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Technical Note

GENERALIZED INTUITIONISTIC FUZZY HYBRID CHOQUET AVERAGING OPERATORS*

Fanyong MENG[1] **Qiang ZHANG**[2]

[1]*School of Management, Qingdao Technological University, Qingdao 266520, China*
mengfanyongtjie@163.com (✉)
[2]*Department of Management and Economics, Beijing Institute of Technology, Beijing 100081, China*
qiangzhang@bit.edu.cn

Abstract

In this study a new hybrid aggregation operator named as the generalized intuitionistic fuzzy hybrid Choquet averaging (GIFHCA) operator is defined. Meantime, some desirable properties are studied, and several important cases are examined. Furthermore, we define the generalized Shapley GIFHCA (GS-GIFHCA) operator, which does not only overall consider the importance of elements and their ordered positions, but also globally reflect the correlations among them and their ordered positions. In order to simplify the complexity of solving a fuzzy measure, we further define the generalized λ−Shapley GIFHCA (GλS-GIFHCA) operator.

Keywords: Multi-attribute group decision making, intuitionistic fuzzy set, Choquet integral, generalized Shapley function, fuzzy measure

1. Introduction

The ordered weighted averaging (OWA) operator (Yager 1988) is a very useful tool for aggregating a finite collection of arguments, whose fundamental aspect is a reordering step in which the input arguments are rearranged in descending order and the weighting vector is merely associated with its ordered position. In the past decades, it has been received more and more attentions. Many extending forms are developed, such as the generalized intuitionistic fuzzy OWA (GIFOWA) operator (Li 2011) and the induced generalized intuitionistic fuzzy OWA (IG-IFOWA) operator (Su et al. 2012). Based on the geometric mean, Xu & Yager (2006) developed the ordered weighted geometric (OWG) operator to aggregate intuitionistic fuzzy information in a similar way as the OWA operator, whilst Xu & Chen (2007) proposed some geometric aggregation operators on interval-valued intuitionistic fuzzy sets (IVIFSs).

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Besides the above mentioned aggregation operators, there is another kind of aggregation operators, which is called the hybrid weighted averaging (HWA) operator (Xu & Da 2003). For this kind of aggregation operators, it does not only consider the ordered position of the argument, but also give the importance of the argument. Since it was first proposed in 2003, some developed hybrid aggregation operators have been proposed, such as the generalized intuitionistic fuzzy hybrid aggregation (GIFHA) operator (Zhao et al. 2012). But all these aggregation operators are based on the assumption that the elements in a set are independent, i.e., they only consider the addition of the importance of individual elements. However, in many practical situations, the elements in a set are usually correlative (Grabisch 1995, 1996). If there exist inter-dependent or correlative characteristics among elements, it is unreasonable to aggregate the values for elements by using additive measures.

The purpose of this paper is to develop a new hybrid aggregation operator named as the generalized intuitionistic fuzzy hybrid Choquet averaging (GIFHCA) operator. It does not only consider the importance of elements and their ordered positions, but also reflect the correlations among them and their ordered positions. In order to overall reflect the interactions among elements, we further define the generalized Shapley GIFHCA (GS-GIFHCA) operator. Meantime, some properties are studied. In order to simplify the complexity of solving a fuzzy measure, we define the generalized λ-Shapley GIFHCA (GλS-GIFHCA) operator.

2. New Generalized Intuitionistic Fuzzy Hybrid Choquet Averaging Operators

Based on the GIFHA operator (Zhao et al. 2012) and the Choquet integral, we shall define the generalized intuitionistic fuzzy hybrid Choquet averaging (GIFHCA) operator, which does not only consider the importance of elements and their ordered positions, but also reflect the correlations among them and their ordered positions.

First, we introduce the well-known Shapley function (Shapley 1953) in game theory. When it is calculated with respect to the fuzzy measure μ on N (Sugeno 1974), we have

$$\varphi_i(\mu, N) = \sum_{S \subseteq N \backslash i} \frac{(n-s-1)!s!}{n!}(\mu(S \cup i) - \mu(S)), \quad \forall i \in N, \tag{1}$$

where s and n respectively denote the cardinalities of S and N. In fact, the Shapley function is an expect value of the overall interactions between the element i and each subset in $N \backslash i$.

