposed to be unlimited. Yet, these assumptions are unrealistic in the disaster context. For example, when vehicle routes are shorter than the planning horizon or when the vehicle fleet is small, some vehicles may take several routes on the same day. Another commonly made assumption in the literature is that each demand point is served once and only once. However, in post-disaster situations, the volume of demands can be bigger than the vehicle's capacity in the affected areas and, therefore, an affected area can be served more than once by different vehicles. This feature is called split delivery in the relevant literature and was first introduced by Dror and Trudeau (1989). Nevertheless, some complexities have limited its use. Most previous works addressed static planning problems. Since the situation after a disaster is very dynamic, and information continually varies over time, it is unreasonable to solve response decisions for a single period. To this end, a multi-period model could be more suitable than a single-period model (Mahootchi and Golmmhammadi 2017). In single-period models, there is no difference between demands occurring in the first hours and those requested in later days of the disaster. Henceforward, while satisfying demands in-time seems very vital in the disaster situation.

According to the literature review on the

routing of relief operations in last-mile distribution and the mentioned research gaps, the present research proffers a multi-period multitrip vehicle routing problem with split delivery (MTVRPSD), which is more realistic for real emergency situations. Split delivery and multi-trip attributes in the model can result in serving large demands using fewer vehicles (Huang et al. 2012). Since the situation after a disaster is very dynamic, and information constantly varies over time, it is unreasonable to solve the first response decisions for a single period. We use a rolling horizon planning approach to cope with the dynamic nature of the problem. The major objective of disaster management is to reduce loss. In order to reduce human loss, unsatisfied demands are minimized on the entire planning horizon. Since the losses in affected areas increase proportionately with time, minimizing the response time is considered as another objective. Given the high volume of needs and limited resources, it is important that those resources be allocated fairly. Therefore, the fair distribution of relief items is considered as the third objective of the problem.

The main contributions of this research are:

- 1) Presenting a comprehensive model that simultaneously captures many aspects relevant to real-life problems, such as multiple depots, split delivery, multitrips, and multiple periods;
- 2) Applying a rolling-horizon approach so that a disaster management center can probably update programs as new information becomes available during our planning horizon and increases the effectiveness of the programs;
- 3) Considering the important objectives of efficiency, effectiveness, and equity; and
- 4) Applying the model to a real-world decision problem.

Table 1 Disaster Vehicle Routing Models **Table 1** Disaster Vehicle Routing Models

3. Problem Description and Formulation

In this study, we consider a post-disaster situation in which one of the most important actions is dispatching different kinds of relief commodities such as food, water, medical supplies, and so forth from relief centers to affected people in order to save lives and reduce human suffering. In this problem, the relief logistics network involves two members, suppliers or distribution centers, and affected areas as demand nodes. Several relevant decisions will be made regarding transportation quantities, delivery quantities, scheduling, and delivery routes.

It is assumed that the location of distribution centers is predetermined, and their accessibility of commodity is limited. Several types of goods with different degrees of importance are utilized as a response to the disaster. In the situation post-disaster, individuals have different conditions so the amount of one resource unit may have a different effect on different individuals. For example, the value of a resource unit for rescuing a severely injured individual is greater than its value for rescuing a less injured individual. Therefore for considering this effect, different demand points and commodities have been given different priorities. These priorities can e.g. be determined based on several external criteria such as the type, severity, or magnitude of the disaster, profile of damaged location, and response strategies and etc. We assume that these priorities can be devised by subject matter experts or disaster planners. Furthermore, this fact is considered that the vehicle fleet is limited to volume and weight constraints, and each vehicle can transport any type of commodities. Each vehicle can execute multiple trips in a single period, and each demand location can be visited multiple times by the same or different vehicles in the same planning period. A vehicle in a trip visits a sequence of demand locations and returns to the depot after completing its trip until the next order is specified. The information related to supply, demand, and network structure is obtained from the assessments made by relief agencies in the affected regions after the disaster occurs.

To cope with the dynamic nature of the problem, the rolling-horizon method presented by Wang, Kopfer, and Kopfer (2013) is applied. Figure 1 depicts the related approach. The entire time of the relief distribution process is subdivided into a series of stages (planning horizons). Each planning horizon consists of $p = 1, 2, \cdots, T$ planning periods. First, an initial plan for the first planning horizon, including *p*, the first period, is designed. The plan for the first planning period in which forecasts are typically more reliable is fixed and only implemented for this period. The plan for the following planning periods $p = 2, \dots, T$ will be actualized in the forthcoming plans as new information , and reliable forecasts become available.

