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ROBUSTNESS OF EQUILIBRIA IN THE GRAPH MODEL FOR CONFLICT RESOLUTION

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Abstract

A novel approach for assessing the robustness of an equilibrium in conflict resolution is presented. Roughly, an equilibrium is robust if it is resilient, or resistant to deviation. Robustness assessment is based on a new concept called Level of Freedom, which evaluates the relative freedom of a decision maker to escape an equilibrium. Resolutions of a conflict can be affected by changes in decision makers' preferences, which may destabilize an equilibrium, causing the conflict to evolve. Hence, a conflict may become long-term and thereby continue to evolve, even after reaching an equilibrium. The new robustness measure is used to rank equilibria based on robustness, to facilitate distinguishing equilibria that are relatively sustainable. An absolutely robust equilibrium is a special case in which the level of freedom is at an absolute minimum for each individual stability definition.

Keywords: Robustness, equilibria, level of freedom, conflict evolution, graph model

1. Introduction

Strategic conflicts can be formally modelled and analyzed using the Graph Model for Conflict Resolution (GMCR) (Kilgour et al. 1987, Fang et al. 1993, Hipel 2009 a,b, Kilgour and Eden 2010, Hipel et al. 2011), which utilizes various stability concepts to determine possible equilibria or resolutions to a given conflict, and thereby obtain strategic insights. It has been observed that some real world conflicts demonstrate that it is possible for conflicts to continue evolving even after reaching an equilibrium (Matbouli et al. 2013, 2014b). Therefore, further analysis of equilibrium robustness is desired in order to gain some insights about the sustainability of a resolution. In this research, robust equilibrium is not necessarily a binary relation; instead, robustness can be viewed as a level in which equilibria are ranked from most robust to least robust. Such a methodology, for example, will make it possible to distinguish between stable states, where, for instance, in the sense of GMCR, a specific Nash equilibrium can be more stable than another Nash equilibrium.

A new formal robustness of equilibrium analysis is introduced within GMCR, which will provide insights for better understanding the evolution of long-term conflicts. Refined stability definitions are presented, which facilitates the evaluation of equilibrium robustness, thereby making conflict resolution more sustainable. In the following sections, a background on strategic long-term conflicts and a review of GMCR is summarized. Then, a formal methodology for analyzing robust stability and ranking procedures to assess equilibrium robustness are proposed. The new strategic approach is applied to a groundwater contamination dispute that took place in Elmira, Ontario, Canada. Subsequently, strategic results and insights obtained when applying robustness of equilibrium analysis are discussed.

2. Background

2.1 Strategic Conflicts

Strategic conflicts are a complex form of decision making (Jeong et al. 2008) where each decision maker (DM) considers his or her options while thinking about other DMs' moves. In such a situation, a DM cannot achieve a desired outcome without carefully anticipating the decisions of opponents. Conflict conditions exist when two or more DMs pursue incompatible goals (Galtung 2008). Modelling and analysis of these interactive decision-making problems have been widely implemented using game-theoretic methodologies, such as GMCR. The development of the graph model started in 1987 by Kilgour et al. (1987), while the first book on the topic was written in 1993 by Fang et al. (1993). It has been widely used around the world in various application areas such as environmental conflicts (Kilgour et al. 2001), energy disputes (Matbouli et al. 2014b), negotiations (Sheikhmohammady et al. 2010), policy design (Zeng et al. 2004), and business applications (Kilgour and Hipel 2005).

2.2 Long-Term Conflicts

The need to study the robustness of equilibria is recognized because some real world conflicts, such as the great Canadian hydroelectric power conflict (Matbouli et al. 2014b), continued to evolve even after reaching a predicted resolution, thereby creating a challenge to modelling and analysis. For the purpose of analyzing such conflicts, a new concept of robustness of equilibria is introduced to make the classification of resolutions possible, and permit the examination of the sustainability of different solution concepts. As Figure 1 shows, after an active conflict reaches an equilibrium, it may or may not transform into a long-term conflict if preferences change. A measure of equilibrium robustness is introduced in the methodology section.

2.3 The Graph Model for Conflict Resolution (GMCR)

GMCR (Kilgour et al. 1987, Fang et al. 1993, Kilgour and Hipel 2005, Hipel 2009 a,b, Kilgour and Eden 2010, Hipel et al. 2011) is a systematic chess-like approach that is used to analyze strategic conflicts. GMCR uses solution concepts inspired by game-theoretic equilibrium definitions in order to model interactions amongst DMs under conflicts. The use of GMCR has several advantages over classical game-theoretic approaches in the analysis of conflicts. First, GMCR is flexible in representing large conflicts, compared to normal or extensive games. Additionally, conflicts represented in GMCR can handle moves that are not only reversible, but also irreversible or common. Moreover, GMCR can represent complex preference structures of DMs, such as cardinal, transitive, and intransitive preferences.