Property 2.1 *Let* $\mu : P(N) \to [0,1]$ *be a fuzzy measure on the finite set N, then* $\varphi_i(\mu, N) \geq 0$ *for each element* $i \in N$ *.*

Property 2.2 *Let* $\mu : P(N) \to [0,1]$ *be a fuzzy measure on the finite set N, then*

$$\sum_{i=1}^{n} \varphi_i(\mu, N) = \mu(N) = 1 . \tag{2}$$

From Properties 2.1 and 2.2, we know $\{\varphi_i(\mu, N)\}_{i \in N}$ is a weighting vector. If μ is additive, namely, $\mu(S) = \sum_{i \in S} \mu(i)$ for any $S \subseteq N$. Then, $\{\varphi_i(\mu, N)\}_{i \in N}$ degenerates to be $\{\mu(i)\}_{i \in N}$, which is an additive weighting vector.

As an extension of fuzzy sets (FSs), IFSs proposed by Atanassov (1986) are characterized by a membership degree, a non-membership degree and a hesitancy degree. IFSs are more powerful in dealing with vagueness and uncertainty than FSs. Let X be a no empty finite set. An IFS A in X is expressed as $A=\{\langle x,u(x),v(x)\rangle \mid x\in X\}$, where $u(x)\in[0,1]$ and $v(x)\in[0,1]$ respectively denote the degrees of membership and non-membership of the element $x\in X$ with the condition $u(x)+v(x)\le 1$. For any $x\in X$, $\pi(x)=1-u(x)-v(x)$ denotes the hesitance degree of the element x to A, which is also called intuitionistic index. If $\pi(x)=0$ for all $x\in X$, then IFS A degenerates to be a fuzzy set.

In order to denote simply, any intuitionistic fuzzy value (IFV) $\tilde{\alpha}$ can be expressed by $\tilde{\alpha}=(a,b)$, where $a\in[0,1]$ and $b\in[0,1]$ respectively denote the degrees of membership and non-membership with the condition $a+b\le 1$. Let Ω be the set of all IFVs in X.

2.1 The GIFHCA Operator

Definition 2.1 *A GIFHCA operator of dimension n is a mapping GIFHCA:* $\Omega^n\to\Omega$, *which has an associated fuzzy measure* μ *defined on the set* $N=\{1,2,...,n\}$, *such that*

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}_{(j)}^{\ \gamma}\right)^{1/\gamma} \qquad (3)$$

where γ > 0, $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ $(\tilde{\beta}_i=n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$, $i=1,2,...,n)$, $\varphi_{\tilde{\alpha}_i}(\rho,B)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $B=\{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$ *(i=1,2,…,n), n is the balancing coefficient, and* $A_{(j)}$ = {(*j*),…,(*n*)} *with* $A_{(n+1)}=\varnothing$.

Remark 2.1 *If* μ *and* ρ *are both additive, then the GIFHCA operator degenerates to be the GIFHA operator (Zhao et al. 2012).*

Remark 2.2 *If each* $\tilde{\alpha}_i=(a_i,b_i)$ (*i*=1,2,…,*n*) *degenerates to be a fuzzy value, namely,* $\tilde{\alpha}_i=(a_i,1-a_i)$ (*i*=1,2,…,*n*), *then the GIFHCA operator reduces to be the generalized fuzzy hybrid Choquet averaging (GFHCA) operator*

$$\mathrm{GFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}_{(j)}^{\ \gamma}\right)^{1/\gamma}. \qquad (4)$$

Remark 2.3 *If* $\gamma\to 0^+$, *then the GIFHCA operator degenerates to be the intuitionistic fuzzy hybrid Choquet geometric mean (IFHCGM) operator*

$$\mathrm{IFHCGM}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\mathop{\otimes}_{j=1}^{n}\tilde{\beta}_{(j)}^{\ \mu(A_{(j)})-\mu(A_{(j+1)})}. \qquad (5)$$

Remark 2.4 *If* $\gamma=1$, *then the GIFHCA operator degenerates to be the intuitionistic fuzzy hybrid Choquet arithmetic averaging (IFHCAA) operator*

$$\mathrm{IFHCAA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}_{(j)}. \qquad (6)$$

Remark 2.5 *If* $\gamma=2$, *then the GIFHCA operator degenerates to be the intuitionistic fuzzy hybrid Choquet arithmetic quadratic averaging (IFHCAQA) operator*

$$\mathrm{IFHCAQA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}_{(j)}^{\ 2}\right)^{1/2}. \qquad (7)$$

Remark 2.6 *If* $\varphi_{\tilde{\alpha}_i}(\rho,B)=1/n$ (*i*=1,2,…,*n*), *then the GIFHCA operator degenerates to be the intuitionistic fuzzy Choquet OWA (IFCOWA) operator*

$$\text{IFCOWA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \left(\mathop{\oplus}_{j=1}^{n} \left(\mu(A_{(j)}) - \mu(A_{(j+1)}) \right) \tilde{\alpha}_{(j)}{}^{\gamma} \right)^{1/\gamma}, \quad (8)$$

where $\tilde{\alpha}_{(j)}$ *is the jth least of the IFV* $\tilde{\alpha}_i$ (i=1, 2,…,n).

