

A MULTI-ATTRIBUTE LARGE GROUP EMERGENCY DECISION MAKING METHOD BASED ON GROUP PREFERENCE CONSISTENCY OF GENERALIZED INTERVAL-VALUED TRAPEZOIDAL FUZZY NUMBERS

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Abstract

In this paper, a new decision making approach is proposed for the multi-attribute large group emergency decision-making problem that attribute weights are unknown and expert preference information is expressed by generalized interval-valued trapezoidal fuzzy numbers (GITFNs). Firstly, a degree of similarity formula between GITFNs is presented. Secondly, expert preference information on different alternatives is clustered into several aggregations via the fuzzy clustering method. As the clustering proceeds, an index of group preference consistency is introduced to ensure the clustering effect, and then the group preference information on different alternatives is obtained. Thirdly, the TOPSIS method is used to rank the alternatives. Finally, an example is taken to show the feasibility and effectiveness of this approach. These method can ensure the consistency degree of group preference, thus decision efficiency of emergency response activities can be improved.

Key words: Generalized interval-valued trapezoidal fuzzy numbers, large group decision making, group preference consistency, emergency response

1. Introduction

Recent years witnessed the frequent occurrence of unconventional emergencies all around the world, for instance, the 2003 SARS epidemic, the 2004 Indonesia tsunami, the 2008 Wenchuan earthquake, etc. In case of unconventional events, emergency experts are often required to map a reasonable plan in a short time (Fu et al. 2012, Liu et al. 2012). Owing to the

fuzziness and complexity of emergencies, decision experts find it a hard task to make accurate judgments on the problem under time pressure, and instead most decision makers tend to express their preferences in fuzzy numbers such as interval numbers (INs), triangular fuzzy numbers (TFNs), intuitionistic fuzzy numbers (IFNs), etc. (Xu and Cai 2013, Kuo and Liang 2012, Chai et al. 2013, Yang and Li 2012).

Ever since the concept of fuzzy set was first proposed by Zadeh in 1965, the fuzzy set theory has been drawing increasing attention from scholars. As a common form of fuzzy sets, interval-valued fuzzy set (IVFS) has found extensive application in various decision makings, scoring a series of achievements in the field of IVFS. Gorzlczany (1987) and Turksen (1996) introduced the concept of interval-valued fuzzy set (IVFS). Afterwards, some operational rules of interval-valued fuzzy numbers (IFNs) were developed, such as interval-valued fuzzy weighted average arithmetic (IVFWAA) operator, interval-valued fuzzy weighted geometric average (IVFWGA) operator, interval-valued fuzzy quadratic-mean (IVFQM) operator, interval-valued fuzzy induced ordered weighted average (IVFIOWA) operator, etc. (Xu 2004, Chen 2011, Chen and Chen 2003). Zhou et al. (2005) improved the traditional TOPSIS method so as to apply it in the decision environment of interval-valued numbers (IFs). Zhang et al. (1999) studied the possibility degree formula of IFs, and proposed an algorithm to rank IFs based on the possibility degree matrix.

Along with the increasing complexity of decision environment, some scholars have been constantly drilling down, putting forward multitudes of novel expression forms of IFs. Wang and Li (1998) brought up the theory of generalized interval-valued trapezoidal fuzzy numbers (GITFNs). Then, several similarity measures between GITFNs are presented (Liu and Wang 2011, Ye 2012, Farhadinia 2014). Chen and Chen (2009) developed a new approach to rank GITFNs, which considers the defuzzified values, making the ranking result more accurate. The emergence of GITFNs provides decision

makers a new form to express their opinions, but few current studies are focused on GITFNs, thereby calling for further extension and breakthrough in this area.

Unconventional emergency is characterized by suddenness, far-reaching influence and severe destruction. Compared with conventional decision makings, emergency decision making involves more relevant departments and experts. Hence emergency decision making possesses the characteristics of large group involvement (a large group decision making is generally defined as one involving 11 or more experts) (Song and Yang 2000). During the process of decision making, due to the differences of decision makers in forms of social status, attitude, knowledge background, etc., preference conflicts among decision makers is inevitable (Xu 2012). Generally speaking, the more decision makers are involved, the higher degree of preference conflict has. Consequently, how to ensure the group preference consensus is of great importance to improve the efficiency of large group decision making. In the existing literatures, how to deal with group preference mainly falls into two types: one is conflict resolution method, i.e., to reach group preference consensus by adjusting decision makers' preferences (Xu et al. 2013, Xu et al. 2014, Xu 2009); the other is weighting method, namely, to achieve a suitable group preference through setting a reasonable weight for each decision maker (Xu 2007, Chen and Liu 2010). Nevertheless, most of the methods are only fit for the conventional group decision making problems, and are rarely applied to large group decision making problems. Faced with this situation, a new approach is presented in this paper, which is to handle the multi-attribute large

group emergency decision problems featuring preference information expressed by GITFNs.

The rest of this paper is organized as follows. In section 2, some basic concepts of GITFNs are reviewed. In section 3, related principles concerning the method are illustrated, mainly covering the clustering method, attribute weighting method and the corresponding concrete decision processes. In section 4, a numerical example is taken to illustrate this method. In the end, a conclusion is drawn about the proposed method.

2. Preliminaries

Definition 1 (Chen 1985) The generalized trapezoidal fuzzy numbers (GTFN) is defined as $a = (a_1, a_2, a_3, a_4; w_a^-)$ (as shown in Figure 1), the membership function $\mu_a(x) : R \rightarrow [0, 1]$ is defined as:

$$\mu_a(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1} w_a^-, & a_1 \leq x \leq a_2, \\ w_a^-, & a_2 \leq x \leq a_3, \\ \frac{x - a_3}{a_4 - a_3} w_a^-, & a_3 \leq x \leq a_4, \\ 0, & a_4 < x. \end{cases}$$

where

$$a_1 \leq a_2 \leq a_3 \leq a_4, \quad w_a^- \in [0, 1].$$

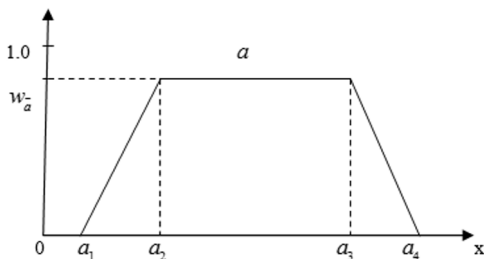


Figure 1 Generalized trapezoidal fuzzy number a

If $-1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$, a is the

normalized generalized trapezoidal fuzzy number (NGTFN). Especially, if $w_a^- = 1$, then a is called trapezoidal fuzzy number (TFN).

