J Syst Sci Syst Eng (Sep 2012) 21(3): 316-336 ISSN: 1004-3756 (Paper) 1861-9576 (Online)
DOI: 10.1007/s11518-012-5198-x CN11-2983/N

DOMINATING ATTITUDES IN THE GRAPH MODEL FOR CONFLICT RESOLUTION

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Abstract

A formal methodology for analyzing the importance of weighing a decision maker's attitudes in a conflict is introduced and applied to the problem of negotiating a fair transfer of a brownfield property. A decision maker's attitudes are expressed in his consideration of his own preferences, as well as those of his opponents. Dominating attitudes are used to suggest that in a circumstance in which a decision maker takes into account multiple perspectives due to his attitudes, he may favor one perspective more heavily. The analysis of a brownfield acquisition conflict illustrates the types of insights that this methodology reveals.

Keywords: Conflict analysis, attitudes, preferences, graph model for conflict resolution, dominating attitudes, brownfields

1. Dominating Attitudes

The weighing of priorities is a key part of decision making, be it in the case of a single decision maker (DM) choosing amongst multiple options, or the interaction of multiple DMs in a conflict situation. Even more important is the consequences one's attitudes and decisions may have on oneself and others in a given conflict. Dominating attitudes, a formal framework with game theoretic roots, are proposed to allow decision analysts to determine the strategic impacts of concentrating on each DM's key attitudes towards himself and others in the process of resolving the conflict. By focusing on the most important attitudes that a DM may have, both the theory and practice of investigating the role of attitudes in conflict resolution are greatly simplified using this new methodology. For example, complex attitudes may exist between environmentalists and industrialists involved in a serious dispute over the proper disposal of industrial wastes. However, the dominating attitude of the environmentalists is that the industrialists make a concerted effort to clean up their wastes, while the industrialists' overriding attitude is that the environmentalists do not significantly endanger their profitability.

Attitudes play an undoubtedly important part in most conflict negotiations. Due to this, their inclusion within the Graph Model for Conflict Resolution (GMCR) (Fang et al. 1993) is necessary for determining conflict outcomes that may not be readily apparent, for deriving new paths to win-win resolutions based on each DM's behavior and for providing clarity to complex negotiations. In the following sections, a new and simple method for improving the implementation of attitudes, referred to as "Dominating Attitudes", is introduced and defined. The reason for this further extension of the attitudes methodology developed by Inohara et al. (2007) is to accommodate the increase in restrictions that the consideration of multiple attitudes can place upon a DM. That is to say, when multiple attitudes are present in a conflict, a reduction in the possible movements by a DM occurs, due to the single DM's consideration of multiple DMs' preferences. Thus, it is reasonable to assume that a DM will allow certain important attitudes to dominate his decision making to permit at least some of his utilities or desires to be fulfilled. One can see that a DM in such a position may make a reasonable trade-off to satisfy particular attitudes and ignore others, if he is forced to do so.

In Section 2, GMCR and four solution concepts for describing human behavior under conflict are formally defined. In Section 3, the framework for attitudes within GMCR, as developed by Inohara et al. (2007), is outlined. Within Section 4, the concept of dominating attitudes is introduced and the corresponding solution concepts are developed. Next, in Section 5, a simple environmental conflict is discussed and analyzed using the analytical methods developed in the first three sections. Finally, in Section 6, conclusions and insights based on the theoretical developments and the case study are discussed.

2. The Graph Model for Conflict Resolution

An overview of game theory is presented in the next subsection. Subsequently, the Graph Model for Conflict Resolution (Fang et al. 1993) is defined along with definitions for calculating stability. Extensions to GMCR are summarized in Section 2.3, while in Section 2.4 the applicability of GMCR and general situations in which it can be used are explained.

2.1 Game Theory in Perspective

Formal methods for analyzing conflicts find much of their common origins in the work of Von Neumann & Morgenstern (1944), who laid down the framework for a general methodology known as game theory. Theoretical research within game theory by Nash (1950, 1951) led to the establishment of the state stability calculation called Nash stability, whereby a DM who cannot make an immediate improvement is likely to remain at a given state. Over the years, a plethora of methods based on game theory have been designed. An informative way to categorize these techniques is according to qualitative and quantitative methods as shown by the left and right branches, respectively, in Figure 1. Qualitative game theory methods can be calibrated using relative preferences in which a DM prefers one state over another, but does not have to say by how much, or is indifferent between them. For instance, when a person is asked whether she would like coffee or tea, she simply has to respond that she prefers to have tea or that she is indifferent between the two options. Qualitative models are thus based upon relative preferences in which a DM may prefer or equally prefer one state with respect to another. For quantitative methods, preferences are represented by real numbers such as dollars or utility values. These techniques often find applications in the fields of economics and science. In general, quantitative models are useful for the analysis of tactical data while qualitative models lend themselves to modeling real world strategic negotiations. As environmental decision making becomes more important, these types of models will find greater applications in engineering problem solving (Riley 2000). Both of these general classes of models have been applied to a wide range of conflict studies, such as environmental conflict analysis including fisheries management (Benchekouroun & Van Long 2002) and forestry (Benchekouroun & Gaudet 2003), as well as historical-political analysis (Inohara et al. 2007).