Definition 1 A graph model for conflict resolution is defined as $G = \langle N, S, \{A_i\}_{i\in N}, \{\succ_i, \prec_i, \sim_i\}_{i\in N}\rangle$, where N is the set of all DMs, S is the set of feasible states, A_i is the set of unilateral moves available for DM_i such that $A_i \subseteq S \times S$, and $\{\succ_i, \prec_i, \sim_i\}$ represents $DM\, i$'s preference relation, such that for any $s, q \in S$, $s \succ_i q$ means state s is more preferred by DM_i than state q, $s \prec_i q$ indicates that state s is less preferred for DM_i than state q, and $s \sim_i q$ means DM_i is indifferent between state s and state q.

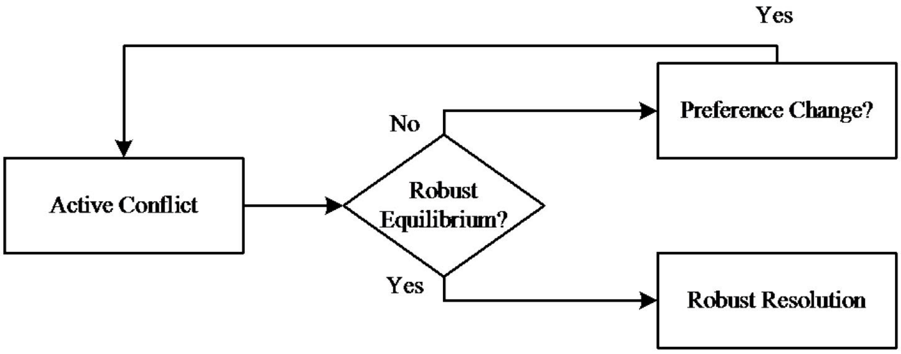

Figure 1 Long-term conflict

Definition 2 In a graph model G, the reachable list for a DM_i (Fang et al. 1993) $i \in N$ from state $s \in S$ denoted by $R_i(s) = \{q \in S : (s,q) \in A_i\}$, is the set of states to which DM_i can move unilaterally from state s. Similarly, the unilateral improvement (UI) list of moves denoted by $R_i^+(s)$, is a subset of $R(s)$ defined by $\{q \in R(s) : q \prec_i s\}$. The lists $R_i^=(s) \subseteq R(s) \ni \{q \in R(s) : q \sim_i s\}$ and $R_i^-(s) \subseteq R(s) \ni \{q \in R(s) : q \succ_i s\}$ are defined analogously such that $R_i^+(s) \cup R_i^-(s) \cup R_i^=(s) = R_i(s)$.

Hence, if there is a move between states s and q such that $s \sim_i q$, the move is considered a unilateral move (UM) available to DM_i between indifferent states, and it is denoted by $R_i^=(s)$. Moreover, in case $s \prec_i q$, the move from s to q is considered a unilateral disimprovement (UD) and it is denoted by $R_i^-(s)$.

2.3.1 Stability Definitions

Primary stability concepts, defined below, are the context in which robustness can be understood in terms of individual stability. These stability concepts include Nash (Nash 1950, 1951), Sequential Sanctioning (SEQ) (Fraser and Hipel 1979, 1984), general metarational (GMR)

(Howard1 1971), and symmetric metarational (Howard 1971). First, the Nash stable states for $i \in N$, $S_i^{Nash} \subseteq S$ are defined as follows:

Definition 3 For $i \in N$, state $s \in S_i^{Nash} \Leftrightarrow R_i^+(s) = \varnothing$.

Thus, a state $s \in S$ is Nash stable for DM_i if and only if (iff) DM_i has no UI from s. In Nash stability, a DM looks one move ahead from the present state, considering only his or her own preference. The next definition, sequential stability (SEQ), extends the consideration for DM_i's foresight to two moves, where the DM takes into account possible sanctions of a UI by other DMs.