Definition 2 (Wang and Li 1998) $\tilde{a} = [\tilde{a}^L, \tilde{a}^U]$ $= [(a_1^L, a_2^L, a_3^L, a_4^L; w_a^-), (a_1^U, a_2^U, a_3^U, a_4^U; w_a^U)]$ is the generalized interval-valued trapezoidal fuzzy number (GITFN) (as shown in Figure 2), where $0 \leq a_1^L \leq a_2^L \leq a_3^L \leq a_4^L \leq 1$, $0 \leq a_1^U \leq a_2^U \leq a_3^U \leq a_4^U \leq 1$, $0 \leq w_a^- \leq w_a^+ \leq 1$, $w_a^- \subset w_a^+$.

From Figure 2, it can be found that GITFN \tilde{a} is made up of the lower values of GITFN \tilde{a}^L and the upper values of GITFN \tilde{a}^U .

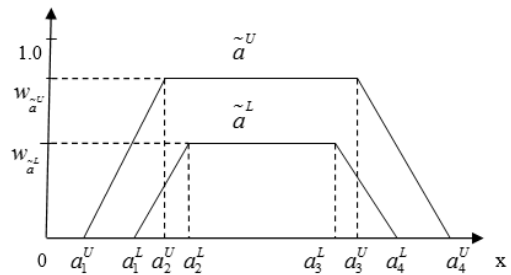


Figure 2 Generalized interval-valued trapezoidal fuzzy number \tilde{a}

Let $\tilde{a} = [\tilde{a}^L, \tilde{a}^U] = [(a_1^L, a_2^L, a_3^L, a_4^L; w_a^-), (a_1^U, a_2^U, a_3^U, a_4^U; w_a^+)]$ and $\tilde{b} = [\tilde{b}^L, \tilde{b}^U] = [(b_1^L, b_2^L, b_3^L, b_4^L; w_b^-), (b_1^U, b_2^U, b_3^U, b_4^U; w_b^+)]$ be two GITFNs, where $0 \leq a_1^L \leq a_2^L \leq a_3^L \leq a_4^L \leq 1$, $0 \leq a_1^U \leq a_2^U \leq a_3^U \leq a_4^U \leq 1$, $0 \leq w_a^- \leq w_a^+ \leq 1$, $w_a^- \subset w_a^+$; $0 \leq b_1^L \leq b_2^L \leq b_3^L \leq b_4^L \leq 1$, $0 \leq b_1^U \leq b_2^U \leq b_3^U \leq b_4^U \leq 1$, $0 \leq w_b^- \leq w_b^+ \leq 1$, $w_b^- \subset w_b^+$.

The operational rules are shown as follows (Wei and Chen 2009):

$$\begin{aligned} \tilde{a} + \tilde{b} &= [(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min(w_{\tilde{a}-L}, w_{\tilde{b}-L})), \\ &\quad (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min(w_{\tilde{a}-U}, w_{\tilde{b}-U}))], \\ \tilde{a} - \tilde{b} &= [(a_1^L - b_1^L, a_2^L - b_2^L, a_3^L - b_3^L, a_4^L - b_4^L; \min(w_{\tilde{a}-L}, w_{\tilde{b}-L})), \\ &\quad (a_1^U - b_1^U, a_2^U - b_2^U, a_3^U - b_3^U, a_4^U - b_4^U; \min(w_{\tilde{a}-U}, w_{\tilde{b}-U}))], \\ \tilde{a} \otimes \tilde{b} &= [(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min(w_{\tilde{a}-L}, w_{\tilde{b}-L})), \\ &\quad (a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min(w_{\tilde{a}-U}, w_{\tilde{b}-U}))], \\ \tilde{a} / \tilde{b} &= [(a_1^L / b_1^L, a_2^L / b_2^L, a_3^L / b_3^L, a_4^L / b_4^L; \min(w_{\tilde{a}-L}, w_{\tilde{b}-L})), \\ &\quad (a_1^U / b_1^U, a_2^U / b_2^U, a_3^U / b_3^U, a_4^U / b_4^U; \min(w_{\tilde{a}-U}, w_{\tilde{b}-U}))], \\ \lambda \tilde{a} &= [(\lambda a_1^L, \lambda a_2^L, \lambda a_3^L, \lambda a_4^L; w_{\tilde{a}-L}), (\lambda a_1^U, \lambda a_2^U, \lambda a_3^U, \lambda a_4^U; w_{\tilde{a}-U})]. \end{aligned}$$

Based on the definition of Euclidean distance and the features of fuzzy numbers (Liu and Wang 2011, Zhang et al. 2012), the similarity between GIFFNs is defined as follows:

Definition 3 Let $\tilde{a} = [\tilde{a}^L, \tilde{a}^U]$ $= [(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{a}-L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{a}-U})]$ and $\tilde{b} = [\tilde{b}^L, \tilde{b}^U] = [(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{b}-L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{b}-U})]$ be two GIFFNs, the similarity between \tilde{a} and \tilde{b} is defined as:

$$\begin{aligned} \rho(\tilde{a}, \tilde{b}) &= 1 - \frac{1}{2} \left\{ \left[\frac{(w_{\tilde{a}-L} a_1^L - w_{\tilde{b}-L} b_1^L)^2 + (w_{\tilde{a}-L} a_2^L - w_{\tilde{b}-L} b_2^L)^2}{4} + \right. \right. \\ &\quad \left. \left. \frac{(w_{\tilde{a}-L} a_3^L - w_{\tilde{b}-L} b_3^L)^2 + (w_{\tilde{a}-L} a_4^L - w_{\tilde{b}-L} b_4^L)^2}{4} \right]^{\frac{1}{2}} + \right. \\ &\quad \left. \left[\frac{(w_{\tilde{a}-U} a_1^U - w_{\tilde{b}-U} b_1^U)^2 + (w_{\tilde{a}-U} a_2^U - w_{\tilde{b}-U} b_2^U)^2}{4} + \right. \right. \\ &\quad \left. \left. \frac{(w_{\tilde{a}-U} a_3^U - w_{\tilde{b}-U} b_3^U)^2 + (w_{\tilde{a}-U} a_4^U - w_{\tilde{b}-U} b_4^U)^2}{4} \right]^{\frac{1}{2}} \right\}. \end{aligned} \tag{1}$$

Theorem 1 The similarity degree

$\rho(\tilde{a}, \tilde{b})$ derived from the Eq.(1) should satisfy the following properties:

$$\begin{aligned} (\mathbf{P}_1) \quad &0 \leq \rho(\tilde{a}, \tilde{b}) \leq 1; \quad (\mathbf{P}_2) \quad \rho(\tilde{a}, \tilde{b}) = 1, \\ &\text{if } \tilde{a} = \tilde{b}; \quad (\mathbf{P}_3) \quad \rho(\tilde{a}, \tilde{b}) = \rho(\tilde{b}, \tilde{a}). \end{aligned}$$

It is obvious that $(\mathbf{P}_1) \sim (\mathbf{P}_3)$ is true, so the proving process will not be presented in this section.

