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LOT SIZING WITH NON-ZERO SETUP TIMES FOR REWORK*

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Abstract

In this paper we consider a single machine multi-product lot scheduling problem in which defective items are produced in any production run of each product. In each cycle after the normal production of each product the machine is setup for the rework of the defectives of the same product and then the rework process starts. We assume that the setup time for the normal production process as well as the rework process is non-zero. Further we consider the waiting time cost of defectives for rework. This paper has two objectives. The first objective is to obtain the economic batch quantity (*EBQ*) for a single product. The second objective is to extend the result of the first objective to the multi-product case. Adopting the common cycle scheduling policy we obtain optimal batch sizes for each product such that the total cost of the system per unit time is minimized.

Keywords: Inventory control, rework, lot scheduling, production control

1. Introduction

Consider the problem of obtaining a low cost schedule for a production system in which a number of products are manufactured on a single facility in a fixed sequence repeatedly from cycle to cycle. This problem is known as economic lot scheduling problem (*ELSP*). There is, at the present time, no algorithm available which solves the problem optimally, and several different types of approaches have been presented in the literature, see Elmaghraby (1978) for a comprehensive literature review through 1978; for recent contribution see (Cook 1980, Dobson 1987, Gallego and Shaw 1997, Glass 1992, Graves 1979, Gunter 1986, Haessler 1979, Haji and Mansuri 1995, Park and Yun 1988, Roundy 1988, and Zipkin 1988) among others.

In the ELSP, it is assumed that a perfectly reliable facility produces items at a fixed production rate and the products produced are all non-defective. But in practice there are many

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situations in which a certain amount of defective products results due to various reasons including poor production quality and material defects. Depending on the proportion of defectives, the amount of optimal batch sizes also varies depending on several cost factors such as setup cost, processing cost, and inventory carrying cost. In a production system where there is no repair facility, defective items are wasted as scrap, and as a result, they lose a big share of profit margin. Researchers that consider rework option are meager (Jamal et al. 2004).

A study by Goyal and Gunashekharan (1990) showed the effect of process control and they ignored the situation of producing defects. The issue of imperfect production and quality control in a lot-sizing problem had been addressed in the literature by Hayek and Salameh (2001), Lee (1992), and Lee and Rosenblatt (1986). Lee et al. (1997) developed a model of batch quantity in a multi-stage production system considering various proportions of defective items produced in every stage, but they did not consider the rework option of the defective items. In a recent paper, Jamal et al. (2004) considering the reworking of defective items for the case of a single product, developed two models for obtaining the economic batch quantity for the single product. In their derivation of EBQ, they ignored the following 3 points; (a) The setup time and setup cost for rework of the defective units, (b) The cost of waiting time of defective units for rework during the production run of the product, and (c) the different inspection costs during production and rework processes. In a recent effort, Haji et al. (2006) investigated the economic production quantity with accumulated rework for a single product. They considered the case in which defective items from each cycle are accumulated until N equal cycles are completed after which all defectives are reworked in a new cycle, called rework cycle.

In this paper we do not ignore the above 3 points. We first, obtain EBQ for the case of a single product. Then, we extend these results for the case of a multi-product single machine system. To do this, we adopt the common cycle time approach for ELSP proposed by Hanssman (1962). This approach is to schedule exactly one lot of each product in a time interval called common cycle (CC) or T . The CC approach always finds a feasible schedule and consists of a very simple procedure. Jones and Inmann (1989) have shown that the CC approach produces optimal and near optimal schedules in many realistic situations. Adopting the CC approach for all products, allowing non-zero setup times for the normal production and rework process for each product, we obtain the optimal common cycle time, hence the optimal batch sizes for all products, which minimize the sum of inventory holding cost, setup cost, production process and inspection costs and waiting time cost of defectives for rework.

2. Assumptions

In this paper, all the standard assumptions of the general *ELSP* hold true (see for example, Johnson and Montgomery 1974). The most relevant assumptions used in this paper are as follows:

- There are n products, all of which must be produced on a single machine, which can make only one product at a time.
- Demand rates for all products are constant, known, and finite.

- Production rates for all products are constant, known, and finite.
- All demands must be filled immediately, so no shortages are permitted.

Furthermore, it is assumed that for product j, $j = 1,...,n$:

- Proportion of defective is constant in each cycle,
- The production rate of non-defectives is greater than the demand rate,
- Scrap is not produced at any cycle,
- No defectives are produced during the rework,
- Production and rework are done using the same resources.