Table 1 Foresight of focal decision maker in different stability conditions

Stability Definitions	Foresight	Focal DM	Opponent DM
Nash (r)	One move	UI	-
Sequential Stability (SEQ)	Two moves	UI	UI
General Metarationality (GMR)	Two moves	UI	UI, UD
Symmetric Metarationality (SMR)	Three moves	UI	UI, UD

Definition 4 For $i \in N$, state $s \in S_i^{SEQ} \Leftrightarrow \forall q \in R_i^+(s), \exists x \in R_{N-i}^+(q) \ni x \precsim_i s.$

A state s is sequentially stable for DM_i iff any UI from state s is sanctioned by a countermove from DM_{N-i}. Note that any Nash stable state is sequentially stable, and hence, $S_i^{Nash} \subseteq S_i^{SEQ}$.

The difference between SEQ and GMR, which is defined below, is that in SEQ, the threat to sanction a UI is credible because it will result in an improved position for the sanctioning DM, whereas, in GMR, the sanctioning DM does not consider his or her own benefit. GMR is a more conservative stability definition; therefore, it is a weaker stability concept than SEQ.

Definition 5 For $i \in N$, state $s \in S_i^{GMR}$ $\Leftrightarrow \forall q \in R_i^+(s), \exists x \in R_{N-i}(q) \ni x \precsim_i s$.

A state $s \in S$ is general metarational (GMR) for DM_i, iff any UI by DM_i from state s to q is sanctioned by a unilateral move (UM) by DM_{N-i} q to x such that state x is not more preferred than state s by DM_i.

When the GMR definition is extended to see if the focal DM is able to recover from sanctioning, the result is the definition of SMR stability, in which the focal DM cannot escape from a sanction.

Definition 6 For $i \in N$, state $s \in S_i^{SMR}$ $\Leftrightarrow \forall q \in R_i^+(s), \exists x \in R_{N-i}(q) \ni x \precsim_i s \wedge \forall h \in R_i^+(x) \ni h \precsim_i s.$

A resolution is reached when a state that is stable for all DMs is reached. Such a state is called an equilibrium. A conflict may have more than one equilibrium.

Definition 7 $s \in S^{equilibrium} \Leftrightarrow s \in S_N^{Stable} \ni S_N^{Stable} = S_N^{Nash} \cup S_N^{SEQ} \cup S_N^{GMR} \cup S_N^{SMR}$

In Table 1, a summary of the numbers and types of moves the focal DM considers in the various stability concepts is given. For example, in Nash stability, the focal DM considers only his or her own possible UIs, while in SEQ the focal DM also considers UIs by opponents. After incorporating robustness in the stability definitions, the contents of Table 1 will be extended to consider additional foresight or moves that are not necessarily UIs.

2.4 Robustness of Equilibria

Although robustness of equilibria is a fresh concept in GMCR, the term robustness of equilibria is not entirely new, as it has been used in game theory settings by Fudenberg et al. (1988), and it has also been suggested that some equilibrium states are more stable than others (Kohlberg and Mertens 1986). Methodologies have been put forward to further refine the definition of equilibrium in order to find the most stable, robust, or likely equilibrium, by assigning probabilistic weights to opponents' strategies to exclude unlikely equilibria.

Examples of robustness of equilibria approaches in game theory include perfect equilibrium (Selten 1975), proper equilibrium (Myerson 1978), strategically stable equilibrium (Kohlberg and Mertens 1986), and robust equilibria of potential games (Ui 2001).

Moreover, robustness can be attributed by flexibility of DMs (Rosenhead et al. 1972, Rosenhead and Mingers 2001, Rosenhead and Wiedemann 1979). In this respect, a DM makes robust decisions by maintaining flexibility against unforeseeable changes in the future.

However, in long-term conflicts in general, and in GMCR in particular, the interest is not to predict the most likely equilibria. Instead, the goal is to find which equilibria are more resilient to change, and sustainable in relative comparison, in other words, which equilibria are more likely to persist despite future uncertainties or gradual preference changes.

Therefore, a fresh concept in conflict resolution, Level of Freedom (LoF) (Matbouli et al. 2014a), was introduced as a new measure in equilibrium robustness. A formal framework for equilibrium analysis is presented in the next section.

3. Methodology

Strategic long-term conflicts take place when two or more interested parties seek incompatible objectives (Galtung 2008). Although equilibria can be seen as resolutions for such conflicts, the tendency of conflicts to evolve can be attributed to many reasons, one of which is the changing of preference. Future changes in preferences can be hard to predict. When a preference change takes place, the stability concepts of individual stabilities of DMs at some states no longer hold. Preferences can change for a number of reasons, for example, a change in goal realization because of external or internal factors. Take, for instance, a conflict arising over the utilization of limited water resources. When the demand of one region increases, more water needs to be drawn, which could put a prior agreement at risk of initiating a conflict.