In spite of the lack of specific quantitative data, one must not be fooled: qualitative techniques are entirely mathematical. Metagame analysis, developed by Howard (1971), was an attempt to make conflict models more intuitive and realistic by using relative preferences in place of cardinal payoffs. To analyze and determine state stabilities and thus overall equilibrium states, Howard introduced the solution concepts of General Metarational (GMR) and Symmetric Metarational (SMR) stability. Both GMR and SMR stabilities assume that opposing DMs may make moves to harm their opponents without considering their own personal risk. These solution concepts, applied under conflict situations, allow decision analysts to contemplate the strategic implications of specific moves. The development of metagame

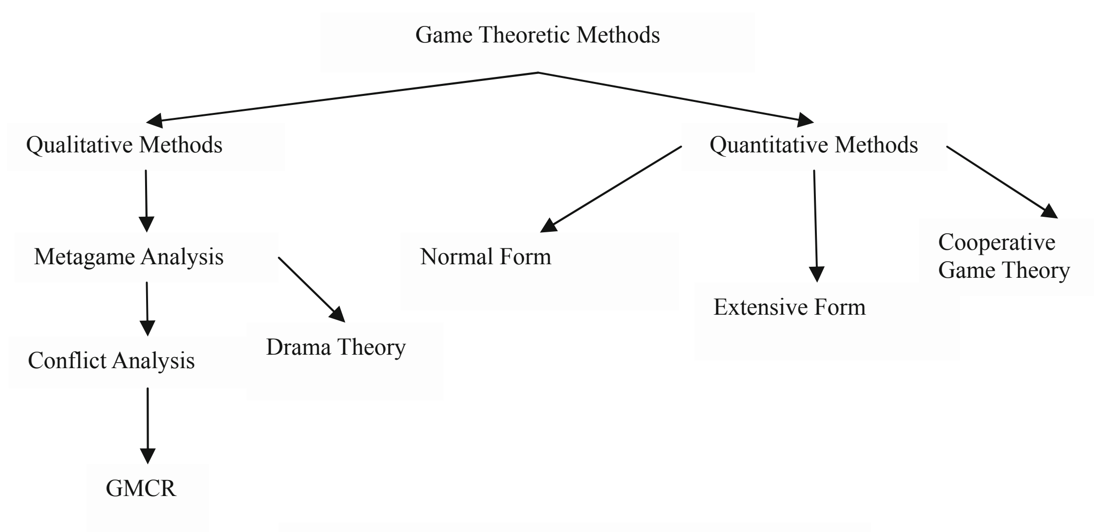

Figure 1 Genealogy of game theoretic methods

analysis has given rise to two non-quantitative branches of conflict analysis, of which Howard (1971, 1994a, 1994b) himself developed the first, drama theory. The second methodology developed from metagame analysis is conflict analysis. Fraser & Hipel (1984) introduced the tableau form for conveniently displaying aconflict model and calculating stability while also introducing two new solution concepts: simultaneous and sequential stability. From conflict analysis, Fang et al. (1993) developed the Graph Model for Conflict Resolution (GMCR) which combines elements of graph theory and game theory in a manner that allows decision analysts to create a visual representation of a given conflict. In two books edited by Hipel (2009), a range of formal conflict models are defined and compared.

2.2 Definitions for the Graph Model

In order to provide the detailed analyses that can be undertaken using GMCR, one must first understand its structure. GMCR is a framework which allows an analyst to lay out a conflict in a logical, mathematical way. In Definition 1, this analytical framework is described in terms of both graph and set theories.

Definition 1 Graph Model for Conflict Resolution (GMCR): The Graph Model of a conflict is defined as the set $(N, S, (A_i), (>_i, \sim_i))$
Where: N is the set of all DMs, such that $|N| \geq 2$ and S represents the set of states in the conflict, where $|S| \geq 2$.

For DM $i \in N$, (S, A_i) constitutes DM i's graph, for which S is the set of all vertices and A_i is the set of all arcs between vertices in the set of states S that can be utilized by DM i. It is assumed that (S, A_i) is a directed graph with no loops or multiple arcs. For example, for $s, t \in S$, and $(s, t) \in A_i$, DM i can unilaterally shift the state of the conflict in one step from state s to state t. The pair of binary operators $(>_i, \sim_i)$ act on S to represent the relative preferences for DM i between two states. For instance, for $s, t \in S$, $s >_i t$ means that DM i prefers state s to t, while $s \sim_i t$ indicates that DM i is indifferent between s and t. Relative preferences are assumed to satisfy the following properties:

1. $>_i$ is asymmetric; hence, for all $s, t \in S$, $s >_i t$ and $t >_i s$ cannot hold true simultaneously.
2. $\sim_i$ is reflexive; therefore, for any $s \in S$, $s \sim_i s$.
3. $\sim_i$ is symmetric; hence, for any $s, t \in$ S, if $s \sim_i t$ then $t \sim_i s$.
4. $(>_i, \sim_i)$ is strongly complete; therefore, for all $s, t \in S$, one of $s >_i t$, $t >_i s$ or $s \sim_i t$ is true.

The four properties that define the nature of relative preferences in GMCR, along with definitions for reachable lists and unilateral improvement lists, make it possible to examine the moves and countermoves of each DM within a conflict as well as the potentially enduring outcomes of the conflict. For DM $i \in N$ at state $s \in S$, DM i's reachable list from state s is defined as the set denoted by $R_i(s)$, and thus constitutes the set of vertices or states in DM i's directed graph G_i, which DM i can reach from s in just one step. A unilateral improvement t of DM i from state s is defined as a state that is in DM i's reachable list from s, $t \in R_i(s)$, and DM i strictly prefers state t to state s, such that $t >_i s$. Accordingly, the set of all unilateral improvements (UIs) of DM i from state s is the set $\{t \in R_i(s) \mid t >_i s \}$, called DM i's unilateral improvement list from state s, and is denoted by $R^+_i(s)$. Let $\varphi_i^{\simeq}(s)$ denote the set of all states that are less preferred to state s or are equally preferred with respect to state s by DM i, that is, $\{x \in S \mid s >_i x \text{ or } s \sim_i x\}$.

Using this framework, the potential moves and countermoves of DMs between various outcomes can be modeled in an effective manner. Within this framework, four of a range of useful solution concepts for describing human behavior under conflict are now defined.

Definition 2 Nash stability (Nash): DM $i \in N$ at state $s \in S$ is Nash stable, denoted by $s \in S_i^{\text{Nash}}$, if and only if $R^+_i(s) = \varphi$. Thus, a state is Nash stable and belongs to the set S_i^{Nash} if it has no UIs from a given state. Nash stability simply means that if a DM has no unilateral improvements from a given state, he or she will stay at that state (Nash 1950, 1951). Simply stated, if a DM cannot make any improvements from the current state of the conflict, he or she will take no action.