3. Notations

In this paper the following notations is used for product j, $j = 1,...,n$:

P_j Production rate, units/year
P_{rj} Production rate of non-defectives, units/year
D_j Demand rate, units/year
C_j Processing cost for each unit of product, \$/unit
Q_j Batch quantity per cycle, units/batch or units/cycle
β_j Proportion of defectives in each cycle
A_{1j} Setup cost for normal production, \$/batch
A_{2j} Setup cost for rework process, \$/batch
H_j Inventory carrying cost, \$/unit/year
W_j Waiting time cost, \$/unit/year
S_{1j} Setup time for normal production, year/setup
S_{2j} Setup time for rework process, year/setup
L_j Inspection cost for normal process, \$/unit
M_j Inspection cost for rework process, \$/unit

4. Model

In this model, we consider a common cycle time T for all products as depicted in Figure 1. Each product is produced only once in each cycle T. We assume all the defective items for each product during cycle T are reworked within the same cycle. After the normal processing time, the machine is immediately set up for the rework of the defective units. At the end of this setup time the rework process starts. For the feasibility of the problem, we assume that for each $j = 1,...,n$.

$$P_j(1-\beta_j) > D_j \quad (1)$$

From Fig.1, we see that during the interval t_{1j} the product j is produced at rate P_j, but the value of non-defectives of this product is produced at rate $P_j(1-\beta_j)$ in the same interval. Hence, the inventory of non-defective units of product j will increase at rate $[P_j(1-\beta_j) - D_j]$. After t_{1j}, immediately the setup time for rework process on defective items produced in t_{1j} starts. Thus, during the interval S_{2j} the inventory of non-defective items decreases at rate $-D_j$. After this setup time the rework process starts on defective items produced in t_{1j}. Therefore, the inventory of non-defective items increases at rate $P_{rj} - D_j$. It is assumed that no defective occurs during the rework process time, i.e., during t_{2j}, because of the careful operation or special attention.

We can easily show that in each cycle, the length of the first phase of production of product j, t_{1j} and the length of the rework processing time of defectives of the same product, t_{2j}, is respectively equal to

$$t_{1j} = \frac{Q_j}{P_j} = \frac{D_j T}{P_j} \quad (2)$$

$$t_{2j} = \frac{\beta_j Q_j}{P_{rj}} = \beta_j \frac{D_j T}{P_{rj}}$$

or from (2)

$$t_{2j} = \frac{\beta_j D_j}{P_{rj}} T = \beta_j \frac{P_j}{P_{rj}} t_{1j} \tag{3}$$

For product j, define the on hand inventory of non-defective items at the end of t_{1j}, at the end of S_{2j}, and at the end of t_{2j} respectively by h_{1j}, h_{2j}, and h_{3j}. Now from Fig.1 we can write

$$\begin{aligned} h_{1j} &= [P_j(1-\beta_j) - D_j]\, t_{1j} \\ &= (P_j - D_j - \beta_j P_j) t_{1j} \end{aligned} \tag{4}$$

and

$$h_{2j} = h_{1j} - D_j S_{2j} \tag{5}$$

$$\begin{aligned} h_{3j} &= h_{2j} + (P_{rj} - D_j) t_{2j} \\ &= h_{1j} - D_j S_{2j} + (P_{rj} - D_j) t_{2j} \end{aligned}$$

or from (3) and (4)

$$\begin{aligned} h_{3j} &= [P_j(1-\beta_j) - D_j] t_{1j} - D_j S_{2j} \\ &\quad + (P_{rj} - D_j) \beta_j \frac{P_j}{P_{rj}} t_{1j} \\ &= (P_j - D_j - \beta_j \frac{P_j}{P_{rj}} D_j) t_{1j} - D_j S_{2j} \end{aligned} \tag{6}$$

The total cost of product j, denoted by K_j, consists of setup costs, processing cost, inspection costs, waiting time cost of defectives for rework, and inventory carrying cost for the product.

Let for product j, $j = 1, ..., n$:

K_{js} = Total set up cost per year,

$\overline{I}_j$ = Average on hand inventory,

K_{jH} = Average inventory carrying cost per year,

Then, we can write

$$K_{js} = (D_j / Q_j)(A_{1j} + A_{2j})$$

or

$$K_{js} = \frac{A_{1j} + A_{2j}}{T} \tag{7}$$

In which, $D_j / Q_j = 1/T$, stands for the number of cycles per year. We also can write

$$K_{jH} = H_j \overline{I}_j \tag{8}$$

where $\overline{I}_j$ is the average inventory of product j and H_j is the inventory holding cost for a unit of product j per unit time (year).