Theorem 2.1 *Let* $\tilde{\alpha}_i = (a_i, b_i)$ (i=1,2,...,n) *be a collection of IFVs in* Ω, *and* μ *be the associated fuzzy measure on* $N = \{1,2,\ldots,n\}$, *then their aggregated value by using the GIFHCA operator is also an IFV, denoted by*

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \left(\left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - (1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma}, 1 - \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \right), \quad (9)$$

where γ>0, $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ ($\tilde{\beta}_i = n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$, $i = 1,2,...,n$), $\varphi_{\tilde{\alpha}_i}(\rho,B)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $B = \{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$ (i=1,2,…,n), n *is the balancing coefficient, and* $A(j)$ = {(j),…,(n)} *with* $A(n+1) = \varnothing$.

Proof. First, we show Equation (9) is an IFV. From $a_{(j)}, b_{(j)} \in [0,1]$ with $a_{(j)} + b_{(j)} \le 1$ and $\varphi_{\tilde{\alpha}_{(j)}}(\rho,B) \ge 0$ for all j=1,2,…,n, we get

$$\left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \in [0,1],$$

$$1 - \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \in [0,1].$$

Namely,

$$\left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - (1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \ge 0,$$

$$1 - \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \ge 0.$$

Since $1 - a_{(j)} \ge b_{(j)}$, $\varphi_{\tilde{\alpha}_{(j)}}(\rho,B) \ge 0$, $\mu(A_{(j)}) - \mu(A_{(j+1)}) \ge 0$ and γ >0 for all j=1,2,…,n, we get

$$\left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - (1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \le \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma}.$$

Thus,

$$1 - \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - b_{(j)}{}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} + \left(1 - \prod_{j=1}^{n} \left(1 - \left(1 - (1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)} \right)^{\gamma} \right)^{\mu(A_{(j)})-\mu(A_{(j+1)})} \right)^{1/\gamma} \le 1.$$

Namely, Equation (9) is an IFV.

Next, we shall show that Equation (9) holds. From the operational laws on IFSs (Xu & Yager 2006), we have

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \left(\mathop{\oplus}_{j=1}^{n} \left(\mu(A_{(j)}) - \mu(A_{(j+1)}) \right) \tilde{\beta}_{(j)}{}^{\gamma} \right)^{1/\gamma}$$
$$= \left(\mathop{\oplus}_{j=1}^{n} \left(\mu(A_{(j)}) - \mu(A_{(j+1)}) \right) \left(n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)\tilde{\alpha}_{(j)} \right)^{\gamma} \right)^{1/\gamma}$$

$$=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\left(1-(1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)},b_{(j)}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{1/\gamma}$$

$$=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\left(\left(1-(1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma},\ 1-\left(1-b_{(j)}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)\right)^{1/\gamma}$$

$$=\left(\mathop{\oplus}_{j=1}^{n}\left(1-\left(1-\left(1-(1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})},\ \left(1-\left(1-b_{(j)}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)\right)^{1/\gamma}$$

$$=\left(1-\prod_{j=1}^{n}\left(1-\left(1-(1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})},\ \prod_{j=1}^{n}\left(1-\left(1-b_{(j)}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}$$

$$=\left(\left(1-\prod_{j=1}^{n}\left(1-\left(1-(1-a_{(j)})^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\ 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-b_{(j)}^{n\varphi_{\tilde{\alpha}_{(j)}}(\rho,B)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right).$$

■

Next, we investigate some desirable properties of the GIFHCA operator.