To define the next set of solution concepts, it is useful to know if a state or states are less or equally preferred to the initial state. Given DM $i \in$ N and states $s, x \in S$, the set of all states that are less preferred or equally preferred to state s by DM i is $\varphi_i^{\simeq}(s) = \{x \in S \mid s \succeq_i x\}$.

Definition 3 Sequential stability (SEQ): For $i \in N$, state s is Sequentially stable for DM i, denoted by $s \in S_i^{SEQ}$, if and only if for all $x \in R^+_i(s)$, $R^+_{N\setminus\{i\}}(x) \cap \varphi_i^{\simeq}(s) \neq \varphi$.

Sequential stability implies that for each unilateral improvement x made by DM i from a starting state s, at least one of its opponents has a sanctioning unilateral improvement from x to z such that DM i prefers state z less than state s (Fraser & Hipel 1984, Fang et al. 1993).

Definition 4 General Metarational stability (GMR): For $i \in N$, state s is General Metarational for DM i, denoted by $s \in S_i^{\text{GMR}}$, if and only if for all $x \in R^+_i(s)$, $R_{N\setminus\{i\}}(x) \cap \varphi_i^{\simeq}(s) \neq \varphi$.

General Metarationality implies that for each unilateral improvement x made by DM i from a starting state s, at least one of its opponents has a sanctioning movement from x to z such that DM i prefers state z less than state s (Howard 1971, 1994a).

Definition 5 Symmetric Metarational stability (SMR): For $i \in N$, state s is Symmetric Metarational for DM i, denoted by $s \in S_i^{\text{SMR}}$, if and only if for all $x \in R^+_i(s)$, there exists $y \in R_{N\setminus i}(x)$ such that $x \succeq_i y$ and for all $z \in R_i(y)$, $x \succeq_i z$.

Symmetrical Metarationality implies that for each unilateral improvement x made by DM i from a starting state s, at least one of its opponents has a sanctioning movement from x to y such that DM i prefers state y less than state s. Further, any attempt by DM i to escape from the sanction, say to some state z, does not yield a state that was more preferred to the original state (Fraser & Hipel 1984). The DM, having looked at these moves and countermoves, will then logically decide to remain at state s, by SMR stability.

2.3 Extensions to the Graph Model

Through the definitions given in the previous subsection, GMCR can be employed to model and analyze real world disputes. Aside from this basic framework of GMCR, various other extensions have been developed in conjunction with GMCR in order to more realistically model a given conflict and to gain additional strategic insights. Research has been undertaken into the development of new methodologies for analyzing coalitions and attitudes (COAT)

(Bernath Walker et al. 2009), strength of preference (Hamouda et al. 2005), unknown preferences (Li et al. 2004), emotions (Obeidi et al. 2005), and fuzzy preferences (Al-Mutairi et al. 2008, Hipel et al. 2011a, Bashar et al. 2012). These advancements in preference research have been applied to natural resource, brownfield redevelopment and waste management conflicts. For example, Bernath Walker et al. (2009) consider the effects of attitudes of DMs in a strategic study of a brownfield redevelopment project which took place in Kitchener, Ontario, Canada. The study of the evolution of conflicts (Li et al. 2005), coalitions (Inohara & Hipel 2008a, 2008b), and misunderstanding (Wang et al. 1989) has also been important in the development of conflict analysis methods. Finally, by expressing GMCR and its various extensions using a matrix formulation (Xu et al. 2009, 2012), an efficient engine can be designed for employment within a decision support system (DSS) for GMCR to permit it to be conveniently applied to real-world disputes. This would permit the current DSS, called GMCR II (Fang et al. 2003a,b, Hipel et al. 2008), to be replaced by the next generation for a DSS which could also take into account attitudes.

2.4 Applicability of the Graph Model

As explained by Hipel et al. (2011b), GMCR was intentionally designed to reflect key characteristics of real-world conflict and has thereby incorporated into its basic structure the capability to:

- summarize in a systematic manner intricate and often perplexing information regarding a conflict into a clear model structure. This procedure provides an insightful perspective of the dispute by concentrating on the fundamentals of the problem within a straightforward, yet instructive, model framework;
- operate when information or data are either very limited or extremely abundant;
- improve the understanding of the conflict being analyzed by means of this type of systems thinking;
- enhance communication by utilizing the structured "graph model language" among stakeholders and interested parties;
- suggest strategic consequences of selecting possible options under conflict in order to choose the best possible decision given the social and strategic constraints existing in light of what others may do to advance their own positions. This will lessen making misinformed decisions having potentially highly detrimental and perhaps irreversible impacts;
- be aware of the strategic implications of other characteristics of the conflict being studied such as coalition formation, preference uncertainty and psychological factors; and,
- carry out informed decisions based on the foregoing sensible modelling and analyses which may bring about win/win resolutions.

Zeng et al. (2007) discuss the advantages of GMCR over classical game theory techniques. In practice, the GMCR methodology can be utilized in the following three main kinds of situations:

- *Analysis and simulation tool for a DM in a conflict, or a DM's agent.* An analysis of the moves and counter-moves based on a DM's potential actions can be carried out, as well as an examination of the possible consequences of certain actions, in order to enhance the DM's position. Evaluations can be undertaken at different points in the conflict. For instance, an analyst may advise the American government about strategic initiatives to implement in tackling climate change over time.

- *Communication and analysis tool in mediation.* A mediator can utilize GMCR II to analyze the potential impacts of DMs' preferences, without confirmation from the DMs concerning which ones correctly describe their preferences. This process can be used to determine options that are beneficial, detrimental, or irrelevant to all parties. If labor and management are negotiating the creation of a new contract, for instance, a mediator may be able to show how only tiny concessions and minimum changes in preferences by each side may bring about a contract that is beneficial to both parties.
- *Analysis tool for a third-party analyst.* Founded on the outcome and evolution of a conflict, an analyst can estimate the DMs' preferences. One can also investigate how the structure of the conflict influenced DMs' behaviour and identify better ways to structure a future conflict. For example, the government of the United States might want to assess the efficacy of Chinese policy in Tibet, even though Tibet is part of China.

In Section 3, attitudes (Inohara et al. 2007) are defined for implementation within GMCR. In order to carry out accurate analyses within GMCR using this concept of DM attitudes, new definitions of moves and solution concepts are presented.