From Fig.1, the average inventory of product j can be written as:

$$\overline{I}_j = \frac{1}{2} \begin{bmatrix} h_{1j}(t_{1j}/T) + (h_{1j} + h_{2j})(S_{2j}/T) \\ +(h_{2j} + h_{3j})(t_{2j}/T) \\ +h_{3j}(T - t_{1j} - S_{2j} - t_{2j})/T) \end{bmatrix}$$

or equivalently

$$\overline{I}_j = \frac{1}{2T} \begin{bmatrix} h_{1j} t_{1j} + (h_{1j} + h_{2j}) S_{2j} + (h_{2j} + h_{3j}) t_{2j} \\ +h_{3j}(T - t_{1j} - S_{2j} - t_{2j}) \end{bmatrix}$$

Substituting t_{1j} and t_{2j} from (2) and (3), h_{1j}, h_{2j} and h_{3j} from (4), (5) and (6) in the above relation we can write

$$\overline{I}_j = \frac{1}{2} \left[P_j - D_j - \beta_j D_j (1 + \beta_j \frac{P_j}{P_{rj}}) \right] \frac{D_j T}{P_j} - \beta_j D_j S_{2j} \tag{9}$$

Thus, from (8) and (9) we have

$$K_{jH} = \frac{H_j}{2} \left[P_j - D_j - \beta_j D_j (1 + \beta_j \frac{P_j}{P_{rj}}) \right] \frac{D_j}{P_j} T - H_j \beta_j D_j S_{2j} \tag{10}$$

Let K_{jo} denote the sum of operation processing cost and inspection cost per year for product j. To obtain K_{jo} first we note that in each cycle time T the cost of operation process and inspection during t_{1j} is equal to

$(C_j + L_j)Q_j$ and the cost of operation process and inspection during t_{2j}, for $\beta_j Q_j$ defective units, is equal to $(C_j + M_j)\beta_j Q_j$. Thus the sum of these two costs in each cycle T is $[C_j(1+\beta_j)+(L_j+M_j\beta_j)]Q_j$. Therefore, the total production process and inspection cost per year is:

$$K_{jo} = [C_j(1+\beta_j) + L_j + M_j\beta_j]D_j \qquad (11)$$

Let, $K_j(T)$ denote the waiting time cost of defective units for rework in a cycle. Then $K_j(T)$ is the product of W_j, waiting cost per unit time (year) of a defective unit for rework, and the area of the shaded part in Fig. 1. That is,

$$K_j(T) = W_j\beta_j Q_j\left(\frac{t_{1j}+t_{2j}}{2} + S_{2j}\right)$$

Substituting (2) and (3) in $K_j(T)$ and noting that $Q = D_j T$, we obtain

$$K_j(T) = W_j\beta_j D_j T\left[\frac{D_j T(P_{rj} + \beta_j P_j)}{2P_j P_{rj}} + S_{2j}\right]$$

or equivalently

$$K_j(T) = W_j\beta_j T\left[(1+\beta_j\frac{P_j}{P_{rj}})\frac{D_j^2 T}{2P_j} + S_{2j}\right]$$

If we denote the total waiting time of defective units in a year by K_{jW}, then

$$K_{jW} = K_j(T)/T,$$

or

$$K_{jW} = W_j\beta_j\left[(1+\beta_j\frac{P_j}{P_{rj}})\frac{D_j^2 T}{2P_j}\right] + W_j\beta_j D_j S_{2j} \qquad (12)$$

Now, K_j, the total cost per unit time (year) for product j, can be written as:

$$K_j = K_{js} + K_{jH} + K_{jo} + K_{jW} \qquad (13)$$

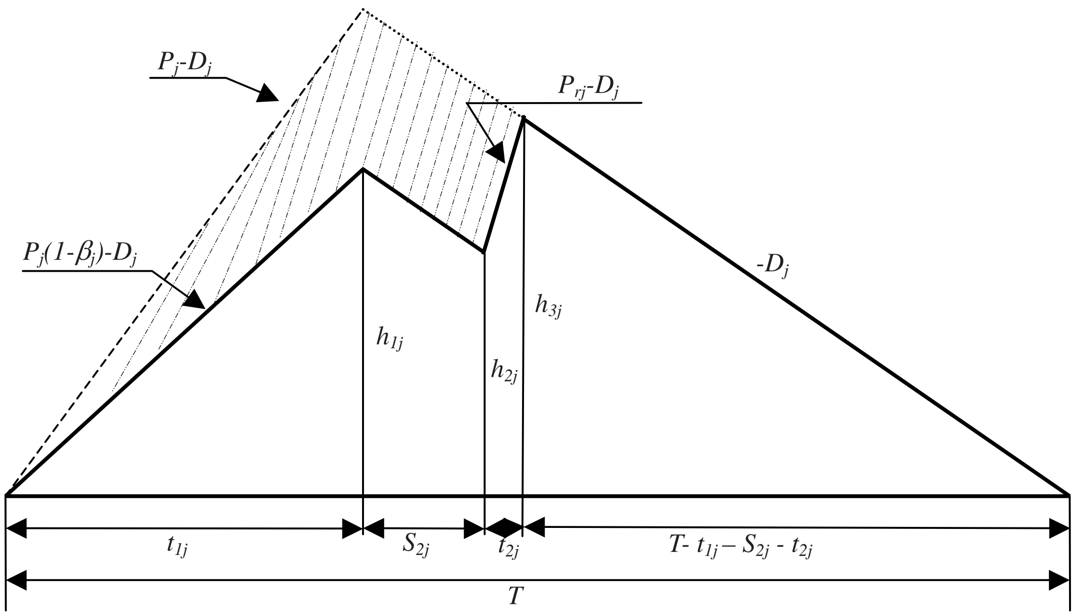

Figure1 On-hand inventory in one cycle

Thus, from (7), (10), (11), (12), and (13) we have

$$
\begin{aligned}
K_j &= (A_{1j} + A_{2j})/T \\
&+ \frac{H_j}{2}\left[P_j - D_j - \beta_j D_j (1+\beta_j \frac{P_j}{P_{rj}})\right]\frac{D_j}{P_j}T \\
&+ W_j\beta_j(1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j^2 T}{2P_j} \\
&+ [C_j(1+\beta_j) + L_j + M_j\beta_j]D_j \\
&- H_j\beta_j D_j S_{2j} + W_j\beta_j D_j S_{2j}
\end{aligned}
$$

5. EBQ for a Single Product

It can easily be shown that the second derivative of K_j with respect to T is positive [see (A1) in appendix]. Thus K_j is a convex function and by letting the first derivative of K_j with respect to T be equal zero, we can obtain the optimal value of T, denoted by $T_{\circ j}$, which minimizes K_j. Therefore from the minimization of K_j with respect to T, we have

$$T_{\circ j} = \sqrt{\frac{2(A_{1j} + A_{2j})P_j}{H_j D_j [P_j - D_j - \beta_j D_j (1+\beta_j \frac{P_j}{P_{rj}})] + W_j \beta_j (1+\beta_j \frac{P_j}{P_{rj}}) D_j^2}}$$

or

$$T_{\circ j} = \sqrt{\frac{2(A_{1j} + A_{2j})P_j / D_j}{H_j(P_j - D_j) - (H_j - W_j)\beta_j D_j (1+\beta_j \frac{P_j}{P_{rj}})}}$$

This value of T_{0j} is the optimal cycle time if it is a feasible solution. In fact, an arbitrary cycle time with length T is a feasible solution if the sum of the setup time and production time of the product in that cycle is not greater than the length of the cycle time *T*. That is, a cycle time *T* is a feasible solution if it satisfies the following relation

$$S_{1j} + S_{2j} + t_{1j} + t_{2j} \le T$$

From (2) and (3) we can write the above relation as:

$$S_{1j} + S_{2j} + (1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j}{P_j}T \le T$$

or equivalently

$$T \ge \frac{S_{1j} + S_{2j}}{1 - (1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j}{P_j}}.$$

Let $T_{mj} = \dfrac{S_{1j} + S_{2j}}{1 - (1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j}{P_j}}$.

Then, since K_j is a convex function, we can obtain the optimal value of T, denoted by T_j^*, as follows:

$T_j^* = T_{0j}$ if $T_{0j} \ge T_{mj}$

$T_j^* = T_{mj}$ if $T_{0j} < T_{mj}$

The optimal batch size for the product *j* is:

$Q_j^* = D_j T_j^*$.