There are three main input parameters for GMCR: DMs, options, and preferences. The reliability of equilibria states, which are the output of GMCR, depends on the quality of the inputs. Because uncertainty is prevalent in real world conflicts, DMs, options, and preferences can be assumed to be characterized by high variability. To account for uncertainty of conflict parameters, there are a number of extensions to the original graph model which account for uncertainty at the present or historical state of a conflict, such as fuzzy preferences (Al-Mutairi et al. 2008, Bashar et al. 2012), and stochastic preferences (Rêgo and Santos 2013). If uncertainty exists in the present conditions of a conflict, future changes are even more uncertain (Pye 1978). The robustness of equilibria method provides insights into the sensitivity of stable states against future changes by considering available moves to be the main factor in challenging the stability of equilibria.

In the proposed enhancement, the information about DMs, options, and preferences is assumed to be complete. However, considering the risk of future changes, some Nash stable states can be more robust than other Nash stable states based on the possibility of deviation from the state on account of future changes.

3.1 Factors in Equilibrium Robustness

Robustness of an equilibrium is a relative measure that is dependent on the characteristics of a particular conflict being studied. The comparison and evaluation of equilibrium robustness is based on three factors: level of freedom (LoF) (Matbouli et al. 2014a), type of individual stability, and relative preference. The thought process that led to the idea of LoF started by looking for possible unilateral escapes from stable states.

3.1.1 Level of Freedom (LoF)

In ongoing long-term conflicts, some equilibrium conditions may remain satisfied even when the preferences of one or more DMs change in a new round. This situation can be linked to the availability of moves; for example, when a DM has no possibility of moving from an equilibrium, the conditions for stability will be maintained.

The level of freedom (LoF) constitutes a rough measure to assess relative resistance to a stability disruption in cases of preference change (Matbouli et al. 2014a). If a DM changes his or her preferences for any reason, a stability condition may not be maintained. Therefore, the concept of LoF evaluates the robustness of stability of a given state for a particular DM, by counting the number of unilateral moves available to the DM. If the focal DM has a high LoF, it means that even a small change in preference would likely disturb the equilibrium. If the LoF is very low, or if it is zero, even a dramatic change in preference will not affect the equilibrium.

Assume that DM_i finds states s_1 and s_2 to be Nash stable. Assume also that from state s_1, there is no unilateral move available to DM_i, while at state s_2 there are three available moves that are disimprovements for DM_i. Now, both states are Nash, but which state is more stable? For the case of preference change for DM_i, his or her stability at state s_1 cannot be affected. For state s_2, however, if the preference change for DM_i makes an available move a unilateral improvement, then the conditions of Nash stability at state s_2 will no longer be valid. Thus, even though both states s_1 and s_2 are Nash stable, state s_1 is more robust than state s_2.

LoF can be calculated in a number of ways. For example, one may count the available number of moves from the present state in one step to other states. Or an analyst may choose to extend the horizon to calculate the number of moves in two or three consecutive steps ahead. However, LoF should be interpreted only relatively. The difference between LoFs of 4 and 5 may not be significant, but larger differences in LoF tend to indicate a significant difference in the robustness of the states. A state is Absolutely Robust for an individual stability concept, as introduced in Section 3.2, when LoF is zero, the absolute minimum for a stability definition.

In this article, a two-step calculation has been chosen for LoF, by counting the number of unilateral moves from the present state, regardless of the type of move: unilateral improvement, disimprovement, or neither. States that are one move ahead from the present state can be endpoints or transitional nodes. There is the possibility to encounter two-step loops that return to the initial state. Since the aim is to find the robustness of a state relative to another state, the count of the number of moves was limited to two sets of moves. The first set of moves is counted from the present state to the next endpoint. The second set of moves is counted based on the number of moves leading to transitional nodes multiplied by two. LoF is defined as follows (Matbouli et al. 2014a):

Definition 8. $LoF_i(s) = 2c - d$, where c is the number of UMs from state s, and d is the number of UMs from state s that lead to a stable state for the DM.

In addition, state s_1 in the above example, which is a Nash stable state with no possible unilateral moves, is called an Absolutely Robust state. Further definitions related to absolute robustness are presented in Section 3.2.