Property 2.3 (*Commutativity*). *Let* $\tilde{\alpha}_i=(a_i,b_i)$ *(i=1,2,…,n) be a collection of IFVs in* Ω*, and* $\tilde{\alpha}'_i=(a'_i,b'_i)$ *(i=1, 2,…,n) be a permutation of* $\tilde{\alpha}_i$*. Then,*

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}'_1,\tilde{\alpha}'_2,...,\tilde{\alpha}'_n). \tag{10}$$

Proof. Let

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}_{(j)}^{\gamma}\right)^{1/\gamma}$$

and

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}'_1,\tilde{\alpha}'_2,...,\tilde{\alpha}'_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\mu(A_{(j)})-\mu(A_{(j+1)})\right)\tilde{\beta}'^{\gamma}_{(j)}\right)^{1/\gamma}.$$

Since $\tilde{\alpha}'_i=(a'_i,b'_i)$ (*i*=1,2,…,*n*) is a permutation of $\tilde{\alpha}_i$, we have $\tilde{\alpha}_{(j)}=\tilde{\alpha}'_{(j)}$ for all *j*=1, 2,…,*n*.

Thus,

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1',\tilde{\alpha}_2',...,\tilde{\alpha}_n').$$ ■

Property 2.4 *(S-weighted comonotonicity). Let* $\tilde{\alpha}_i=(a_i,b_i)$ *and* $\tilde{\beta}_i=(c_i,d_i)$ *(i=1,2,...,n) be two collections of IFVs. If* $a'_{(j)}\le c'_{(j)}$ *and* $b'_{(j)}\ge d'_{(j)}$ *for all j=1,2,...,n, then*

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)\le\text{GIFHCA}_{\mu,\varphi}(\tilde{\beta}_1,\tilde{\beta}_2,...,\tilde{\beta}_n),$$

where $\tilde{\alpha}'_{(j)}=(a'_{(j)},b'_{(j)})$ *and* $\tilde{\beta}'_{(j)}=(c'_{(j)},d'_{(j)})$ *respectively represent the jth least of the Shapley weighted IFVs* $\tilde{\alpha}'_i=n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$ *and* $\tilde{\beta}'_j=n\varphi_{\tilde{\beta}_i}(\rho,C)\tilde{\beta}_i$ $(i=1,2,...,n)$*,* $\varphi_{\tilde{\alpha}_i}(\rho,B)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $B=\{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$*, and* $\varphi_{\tilde{\beta}_j}(\rho,C)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $C=\{\tilde{\beta}_i\}_{i=1,...,n}$ *for* $\tilde{\beta}_i$*.*

Proof. From Theorem 2.1, we get

$$\text{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\left(1-\prod_{j=1}^{n}\left(1-\left(a'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\ 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-b'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$

and

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\beta}_1,\tilde{\beta}_2,...,\tilde{\beta}_n)$$
$$=\left(\left(1-\prod_{j=1}^{n}\left(1-\left(c'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$
$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right).$$

By $a'_{(j)} \le c'_{(j)}$, $b'_{(j)} \ge d'_{(j)}$, $\mu(A_{(j)})-\mu(A_{(j+1)}) \ge 0$ and $\gamma > 0$, we have

$$\left(1-\prod_{j=1}^{n}\left(1-\left(a'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}$$
$$\le\left(1-\prod_{j=1}^{n}\left(1-\left(c'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}, \quad (11)$$
$$1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-b'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}$$
$$\ge 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}. \quad (12)$$

From Equations (11), (12) and the relationship between IFVs (Xu & Yager 2006), we get

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) \le \mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\beta}_1,\tilde{\beta}_2,...,\tilde{\beta}_n). \quad ■$$

Property 2.5 *(S-weighted idempotency) Let* $\tilde{\alpha}_i=(a_i,b_i)$ *(i=1,2,…,n) be a collection of IFVs in* Ω. *If* $\tilde{\alpha}'_i=\tilde{\alpha}=(a,b)$ *for all i=1,2,…,n, then*

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\tilde{\alpha}, \quad (13)$$

where $\tilde{\alpha}'_i$ *is the Shapley weighted IFV* $\tilde{\alpha}'_i = n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i \quad (i=1,2,...,n)$.

Proof. From Theorem 2.1, we have

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)$$
$$=\left(\left(1-\prod_{j=1}^{n}\left(1-\left(a'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$
$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-b'_{(j)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$
$$=\left(\left(1-\prod_{j=1}^{n}\left(1-a^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$
$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-(1-b)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$
$$=\left(\left(1-\left(1-a^{\gamma}\right)^{\sum_{j=1}^{n}\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$
$$\left.1-\left(1-\left(1-(1-b)^{\gamma}\right)^{\sum_{j=1}^{n}\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$
$$=\left(\left(1-\left(1-a^{\gamma}\right)\right)^{1/\gamma},1-\left(1-\left(1-(1-b)^{\gamma}\right)\right)^{1/\gamma}\right)$$
$$=\left(\left(a^{\gamma}\right)^{1/\gamma},1-\left((1-b)^{\gamma}\right)^{1/\gamma}\right)$$
$$=(a,b)$$
$$=\tilde{\alpha}. \quad ■$$