Thus when there is only a single product *j* then for $T_{0j} \ge T_{mj}$

$$Q_j^* = \sqrt{\frac{2(A_{1j} + A_{2j})P_j D_j}{H_j(P_j - D_j) - (H_j - W_j)\beta_j (1+\beta_j \frac{P_j}{P_{rj}})D_j}}$$

and for $T_{0j} < T_{mj}$

$$Q_j^* = D_j T_j^* = D_j T_{mj} = D_j \frac{S_{1j} + S_{2j}}{1 - (1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j}{P_j}}$$

In the next section we adopt the common

cycle time approach and obtain the *EBQ* for the multi-product case.

6. Multi-Product Case

In the case of multi-product, the total cost per year for all products, K can be written as:

$$K = \sum_{j=1}^{n} K_j$$

or

$$\begin{aligned} K = \sum_{j=1}^{n} \Big\{ & (A_{1j} + A_{2j}) \big/ T \\ & + \frac{H_j}{2} \left[P_j - D_j - \beta_j D_j (1 + \beta_j \frac{P_j}{P_{rj}}) \right] \frac{D_j}{P_j} T \\ & + W_j \beta_j (1 + \beta_j \frac{P_j}{P_{rj}}) \frac{D_j^2 T}{2P_j} \\ & + [C_j (1 + \beta_j) + L_j + M_j \beta_j] D_j \\ & - H_j \beta_j D_j S_{2j} + W_j \beta_j D_j S_{2j} \Big\} \end{aligned} \quad (14)$$

One can easily show that the second derivative of K with respect to T is positive. Hence K is a convex function. Therefore, letting the first derivative of K with respect to T to be equal to zero, we can obtain the optimal value of T, denoted by T_0, which minimizes K. Thus, letting the first derivative of K equal to zero, we obtain

$$T_0 = \sqrt{\frac{2\sum_{j=1}^{n} (A_{1j} + A_{2j}) P_j / D_j}{\sum_{j=1}^{n} \left\{ H_j [P_j - D_j] - [H_j - W_j] \beta_j D_j (1 + \beta_j \frac{P_j}{P_{rj}}) \right\}}} \quad (15)$$

Note that for the case $\beta_j = 0$, j=1, 2,…,n, (15) gives the optimal value of the common cycle time for the case that no defectives are produced (Johnson and Montgomery 1974). That is,

$$T = \sqrt{2\sum_{j=1}^{n} A_{1j} \Big/ \left(\sum_{j=1}^{n} H_j D_j [1 - (D_j / P_j] \right)} \quad (16)$$

The value of T_0 in (15) is the optimal cycle time if it is a feasible solution. In fact, an arbitrary cycle time with length T is a feasible solution if the sum of the setup times and production times of all products in that cycle is not greater than the length of the cycle time T. That is, a cycle time T is a feasible solution if it satisfies the following relation

$$\sum_{j=1}^{n} (S_{1j} + S_{2j} + t_{1j} + t_{2j}) \le T \quad (17)$$

Now from (2) and (3) we can write (17) as:

$$T \ge \frac{\sum_{j=1}^{n} (S_{1j} + S_{2j})}{\left[1 - \sum_{j=1}^{n} (1 + \beta_j \frac{P_j}{P_{rj}}) \frac{D_j}{P_j} \right]} \quad (18)$$

let

$$T_m = \frac{\sum_{j=1}^{n} (S_{1j} + S_{2j})}{\left[1 - \sum_{j=1}^{n} (1 + \beta_j \frac{P_j}{P_{rj}}) \frac{D_j}{P_j} \right]} \quad (19)$$

Then, since K is a convex function, we can obtain the optimal value of T, denoted by T^*, as follows:

$$T^* = T_0 \qquad \text{if} \qquad T_0 \ge T_m$$
$$T^* = T_m \qquad \text{if} \qquad T_0 < T_m$$

Finally, the optimal batch size for product j, $j = 1,...,n$, is:

$$Q_j^* = D_j T^*.$$

Thus, for $T_0 \ge T_m$, $Q_j^* = D_j T_0, j =$ 1,2, …, n, that is, for $j = 1,2,...,n$, we have

$$Q_j^* = D_j \sqrt{\frac{2\sum_{j=1}^{n}(A_{1j}+A_{2j})P_j / D_j}{\sum_{j=1}^{n}\left\{H_j(P_j-D_j)-(H_j-W_j)\beta_j D_j(1+\beta_j \frac{P_j}{P_{rj}})\right\}}}$$

and for $T_0 < T_m$, we have

$$Q_j^* = D_j T_m,\ j=1,2,...,n,$$

or

$$Q_j^* = \frac{D_j \sum_{j=1}^{n}(S_{1j}+S_{2j})}{\left[1-\sum_{j=1}^{n}(1+\beta_j \frac{P_j}{P_{rj}})\frac{D_j}{P_j}\right]},\ j=1,2,...,n$$

7. Example

In this section we provide an example to show how the model determines the optimal order quantities for all products. In this problem there are four different items which are produced on a single machine. The constant proportions of defectives, β_j , j=1,2,3, and 4, are 0.05, 0.05, 0.04 and 0.04 respectively. Other parameters of problem are given in the Table 1.