Moreover, calculating LoF for each DM at every state can produce some interesting properties, some of which are given below:

1. If a state s has an $LoF_i(s) = 1$, then s is Nash stable for DM_i iff $s \succ_i R_i(s)$.
2. If for a state s the number of feasible states is not greater than $LoF_i(s) + 1$, then DM_i can move unilaterally from state s to any state in the conflict.
3. If $LoF_i(s) = 0$ and $R_i(q) = s$, then the move from q to s is irreversible.
4. If, for all states $s \in S$, $LoF_i(s) = 0$, then DM_i has no choices in the conflict.
5. If $LoF_i(s) = 0$, then s is Nash, SEQ, GMR, and SMR stable for DM_i.

3.1.2 Types of Individual Stability

There are four types of stability considered for robustness of equilibria analysis: Nash, SEQ, GMR and SMR. It is generally accepted that Nash and SEQ stability concepts are stronger than GMR and SMR, because of the sociological assumption that supports each stability definition. In Nash, the DM is assumed to move unilaterally when a better outcome is achieved. In SEQ, the DM avoids making a UI, fearing a credible threat, which will put his or her opponent in a better position while harming the focal DM. On the other hand, in GMR and SMR stable states, the focal DM is assumed to abstain from making a unilateral improvement to avoid a less credible sanctioning. In GMR and SMR, the threat of sanctioning assumes that an opponent DM will harm his or her own position in order to put the focal DM in a worse state. This assumption is less likely than the general assumption that in strategic conflicts, DMs seek to improve their outcome and not the reverse.

In order to rank individual stabilities, LoF is measured in two separate categories: strong stability concepts (Nash and SEQ), and weak stability concepts (GMR and SMR). Initially, all Nash and SEQ states are assumed to be more robust than GMR and SMR states. In addition, within each category, the lower the LoF, the higher the robustness ranking of individual stability with respect to the focal DM.

Table 2 Foresight of focal decision maker in different robust stability conditions

Stability Definitions	Foresight	Focal DM	Opponent DM
Robust Nash (RNash)	One move	UI, UD	-
Robust Sequential Stability (RSEQ)	Two moves	UI, UD	UI
Robust General Metarationality (RGMR)	Two moves	UI, UD	UI, UD
Robust Symmetric Metarationality (RSMR)	Three moves	UI, UD	UI, UD

3.1.3 Preferences

Since LoF assesses the possibility of deviation in case of preference change, it is useful to consider preferences when ranking equilibrium robustness. Assume that a DM i has the same LoF at two stable states of the same type. To determine which state is more robust in this case, it is plausible to compare the preferences of both states. The state that is more preferred to the focal DM is considered more robust for the respective DM.

3.2 Robust Individual Stabilities

A robust state is a particular case of stability robustness where LoF is equal to the absolute minimum for each stability type. An absolutely robust equilibrium is the state that all DMs find to be robust stable. The robust stable states– Nash, SEQ, GMR, and SMR–are defined below.

Definition 9 $s \in S_i^{RNash} \Leftrightarrow R_i(s) = \varnothing$.

Note that state s is a Robust Nash (RNash Stable) iff DM_i has no UM from s . Thus $S_i^{RNash} \subseteq S_i^{Nash}$, i.e. s robust Nash is Nash. Clearly s is Robust Nash iff $LoF_i(s) = 0$.

Definition 10 $s \in S_i^{RSEQ} \Leftrightarrow \forall q \in R_i^+(s)$, $\exists x \in R_{N-i}^+(q) \ni x \precsim_i s \wedge x \in S_i^{Stable}$.

In SEQ stability, a DM abstains from making a UI to avoid a credible sanctioning by the opponent. For Robust SEQ (RSEQ), there is at most one possible move that is the UI, and this UI leads to an end point (stable state), not a transitional state. State s is Robust SEQ iff $LoF_i(s) \le 1$.

Definition 11 $s \in S_i^{RGMR} \Leftrightarrow \forall q \in R_i^+(s)$ $\exists x \in R_{N-i}(q) \ni x \precsim_i s \wedge x \in S_i^{Stable}$.

Likewise, in Robust GMR (RGMR), the number of moves $LoF_i(s) \le 1$ is similar to the LoF of RSEQ, except that the threat of sanctioning in RGMR is less credible than in RSEQ.

Definition 12 $s \in S_i^{SMR} \Leftrightarrow \forall q \in R_i^+(s)$, $\exists x \in R_{N-i}(q) \ni x \precsim_i s \wedge \forall h \in R_i^+(x) \ni h \precsim_i s \wedge x \in S_i^{Stable}$.

The focal DM, who finds a state to be SMR stable, considers two moves ahead from the present state, so there are at most 2 unilateral moves. Thus, a Robust SMR state has a $LoF_i(s) \le 2$.