3. Attitudes within the Graph Model for Conflict Resolution

In order to effectively model conflict under situations in which a DM acts or may act in a manner that is not just for his own advancement, the attitudes framework can be applied. Developed by Inohara et al. (2007), the attitudes structure allows decision analysts to determine the strategic impact on a conflict outcome that may arise when a DM takes other DMs' preferences into account. Definitions 6 to 13 provide a formal structure which can be used to analyze conflicts in which attitudes are at play and may have strategic impacts on the outcomes.

Definition 6 (Attitudes): For DMs $i, j \in N$, let $E_i = \{+, 0, -\}^N$ represent the set of attitudes of DM i. An element $e_i \in E_i$ is called the attitudes of DM i for which $e_i = (e_{ij})$ is the list of attitudes of DM i towards DM j for each $j \in N$ where $e_{ij} = \{+, 0, -\}$. e_{ij} is referred to as the attitude of DM i to DM j where the values $e_{ij} = +$, $e_{ij} = 0$ and $e_{ij} = -$ indicate that DM i has a positive, neutral and negative attitude towards DM j, respectively.

A social network is the pair $(N, (e_i))$ where N represents the set of DMs within the conflict. The solution concepts presented earlier in Definitions 2 to 5 are now expanded to explicitly account for DMs' attitudes and, hence, are appropriately called relational solution concepts. Prior to providing these stability definitions, a range of preference structures and special types of movements among states must first be explained as originally defined by Inohara et al. (2007).

Definition 7 Devoting preference (DP): The devoting preference of DM $i \in N$ with respect to DM $j \in N$ is denoted by $\mathbf{DP}_{ij}$, such that for s, $t \in$ S, s $\mathbf{DP}_{ij}$ t if and only if $s \succeq_j t$. That is, if DM i

has a devoting preference for state s to state t with respect to DM j (s $\mathbf{DP}_{ij}$ t), then s must be more preferred or equally preferred to state t for DM j ($s \geq_j t$). That is to say, if a DM i has a devoting preference with respect to a DM j, then i prefers any state that is more preferred for j.

Definition 8 (Aggressive preference (AP)): The aggressive preference of DM $i \in N$ with respect to DM $j \in N$ is $NE(\geq_j)$, denoted by $\mathbf{AP}_{ij}$, where $NE(\geq_j)$, is defined as follows: for s, $t \in$ S, s $NE(\geq_j)$ t if and only if $s \geq_j t$ is not true. That is, for s, $t \in$ S, s $\mathbf{AP}_{ij}$ t if and only if s $NE(\geq_j)$ t (if and only if $t <_j s$ under completeness of $\geq_j$). In contrast to the devoting preference, the aggressive preference is such that if DM i has an aggressive preference (s $\mathbf{AP}_{ij}$ t), then state s must be less preferred to state t by DM j ($s >_j t$). This can be expressed as the idea that if a DM i has an aggressive preference with respect to a DM j, then i prefers any state that is less preferred for j.

Using these concepts, as well as an indifference preference represented by **I**, relational preference can be determined.

Definition 9 (Relational preference): The relational preference $\mathbf{RP}(e)_{ij}$ of DM $i \in N$ with respect to DM $j \in N$ at e is defined as follows: $\mathbf{RP}(e)_{ij} = \{\mathbf{DP}_{ij}$ if $e_{ij} = +$; $\mathbf{AP}_{ij}$ if $e_{ij} = -$; $\mathbf{I}_{ij}$ if $e_{ij} = 0\}$, where $\mathbf{I}_{ij}$ denotes that DM i is indifferent with respect to j's preference and, hence, s $\mathbf{I}_{ij}$ x means that DM i's preference between state s and x is not influenced by DM j's preference.

Here, the types of preferences are matched with the three different attitudes to create a relational preference that reflects the attitudes of DM i towards all other DMs in the conflict.

Definition 10 Total Relational Preference (TRP): The total relational preference of DM $i \in N$ at e is defined as the ordering $\mathbf{TRP}(e)_i$ such that for s, $t \in S$, s $\mathbf{TRP}(e)_i$ t if and only if s $\mathbf{RP}(e)_{ij}$ t for all $j \in N$.

The Total Relational Preference orders the states with respect to the relational preferences. A state satisfies a total relational preference for the situation in which it is a relational preference for DM i according to the attitudes of DM i towards all of the DMs in the conflict.

Definition 11 Total Relational Reply (TRR): The total relational reply list of DM $i \in N$ at e for state $s \in S$ is defined as the set $\{t \in R_i(s)\backslash\{s\} \mid t \, \mathrm{TRP}(e)_i \, s\} \subseteq R_i(s)\backslash\{s\}$, denoted by $\mathrm{TRR}(e)_i(s)$.

The set of total relational replies is analogous to the set of unilateral improvements in a regular analysis, because in order to belong to the set of total relational replies from a given state a, the state must be reachable from a and be a total relational preference to a.

Definition 12 Relational Nash stability (RNash): For $i \in N$, state $s \in S$ is Relational Nash stable at e for DM i, denoted by $s \in S_i^{RNash}(s)$, if and only if $\mathbf{TRR}(e)_i\ (s) = \{s\}$. Relational Nash stability parallels regular Nash stability in that a state is Relational Nash stable when there are no TRRs from a given state, except for the state itself. In regular Nash stability, Definition 1, a state is stable if there are no UIs from that state; this is comparable to the lack of TRRs needed to conclude that a state is Relational Nash stable.

To define the following relational solution concepts, it is necessary to define $R\varphi^{\simeq}(e)_i(s)$. The symbol $R\varphi^{\simeq}(e)_i(s)$ is an analogue of $\varphi_i^{\simeq}(s)$ given in Section II. Hence, $R\varphi^{\simeq}(e)_i(s)$ is the set $\{t \in S \mid \mathrm{NE}(t\ \mathbf{TRP}(e)_i\ s)\}$ of all states that are not relationally preferred to s by DM i under attitude e. Note that $\mathrm{NE}(t\ \mathbf{TRP}(e)_i\ s)$ means that "t $\mathbf{TRP}(e)_i$ s" is not true.