Corollary 2.1 *Let* $\tilde{\alpha}_i=(a_i,b_i)$ *(i=1,2,…,n) be a collection of IFVs in* Ω. *If* $\tilde{\alpha}_i=\tilde{\alpha}=(a,b)$ *and* $\varphi_{\tilde{\alpha}_i}(\rho,B)=1/n$ *for all i=1,2,…,n, then*

$$\mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\tilde{\alpha}.$$

Property 2.6 *(S-weighted boundary) Let* $\tilde{\alpha}_i=(a_i,b_i)$ *(i=1,2,…,n) be a collection of IFVs in* Ω, *then*

$$\tilde{\beta}_{(1)} \le \mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) \le \tilde{\beta}_{(n)}, \quad (14)$$

where $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ ($\tilde{\beta}_i = n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$, $i=1,2,...,n$).

Proof. From Property 2.4, we have

$$\left(\left(1-\prod_{j=1}^{n}\left(1-c_{(1)}{}^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$

$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d_{(1)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$

$$\leq \mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)$$

$$\leq\left(\left(1-\prod_{j=1}^{n}\left(1-c_{(n)}{}^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$

$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d_{(n)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right),$$

where $\tilde{\beta}_{(1)}=(c_{(1)},d_{(1)})$ and $\tilde{\beta}_{(n)}=(c_{(n)},d_{(n)})$. By Property 2.5, we get

$$\left(\left(1-\prod_{j=1}^{n}\left(1-c_{(1)}{}^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$

$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d_{(1)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$

$$=\tilde{\beta}_{(1)},$$

$$\left(\left(1-\prod_{j=1}^{n}\left(1-c_{(n)}{}^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma},\right.$$

$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-d_{(n)}\right)^{\gamma}\right)^{\mu(A_{(j)})-\mu(A_{(j+1)})}\right)^{1/\gamma}\right)$$

$$=\tilde{\beta}_{(n)}.,$$

Thus, $\tilde{\beta}_{(1)}\leq \mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)\leq\tilde{\beta}_{(n)}$. ■

Corollary 2.2 *Let* $\tilde{\alpha}_i=\left(a_i,b_i\right)$ *(i=1,2,...,n) be a collection of IFVs in* Ω*, then*

$$(0,1)\leq \mathrm{GIFHCA}_{\mu,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)\leq n\tilde{\alpha}^{+}, \quad (15)$$

where $\tilde{\alpha}^{+}=\left(\max_i a_i,\min_i b_i\right)$.

Proof. Since $0\leq\varphi_{\tilde{\alpha}_i}(\rho,B)\leq 1$, we get $(0,1)\leq n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i\leq n\tilde{\alpha}^{+}$ for all i=1,2,…,n. From Properties 2.4 and 2.5, it is not difficult to get the result. ■

2.2 The Generalized Shapley GIFHCA Operator

Although the GIFHCA operator can reflect the interactions among elements, it only gives the relationship between the combinations $A_{(j)}$ and $A_{(j+1)}$ (j=1,2,…,n) w.r.t. the associated fuzzy measure μ. In order to overall reflect the interactions between combinations $A_{(i)}$ (i=1,2,…, n), we further define the generalized Shapley GIFHCA (GS-GIFHCA) operator. First, we introduce the generalized Shapley function proposed by Marichal (2000) in game theory as follows:

$$\Phi_S(\mu,N)=\sum_{T\subseteq N\setminus S}\frac{(n-s-t)!t!}{(n-s+1)!}(\mu(S\cup T)-\mu(T)),\quad \forall S\subseteq N, \quad (16)$$

where N={1,2,…,n}, s, t and n are the cardinalities of S, T and N, respectively.

When s=1, Equation (16) degenerates to be the Shapley function. By Equation (16), we know it is an expect value of the overall interactions between the coalition S and each subset of $N\setminus S$. From the definition of fuzzy measures (Sugeno 1974), it is not difficult to know that Φ given by Equation (16) is also a fuzzy measure.