According to these explanations, this section provides a multi-objective mixed-integer linear programming formulation for the proposed MTVRPSD.

3.1 Sets, Parameters, and Variables

3.1.1 Sets/Indices

- *NS*: Set of all nodes indexed by $i, j \in NS$, $N = |NS|$
- *DN*: Set of demand nodes indexed by *^k*, *^k* $\in DN, K = |DN|$
- *DC*: Set of distribution centres indexed by *d*, *d* ∈ *DC*, *D* = $|DC|$
- *VS*: Set of vehicles indexed by $v \in VS$, *V* $=$ $|VS|$
- *CS*: Set of commodity types indexed by *b* \in *CS*, $C = |CS|$
- *TS*: Set of periods indexed by *t*, $t \in TS$, $T = |TS|$
- *RS*: Set of trips any vehicle makes indexed by $r \in R$, $R = |RS|$

Figure 1 The Rolling Horizon Approach with Fixed Interval (Wang and Kopfer 2013).

3.1.2 Parameters

M_{big}: A large positive number

3.2 Sets, Parameters, and Variables

3.2.1 Sets/Indices

- *NS*: Set of all nodes indexed by $i, j \in NS$, $N = |NS|$
- *DN*: Set of demand nodes indexed by *^k*, *^k* $\in DN, K = |DN|$
- *DC*: Set of distribution centres indexed by *d*, *d* ∈ *DC*, *D* = $|DC|$
- *VS*: Set of vehicles indexed by $v \in VS$, *V*
- $=$ $|VS|$ *CS*: Set of commodity types indexed by *^b*

$$
E.S: \in CS, C = |CS|
$$

- *TS*: Set of periods indexed by *t*, $t \in TS$, $T = |TS|$
- *RS*: Set of trips any vehicle makes indexed by $r \in R$, $R = |RS|$

3.2.2 Parameters

δ*ijt*: accessibility to arcs *i* and *j* in period *t*

3.2.4 Mathematical Model *C K T*

Min
$$
Z_1 = \sum_{b=1}^{C} \sum_{k=1}^{N} \sum_{t=1}^{I} P_{kt} P_{bkt} U d_{bkt}
$$
 (1)

Min
$$
Z_2 = \sum_{t=1}^{T} \sum_{r=1}^{R} \sum_{v=1}^{V} \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ijt} Z_{vrijt}
$$
 (2)

Min
$$
Z_3 = \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{k=1}^{K} \sum_{b=1}^{C} |f_{bkt} - f_{bk \cdot t}|
$$
 (3)

$$
\sum_{v=1}^{V} \sum_{r=1}^{R} Q_{bdvrt} \le S_{bdt},
$$

$$
\forall b \in CS, d \in DC, t \in TS \quad (4)
$$

$$
\sum_{b=1}^{C} C_b \cdot Q_{bdort} \le CC,
$$

\n
$$
\forall d \in DC, r \in RS, v \in VS, t \in TS \quad (5)
$$

\n
$$
\sum_{b=1}^{C} W_b \cdot Q_{bdort} \le CW,
$$

 $\forall d \in DC, r \in RS, v \in VS, t \in TS$ (6)

$$
Q_{bdvrt} \leq M_{big} \sum_{k=1}^{K} Z_{vrdkt},
$$

\n
$$
\forall b \in CS, d \in DC, r \in RS, v \in VS, t \in TS
$$

\n
$$
Z_{vrijt} \leq M_{big}.\delta_{ijt},
$$

\n
$$
\forall i, j \in NS, v \in VS, r \in RS, t \in TS
$$

\n
$$
\sum_{i=1}^{N} Z_{vrikt} - \sum_{i=1}^{N} Z_{vrkit} = 0,
$$

\n
$$
\forall k \in DN, r \in R, v \in VS, t \in T
$$

\n
$$
\sum_{j=1}^{K} Z_{vrdkt} \leq 1,
$$

\n
$$
\forall d \in DC, r \in RS, v \in VS, t \in TS
$$

\n
$$
\sum_{j=1}^{N} \sum_{i=1}^{R} t_{ijt}.\overline{Z}_{vrijt} \leq t_{max},
$$