3. Principle of the Decision Method

3.1 Problem Description

For an emergency decision problem, let $E = \{e_1, e_2, \dots, e_p\}$ be the set of alternatives, $X = \{x_1, x_2, \dots, x_M\}$ be the set of decision makers, $G = \{g_1, g_2, \dots, g_N\}$ be the set of attributes. Decision makers are required to express their preference information in the form of GIFFNs. However as decision makers find it hard to give attribute values by GIFFNs directly, they usually choose to give evaluation values in linguistic form first, for example, ‘‘Good’’, ‘‘Very Good’’, ‘‘Bad’’, etc. Then these linguistic values are converted into GIFFNs. The conversion relationship between linguistic variable and GIFFNs is shown in Table 1 below (Wei and Chen 2009). For alternative l , decision maker i gives his/her preference over attribute j independently, which is defined as $\tilde{a}_{ij}^l, j = 1, 2, \dots, N$. The GIFFNs vector of $\tilde{A}_i^l = (\tilde{a}_{i1}^l, \tilde{a}_{i2}^l, \dots, \tilde{a}_{iN}^l)$ is defined as the preference vector of decision maker i over alternative l .

Table 1 Conversion relationship between linguistic variables and GIFFNs

Linguistic variables	GIFFNs	Linguistic variables	GIFFNs
Extremely Good (EG)	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	Bad (B)	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]
Very Very Good (VVG)	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	Very Bad (VB)	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]
Very Good (VG)	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	Very Very Bad (VVB)	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]
Good (G)	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	Extremely Bad (EB)	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]
Fair (F)	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]	-	-

3.2 Clustering of Expert Preferences

3.2.1 Closure Fuzzy Clustering Algorithm of GIFFNs

The preference vectors of M decision makers provide over alternative l form a set, which is defined as Ω_l . Closure fuzzy clustering algorithm (Xu et al. 2008) is adopted for clustering the expert preferences in Ω_l . The specific steps are as follows:

Step 1 The number of preference vectors in Ω_l is M , and the Eq.(2) is taken as the association degree formula to establish the association matrix over Ω_l , which is expressed as $R_l = (z_{i_1 i_2}^l)_{M \times M}$. $z_{i_1 i_2}^l$ is the preference association coefficient over alternative l between decision makers i_1 and i_2 , as shown in Eq.(2):

$$z_{i_1 i_2}^l = \frac{1}{N} \sum_{j=1}^N \rho(\tilde{a}_{i_1 j}^l, \tilde{a}_{i_2 j}^l), \quad (2)$$

where

$i_1, i_2 = 1, 2, \dots, M$. The similarity measure of $\rho(\tilde{a}_{i_1 j}^l, \tilde{a}_{i_2 j}^l)$ is shown in the Eq.(1).

Step 2 Let $R_l^2 = R_l \circ R_l = (z_{i_1 i_2}^{-l})_{M \times M}$ and $z_{i_1 i_2}^{-l} = \max_k \{\min\{z_{i_1 k}^l, z_{k i_2}^l\}\}$. R_l^2 is called the composition matrix of R_l . After k times of compositions, it is as below:

$$R_l \rightarrow R_l^2 \rightarrow R_l^4 \dots \rightarrow R_l^{2^k} \rightarrow \dots$$

A positive integer k can be obtained, such that $R_l^{2^k} = R_l^{2^{(k+1)}}$, in which $R_l^{2^k}$ is the equivalent correlation matrix of R_l , defined as

$$t(R_l) = (z_{i_1 i_2}^{-l*})_{M \times M}.$$

Step 3 According to the value of $z_{i_1 i_2}^{-l*}$ in $t(R_l)$, the λ -cutting matrix of $t(R_l)$ is constructed, which is defined as $R_l^{\lambda} = (\lambda_l z_{i_1 i_2}^{-l*})_{M \times M}$, where

$$\lambda_l z_{i_1 i_2}^{-l*} = \begin{cases} 0, & z_{i_1 i_2}^{-l*} < \lambda_l, \\ 1, & z_{i_1 i_2}^{-l*} \geq \lambda_l. \end{cases}$$

λ_l is the confidence level of R_l , $\lambda_l \in [0, 1]$.

Step 4 If the corresponding elements in both the i_{1th} line (column) and the i_{2th} line

(column) of $R_l^{\lambda_l}$ are equal, the preference vectors $\tilde{A}_{i_1}^l$ and $\tilde{A}_{i_2}^l$ can be placed into the same aggregation. According to the λ -cutting matrix $R_l^{\lambda_l}$, all the preference vectors in Ω_l can be divided into several aggregations.

3.2.2 Determination of the Confidence Level

In order to ensure the consistency degree of group preference, group preference consistency index is introduced to determine the value of λ_l .

After clustering operation, preference vectors in Ω_l are divided into K_l aggregations. C_k^l is the k th aggregation in Ω_l ($k=1,2,\dots,K_l$), and the number of expert preference vectors in C_k^l is defined as

$$r_k^l, \sum_{k=1, k \in \Omega_l}^{K_l} r_k^l = M.$$

Assume that every decision maker is of equal importance, the aggregation preference consistency index of C_k^l is defined as follows:

Definition 4 Aggregation preference consistency index of C_k^l is:

$$h_k^l = \frac{2}{r_k^l(r_k^l - 1)} \sum_{\substack{i_1, i_2=1, i_1 > i_2 \\ i_1, i_2 \in C_k^l}}^{r_k^l} \left[\frac{1}{N} \sum_{j=1}^N \rho(\tilde{a}_{i_1, j}^l, \tilde{a}_{i_2, j}^l) \right], \quad (3)$$

where $\rho(\tilde{a}_{i_1, j}^l, \tilde{a}_{i_2, j}^l)$ is defined as the preference consistency of alternative l over attribute j between decision maker i_1 and i_2 , $i_1, i_2 = 1, 2, \dots, r_k^l$.