In the definitions above, it can be observed that UDs are taken into consideration when performing robust stability analysis (see Table 2). The number of moves considered ahead is no different from regular stability definitions as seen in Table 1. However, there are more types of future moves considered.

3.3 Ranking of Robustness of Equilibria

In order to relatively rank equilibria robustness, two steps of the ranking are performed. First, each individual stability that results in equilibrium is ranked from an individual DM's perspective. Then, the overall ranking is ordered based on individual ranking of stable states.

3.3.1 Ranking of Individual Stability

The interest here is only to rank states that represent equilibria. So this is kind of a reverse process. After individual stability analysis is performed, and equilibria points are defined, we go back to individual stability states and select only those pertaining to equilibria.

For each DM, rank the individual states based on LoF in two groups: one is the group of Nash and SEQ stable states, and the other group consists of GMR and SMR stable states. For each group, the states are ranked from most robust to least robust, with the state having the lowest LoF regarded as the most robust. Then combine the ranking of both groups by assigning all Nash and SEQ stable states a higher order of robustness than GMR and SMR states. Ties in ranking are acceptable at this stage. Assign an order number for each individual state from each DM's perspective as seen in the first section of Table 3.

Table 3 Ranking of equilibria robustness

Decision Makers	Equilibria States s_1	s_2	$\cdots$	s_n
DM_1	$Rank_1(s_1)$	$Rank_1(s_2)$	$Rank_1(\ldots)$	$Rank_1(s_n)$
DM_2	$Rank_2(s_1)$	$Rank_2(s_2)$	$Rank_2(\ldots)$	$Rank_2(s_n)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
DM_N	$Rank_N(s_1)$	$Rank_N(s_2)$	$Rank_N(\ldots)$	$Rank_N(s_n)$
Overall Ranking	$\sum_{i=1}^{N}(Rank_N(s_1))$	$\sum_{i=1}^{N}(Rank_N(s_2))$	$\sum_{i=1}^{N}(Rank_N(s_{\ldots}))$	$\sum_{i=1}^{N}(Rank_N(s_n))$

3.3.2 Overall Ranking of Equilibria Robustness

For each state, combine the order number given from each DM's perspective (see Table 3). The states are ranked based on the summation of ranks for each state from most robust to least robust. The lower the order of ranking, the higher the robustness. For ease of interpretation, readjust the order number with the lowest order starting at 1. At this stage, in case of a tie in ranking, we investigate the preferences according to either methodology discussed in Section 3.1.3. However, if the equilibrium states have exactly the same LoF and type of stability, then ranking equilibrium robustness based on LoF is not currently possible. This is because the possibility of deviation from equilibria will be identical among equilibrium states.

3.4 Insights from Level of Freedom

The utilization of LoF can provide interesting insights that not only rank equilibria according to robustness, but also highlight general assessments about the robustness of stable states for DMs in a conflict.

Theorem 3.1 *Suppose that, for* DM_i *,* $\sum_{s\in S} LoF_i(s) \le n$*, where* n *is the total number of* *feasible states. Then, there is at least one state that is Nash, and RNash for* DM_i.

Proof. If the total LoF_i is less than the number of feasible states, then one or more states must have $LoF_i = 0$. ■

Theorem 3.2 *For a 2-DM conflict* $\{i, j\}$ *, suppose* DM_j *has* $LoF_j(q) = 0$ *. Then,* DM_j *cannot sanction any move by* DM_i *to state* q

Proof. For a 2-DM conflict $\{i, j\}$, suppose DM_j has $LoF_j(q) = 0$. Then, DM_j cannot sanction any move by DM_i to state q. ■

Theorem 3.3 *If state* s *is RNash for* DM_i*, the state* s *is also RSEQ for* $DM\ i$.

Proof. If state s is RNash for DM_i , then Definition 10 for RSEQ is satisfied, because $R_i(s) = \varnothing$. ■

Theorem 3.4 *If state* s *is RSEQ for* DM_i*, the state* s *is also RGMR for* DM_i.

Proof. If state s is RSEQ for DM_i , then Definition 11 for RGMR is satisfied, since $R_i^+(s) \subseteq R_i(s)$. ■

Theorem 3.5 *If state* s *is RNash for* DM_i*, the state* s *is also RSMR for* DM_i.