Definition 2.2 *A GS-GIFHCA operator of dimension n is a mapping GS-GIFHCA:* $\Omega^n\rightarrow\Omega$*, which has the associated generalized Shapley function Φ w.r.t. the fuzzy measure* μ *on N* = {1,2,...,n}*, defined by*

$$\text{GS-GIFHCA}_{\Phi,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\left(\mathop{\oplus}_{j=1}^{n}\left(\Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)\right)\tilde{\beta}_{(j)}{}^{\gamma}\right)^{1/\gamma}, \quad (17)$$

where γ>0, $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ ($\tilde{\beta}_i = n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$, $i=1,2,...,n$), $\varphi_{\tilde{\alpha}_i}(\rho,B)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $B=\{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$ (*i*=1,2,…,*n*), *n is the balancing coefficient, and A*(*j*) = {(*j*),…,(*n*)} *with A*(*n*+1) = $\varnothing$.

Remark 2.7 *If* $\gamma \to 0^+$, *then the GS-GIFHCA operator degenerates to be the generalized Shapley intuitionistic fuzzy hybrid Choquet geometric mean (GS-IFHCGM) operator*

$$\text{GS-IFHCGM}_{\Phi,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \bigotimes_{j=1}^{n} \tilde{\beta}_{(j)}^{\ \Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)}. \quad (18)$$

Remark 2.8 *If* $\gamma = 1$, *then the GS-GIFHCA operator degenerates to be the generalized Shapley intuitionistic fuzzy hybrid Choquet arithmetic averaging (GS-IFHCAA) operator*

$$\text{GS-IFHCAA}_{\Phi,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \bigoplus_{j=1}^{n}\left(\Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)\right)\tilde{\beta}_{(j)}. \quad (19)$$

Remark 2.9 *If* $\varphi_{\tilde{\alpha}_i}(\rho,B)=1/n$ (*i*=1,2,…,*n*), *then the GS-GIFHCA operator degenerates to be the generalized Shapley intuitionistic fuzzy Choquet OWA (GS-IFCOWA) operator*

$$\text{GS-IFCOWA}_{\Phi,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n) = \left(\bigoplus_{j=1}^{n}\left(\Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)\right)\tilde{\alpha}_{(j)}^{\ \gamma}\right)^{1/\gamma}, \quad (20)$$

where $\tilde{\alpha}_{(j)}$ *is the jth least of the IFV* $\tilde{\alpha}_i$ (*i*=1, 2,…,*n*).

Theorem 2.2 *Let* $\tilde{\alpha}_i = (a_i, b_i)$ (*i*=1,2,…,*n*) *be a collection of IFVs in Ω, and Φ be the associated generalized Shapley index w.r.t. the fuzzy measure* μ *on N*={1,2,..,*n*}. *Then their aggregated value by using the GS-GIFHCA operator is also an IFV, expressed by*

$$\text{GS-GIFHCA}_{\Phi,\varphi}(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)$$
$$=\left(\left(1-\prod_{j=1}^{n}\left(1-\left(1-(1-a_{(j)})^{n\varphi_{(j)}(\rho,B)}\right)^{\gamma}\right)^{\Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)}\right)^{1/\gamma},\right.$$
$$\left.1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-b_{(j)}^{\ n\varphi_{(j)}(\rho,B)}\right)^{\gamma}\right)^{\Phi_{A_{(j)}}(\mu,N)-\Phi_{A_{(j+1)}}(\mu,N)}\right)^{1/\gamma}\right), \quad (21)$$

where γ>0, $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ ($\tilde{\beta}_i = n\varphi_{\tilde{\alpha}_i}(\rho,B)\tilde{\alpha}_i$, $i=1,2,...,n$), $\varphi_{\tilde{\alpha}_i}(\rho,B)$ *is the Shapley value w.r.t. the fuzzy measure* ρ *on* $B=\{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$ (*i*=1,2,…,*n*), *n is the balancing coefficient, and A*(*j*) = {(*j*),…,(*n*)} *with A*(*n*+1) = $\varnothing$.

Proof. Since the generalized Shapley index Φ is a fuzzy measure, by Theorem 2.1 it is not difficult to the conclusion. ■

All properties studied in section 2.1 still hold for the GS-GIFHCA operator, we here no longer discuss them in detail.

2.3 A Special Case of the GS-GIFHCA Operator

Since the fuzzy measure is defined on the power set, it makes the problem exponentially complex. Thus, it is not easy to get the fuzzy measure of each combination in a set when it is large. In order to reflect the interactions among elements and simplify the complexity of solving a fuzzy measure, the λ-fuzzy measure proposed by Sugeno (1974) seems to well deal with this issue, expressed by

$$g_\lambda(A\cup B) = g_\lambda(A)+g_\lambda(B)+\lambda g_\lambda(A)g_\lambda(B), \quad (22)$$

where $\lambda > -1$, and $A, B \subseteq N$ with $A \cap B = \varnothing$.