The interval of h_k^l is $[0,1]$, the larger the value of h_k^l is, the higher degree of aggregation preference consistency C_k^l has. Only when the number of decision makers in C_k^l is more than one, the clustering result of C_k^l is considered meaningful, if there is only one decision maker in C_k^l , we define that $h_k^l = 0$.

Definition 5 Group preference consistency index of Ω_l is defined as

$$h_l = \sum_{k=1}^{K_l} \frac{r_k^l}{M} \cdot h_k^l. \quad (4)$$

Similarly, the interval of h_l is $[0,1]$, the bigger the value of h_l is, the higher degree of group preference consistency Ω_l has.

The concrete operations in determining the value of λ_l are as follows:

The feasible region of λ_l is known as $\lambda_l \in [0,1]$. According to the value of $\bar{z}_{i_1 i_2}^{l*}$ in $t(R_l)$, the feasible region of λ_l is divided into several interval values first, and each interval value corresponds to a clustering result. Then, all the possible clustering situations are identified, and the value of h_l in each situation are calculated. Finally, the interval value of λ_l is taken, from which the maximum value of h_l is obtained as the ideal confidence level of R_l .

3.2.3 Determination of Alternative Group Preference

Based on the ideal clustering result of Ω_l , the aggregation preference of C_k^l is:

$$\begin{aligned} \tilde{Y}_k^l &= (\tilde{y}_{k1}^l, \tilde{y}_{k2}^l, \dots, \tilde{y}_{kN}^l) \\ &= \left(\frac{1}{r_k^l} \sum_{i=1; i \in C_k^l}^{r_k^l} \tilde{a}_{i1}^l, \frac{1}{r_k^l} \sum_{i=1; i \in C_k^l}^{r_k^l} \tilde{a}_{i2}^l, \dots, \frac{1}{r_k^l} \sum_{i=1; i \in C_k^l}^{r_k^l} \tilde{a}_{iN}^l \right). \end{aligned} \quad (5)$$

The alternative group preference is obtained by the Eq.(6):

$$\begin{aligned} \tilde{Y}_l &= (\tilde{y}_{l1}, \tilde{y}_{l2}, \dots, \tilde{y}_{lN}) \\ &= \left(\sum_{k=1, k \in \Omega_l}^{K_l} \frac{r_k^l}{M} \tilde{y}_{k1}^l, \sum_{k=1, k \in \Omega_l}^{K_l} \frac{r_k^l}{M} \tilde{y}_{k2}^l, \dots, \sum_{k=1, k \in \Omega_l}^{K_l} \frac{r_k^l}{M} \tilde{y}_{kN}^l \right). \end{aligned} \quad (6)$$

3.3 Calculation Method for Attribute Weights

We proposed an attribute weighting method based on the theory of differentiation driven (Guo 2002). This theory requires to take the maximum differentiation degree between alternatives as the goal of assigning weights to all attributes so as to make alternatives ranking more easily.

Definition 6 The similarity of alternative group preference over attribute j is defined as:

$$\psi_j = \frac{2}{P(P-1)} \sum_{l_1, l_2=1, l_1 > l_2}^P \rho(\tilde{y}_{l_1j}, \tilde{y}_{l_2j}), \quad (7)$$

where $\rho(\tilde{y}_{l_1j}, \tilde{y}_{l_2j})$ is the group preference similarity between alternative l_1 and l_2 over attribute j , the similarity measurement of $\rho(\tilde{y}_{l_1j}, \tilde{y}_{l_2j})$ is shown in the Eq.(1), $l_1, l_2 = 1, 2, \dots, P$.

The interval of ψ_j is $[0,1]$, the larger the value of ψ_j is, the higher degree of alternative group preference similarity over attribute j is.

Let attribute weights be defined as $\omega = (\omega_1, \omega_2, \dots, \omega_N)^T$. In order to improve the group preference differentiation between alternatives, an optimization model is constructed to minimize the weighted sum of squared over all attributes as follows:

$$\begin{aligned} \min H(\omega_j) &= \sum_{j=1}^N (\omega_j \psi_j)^2 & (M-1) \\ \text{s.t.} & \begin{cases} \omega_j > 0, \\ \sum_{j=1}^N \omega_j = 1. \end{cases} \end{aligned}$$

The Lagrangian function is constructed as follows:

$$L(\omega_j, \lambda) = \sum_{j=1}^N (\omega_j \psi_j)^2 + 2\lambda (\sum_{j=1}^N \omega_j - 1).$$

Let $\nabla L(\omega_j, \lambda) = 0$, it is obtained:

$$\frac{\partial L}{\partial \omega_j} = \omega_j \psi_j^2 + \lambda = 0,$$

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^N \omega_j - 1 = 0.$$

By solving the equations above, the result of ω_j is:

$$\omega_j = \frac{1}{\psi_j^2} / \sum_{j=1}^N \frac{1}{\psi_j^2}, \quad j = 1, 2, \dots, N. \quad (8)$$

In conclusion, specific steps of this approach are as follows:

Step 1 Expert preference information in the form of GIFFNs is presented.

Step 2 By using the closure fuzzy clustering algorithm, the clustering results of different alternatives are obtained, and the group preference for each alternative are achieved by the Eq.(5~6).

Step 3 Attribute weights are calculated by the Eq.(7~8).

Step 4 The TOPSIS method is used for ranking alternatives, the Eq.(9) is used to compute the score of each alternative, and then all alternatives are ranked.

$$S(\tilde{Y}_l) = \frac{\sum_{j=1}^N \omega_j \rho(\tilde{y}_{lj}, \tilde{y}_j^+)}{\sum_{j=1}^N \omega_j \rho(\tilde{y}_{lj}, \tilde{y}_j^+) + \sum_{j=1}^N \omega_j \rho(\tilde{y}_{lj}, \tilde{y}_j^-)}. \quad (9)$$

The similarity measurement of $\rho(\tilde{y}_{lj}, \tilde{y}_j^+)$ is shown in the Eq.(1). \tilde{y}_j^+ and \tilde{y}_j^- is the upper and lower limit for group preference over attribute j respectively. According to the

Definition 2 and Table 1, they are set as follows:

$$\tilde{y}_j^+ = [(1.00, 1.00, 1.00, 1.00; \max(w_{y_j}^-)), (1.00, 1.00, 1.00, 1.00; \max(w_{y_j}^+))],$$

$$\tilde{y}_j^- = [(0.00, 0.00, 0.00, 0.00; \min(w_{y_j}^-)), (0.00, 0.00, 0.00, 0.00; \min(w_{y_j}^+))].$$

4. Example Analysis

In early January 2014, a forest fire broke out along the Jing-Zhu expressway in south part of Hunan Province. In order to control the raging fire, local emergency management agency worked out 3 alternatives against fire, as shown below:

Closing the highway entirely and putting out the fire with large machine (e_1);

Closing the highway partly and putting out the fire with small machine (e_2);

Closing the highway partly and putting out the fire manually (e_3).