Definition 13 (Relational Sequential Stability (RSEQ)): For $i \in N$, state $s \in S$ is Relational Sequentially stable at e for DM i, denoted by $s \in S_i^{RSEQ}(s)$, if and only if for all $x \in \mathbf{TRR}(e)_i(s) \backslash \{s\}$, $\mathrm{TRR}_{N\backslash i}(x) \cap R\varphi^{\tilde{=}}(e)_i(s) \neq \varphi$.

Relational Sequential stability parallels regular Sequential stability in that a state is RSEQ stable when for every TRR a DM has from a given state, there exists a credible move by the set of all other DMs that can relationally sanction the DM's TRR, similar to the definition of SEQ in Definition 3.

Definition 14 Relational General Metarational stability (RGMR): For $i \in N$, state $s \in S$ is Relational General Metarational at e for DM i, denoted by $s \in S_i^{RGMR}(s)$, if and only if for all $x \in \mathbf{TRR}(e)_i(s) \backslash \{s\}$, $R_{N\backslash i}(x) \cap R\varphi^{\tilde{=}}(e)_i(s) \neq \varphi$.

Relational General Metarationality mirrors Rational Sequential stability in that it looks two moves ahead. Stability by RSEQ is dependent upon the relational sanctions of DMs opposing the DM who makes the first move, *i*. If TRR lists are seen as analogous to UI lists, the use and definition of RGMR and RNash stabilities become more apparent. In other words, a state *a* is RGMR stable for a DM *i* if for every TRR away from state *a*, another DM belonging to the set *N-i* can make a unilateral move from the new state to one that is not a TRP for DM *i.*

Definition 15 Relational Symmetric Metarational stability (RSMR):

For $i \in N$, state $s \in S$ is Relational Symmetric Metarational at e for DM i, denoted by $s \in S_i^{RSMR}(s)$, if and only if for all $x \in \mathbf{TRR}(e)_i(s) \backslash \{s\}$, there exists $y \in R_{N\backslash i}(x) \cap R\varphi^{\tilde{=}}(e)_i(s)$ such that $z \in R\varphi^{\tilde{=}}(e)_i(s)$ for all $z \in R_i(y)$.

By Definition 15, a state is RSMR stable if the state is RGMR stable and there exists no move on the part of the DM to escape from her opponent's sanction. RSMR takes RGMR stability and looks one move ahead.

4. Dominating Attitudes

Dominating attitudes represent a new and simple way to group a DM's set of attitudes into sets of most important and least important attitudes. For example, in a large conflict, a given DM may have attitudes towards many DMs, but he may feel most strongly about certain DMs whom he has dealt with on numerous occasions. The utility of creating sets this way is twofold: first of all, it will improve the accuracy of conflict modeling by providing flexibility to the attitudes framework and secondly, it will allow realistic solution concepts to be defined. The definition of "dominating attitudes", given below, is a simple partition of the attitudes defined previously in Definition 6.

Definition 16 (Dominating attitudes): For $i \in N$, with attitudes $e_i = \{e_{ii}, e_{ij},...,e_{in}\}$, the set of dominant attitudes considered most consequential is denoted as $e_i^d \subseteq e_i$, where $e_i^d = \{e_{ik},...,e_{im}\}$. The set of non-dominating attitudes considered least consequential is denoted as $e_i^{nd} \subseteq e_i$, where $e_i^{nd} = (e_i - e_i^d) - e_i^*$, where e_i^* is the set of indifferent attitudes.

Using these sets of attitudes, as defined in Definition 16, four new solution concepts are developed based on the relational definitions given in Definitions 12 to 15.

Definition 17 Weak Relational Nash stability (WRNash): For $i \in N$, state $s \in S$ is weak relational Nash stable at $e_i = e_i^d \cup e_i^{nd} \cup e_i*$ for DM i, denoted by $s \in S_i^{WRNash}(s)$, if and only if $\mathbf{TRR}(e_i)(s) = \{s\}$ and there exists $y \in S$ such that $\mathbf{TRR}(e_i^d)(s) = \{y, s\}$.

By Definition 17, if a state is RNash stable, but there exists a total relational reply that satisfies its set of ranked attitudes, e_i^r, then the RNash stability is "weak", as DM i may have a strong temptation to satisfy one or more of these dominant attitudes.

Definition 18 Weak Relational Sequential stability (WRSEQ): For $i \in N$, state $s \in S$ is weak relational sequential stable at $e_i = e_i^d \cup e_i^{nd} \cup e_i*$ for DM i, denoted by $s \in S_i^{WRSEQ}(s)$, if and only if for all $x \in \mathbf{TRR}(e_i)(s)$, there exists $y \in \mathbf{TRR}_{N-i}(x)$ such that $s\mathbf{TRP}(e_i^{nd})y$ and $NE(s\mathbf{TRP}(e_i^d)y)$.

In Definition 18, a state is seen to be Weak Relational Sequentially stable if after DM i makes a Total Relational Reply according to her attitudes, the opposing DMs can only make TRRs that are sanctions according to her non-dominating attitudes. That is to say, by her dominating attitudes the final state is more preferred and thus she may risk making the TRR.

Definition 19 Weak Relational General Metarational stability (WRGMR): For $i \in N$, state $s \in S$ is weak relational general metarational at $e_i = e_i^d \cup e_i^{nd} \cup e_i*$ for DM i, denoted by $s \in S_i^{WRGMR}(s)$, if and only if for all $x \in \mathbf{TRR}(e_i)(s)$, there exists $y \in \mathbf{R}_{N-i}(x)$ such that $s\mathbf{TRP}(e_i^{nd})y$ and $NE(s\mathbf{TRP}(e_i^d)y)$.

In Definition 19, a state is seen to be Weak Relational Sequentially stable if after DM i makes a Total Relational Reply according to her attitudes, the opposing DMs can only make moves that are sanctions according to her non-dominating attitudes. That is to say, by her dominating attitudes the final state is more preferred and thus she may risk making the TRR.