\n
$$
\forall v \in VS, t \in TS
$$

\n
$$
\forall i, j \in NS, d \in DC, r \in RS, v \in VS, t \in TS
$$

\n
$$
\forall i, j \in NS, d \in DC, r \in RS, v \in VS, t \in TS
$$

\n
$$
\sum_{v=1}^{V} \sum_{r=1}^{R} Y_{bkvrt} + Ud_{bkt} = d_{bkt}, \forall b \in CS, t \in TS
$$

\n
$$
\sum_{v=1}^{V} \sum_{r=1}^{R} Y_{bkvrt} + Ud_{bkt} = d_{bkt}, \forall b \in CS, k \in DN,
$$

\n
$$
r \in RS, v \in VS, t \in TS
$$

\n
$$
\sum_{k=1}^{N} Y_{bkvrt} = Q_{bdvrt}, \forall d \in DC, b \in CS, t \in TS
$$

\n
$$
\sum_{k=1}^{K} Y_{bkvrt} = Q_{bdvrt}, \forall d \in DC, b \in CS, t \in TS
$$

\n
$$
r \in RS, v \in VS, t \in TS
$$

\n
$$
\sum_{k=1}^{K} Y_{bkvrt} = Q_{bdvrt}, \forall d \in DC, b \in CS, t \in TS
$$

\n

 $snt_{kvrt} ≥ st_{rot} + t_{dkt} - M_{big}$.(1 − Z_{vrdkt}), ∀*d* ∈ *DC*, *k* ∈ *DN*, *r* ∈ *RS*, *v* ∈ *VS*, *t* ∈ *TS* (18) $snt_{kvt} \leq st_{rot} + t_{dkt} + M_{big}$. $(1 - Z_{vrdkt})$,

$$
\forall d \in DC, k \in DN, r \in RS, v \in VS, t \in TS
$$
\n
$$
(19)
$$
\n
$$
snt_{k \cdot vt} \geq snt_{k \cdot vt} + t_{k k \cdot t} - M_{big} \cdot (1 - Z_{v r k k \cdot t}),
$$
\n
$$
\forall k \neq k' \in DN, r \in RS, v \in VS, t \in TS
$$
\n
$$
(20)
$$
\n
$$
snt_{k \cdot vrt} \leq snt_{k \cdot vt} + t_{k k \cdot t} + M_{big} \cdot (1 - Z_{v r k k \cdot t}),
$$
\n
$$
\forall k \neq k' \in DN, r \in RS, v \in VS, t \in TS
$$
\n
$$
(21)
$$
\n
$$
snt_{k \cdot vt} \geq M_{big} \left(1 - \sum_{j=1}^{N} Z_{v r j k t}\right), \forall k \in DN,
$$

$$
r \in RS, \ v \in VS, \ t \in TS \quad (22)
$$

$$
st_{1vt} = 0, \quad \forall v \in VS, \ t \in TS \tag{23}
$$

 $snt_{dvrt} = st_{rot}$,

$$
\forall d \in DC, r \in RS, \ v \in VS, \ t \in TS \quad (24)
$$

$$
et_{r-1vt} = str_{vt}, \quad \forall r \in RS, \ v \in VS, \ t \in TS
$$
\n
$$
et_{rot} = str_{vt} + \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ijt} \cdot Z_{orijt},
$$
\n
$$
\forall r \in RS, \ v \in VS, \ t \in TS \quad (26)
$$

$$
Y, Q, Ud \ge 0 \& Integer; snt, st, et, f \ge 0;
$$

$$
Z \in \{0, 1\} \quad (27)
$$

The first objective function (1) minimizes the total amount of unsatisfied demand weighted by the priorities assigned to each node and commodity. The second objective function (2) attempts to minimize the total travel time to ship commodities to demand points. The third objective function (3) minimizes the sum of the absolute deviations of a fraction of unsatisfied demands between demand points to fairly allocate recourses. This objective is nonlinear, and the equivalent linear equations can be formulated as follows:

$$
\text{Min } Z_3 = (r_{bkkt} + s_{bkkt}) \tag{28}
$$

s.t.