14 decision makers were organized to independently evaluate the attributes implementation effect of these alternatives, each alternatives need to be evaluated from 3 attributes: Emergency effects (g_1), Emergency cost (g_2), Emergency response speed (g_3). The decision making preference information based on GIFFNs are shown in Table 2 below (expert preference information based on linguistic has been elided for brevity).

Table 2 Decision making preference information in GIFFNs

E	No.	g_1	g_2	g_3
e_1	\tilde{A}_1	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]
	\tilde{A}_2	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_3	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]
	\tilde{A}_4	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_5	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_6	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]
	\tilde{A}_7	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_8	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]
	\tilde{A}_9	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]

	\tilde{A}_{10}^1	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_{11}^1	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]
	\tilde{A}_{12}^1	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]
	\tilde{A}_{13}^1	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]
	\tilde{A}_{14}^1	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]
	\tilde{A}_1^2	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_2^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_3^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_4^2	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_5^2	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_6^2	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
e_2	\tilde{A}_7^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_8^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]
	\tilde{A}_9^2	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_{10}^2	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.17,0.22,0.36,0.42; 0.8), (0.17,0.22,0.36,0.42; 1.0)]
	\tilde{A}_{11}^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_{12}^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.32,0.41,0.58,0.65; 0.8), (0.32,0.41,0.58,0.65; 1.0)]
	\tilde{A}_{13}^2	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_{14}^2	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]

e_3	\tilde{A}_1^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]
	\tilde{A}_2^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]
	\tilde{A}_3^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]
	\tilde{A}_4^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]
	\tilde{A}_5^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]
	\tilde{A}_6^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_7^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_8^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]
	\tilde{A}_9^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(0.58,0.63,0.8,0.86; 0.8), (0.58,0.63,0.8,0.86; 1.0)]
	\tilde{A}_{10}^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_{11}^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.00,0.00,0.02,0.07; 0.8), (0.00,0.00,0.02,0.07; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]
	\tilde{A}_{12}^3	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]	[(0.72,0.78,0.92,0.97; 0.8), (0.72,0.78,0.92,0.97; 1.0)]
	\tilde{A}_{13}^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.04,0.10,0.18,0.23; 0.8), (0.04,0.10,0.18,0.23; 1.0)]	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]
	\tilde{A}_{14}^3	[(0.93,0.98,1.00,1.00; 0.8), (0.93,0.98,1.00,1.00; 1.0)]	[(0.00,0.00,0.00,0.00; 0.8), (0.00,0.00,0.00,0.00; 1.0)]	[(1.00,1.00,1.00,1.00; 0.8), (1.00,1.00,1.00,1.00; 1.0)]

The preference vectors that M decision makers provided for each alternative forms 3 sets, which are defined as $\Omega_1, \Omega_2, \Omega_3$ in sequence. Take Ω_1 as an example, the operational approach for determining the ideal

confidence level of λ_1 is demonstrated as follows:

- (1) The equivalent correlation matrix $t(R_j)$ is constructed, and shown as Eq.(10).

$$t(R_1) = \begin{bmatrix} 1.0000 & 0.8546 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 0.8724 & 0.8724 & 0.8546 & 0.8926 & 0.7489 & 0.7942 & 0.7489 \\ 0.8546 & 1.0000 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 0.8546 & 0.8546 & 0.8691 & 0.8546 & 0.7489 & 0.7942 & 0.7489 \\ 0.7942 & 0.7942 & 1.0000 & 0.7942 & 0.8080 & 0.7942 & 0.8080 & 0.7942 & 0.7942 & 0.7942 & 0.7942 & 0.7489 & 0.8924 & 0.7489 \\ 0.8531 & 0.8531 & 0.7942 & 1.0000 & 0.7942 & 0.8372 & 0.7942 & 0.8531 & 0.8531 & 0.8531 & 0.8531 & 0.7489 & 0.7942 & 0.7489 \\ 0.7942 & 0.7942 & 0.8080 & 0.7942 & 1.0000 & 0.7942 & 0.9084 & 0.7942 & 0.7942 & 0.7942 & 0.7942 & 0.7489 & 0.8080 & 0.7489 \\ 0.8372 & 0.8372 & 0.7942 & 0.8372 & 0.7942 & 1.0000 & 0.7942 & 0.8372 & 0.8372 & 0.8372 & 0.8372 & 0.7489 & 0.7942 & 0.7489 \\ 0.7942 & 0.7942 & 0.8080 & 0.7942 & 0.9084 & 0.7942 & 1.0000 & 0.7942 & 0.7942 & 0.7942 & 0.7942 & 0.7489 & 0.8080 & 0.7489 \\ 0.8724 & 0.8546 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 1.0000 & 0.9007 & 0.8546 & 0.8724 & 0.7489 & 0.7942 & 0.7489 \\ 0.8724 & 0.8546 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 0.9007 & 1.0000 & 0.8546 & 0.8724 & 0.7489 & 0.7942 & 0.7489 \\ 0.8546 & 0.8691 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 0.8546 & 0.8546 & 1.0000 & 0.8546 & 0.7489 & 0.7942 & 0.7489 \\ 0.8926 & 0.8546 & 0.7942 & 0.8531 & 0.7942 & 0.8372 & 0.7942 & 0.8724 & 0.8724 & 0.8546 & 1.0000 & 0.7489 & 0.7942 & 0.7489 \\ 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 1.0000 & 0.7489 & 0.9624 \\ 0.7942 & 0.7942 & 0.8924 & 0.7942 & 0.8080 & 0.7942 & 0.8080 & 0.7942 & 0.7942 & 0.7942 & 0.7942 & 0.7489 & 1.0000 & 0.7489 \\ 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.7489 & 0.9624 & 0.7489 & 1.0000 \end{bmatrix}$$

(10)

(2) The feasible region of λ_1 is $[0,1]$, it depends on the element value of $z_{i_1 i_2}^{-1*}$ in $t(R_1)$, the region is divided into 14 portions as follows: $(0.9624, 1.0000]$; $(0.9084, 0.9624]$; $(0.9007, 0.9084]$; $(0.8926, 0.9007]$; $(0.8924, 0.8926]$; $(0.8724, 0.8924]$; $(0.8691, 0.8724]$; $(0.8546, 0.8691]$; $(0.8531, 0.8546]$; $(0.8372, 0.8531]$; $(0.8080, 0.8372]$; $(0.7942, 0.8080]$; $(0.7489, 0.7942]$; $(0.0000, 0.7489]$.