Definition 20 Weak Relational Symmetric Metarational stability (WRSMR): For $i \in N$, state $s \in S$ is weak relational symmetric metarational at $e_i = e_i^d \cup e_i^{nd} \cup e_i*$ for DM i, denoted by $s \in S_i^{WRSMR}(s)$, if and only if for all $x \in \mathbf{TRR}(e_i)(s)$, there exists $y \in \mathbf{R}_{N-i}(x)$ such that $s\mathbf{TRP}(e_i)y$ and for all $z \in \mathbf{R}_i(s)$, $s\mathbf{TRP}(e_i^{nd})z$ and $NE(s\mathbf{TRP}(e_i^d)z)$.

From Definition 20, if a state is RSMR stable by Definition 13 and DM i's escape from the sanction is less preferred to the original state by only her non-dominating attitudes, the state is WRSMR stable. Thus, the DM may decide that it is in her own interest to make a move in spite of the sanction.

In this development of "dominating attitudes" definitions, the authors have considered the importance of weighing the DMs' individual attitudes. There is, however, another approach which leads to the same result through a slightly different path. In this alternate case, a DM considers giving different weights to particular DMs in the conflict and then calculating which moves satisfy his attitudes towards the more important DMs. In this sense, the solution concepts are the same, only now the decision analyst does not need to categorize the particular attitudes; the categorization of one DM's views of other DMs does this. In Section 5, these new definitions of dominating attitudes, as well as the definitions for GMCR and Attitudes discussed earlier, are applied to the problem of a brownfield redevelopment in Kitchener, Canada.

5. Redevelopment of a Brownfield Property in Kitchener, Ontario, Canada

In this section, an environmental conflict over the purchase of an abandoned factory on a brownfield location will be examined using this new preference structure discussed above. A brownfield is defined as land that has been polluted through industrial or commercial activities (US EPA 1997; Hipel & Bernath Walker 2012). The redevelopment of brownfields, which are properties that are perceived to be environmentally contaminated (US EPA 1997), provides an opportunity for communities and businesses to improve their quality of life and commerce (Bernath Walker et al. 2009, 2010, Greenberg et al. 2000, Greenberg & Lewis 2000). Local governments, which are of course aware of this, are thus under pressure to encourage redevelopment of these lands in order to benefit local interests.

The objective of using a formal systems model to analyze this particular conflict is not intended to be predictive. Rather, this particular type of conflict, which is a commonly occurring one in brownfield redevelopment, is used to demonstrate how the approach which is newly proposed and described in this paper works in applications and is useful for generating new ideas for negotiation strategies. For comparative analyses of a range of approaches, including game theoretic methods, for forecasting decisions in conflicts, the reader can refer to the interesting research of Green (2002, 2005) and Green & Armstrong (2007) as well as the references contained therein. As documented in their papers, these authors carry out actual experiments to compare their experimental forecast with those obtained using game theory methods.

5.1 Brownfield Acquisition Conflict

Walker et al. (2007) developed a three-step process outlining the redevelopment of brownfield properties beginning with the important step of property acquisition, followed by remediation and redevelopment. This process is based on case studies of private brownfield renovations and, in particular, focuses on the conversion of the Kaufman Shoe Factory, located in Kitchener, Ontario, Canada, into condominiums called the Kaufman Lofts. In the property acquisition step of the Kaufman renovation, the current property owner, the local government and the potential developer were all involved. The current property owner (PO) and the buyer or property developer (D) needed to negotiate a market-value price for the property, while the city government (CG) had to decide whether to offer incentives to the developer to take on the blighted property. In the case of the Kaufman Lofts and many other brownfield properties throughout the developed world, government involvement is essential in order to not only fund the clean-up of the property, but to improve the living conditions of the residents in the area. Often this is done using a Tax Incremental Financing Approach (US EPA 2010).

The option form of this conflict is shown in Table 1, where each option is either marked with a “Y” or an “N”, denoting that either “Yes” the option is selected, or “No” it isn’t, respectively. The state identifications (IDs) are derived by decimalizing the binary entries in each column. As expected, the Property Owner (PO) has the opportunity to sell the land at a high or low value, the City Government (CG) has the option to offer incentives or not, and the Developer (D) has the option to purchase the property or not. The bottom row of Table 1 is made up of the corresponding states labels. For example, the 3rd state from the left is state "2" in which PO is selling low, CG is not providing incentives and D is not purchasing the property.

Given this set of feasible states in Table 1 shown in option form, it is necessary to develop ordinal preference rankings of the states, from the perspective of each DM. For example, CG would prefer that PO sell the property at either price, as opposed to not selling it, and that D purchases the property so that the land will be used. Using this type of logical ordering, the preferences are developed, as shown in Table 2. Here, the states are listed from most to least preferred, when reading from left to right, while equally preferred states are grouped together in parentheses. For example, PO prefers states 13 and 9 equally to each other and more than any other state.

Next, the conflict is put into tableau form, as first shown by Fraser & Hipel (1984), by placing each preference ranking in a table with the unilateral improvements available to the given DM at a particular state shown below the state. Each state is tested for Nash and Sequential stability according to Definitions 2 and 3, laid out at the beginning of this article, as can be seen in Table 3. For example, state 6 is SEQ stable for D, by Definition 3, as follows: RD+(6) = 14, RN-D+(14) = {8, 9, 10, 12, 13} and 6 >D {8, 9, 10, 12, 13}; thus, state 6 is SEQ for D. When looking at likely outcomes, there is only one equilibrium state, state 1. At state 1, PO offers to sell the property at a high price, CG offers no incentives and D does not buy the property. From the standpoint of CG and D, this is not a successful negotiation outcome. Thus, it

Table 1 Acquisition conflict in option form

PO	Sell High	N	Y	N	N	Y	N	N	Y	N	N	Y	N
	Sell Low	N	N	Y	N	N	Y	N	N	Y	N	N	Y
CG	Incentives	N	N	N	Y	Y	Y	N	N	N	Y	Y	Y
D	Buy	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y
	State IDs	0	1	2	4	5	6	8	9	10	12	13	14