$$
f_{bkt} - f_{bkt} = r_{bkk \cdot t} - s_{bkk \cdot t}, \ \forall \ k, \ k' \in DN,
$$

$$
b \in CS, t \in TS \quad (29)
$$

$$
r_{bk \cdot t} \ge 0, \ s_{bk \cdot t} \ge 0 \tag{30}
$$

Constraint (4) ensures that the amount of dispatched goods from each distribution center does not exceed the commodities available at the distribution center. Constraints (5) and (6) are related to the vehicle capacity and ensure that the commodity assigned for transport does not exceed the vehicle volume and weight capacity. Constraint (7) indicates that if vehicle *v* in trip *r* in period *t* does not leave the depot, i.e. $\sum_{k=1}^{K} Z_{vrdkt} = 0$, then the amount of commodity *b* loaded on vehicle *v* will be zero. Constraint (8) restricts the travel of each vehicle to existing arcs. Constraint (9) states that the vehicle that has arrived at each demand node must also leave this node. Constraint (10) implies that it is possible for the vehicle not to leave the depot. Constraint (11) expresses that the total travel time of all trips assigned to any vehicle in this period cannot exceed the available working hours in a period. Constraint (12) indicates that the vehicle will meet nodes if it is left depot. Constraint (13) forbids the vehicle to dispatch from one depot to another. Constraint (14) determines the unsatisfied demand of any commodity type at each demand node in a period of time. Constraint (15) denotes that only nodes visited by vehicle *v* in trip *r* in period *t* can receive the commodity. Constraint (16) indicates that the amount of assigned commodity to demand points must be equal to the load of the vehicle. Constraint (17) finds the fraction of unsatisfied demand in each demand node. Constrains (18)-(22) concern the arrival time at demand nodes. Constrains (18) and (19) define the arrival time at node *k* if the vehicle arrives from the distribution center whereas the constraints (20) and (21) define this amount if the vehicle arrives from the demand point. Constraint (23) indicates that the start time of the first trip of each vehicle is assumed to be zero. Constraint (24) states the arrival time at DC (depot). Constraint (25) expresses the start of the next trip when a previous trip ends. Constraint (26) calculates the end time of each trip. Finally, feasible regions for variables are defined by Constraint (27).

The value of M_{big} should be adequately large, but not so larger to avoid potential round off error. On the other hand, too small values for the coefficients can result in infeasibility of their respective constraints, and making the entire model infeasible. Therefore, finding a bound for M_{big} is essential for solving the model.

In constraint (7), *Qbdvrt* ≤ $M_{big} \sum_{k=1}^{K} Z_{\mathit{ordkt}}$, that compute the amount of commodity loaded on vehicle, we have $\frac{Q_{bdvrt}}{\sum_{k=1}^{K} Z_{vrdkt}}$ ≤ *Mbig*. According to $0 \leq Q_{bdvrt} \leq Q$, (*Q* is the maximum quantity of a commodity that can be loaded on each vehicle, depending on the volume and weight capacity of the vehicle) so it suffices to take Q ≤ *M*_{big}. Since Z_{tript} ∈ {0, 1}, therefore M_{big} take at most the value of 1 in constraints (8) and (12). According to constraints (16), *Ybkvrt* denote the amount of commodity type *b* that vehicle *v* in trip *r* in period *t* assigns to node *k* so they are non-negative and less than or equal to the maximum value of nodes demand. So $max \{d_{bkt}\}\leq M_{big}$.

Rearranging constraints (18), we have $snt_{kvrt} \geq st_{rvt} + t_{dkt} - M_{big}$. (1 – Z_{vrdkt}). When $Z_{\text{origt}} = 0$ we have $M_{\text{big}} \geq st_{\text{rot}} + t_{\text{dkt}} - snt_{\text{kort}}.$ We know that $0 \leq st_{rot}$, $snt_{kvt} \leq t$, therefore $M_{big} \ge t + \max\left\{t_{ijt}\right\}$. For constraints (19-22), we may derive similar bounds for *Mbig*. Maximum of upper bounds calculated for *Mbig* is selected as bound of *Mbig*.

4. Solution Procedure

The multi-objective mathematical programming consists of a number of objectives that are supposed to be optimized, and there is no single optimal solution that is able to optimize all the objective functions simultaneously. In this type of problem, the optimal solution is replaced with efficient (Pareto optimal) solutions. Effective solutions are those which cannot be improved by any of the objective functions without deteriorating other objective functions. The set of Pareto optimal solutions is referred to as a Pareto set.