If $\lambda_1 \in (0.9624, 1.0000]$, the λ -cutting matrix of $R_1^{\lambda_1} = (\lambda_1 z_{i_1 i_2}^{-1*})_{M \times M}$ is

$$R_1^{\lambda_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

According to the clustering theorem proposed in Section 3.2, expert preference

vectors in Ω_1 are grouped into 14 aggregations:

$$\{\tilde{A}_1^1\} \{\tilde{A}_2^1\} \{\tilde{A}_3^1\} \{\tilde{A}_4^1\} \{\tilde{A}_5^1\} \{\tilde{A}_6^1\} \{\tilde{A}_7^1\} \\ \{\tilde{A}_8^1\} \{\tilde{A}_9^1\} \{\tilde{A}_{10}^1\} \{\tilde{A}_{11}^1\} \{\tilde{A}_{12}^1\} \{\tilde{A}_{13}^1\} \{\tilde{A}_{14}^1\}$$

If $\lambda_1 \in (0.9084, 0.9624]$, the λ -cutting

matrix of $R_1^{\lambda_1} = (\lambda_1 z_{i_1 i_2}^{-1*})_{M \times M}$ is

$$R_1^{\lambda_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

According to the clustering theorem proposed in Section 3.2, expert preference vectors in Ω_1 are grouped into 13 aggregations:

$$\{\tilde{A}_1^1\} \{\tilde{A}_2^1\} \{\tilde{A}_3^1\} \{\tilde{A}_4^1\} \{\tilde{A}_5^1\} \{\tilde{A}_6^1\} \{\tilde{A}_7^1\} \\ \{\tilde{A}_8^1\} \{\tilde{A}_9^1\} \{\tilde{A}_{10}^1\} \{\tilde{A}_{11}^1\} \{\tilde{A}_{12}^1, \tilde{A}_{14}^1\} \{\tilde{A}_{13}^1\}$$

Likewise, 14 possible clustering results

over Ω_1 in total can be obtained. By the preference consistency h_1 are obtained, as Eq.(3~4), all the possible clustering results shown in Table 3. over Ω_1 and the corresponding group

Table3 Group preference consistency for possible clustering results over Ω_1

λ_1	Ω_1	h_1
(0.9624,1.0000]	$\{\tilde{A}_1\} \{\tilde{A}_2\} \{\tilde{A}_3\} \{\tilde{A}_4\} \{\tilde{A}_5\} \{\tilde{A}_6\} \{\tilde{A}_7\}$ $\{\tilde{A}_8\} \{\tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{11}\} \{\tilde{A}_{12}\} \{\tilde{A}_{13}\} \{\tilde{A}_{14}\}$	0.0000
(0.9084,0.9624]	$\{\tilde{A}_1\} \{\tilde{A}_2\} \{\tilde{A}_3\} \{\tilde{A}_4\} \{\tilde{A}_5\} \{\tilde{A}_6\} \{\tilde{A}_7\}$ $\{\tilde{A}_8\} \{\tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{11}\} \{\tilde{A}_{12}, \tilde{A}_{14}\} \{\tilde{A}_{13}\}$	0.1375
(0.9007,0.9084]	$\{\tilde{A}_1\} \{\tilde{A}_2\} \{\tilde{A}_3\} \{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\}$ $\{\tilde{A}_8\} \{\tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{11}\} \{\tilde{A}_{12}, \tilde{A}_{14}\} \{\tilde{A}_{13}\}$	0.2673
(0.8926,0.9007]	$\{\tilde{A}_1\} \{\tilde{A}_2\} \{\tilde{A}_3\} \{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\}$ $\{\tilde{A}_8, \tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{11}\} \{\tilde{A}_{12}, \tilde{A}_{14}\} \{\tilde{A}_{13}\}$	0.3959
(0.8924,0.8926]	$\{\tilde{A}_1, \tilde{A}_{11}\} \{\tilde{A}_2\} \{\tilde{A}_3\} \{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\}$ $\{\tilde{A}_6\} \{\tilde{A}_8, \tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{12}, \tilde{A}_{14}\} \{\tilde{A}_{13}\}$	0.5234
(0.8724,0.8924]	$\{\tilde{A}_1, \tilde{A}_{11}\} \{\tilde{A}_2\} \{\tilde{A}_3, \tilde{A}_{13}\} \{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\}$ $\{\tilde{A}_6\} \{\tilde{A}_8, \tilde{A}_9\} \{\tilde{A}_{10}\} \{\tilde{A}_{12}, \tilde{A}_{14}\}$	0.6509
(0.8691,0.8724]	$\{\tilde{A}_1, \tilde{A}_8, \tilde{A}_9, \tilde{A}_{11}\} \{\tilde{A}_2\} \{\tilde{A}_3, \tilde{A}_{13}\}$ $\{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\} \{\tilde{A}_{10}\} \{\tilde{A}_{12}, \tilde{A}_{14}\}$	0.6324
(0.8546,0.8691]	$\{\tilde{A}_1, \tilde{A}_8, \tilde{A}_9, \tilde{A}_{11}\} \{\tilde{A}_2, \tilde{A}_{10}\} \{\tilde{A}_3, \tilde{A}_{13}\}$ $\{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\} \{\tilde{A}_{12}, \tilde{A}_{14}\}$	0.7566
(0.8531,0.8546]	$\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_8, \tilde{A}_9, \tilde{A}_{10}, \tilde{A}_{11}\} \{\tilde{A}_3, \tilde{A}_{13}\}$ $\{\tilde{A}_4\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\} \{\tilde{A}_{12}, \tilde{A}_{14}\}$	0.7199
(0.8372,0.8531]	$\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_4, \tilde{A}_8, \tilde{A}_9, \tilde{A}_{10}, \tilde{A}_{11}\}$ $\{\tilde{A}_3, \tilde{A}_{13}\} \{\tilde{A}_5, \tilde{A}_7\} \{\tilde{A}_6\} \{\tilde{A}_{12}, \tilde{A}_{14}\}$	0.7742