Table 2 Preference ranking for acquisition conflict

PO	(13	9)	(1	5)	(0	4	8	12)	(2	6	10)	14
CG	10	14	9	13	(0	1	2	8)	(4	5	6	12)
D	14	6	10	(0	2	4	8	12)	5	1	13	9

Table 3 Tableau form of static analysis

PO	X R (13	x r 9)	E r (1	x r 5)	x u (0 1	x u 4 5	x u 8 9	x u 12) 13	x u (2 1 0	x u 6 5 4	x u 10) 9 8	x u 14 13 12
CG	R 10	s 14 10	r 9	u 13 9	r (0	r 1	r 2	r 8)	u (4 0	u 5 1	u 6 2	u 12) 8
D	R 14	s 6 14	r 10	r (0	u 2 10	r 4	r 8	r 12)	r 5	r 1	u 13 5	u 9 1

is in the interest of both parties to see how they might best negotiate in order to reach a better outcome for the developer and greater community.

5.2 Attitudes in Brownfield Acquisition Conflict

In order to reach a conflict outcome where both CG and D are satisfied, both DMs need to make an effort to help each other. For example, CG would benefit the community by trying to do those things that would entice D to purchase the property, namely by providing incentives. At the same time, CG must make a commitment to preserving taxpayer monies and staying on budget. In this case, the attitudes would be formally written as shown in Table 4.

As there are no TRRs, by Definition 9, for CG for any of the 14 states, each state is RNash stable for CG, by Definition 12. This is illustrated in Table 5, as none of the unilateral movements available to CG are bolded. This leads to a whole new profile of equilibria for our brownfield acquisition conflict, as shown in Table 6.

In order to apply this new perspective to the conflict, it is necessary to examine the preference information as well as the reachable lists for all the DMs. As CG has positive attitudes towards itself and D, all of CG's moves must satisfy both CG's and D's preferences in order to be a credible move, by Definition 9. In Table 5 below, a list of all the states in the conflict is given. Below each state in this list is a set of states that are reachable by CG from the corresponding state in the top row. Those states that are preferred by CG and D, and are thus Totally Relationally Preferred by CG according to Definition 8, would be marked in bold.However, there are no moves that satisfy these attitudes.

As there are no TRRs, by Definition 9, for CG for any of the 14 states, each state is RNash stable for CG, by Definition 12. This is illustrated in Table 5, as none of the unilateral movements available to CG are bolded. This leads to a whole new profile of equilibria for our brownfield acquisition conflict, as shown in Table 6.

Table 4 Attitudes in brownfield negotiation

	PO	CG	D
PO	+	0	
CG	0	+	+
D	0	0	+

Table 5 Unilateral movements and Total Relational Replies (bolded) for CG

States	0 1 2 4 5 6 8 9 10 12 13 14
CG's Moves	4 5 6 0 1 2 12 13 14 8 9 10

Table 6 Tableau form of attitude analysis – Case 1

PO	X R (13	x r 9)	E r (1	E r 5)	x u (0 1	x u 4 5	x u 8 9	x u 12) 13	x u (2 1 0	x u 6 5 4	x u 10) 9 8	x u 14 13 12
CG	rr 10	rr 14	rr 9	rr 13	rr (0	rr 1	rr 2	rr 8)	rr (4	rr 5	rr 6	rr 12)
D	R 14	s 6 14	r 10	r (0	s 2 10	r 4	r 8	r 12)	r 5	r 1	u 13 5	u 9 1

All of CG's states are RNash stable, denoted "rr", by Definition 12, as the only states available to move from are the states themselves due to the restriction of CG considering multiple attitudes. As shown in Table 6, the new equilibrium is state 5 where PO tries to sell at a high price, CG offers incentives and D does nothing. For example, state 0 is RNash stable for CG as $TRR_{CG}(0) = \{0\}$ and therefore, $0 \in S_{CG}^{RNash}$. As this outcome is not advantageous to either D or CG, it is worth looking at what attitudes must change on the part of D, instead of CG, to arrive at a better outcome. These new possible attitudes are expressed in Table 7 below, much as they were in Table 6 above.

Given this new set of DM attitudes, D will have to behave in a manner that satisfies not only her own preferences, but those of CG. Thus, D will need to determine if her possible moves satisfy her Total Relational Preference (TRP) according to Definition 10, before she can be certain of her Total Relational Replies (TRRs). In Table 8 below, the TRRs of D for the set of attitudes given in Table 7 are shown. The top row of the table is the complete set of states in this conflict. Below each state is the set of unilateral movements available to D from the state. Of these movements, those that are bolded represent D's Total Relational Replies.

Given this new set of Total Relational Replies on the part of D, a whole new static analysis can be done, as shown in the tableau form of the conflict given in Table 9. As can be seen, there are now two equilibrium states.

With these new attitudes, there are now moves that can be undertaken to find new equilibria. From state 6, D can still move to state 14, even though she is under attitudes. State 6 is RSEQ stable according to Definition 13 because $14 \in TRR_{CG}(6)$ means that 6 cannot be RNash, by Definition 12. As $5, 4 \in TRR_{PO}(14)$ and $6TRP_{CG}5$, state 6 is thus RSEQ stable.

More generally, as can be seen in Table 9, there are now two equilibria at states 1 and 9. At state 1, PO attempts to sell the property at the highest price, CG does not offer incentives and D does not purchase it. At state 9, however, PO puts the property up for sale at the highest price, CG does not offer incentives and D does purchase it. Although it may seem optimistic to

Table 7 Attitudes in brownfield negotiation – Case II

	PO	CG	D
PO	+	0	0
CG	0	+	0
D	0	+	+

Table 8 Unilateral movements and Total Relational Replies (bolded) for D

States	0 1 2 4 5 6 8 9 10 12 13 14
D's Moves	8 9 **10** 12 13 **14** 0 1 2 4 5 6

Table 9 Tableau form of attitude analysis – Case 2

PO	x r (13	E r 9)	E R (1	x r 5)	x u (0 1	x u 4 5	x u 8 9	x u 12) 13	x u (2 1 0	x u 6 5 4	x u 10) 9 8	x u 14 13 12
CG	r 10	s 14 10	r 9	u 13 9	r (0	r 1	r 2	r 8)	u (4 0	u 5 1	u 6 2	u 12) 8
D	rr 14	rs 6 14	rr 10	rr (0	rs 2 10	rr 4	rr 8	rr 12)	rr (1	rr 5)	rr 13	rr 9

have such a state as an equilibrium, it is unlikely that this state would be reached. The reason for this is that in order to reach state 9, moves must be made from the status quo state, 0. From 0 the only credible move available is for PO to move to state 1. As state 1 is an equilibrium, it is likely that the conflict will stay at this point.