Proof. If state s is RNash for DM_i , then Definition 12 for RSMR is satisfied, because $R_i(s) = \varnothing$. ■

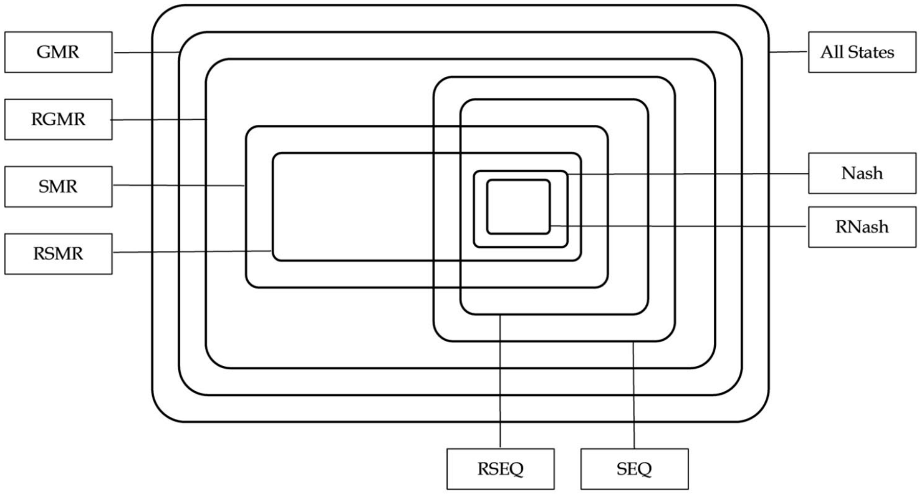

Figure 2 Interrelationships between robust and standard stability concepts

The above interrelationships among robust individual stabilities are similar to the standard stability concepts (Fang et al. 1993). Moreover, it is also noteworthy that each robust stability definition is a subset of the original corresponding stability definitions, as depicted in Figure 2. For example, if state s is RNash, then state s is also Nash stable, because $S^{RNash} \subseteq S^{Nash}$.

4 Case Study: Elmira Groundwater Contamination Conflict

In 1989, contamination was discovered in the drinking water source of Elmira, which is a Canadian town located in southwestern Ontario, about 100 km east of Toronto. The contamination was attributed to chemical discharges released by Uniroyal Chemical Ltd. (UR), which ran a chemical plant. The Ontario Ministry of Environment (MoE) issued a control order demanding UR to remediate the water pollution. UR appealed the control order issued by MoE, while the local government (LG) insisted that the MoE enforce its initial control order without modification of the original control order (Hipel et al. 1993, Mehta and Oullet 1995). Eventually, MoE modified its control order, and UR accepted it.

4.1 Background

The events that followed the discovery of the water contamination at Elmira are modelled using the Graph Model Conflict Resolution. This water conflict has been modelled previously by Hipel et al. (1993) using GMCR. The same conflict is modified by removing LG from the model. This seems plausible because the LG did not have much say in the conflict other than insisting to MoE to keep its original control order. In addition, the deal between MoE and UR came as a surprise to LG.

4.2 Modelling of Elmira Groundwater Contamination Conflict

This conflict is modelled at the point where UR appealed the control order. The MoE has two options: modify the control order to appease UR or not. UR, on the other hand, could delay responding to the control order, accept it, or abandon the plant. A summary of the DMs and their options is given in table 4. The feasible states are also shown in table 4. The letter "Y" in the table indicates "Yes" and "N" indicates "No" for each corresponding option. Figure 3 represents the integrated graph model of the conflict, where available moves for each DM are shown.

Table 4 Elmira conflict: decision makers, options and states

DMs and	States				
Options	1	2	3	4	5
MoE					
1.Modify	N	Y	N	Y	-
UR					
2.Delay	Y	Y	N	N	N
3. Accept	N	N	Y	Y	N
4.Abandon	N	N	N	N	Y

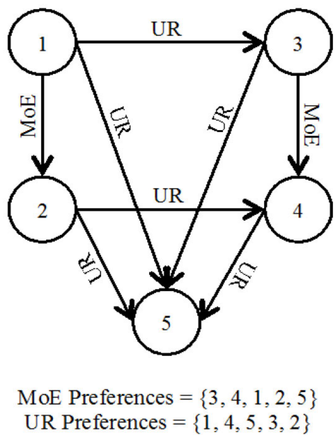

Figure 3 Graph Model for the Elmira water conflict

4.3 Stability Analysis

Stability analysis is performed in Table 5, where (r) indicates a rational state (Nash), and (u) indicates an unstable state. For MoE, all states are Nash stable, because MoE has no UIs from any state. UR finds all states to be stable except states 3 and 2. UDs are shown but not used in regular stability analysis; however, they will be used in calculating LoF. Overall, the analysis of the model results in three equilibria: states 1, 4, and 5. Analysis of equilibria robustness is presented in the following section.