(0.8080, 0.8372]	$\{\tilde{A}_1^1, \tilde{A}_2^1, \tilde{A}_4^1, \tilde{A}_6^1, \tilde{A}_8^1, \tilde{A}_9^1, \tilde{A}_{10}^1, \tilde{A}_{11}^1\}$ $\{\tilde{A}_3^1, \tilde{A}_{13}^1\} \{\tilde{A}_5^1, \tilde{A}_7^1\} \{\tilde{A}_{12}^1, \tilde{A}_{14}^1\}$	0.8137
(0.7942, 0.8080]	$\{\tilde{A}_1^1, \tilde{A}_2^1, \tilde{A}_4^1, \tilde{A}_6^1, \tilde{A}_8^1, \tilde{A}_9^1, \tilde{A}_{10}^1, \tilde{A}_{11}^1\}$ $\{\tilde{A}_3^1, \tilde{A}_5^1, \tilde{A}_7^1, \tilde{A}_{13}^1\} \{\tilde{A}_{12}^1, \tilde{A}_{14}^1\}$	0.7883
(0.7489, 0.7942]	$\{\tilde{A}_1^1, \tilde{A}_2^1, \tilde{A}_4^1, \tilde{A}_6^1, \tilde{A}_8^1, \tilde{A}_9^1, \tilde{A}_{10}^1,$ $\tilde{A}_{11}^1, \tilde{A}_3^1, \tilde{A}_5^1, \tilde{A}_7^1, \tilde{A}_{13}^1\} \{\tilde{A}_{12}^1, \tilde{A}_{14}^1\}$	0.7155
(0.0000, 0.7489]	$\{\tilde{A}_1^1, \tilde{A}_2^1, \tilde{A}_3^1, \tilde{A}_4^1, \tilde{A}_5^1, \tilde{A}_6^1, \tilde{A}_7^1,$ $\tilde{A}_8^1, \tilde{A}_9^1, \tilde{A}_{10}^1, \tilde{A}_{11}^1, \tilde{A}_{12}^1, \tilde{A}_{13}^1, \tilde{A}_{14}^1\}$	0.6370

(3) According to Table 3, it can be concluded that the maximum value of h_i on the condition of $\lambda_1 \in (0.8080, 0.8372]$ can be obtained, which corresponds to the ideal clustering result over Ω_1 . So $(0.8080, 0.8372]$

is selected as the optimal interval value of λ_1 .

Similarly, the optimal confidence level λ_l for other sets can also be achieved. The ideal clustering results for all sets are shown in Table 4 below:

Table 4 Clustering results for all sets

Ω_l	λ_l	C_k^l	\tilde{A}_i^l	\tilde{Y}_k^l
Ω_1	(0.8080, 0.8372]	C_1^1	$\tilde{A}_1^1, \tilde{A}_2^1, \tilde{A}_4^1, \tilde{A}_6^1,$ $\tilde{A}_8^1, \tilde{A}_9^1, \tilde{A}_{10}^1, \tilde{A}_{11}^1$	$((0.7200, 0.7700, 0.9000, 0.9462; 0.8), (0.7200, 0.7700, 0.9000, 0.9462; 1.0))$ $[(0.3512, 0.3925, 0.5025, 0.5563; 0.8), (0.3512, 0.3925, 0.5025, 0.5563; 1.0)]$ $[(0.3750, 0.4313, 0.5675, 0.6262; 0.8), (0.3750, 0.4313, 0.5675, 0.6262; 1.0)]$
		C_2^1	$\tilde{A}_3^1, \tilde{A}_{13}^1$	$((0.0000, 0.0000, 0.0200, 0.0700; 0.8), (0.0000, 0.0000, 0.0200, 0.0700; 1.0))$ $[(0.5200, 0.5950, 0.7500, 0.8100; 0.8), (0.5200, 0.5950, 0.7500, 0.8100; 1.0)]$ $[(0.3200, 0.4100, 0.5800, 0.6500; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1.0)]$
		C_3^1	$\tilde{A}_5^1, \tilde{A}_7^1$	$((0.1700, 0.2200, 0.3600, 0.4200; 0.8), (0.1700, 0.2200, 0.3600, 0.4200; 1.0))$ $[(0.7900, 0.8150, 0.9000, 0.9300; 0.8), (0.7900, 0.8150, 0.9000, 0.9300; 1.0)]$ $[(0.5800, 0.6300, 0.8000, 0.8600; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1.0)]$
		C_4^1	$\tilde{A}_{12}^1, \tilde{A}_{14}^1$	$((0.0000, 0.0000, 0.0200, 0.0700; 0.8), (0.0000, 0.0000, 0.0200, 0.0700; 1.0))$ $[(0.0000, 0.0000, 0.0200, 0.0700; 0.8), (0.0000, 0.0000, 0.0200, 0.0700; 1.0)]$ $[(0.0200, 0.0500, 0.1000, 0.1500; 0.8), (0.0200, 0.0500, 0.1000, 0.1500; 1.0)]$
Ω_2	(0.9315, 0.9451]	C_1^2	$\tilde{A}_1^2, \tilde{A}_3^2, \tilde{A}_4^2, \tilde{A}_6^2, \tilde{A}_7^2,$ $\tilde{A}_9^2, \tilde{A}_{10}^2, \tilde{A}_{11}^2, \tilde{A}_{13}^2, \tilde{A}_{14}^2$	$((0.7620, 0.8010, 0.9080, 0.9460; 0.8), (0.7620, 0.8010, 0.9080, 0.9460; 1.0))$ $[(0.8320, 0.8710, 0.9480, 0.9740; 0.8), (0.8320, 0.8710, 0.9480, 0.9740; 1.0)]$ $[(0.6370, 0.6940, 0.8400, 0.8930; 0.8), (0.6370, 0.6940, 0.8400, 0.8930; 1.0)]$
		C_2^2	$\tilde{A}_2^2, \tilde{A}_5^2$	$((0.8600, 0.8900, 0.9600, 0.9850; 0.8), (0.8600, 0.8900, 0.9600, 0.9850; 1.0))$ $[(0.1700, 0.2200, 0.3600, 0.4200; 0.8), (0.1700, 0.2200, 0.3600, 0.4200; 1.0)]$ $[(0.7200, 0.7800, 0.9200, 0.9700; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1.0)]$
		C_3^2	$\tilde{A}_8^2, \tilde{A}_{12}^2$	$((0.7200, 0.7800, 0.9200, 0.9700; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1.0))$ $[(0.7200, 0.7800, 0.9200, 0.9700; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1.0)]$ $[(0.3200, 0.4100, 0.5800, 0.6500; 0.8), (0.3200, 0.4100, 0.5800, 0.6500; 1.0)]$