In order to solve this dilemma, it is worth looking at how a DM may perhaps favour certain attitudes over others. This favouring, called dominating attitudes, is a way for the different DMs to test the strength of their stabilities, according to the definitions in Section 3.

In Table 10, the dominant attitudes for both CG and D are shown. Here we can see that both D and CG hold positive attitudes towards themselves and each other but that the dominating attitudes they have are for each other. The dominating attitudes are marked with a superscript "d", while the non-dominating attitudes are marked with a superscript "nd". Using this attitude information, it is possible to determine the TRRs of both CG and D. As previously, the movements for CG and D are shown in Table 11. Again, the movements that satisfy a DM's complete set of attitudes are in bold. In addition, however, states that satisfy dominant attitudes, but not the non-dominating attitudes, are denoted by an asterisk (*).

Applying these new movements to the conflict laid out previously results in new equilibria, as illustrated in Table 12. Applying the new dominant attitudes definitions, there are states that are "rr" or RNash stable, by Definition 12, and those that are "wrr" or Weak RNash stable, by Definition 17. For example, at state 2 $TRR_{CG}(2) = \{2\}$, meaning that the only Total Relational Reply from state 2 is itself, and thus the state is RNash for CG by Definition 10. However, as there exists a movement that satisfies e_{CG}^{d}, in this case CG's devoting attitude towards D, the state is WRNash. Specifically, $TRR(e_{CG})(2) = \{2\}$ and there exists a state, 6, such that $TRR(e_{CG}^{d})(2) = \{6, 2\}$. Thus, state 2 is WRNash for CG..

Table 10 Dominant attitudes in brownfield negotiation

	PO	CG	D
PO	+	0	0
CG	0	$+^{nd}$	$+^{d}$
D	0	$+^{d}$	$+^{nd}$

Table 11 Unilateral movements, Weak TRRs (*) and TRRs (bolded) for CG and D

States	0	1	2	4	5	6	8	9	10	12	13	14
CG's Moves	4	5	6*	0	1	2	12	13	14*	8	9	10
D's Moves	8	9*	**10**	12	13*	**14**	0	1	2	4	5	6

Table 12 Tableau form analysis of Unilateral movements and Total Relational Replies for CG and D

	E	WE	WE	WE	x	x	x	x	x	x	x	x
	r	r	r	r	u	u	u	u	u	u	u	u
PO	(13	9)	(1	5)	(0	4	8	12)	(2	6	10)	14
					1	5	9	13	1	5	9	13
									0	4	8	12
CG	wrr	rr	wrr	rr	rr	rr	wrr	rr	rr	rr	rr	rr
	10	14	9	13	(0	1	2	8)	(4	5	6	12)
	14*		13*				6*					
	rr	rs	rr	rr	rs	rr	rr	rr	wrr	wrr	rr	rr
D	14	6	10	(0	2	4	8	12)	5	1	13	9
		14			10				13*	9*		

In Table 12, the conflict is illustrated in tableau form. As can be seen, there are four equilibria, all corresponding to states that are strongly preferred by PO: 13, 9, 1 and 5. Only one of these states, state 13, is enforced by "strong" solution concepts, meaning that there are no dominant moves that could be used by any of the DMs to move away from the state. Thus, state 13 is quite stable compared to the other equilibria under the given conditions. At this state, CG makes the sacrifice of offering incentives and D agrees to purchase the property at a higher price. To reach this relatively stable state, both DMs have to make an agreement to move together to the stronger and more advantageous outcome. Such a movement, called an equilibrium jump (Kilgour et al. 2001), is a way for DMs to act as a temporary coalition in order to move to a state that is advantageous for all coalition members while not risking that their coalition partner(s) will not come through.

6. Conclusions

Through the careful application of attitudes in a logical manner and through the consideration of dominating and non-dominating attitudes, it is possible to develop new win-win resolutions to resolving brownfield conflicts. The importance of the dominating attitudes framework is that it enhances the flexibility of the original attitudes analysis procedure developed by Inohara et al. (2007) and creates possibilities for new moves and countermoves that better mimic human behaviors. Specifically, within the context of the brownfield acquisition problem in Section 5, two DMs are able to work together and act in tandem to achieve a resolution that benefits not only themselves but the community and environment at large. Such techniques and results can also be applied to military and political alliances as well as international environmental agreements – thus their development is essential, not just as a mathematical exercise but for the advancement of formal approaches to describing human behavior under conflict.

In this paper, the authors proposed and described a new approach for generating ideas for negotiation strategies, and showed through an application how the newly proposed approach works. Indeed, as explained in Section 2.3, GMCR can be extended to handle a rich range of types of conflict. Additionally, in Section 2.4, the inherent capabilities of GMCR are outlined followed by an explanation of general situations in which it can be usefully applied. The next step of the attitudes research is to test the usefulness of our approach against that of others and, moreover, to validate the results of the analyses through experimentation when it is used as a predictive tool.

Acknowledgements

The authors are grateful to the Centre for International Governance Innovation (CIGI) for financially supporting Dr. Sean Bernath Walker during his PhD studies in Systems Design Engineering at the University of Waterloo (UW) under the project entitled Multiple Participant – Multiple Objective Decision Making in International Governance, headed by K.W. Hipel, and also partially funded by the UW Faculty of Engineering. The Natural Sciences and Engineering Research Council (NSERC) of Canada also funded some of the work. The authors especially appreciate receiving helpful suggestions from anonymous referees and the Editor of JSSSE for improving the quality of their paper.

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