An appropriate method for finding the Pareto set is the ε -constraint method. In this method, an objective function is optimized while other functions are considered as constraints. The problem is stated as follows:

min
$$
f_1(\bar{x})
$$

subject to $f_2(\bar{x}) \le e_2$, $f_3(\bar{x}) \le e_3$,
..., $f_p(\bar{x}) \le e_p$ (31)

Where subscript *p* indicates, the number of competing objectives functions of the problem and \bar{x} refers to the vector of decision variables. In (31), it is assumed that all *p* objective functions should be minimized. Values *ei* are calculated according to two points in the objective space: Utopia point, where all objectives are simultaneously at their best possible values, and Nadir point is a point in the objective space where all objective functions are simultaneously at their worst values.

Although the ε -constraint method has several advantages, there are some problems. One problem is the proper selection of ε so that its proper value would not be specified before the run. If the value of ε increases, the number of steps to implement the model increases significantly. Therefore, this method requires more input from the user (Aghajani et al. 2020).

Mavrotas has introduced an improved ε constraint method called the "augmented ε constraint method" (AUGMECON) that generates only efficient Pareto optimal solutions and avoids inefficient ones (Mavrotas 2009).

In order to better explain and apply the AUGMECON method, the following steps should be followed:

i. Calculating the Pay-off table using Lexi-

cographic optimization:

$$
\Phi = \begin{pmatrix} f_1^*(\bar{x}_1^*) & \cdots & f_p(\bar{x}_1^*) \\ \vdots & \ddots & \vdots \\ f_1(\bar{x}_p^*) & \cdots & f_p^*(\bar{x}_p^*) \end{pmatrix}
$$
(32)

The payoff table is calculated as follows: The objective function with a higher priority is selected and optimized accordingly. For example, let $f_1^*(\bar{x}_1^*)$ be the optimal solution of the objective function with a higher priority. In the next step, the objective function f_2 is minimized by adding the constraint $f_1 = f_1^*(\bar{x}_1^*)$ to the initial constraints of the model. If the problem has the third objective function, then this objective function is optimized by adding two constraints $f_1 = f_1^*(\bar{x}_1^*)$ and $f_2 = f_2^*(\bar{x}_2^*)$. This process continues until the optimization of the last objective function.

ii. Determining the range of objective functions:

$$
f_i^U \le f_p(\bar{x}) \le f_i^{SN}
$$

$$
f_i^{SN} = \max\{f_i(\bar{x}_1^*) \cdot \cdot \cdot, f_i^*(\bar{x}_i^* \%) \cdot f_i(\bar{x}_p^*)\}
$$

$$
f_i^U = f_i^*(\bar{x}_i^*) \qquad (33)
$$

iii. Dividing the range of objective functions:

$$
e_{i,ni} = f_i^{SN} - \left(\frac{f_i^{SN} - f_i^U}{q_i}\right) \times ni
$$

$$
ni = 0, 1, \dots, q_i \quad (34)
$$

where q_i is the number of intervals used to divide the range of *i*th objective function.

iv. Solving single objective optimization on the base of *ei*,*ni*:

Min
$$
F(x) = f_1(x)
$$

- $r_1 * eps * (\frac{s_2}{r_2} + \frac{s_3}{r_3} + \dots, \frac{s_p}{r_p})$ (35)

subject to
$$
f_i(x) + s_i = e_{i,ni}
$$

$$
i=2,\cdots,p\quad(36)
$$

$$
s_i \in R^+ \tag{37}
$$

where s_2 , \cdots , s_p are the introduced slack variables for Constraint (36) of the problem and *ri* is the range of the *i*th objective function calculated from the payoff table (the difference between the best and worst *i*th objective functions). The value of eps is a small number, usually between 10^{-6} and 10^{-3} . Interested readers are referred to (Mavrotas 2009) for more details on this method.

5. Case Description

The proposed model is applied to a real-world case for earthquakes in Tehran. Tehran, the capital city of Iran, has been affected by strong earthquakes throughout its history, the last one occurring in 1830. The city is now known as one of the most vulnerable urban areas to potential earthquakes. Tehran has become the political, cultural, and commercial centre of Iran. Now, it has nearly 9-million population (about 12 million in the daytime) and is the most populated city in Iran. Figure 2 shows the 22 regions of the Tehran municipality. Region 4, located in Northeastern Tehran, has been selected for the case study of this research. This region that includes nine districts covers 10% of the total area of Tehran $(61,288,367 \text{ m}^2)$, with about 11% of the population of Tehran (919,001 people) inhabiting there. This region is threatened by two faults: Shian-Kousar and Narmak. Demand nodes and distribution centers in this case study are mapped in Figure 3.