Ω_3	0.9515, 0.9891]	C_1^3	$\tilde{A}_1^3, \tilde{A}_4^3, \tilde{A}_5^3,$ $\tilde{A}_8^3, \tilde{A}_{11}^3, \tilde{A}_{14}^3$	$((0.9417, 0.9833, 1.0000, 1.0000; 0.8), (0.9417, 0.9833, 1.0000, 1.0000; 1.0))$ $[(0.0000, 0.0000, 0.000, 0.0000; 0.8), (0.0000, 0.0000, 0.000, 0.0000; 1.0)]$ $[(1.0000, 1.0000, 1.0000, 1.0000; 0.8), (1.0000, 1.0000, 1.0000, 1.0000; 1.0)]$
		C_2^3	$\tilde{A}_2^3, \tilde{A}_3^3, \tilde{A}_{13}^3$	$((0.9300, 0.9800, 1.0000, 1.0000; 0.8), (0.9300, 0.9800, 1.0000, 1.0000; 1.0))$ $[(0.0400, 0.1000, 0.1800, 0.2300; 0.8), (0.0400, 0.1000, 0.1800, 0.2300; 1.0)]$ $[(0.9300, 0.9800, 1.0000, 1.0000; 0.8), (0.9300, 0.9800, 1.0000, 1.0000; 1.0)]$
		C_3^3	$\tilde{A}_6^3, \tilde{A}_{10}^3, \tilde{A}_{12}^3$	$((1.0000, 1.0000, 1.0000, 1.0000; 0.8), (1.0000, 1.0000, 1.0000, 1.0000; 1.0))$ $[(0.0000, 0.0000, 0.0000, 0.0000; 0.8), (0.0000, 0.0000, 0.0000, 0.0000; 1.0)]$ $[(0.7200, 0.7800, 0.9200, 0.9700; 0.8), (0.7200, 0.7800, 0.9200, 0.9700; 1.0)]$
		C_4^3	$\tilde{A}_7^3, \tilde{A}_9^3$	$((1.0000, 1.0000, 1.0000, 1.0000; 0.8), (1.0000, 1.0000, 1.0000, 1.0000; 1.0))$ $[(0.0000, 0.0000, 0.0200, 0.0700; 0.8), (0.0000, 0.0000, 0.0200, 0.0700; 1.0)]$ $[(0.5800, 0.6300, 0.8000, 0.8600; 0.8), (0.5800, 0.6300, 0.8000, 0.8600; 1.0)]$

(4) Group preference vectors for all alternatives are obtained by the Eq.(5~6):

$$\tilde{Y}_1 = \left([(0.4357, 0.4714, 0.5714, 0.6207; 0.8), (0.4357, 0.474, 0.5714, 0.6207; 1.0)] [(0.3878, 0.4257, 0.5257, 0.5765; 0.8), (0.3878, 0.4257, 0.5257, 0.5765; 1.0)] [(0.3457, 0.4022, 0.5357, 0.5950; 0.8), (0.3457, 0.4022, 0.5357, 0.5950; 1.0)] \right)$$

$$\tilde{Y}_2 = \left([(0.7700, 0.8107, 0.9171, 0.9550; 0.8), (0.7700, 0.8107, 0.9171, 0.9550; 1.0)] [(0.7214, 0.7650, 0.8600, 0.8943; 0.8), (0.7214, 0.7650, 0.8600, 0.8943; 1.0)] [(0.6036, 0.6657, 0.8143, 0.8693; 0.8), (0.6036, 0.6657, 0.8143, 0.8693; 1.0)] \right)$$

$$\tilde{Y}_3 = \left([(0.9600, 0.9886, 1.0000, 1.0000; 0.8), (0.9600, 0.9886, 1.0000, 1.0000; 1.0)] [(0.0086, 0.0214, 0.0500, 0.0893; 0.8), (0.0086, 0.0214, 0.0500, 0.0893; 1.0)] [(0.8650, 0.8957, 0.9543, 0.9736; 0.8), (0.8650, 0.8957, 0.9543, 0.9736; 1.0)] \right)$$

(5) Based on the Eq.(7~8), attribute weights can be obtained: $\omega = (0.27, 0.47, 0.26)^T$. The Eq.(9) is used to compute the TOPSIS score of each

alternative: $S(\tilde{Y}_1) = 0.4909$, $S(\tilde{Y}_2) = 0.7494$, $S(\tilde{Y}_3) = 0.5199$, $S(\tilde{Y}_2) > S(\tilde{Y}_3) > S(\tilde{Y}_1)$. Thus e_2 is the preferred alternative, namely Closing the highway partly and putting out the fire with small machine.

After implementing this alternative, the fire was controlled effectively, the circumjacent people's life and highway safety were well protected. From the example above, it can be concluded that the method proposed in this paper is rational and feasible. Based on the clustering result of each alternative, the consistency degree of group preference h_l is computed by the Eq.(3~4): $h_1 = 0.8137$, $h_2 = 0.9031$, $h_3 = 0.9960$. As all the values of h_l are the maximal ones that can be obtained for each alternative, the clustering results over all alternatives are deemed as ideal. Based on the value of attribute obtained by Eq.(7~8), TOPSIS scores for each alternatives can be achieved: $S(\tilde{Y}_1) = 0.4909$, $S(\tilde{Y}_2) = 0.7494$, $S(\tilde{Y}_3) = 0.5199$. From the value of $S(\tilde{Y}_l)$, it is observed that the differentiation degree between two alternatives is obvious, so all alternatives can be sorted easily.

5. Conclusion

In this paper, a new group decision method is put forward for multi-attribute large group emergency decision-making problems characterized by unknown attribute weights as well as preference information expressed by GIFFNs. The highlights of this method are as follows: (1) Based on the characteristics of GIFFNs, the formula for similarity degree between GIFFNs is given, which is used for clustering expert preference information by means of fuzzy clustering method, and helps to achieve the purposes of scaling down preference information and reducing the difficulty of decision making. (2) An index of group preference consistency is introduced, by which the confidence level of each alternative is determined and the ideal clustering result for each alternative is confirmed. This enables the consistency degree of group preference reach to maximum. (3) Aiming at minimizing the weighted sum of squared alternative group preference consistency over all attributes, an optimization model is established to solve the attribute weights, which helps the alternative to achieve a high differentiation degree, and make alternatives ranking more easily.

In the future study, this approach can be further developed and extended, making it suitable for solving the emergency decision making problems featuring multi-department & multi-stage.

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