It is apparent that if $\lambda = 0$, then g_λ is an additive measure, which means there is no interaction between coalitions *A* and *B*. If $\lambda > 0$, then g_λ is a superadditive measure, which indicts there exists complementary interaction between coalitions *A* and *B*. If $-1 < \lambda < 0$, then g_λ is a subadditive measure, which shows there exists redundancy interaction between coalitions *A* and *B*.

For the finite set *N*, the λ-fuzzy measure g_λ can be equivalently expressed by

$$g_\lambda(A) = \begin{cases} \dfrac{1}{\lambda}\left(\prod_{i\in A}[1+\lambda g_\lambda(i)]-1\right) & \text{if } \lambda \neq 0 \\ \sum_{i\in A} g_\lambda(i) & \text{if } \lambda = 0 \end{cases}. \quad (23)$$

From $\mu(N) = 1$, we know λ is determined by

$$\prod_{i\in N}[1+\lambda g_\lambda(i)] = 1+\lambda. \quad (24)$$

So when each $g_\lambda(i)$ is given, we can get the value of λ. From Equation (24), for the set *N* with *n* elements we only need *n* values to get a λ-fuzzy measure, which much simplifies the complexity of solving a fuzzy measure. Furthermore, if $\sum_{i=1}^{n} g_\lambda(i) = 1$, then $\lambda = 0$.

Similar to Definition 2.2, we further define the generalized λ-Shapley GIFHCA (GλS-GIFHCA) operator as follows:

Definition 2.3 *A GλS-GIFHCA operator of dimension n is a mapping GλS-GIFHCA:* $\Omega^n \to \Omega$, *which has the associated generalized Shapley index Φ w.r.t. the λ-fuzzy measure* g_λ *on* $N = \{1, 2, ..., n\}$, *such that*

$$\text{G}\lambda\text{S-GIFHCA}_{\Phi,\varphi}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left(\mathop{\oplus}_{j=1}^{n}\left(\Phi_{A_{(j)}}(g_\lambda, N) - \Phi_{A_{(j+1)}}(g_\lambda, N)\right)\tilde{\beta}_{(j)}^{\gamma}\right)^{1/\gamma}, \quad (25)$$

where γ>0, $\tilde{\beta}_{(j)}$ *is the jth least of the Shapley weighted IFV* $\tilde{\beta}_i$ ($\tilde{\beta}_i = n\varphi_{\tilde{\alpha}_i}(g_{\lambda'}, B)\tilde{\alpha}_i$, $i = 1, 2, ..., n$), $\varphi_{\tilde{\alpha}_i}(g_{\lambda'}, B)$ *is the Shapley value w.r.t. the λ-fuzzy measure* $g_{\lambda'}$ *on* $B = \{\tilde{\alpha}_i\}_{i=1,...,n}$ *for* $\tilde{\alpha}_i$ (*i*=1, 2,…,*n*), *n is the balancing coefficient, and A*(*j*) = {(*j*),…,(*n*)} *with A*(*n*+1) = $\varnothing$.

As an important case of fuzzy measures, the aggregated value of $\tilde{\alpha}_i = (a_i, b_i)$ (*i*=1,2,…,*n*) by using the GλS-GIFHCA operator is also an IFV, and it satisfies all properties studied in section 2.1.

3. Conclusion

Based on the Choquet integral and the Shapley function, we develop a new intuitionistic fuzzy aggregation operator, which does not only consider the importance of the elements, but also reflect the interactive characteristics among them. Meantime, some desirable properties are studied, such as commutativity, S-weighted comonotonicity, S-weighted idempotency and S-weighted boundary. In order to overall reflect the interactions among elements, we define the generalized Shapley generalized intuitionistic fuzzy hybrid Choquet averaging (GS-GIFHCA) operator, which is also an IFV. Since the fuzzy measure is defined on the power set, it makes the problem exponentially complex. Thus, we further present the generalized λ-Shapley generalized intuitionistic fuzzy hybrid Choquet averaging (GλS-GIFHCA) operator by using the λ-fuzzy measure, which much simplifies the complexity of solving a fuzzy measure.