In this case we have:

$$P_1(1-\beta_1)-D_1 = 23000 \ge 0$$
$$P_2(1-\beta_2)-D_2 = 19500 \ge 0$$
$$P_3(1-\beta_3)-D_3 = 37250 \ge 0$$
$$P_4(1-\beta_4)-D_4 = 27100 \ge 0$$

and the idle time of the machine in each cycle is:

$$1-\sum_{j=1}^{4}(1+\beta_j \frac{P_j}{P_{rj}}).\frac{D_j}{P_j} = 0.1524 \ge 0$$

Then from (15) we can find that $T_0 = 0.2051$. On the other hand we obtain from (19) that $T_m = 0.2165$. Since $T_0 \le T_m$, then we will have $T^* = 0.2165$. Thus the associated *EBQ*s in this case are as follows:

$$Q_1^* = D_1.T^* = 5500\times 0.2165 = 1190.75$$
$$Q_2^* = D_2.T^* = 4500\times 0.2165 = 974.25$$
$$Q_3^* = D_3.T^* = 15000\times 0.2165 = 3247.5$$
$$Q_4^* = D_4.T^* = 6500\times 0.2165 = 1407.25$$

Finally the optimal total cost in this case is: $K^* = 3471237.562$.

8. Conclusion

In the general economic lot size scheduling (*ELSP*), it has been assumed that the items produced are non-defective and do not need any rework. In this paper, we address the rework of defective items in a multi-product single machine system. Adopting the common cycle time approach for all products, allowing non-zero set up times for normal production and

Table 1 Characteristics of four items to be produced on a single machine

j	P_j	P_{rj}	D_j	C_j	A_{1j}	A_{2j}	H_j	W_j	S_{1j}	S_{2j}	L_j	M_j
1	30000	45000	5500	50	700	1000	10	30	0.001	0.002	10	15
2	25000	35000	4500	200	500	1500	40	55	0.002	0.004	15	22.5
3	55000	80000	15000	50	800	2000	10	25	0.005	0.010	5	7.5
4	35000	50000	6500	150	1200	2100	30	50	0.003	0.006	20	30

rework process for each product, we obtain the optimal common cycle time, hence the optimal batch sizes for all products, which minimizes the sum of inventory holding cost, setup cost, production process and inspection cost and waiting time cost of defectives for reworks.

Appendices

To show that *K*,

$$K=\sum_{j=1}^{n}\Big\{\frac{(A_{1j}+A_{2j})}{T}$$
$$+\frac{H_j}{2}\left[P_j-D_j-\beta_j D_j(1+\beta_j\frac{P_j}{P_{rj}})\right]\frac{D_j}{P_j}T$$
$$+W_j\beta_j(1+\beta_j\frac{P_j}{P_{rj}})\frac{D_j^2 T}{2P_j}$$
$$+[C_j(1+\beta_j)+L_j+M_j\beta_j]D_j$$
$$-H_j\beta_j D_j S_{2j}+W_j\beta_j D_j S_{2j}\Big\}$$

is a convex function we take the first and second derivative of *K* with respect to *T*.

The first derivative of K with respect to *T* is

$$\frac{\partial K}{\partial T}=\frac{-\sum_{j=1}^{n}(A_{1j}+A_{2j})}{T^2}+\sum_{j=1}^{n}(P_j-D_j)H_j\frac{D_j}{2P_j}$$
$$-\sum_{j=1}^{n}(H_j-W_j)\beta_j(1+\beta_j\frac{P_j}{P_{rj}})\frac{D_j^2}{2P_j}$$

The second derivative of *K* with respect to *T* is

$$\frac{\partial^2 K}{\partial T^2}=\sum_{j=1}^{n}2(A_{1j}+A_{2j})\Big/T^3$$

From this relation it is clear that the second derivative of *K* for all values of *T >0* is positive which implies that *K* is a convex function and has a unique solution.

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