Table 5 Elmira conflict: decision makers, moves, and states

DMs and Moves	States				
Equilibrium States	X	E	E	X	E
MoE					
Preferences	3	4	1	2	5
Stability	r	r	r	r	r
UIs					
UDs	4		2		
UR					
Preferences	1	4	5	3	2
Stability	r	r	r	u	u
UIs				5	4
					5
UDs	3	5			
	5				

4.4 Analysis of Equilibria of Elmira Water Conflict

For the three equilibrium states $\{s_1, s_4, s_5\}$, LoF is calculated for MoE and UR at each state as shown in Table 6. On the right column of Table 6, states are ranked based on robustness for each DM respectively. Then, in Table 7 the overall equilibria robustness is represented. It shows that state s_5 is the most robust state. Also, it meets the requirement for the special case of absolutely robust equilibrium. This means that if the conflict reaches state s_5, it is not possible to destabilize the conflict, even in the case of preference changes, because there are no moves available for any DM. State s_5 represents the situation where UR abandons the plant. According to the model this is an irreversible move for UR, and MoE has no moves from s_5.

The least robust state is s_1, which represents the situation where MoE refuses to modify its control order, and UR delays. This state clearly cannot be a permanent resolution.

Finally, state s_4 represents the situation that took place, where MoE modifies the control order, and UR accepts it, which is less robust than state s_5, because if UR changes its preference, it can abandon the project. This scenario is possible, for example, if market conditions become unfavourable for UR causing it to lose profits. In such a case, UR may change its preference and prefer state s_5 over s_4, which will disturb the stability conditions at state s_4.

In addition, see Figure 4, which shows available moves from each equilibrium state. Looking at the graph, it can be inferred that the results of LoF are consistent with possible escapes as shown in the graph. For state 1, there seems to be many possible escapes compared to states 4 and 5. It can be deduced from the graph that state 5 is the most robust equilibrium because there is no way any DM can move away from this equilibrium. State 1, on the other hand, has the highest possibility of escape among equilibria states.

Table 6 Levels of freedom and individual stability ranking

DMs	Level of Freedom			Robustness Ranking of Individual Stability
	s_1	s_4	s_5	
MoE	1	0	0	$\{s_4, s_5\} > \{s_1\}$
UR	3	1	0	$\{s_5\} > \{s_4\} > \{s_1\}$

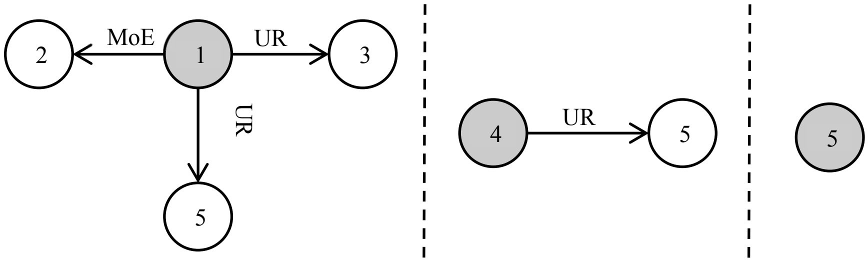

Figure 4 Possible moves from equilibria states

Table 7 Overall ranking of equilibria robustness for the Elmira conflict

DMs	Ranking of Equilibria		
	s_1	s_4	s_5
MoE	2	1	1
UR	3	2	1
Overall Ranking	5	3	2
Adjusted Ranking	3	2	1
State Ranking	$\{s_5\} > \{s_4\} > \{s_1\}$		

5 Conclusions

In this paper, a formal analysis of robustness of equilibria is presented to provide insights on resiliency of equilibria to change in preference. The result can be insightful as seen in the case study of the Elmira groundwater contamination conflict. This approach provides an interesting view on the possibility of deviation from equilibrium. The essence of the new approach is the concept of LoF. GMCR defines stability based on moves and preferences, and a DM is confined to available moves, but his or her preferences can change. Perception of goals over time can cause preferences to change. Also, external factors such as increased demand or natural phenomena may provide opportunities to disturb equilibria. Ranking of equilibrium robustness is helpful to the analyst who is interested in ascertaining which equilibria are more sustainable than others. This could be especially useful for third party interveners attempting to resolve a long-term conflict. This research is the first to introduce a systematic approach to robustness analysis within the graph model for conflict resolution.

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