When there exist interactive characteristics among experts and attributes in some multi-attribute group decision making problems under intuitionistic fuzzy environment, the aggregation operators based on additive measures can not be used. However, we can apply the introduced Choquet intuitionistic fuzzy aggregation operator, which can well deal with the interactions among experts and attributes. Similar to other decision-making procedures, if the fuzzy measures on expert set and attribute set are known, we can use the GS-GIFHCA operator to develop a new method to intuitionistic fuzzy multi-attribute group decision making; if we only have the importance of each expert and each attribute, then we can apply the GλS-GIFHCA operator to develop a new method to intuitionistic fuzzy multi-attribute group decision making. It is worth pointing out that our method can be seen as an extension of some decision-making approaches, which are based on additive measures.

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References

[1] Atanassov, K.T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20 (1): 87-96

[2] Grabisch, M. (1995). Fuzzy integral in multicriteria decision making. Fuzzy Sets Syst., 69 (3): 279-298

[3] Grabisch, M. (1996). The application of fuzzy integrals in multicriteria decision making. Eur. J. Oper. Res., 89 (3): 445-456

[4] Li, D.F. (2011). The GOWA operator based approach to multiattribute decision making using intuitionistic fuzzy sets. Math. Comput. Model., 53 (5-6): 1182-1196

[5] Marichal, J.L. (2000). The influence of variables on pseudo-Boolean functions with applications to game theory and multi-criteria decision making. Discrete Appl. Math., 107 (1-3): 139-164

[6] Shapley, L.S. (1953). A Value for *N*-person Game. Princeton University Press, Princeton

[7] Su, Z.X., Xia, G.P., Chen, M.Y. & Wang, L. (2012). Induced generalized intuitionistic fuzzy OWA operator for multi-attribute group decision making. Expert Syst. Appl., 39 (2): 1902-1910

[8] Sugeno, M. (1974). Theory of fuzzy integral and its application. Doctorial Dissertation. Tokyo Institute of Technology

[9] Xu, Z.S. & Chen, J. (2007). On geometric aggregation over interval-valued intuitionistic fuzzy information. F. S. K. D., Haikou, 466-471, August 2007

[10] Xu, Z.S. & Da, Q.L. (2003). An overview of operators for aggregating information. Int. J. Intell. Syst., 18 (9): 953-969

[11] Xu, Z.S. & Yager, R.R. (2006). Some geometric aggregation operators based on intuitionistic fuzzy sets. Int. J. Gen. Syst., 35 (4): 417-433

[12] Yager, R.R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. IEEE Trans. Systems, Man, Cybernet., 18 (1): 183-190

[13] Zhao, H., Xu, Z.S., Ni, M.F. & Liu, S.H. (2010). Generalized aggregation operators for intuitionistic fuzzy sets. Int. J. Intell. Syst., 25 (1): 1-30

Fanyong Meng is a lecturer in School of

Management, Qingdao Technological University, Qiangdao, China. He received a Ph.D. degree in School of Management and Economics, Beijing Institute of Technology, Beijing, China in 2011. At present, he has contributed over 40 journal articles to professional journals such as Technological and Economic Development of Economy, Knowledge-Based Systems, Fuzzy Optimization and Decision Making, Applied Mathematics Letters, International Journal of Fuzzy Systems, Journal of Intelligent and Fuzzy Systems, International Journal of Computational Intelligence Systems, Journal of Systems Science and Complexity. He is also a paper reviewer of some professional journals such as Information Sciences, Knowledge-Based Systems, Fuzzy Optimization and Decision Making, Group Decision and Negotiation, Journal of Intelligent and Fuzzy Systems and Computers & Industrial Engineering. His current research interests include fuzzy mathematics, decision making and games theory.

Qiang Zhang is a professor in School of Management and Economics, Beijing Institute of Technology, Beijing, China. He received a PhD degree in School of Traffic and Transportation, Southwest Jiaotong University, Sichuan, China in 1999. At present, he has contributed over 200 journal articles to professional journals such as Fuzzy Sets and Systems, European Journal of Operational Research, International Journal of Approximate Reasoning, Expert Systems with Applications, Knowledge-Based Systems, Technological and Economic Development of Economy, International Journal of Computational Intelligence Systems, Fuzzy Optimization and Decision Making. He is also a paper reviewer of many professional journals such as Fuzzy Sets and Systems, International Journal of Approximate Reasoning, European Journal of Operational Research, Fuzzy Optimization and Decision Making, Information Sciences, and Knowledge-Based Systems, and he is an associate editor of Journal of Intelligent and Fuzzy Systems. His current research interests include management decisions in quantitative theory and method, the modern logistics and supply chain management, uncertain system theory and application.