In 2000, the Japan International Cooperation Agency (JICA) conducted a comprehensive study on Tehran seismic disaster prevention and management (JICA 2000). In the JICA report, damage estimations and refugee populations are provided for each district. With regard to this report and available online

Figure 2 The 22 Regions of Tehran

Figure 3 Region 4 of Tehran City along with the Location of Demand Nodes and Distribution Centers

data on population, the demand for commodities at each district based on the number of refugees is determined, as shown in Table 2. Travel times on network links have been computed using the Tehran Navigation Website (http://map.tehran.ir/). Without loss of generality, a package of relief items, including water, food, and medicine, is assumed to be sent to demand points. Needs for demand points, at which the distribution center that located is supplied by the same distribution center. The weight and volume details for the package and weight and volume capacity for the vehicle are demonstrated in Table 3. Table 4 presents the inventory of commodities in distribution centers in different time periods. There are five vehicles in distribution centers to transport commodities to demand nodes.

The entire time horizon is set to the first three days (the first 72 h of a disaster relief effort). The working time for each day is set

Table 4 The Amounts of Commodities Provided by Distribution Centers

to be 18 hours. According to the rolling horizon approach, the entire time horizon is subdivided into eight planning horizons (stages), and each planning horizon consists of two periods. The unit length of a time interval is given by 6 hours (i.e., three periods in a day). The maximum number of routes that a vehicle can take by period is limited to 2. Also, the maximum working time of a vehicle in each period is considered 360 minutes.

6. Numerical Results

This section presents the numerical results and behavior of the proposed model. The mathematical model was solved by GAMS/Cplex on a laptop computer with a processing speed of 2.6 GHz and 6 GB of RAM under Windows 8 Professional.

To solve the multi-objective problem proposed in this study, we used the augmented epsilon constraint method. The Pareto solutions generated by AUGMECON and ε -constraint method are presented in Figure 4. The comparison of these solutions shows that the Pareto front obtained from AUGMECON is more efficient.

In each stage of the rolling horizon-based system, the model is solved based on the input data. Tables 5 and 6 indicate the optimal model decisions in the first stage of the rolling horizon. Table 5 shows the amount of commodity loaded on vehicles over different trips in each period. The distribution plan includes routes, delivery amounts, and schedules of distribution for this stage present in Table 6. For example, the first row of Table 6 shows that *V*¹ in the first period takes two trips. In the first trip, the vehicle departed from depot located at node 3, arrived at node 2 within 14 min to deliver 10 commodities, arrived at node 4 after 27 min to deliver 270 commodities, then reached at node 5 after 39 min to deliver 310 commodities, arrived at node 7 after 52 min to deliver 10 commodities, and finally returned to node 3 after 68 min. The distribution plan of vehicle 1 for the first period is depicted in Figure 5. The dual placed under each node represent the amounts of goods being delivered and the arrival time to them.

According to the distribution plan in Table 6, the arcs priority used in the transportation network is reported in Table 7 based on the weight percent of the commodities passing through these arcs. Results show that all arcs of network are not similarly important in the response because some arcs are used more for the commodity transportation. For instance, more than 58% commodities are transported through the three arcs 4-5, 8-9 and 3-7. In this way, relief organizations can identify emergency roads in the network and make them resistant.

Figure 6 shows the unsatisfied demand ration for each demand node belonging to three models as follows: the proposed model (case I), the proposed model without split delivery (case II), and the model in which each vehicle performs a single trip (case III). It is evident that the value of this criterion in the model expressed for all nodes is minimal.

The effect of the number of vehicles on unsatisfied demand in these three models is illustrated in Figure 7. As expected, with the increase in the number of vehicles, the amount of unsatisfied demand is reduced. In case I and II, after $V = 4$, the amount of unsatisfied demand does not decrease significantly. Since hiring a new vehicle requires consider-

Figure 4 Comparison of the Pareto front of AUGMECON and ε-Constraint Method

Table 5 The Amount of Commodity Loaded on Vehicles on the First Stage

		rη		Υņ		rη		r
$t=1$	600	460	560		380		510	600
$t=2$	600	600	600	600	600		450	600

Figure 5 The Distribution Plan of V_1 for $t = 1$

Figure 6 Unsatisfied Demand Ration in Each Node

Figure 7 Impact of the Number of Relief Vehicles on the Unsatisfied Demand

able costs, decision-maker would decide to hire a new vehicle only if there is a high benefit of hiring another vehicle in terms of the amount of unsatisfied demand.

In Figure 7, we also observe that the unmet demand in the model expressed in this paper is less than that of the other two models for a specific number of vehicles. Therefore, when each vehicle performs multiple trips and each demand node can be served more than once, more demand can be met with a smaller number of vehicles. Such a model can be very useful in disaster situations where resources such as vehicles are limited, and the aim is providing the greatest service to affected individuals.

We investigate the effect of increasing the inventory compared to the current situation

on the percentage of increase in the satisfied demand. Figure 8 illustrates the change in %satisfied demand for inventory increase = 10%, 20%, ··· , 60%for cases I, II and III. According to Figure 8, while having more inventories increases the percentage of demand covered in all cases, after inventory increase = 40%, %satisfied demand does not increase significantly. Based on this analysis, managers can effectively plan for sufficient inventory. In Figure 8, we also observe that in the case I, i.e., the proposed model in this paper, the percentage of increasing demand coverage is considerably more than cases II and III (by nearly 20% even for inventory increase $= 30\%$).

Figure 9 highlights the effect of equity on the distribution of relief supplies. To this end,

Figure 8 Impact of the Inventory on Satisfied Demand

7. Conclusion and Future Research

two cases are designed, and their outcomes are compared. The first case (Model I) includes the model proposed in this study, and the second case (Model II) neglects the objective of equity. The total unsatisfied demand in both models is 435 units, and all of the DNs have the same priority in receiving relief items. It is clear that in Case B, the unsatisfied demand ratio varies significantly between regions. Some districts that are far away from the distribution centres, such as District 4, receive no goods or a small percentage of its demand is met, while districts near distribution centres, such as the Districts 7 and 8, have a higher chance to be satisfied in each period. In Model I, demand satisfaction is almost the same in all districts. The largest difference is 0.01% between different regions, while this value is 1 for Model II. Comparing the results of these two models demonstrates that the outputs of the model significantly depend on the equity objective. Since there are great demands for various relief supplies once disasters occur and relief agencies are confronted with limited resources, it is typically very difficult, if not impossible, to satisfy the entire relief demand immediately. Unfair allocation policies may cause social chaos, such as ransack and even robbery of relief supplies. In these situations, it is important to consider fairness alongside the goals of efficiency and effectiveness.

In this study, we presented a model for MTVRPSD that simultaneously tackles more realistic aspects such as multi-depot, possible of several trips by one vehicle, servicing each disaster area more than once (split-delivery), multi-period, and using multiple conflicting goals. A new multi-objective mixed-integer linear programming model is proposed to formulate the problem. Our model includes efficiency, effectiveness, and equity as the objective functions for emergency logistics management. We employ the augmented epsilon method for multi-objective optimization and the efficient construction of the Pareto curve. To capture the dynamic aspects of the problem, we utilized a rolling horizon approach that can receive updated data and adjust the logistics plan during the response. A practical case study is also presented. The numerical results indicate that the simultaneous consideration of these attributes will improve the effectiveness of distribution efforts. Based on the results, attributes in the model can serve large demands using fewer vehicles. This feature in the disaster situation is highly beneficial due to resource constraints. Similarly, the results highlight the influence of the equity objective on the fair distribution of resources.

Finally, we suggest the following directions for future studies: 1) To solve large-scale problems within a reasonable time; a suitable solution procedure should be designed to obtain high-quality solutions within short run times;

Figure 9 Effect of Equity on the Distribution of Relief Supplies

2) in the real world, some parameters of the model may not be deterministic. Therefore, parameters and data should consider uncertainty; 3) Further operations such as evacuation problem (in which people need to be quickly transported to safe areas or medical facilities) as well as the determination of the location of distribution centers and the required inventory quantities considering various factors (such as safety and transportation infrastructure) can be incorporated; and 4) other goals such as reliability routes and social costs can be considered.

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