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Technical Note

AN APPROACH TO GROUP DECISION MAKING BASED ON INTERVAL FUZZY PREFERENCE RELATIONS *

Yunliang JIANG [1,2]

[1]*School of Information Engineering, Huzhou University, Huzhou, 313000, China*

[2]*Institute of Artificial Intelligence, Zhejiang University, Hangzhou, 310027, China*

jylsy@cs.zju.edu.cn (✉)

Abstract

In this paper, we investigate group decision making problems where the decision information given by decision makers takes the form of interval fuzzy preference relations. We first give an index to measure the similarity degree of two interval fuzzy preference relations, and utilize the similarity index to check the consistency degree of group opinion. Furthermore, we use the error-propagation principle to determine the priority vector of the aggregated matrix, and then develop an approach to group decision making based on interval fuzzy preference relations. Finally, we give an example to illustrate the developed approach.

Keywords: Group decision making, interval fuzzy preference relation, similarity index, aggregation

1. Introduction

This Decision making in group setting is a prominent area of research in normative decision theory, which has been widely studied (Arrow 1966, Bouchon-Meunier and Foulloy et al. 2003, Fodor and Roubens 1994, Hwang and Yoon 1981, Kacprzyk and Fedrizzi 1990, Xu 2004, Yager and Kacprzyk 1997). A group decision making problem generally consists of identifying the most desirable alternative from a given discrete set of n alternatives $X = \{x_1, x_2, \cdots, x_n\}$ based on the preference information given by multiple decision makers. In the process of decision making, each decision maker usually needs to provide his/her comparative preferences for the given n alternatives with respect to a single criterion, and constructs a fuzzy preference relation (Xu 2004) $B = (b_{ij})_{n\times n}$ whose element b_{ij} represents the preference degree of the alternative x_i to the alternative x_j, and satisfies $b_{ij} \in [0,1]$, $b_{ij} + b_{ji} = 1$, $b_{ii} = 0.5$, where $b_{ii} = 0.5$ indicates indifference between x_i and x_j; $r_{ij} > 0.5$ indicates that x_i is preferred to x_j , especially, $r_{ij} = 1$ indicates that x_i is absolutely preferred to x_j ; $r_{ij} < 0.5$

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indicates that x_j is preferred to x_i, especially, $r_{ij} = 0$ indicates that x_j is absolutely preferred to x_i.

Up to now, considerable research has been conducted on fuzzy preference relations with crisp numbers (Chiclana and Herrera et al. 2001, Chiclana and Herrera et al. 1998, Herrera-Viedma and Herrera et al. 2004, Kacprzyk 1986, Lipovetsky and Michael 2002, Nurmi 1981, Orlovski 1978, Tanino 1984, Xu and Geng et al. 2000, Xu and Geng et al. 2001, Xu and Da 2005, Xu 2001a, Xu 2004, Xu 1999). In many real-life situations, however, the decision makers may provide their preferences by means of interval fuzzy preference relations (Xu 2001b) whose elements are expressed in interval numbers rather than crisp numerical ones because of time pressure, lack of knowledge or data, and their limited attention and information processing capabilities. Thus, it is necessary to pay attention to this issue.

In this paper, we shall develop a practical approach to group decision making based on interval fuzzy preference relations. To do so, the remainder of this paper is organized as follows. Section 2 gives an index to measure the similarity degree of two interval fuzzy preference relations, and utilizes the similarity index to check the consistency degree of group opinion. Section 3 introduces a result, which shows that the aggregated matrix of multiple interval fuzzy preference relations is also an interval fuzzy preference relation. Section 4 uses the error error-propagation principle to determine the priority vector of the aggregated matrix, and then develops a practical approach to group decision making based on interval fuzzy preference relations. Section 5 gives an example to illustrate the developed approach, and finally, Section 6 concludes the paper.

2. Similarity Measures

Consider a decision making problem, let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete set of alternatives, a decision maker compares each pair of x_i and x_j, and constructs an interval fuzzy preference relation (Xu 2001b) $B = (b_{ij})_{n\times n}$, where $b_{ij} = [b_{ij}^-, b_{ij}^+]$, where b_{ij}^- and b_{ij}^+ are the lower and upper limits, respectively, and

$$b_{ij}^- + b_{ji}^+ = b_{ij}^+ + b_{ji}^- = 1,\ \ b_{ij}^+ \geq b_{ij}^- \geq 0,$$
$$b_{ii}^- = b_{ii}^+ = 0.5,\ \ i, j = 1, 2, \cdots, n.$$

In the following, we give an index to measure the similarity degree of two interval fuzzy preference relations.

Note: the lower limits of interval numbers discussed throughout this paper are non-negative.

We first introduce the concept of the similarity degree of two interval numbers.

Definition 1 *Let* $a = [a^-, a^+]$ *and* $b = [b^-, b^+]$ *be two interval numbers, then, we call*

$$s(a,b) = 1 - \frac{\left|a^- - b^-\right| + \left|a^+ - b^+\right|}{a^- + a^+ + b^- + b^+}$$

the similarity degree of a *and* b.

Based on the similarity measure between two interval numbers, below we define the concept of the similarity degree of two interval fuzzy preference relations:

Definition 2 *Let* $A = (a_{ij})_{n\times n}$ *and* $B = (b_{ij})_{n\times n}$ *be two interval fuzzy preference relations, then we call*

$$s(A,B) = \sum_{i=1}^{n}\sum_{j=1}^{n} s(a_{ij}, b_{ij})$$

the similarity degree of A *and* B.

By using definition 2, in the following, we introduce the similarity index of two interval fuzzy preference relations:

Definition 3 *Let* A *and* B *be two interval fuzzy preference relations, then we call*

$$si(A,B) = s(A,B)/n^2 \quad (1)$$

the similarity index of A *and* B.

From the above definitions, we can draw the following results easily:

Theorem 1 *Let* A *and* B *be three interval fuzzy preference relations, then we have*

1) $0 \le si(A,B) \le 1$*;*
2) $si(A,B) = 1$ *if and only if* $A = B$*;*
3) $si(A,B) = si(B,A)$.

3. The Aggregation of Interval Fuzzy Preference Relations

For convenience, we first introduce the operational laws of interval numbers and a formula for the comparison between two interval numbers:

Let $a = [a^-, a^+]$ and $b = [b^-, b^+]$ be two interval numbers, and $\mu > 0$, then (Xu and Zhai 1992):

1) $[a^-, a^+] + [b^-, b^+] = [a^- + b^-, a^+ + b^+]$;
2) $[a^-, a^+] \cdot [b^-, b^+] = [a^- b^-, a^+ b^+]$;
3) $\mu[a^-, a^+] = [\mu a^-, \mu a^+]$;
4) $1/[a^-, a^+] = [1/a^+, 1/a^-]$.

Definition 4 (Xu 2004) *Let* $a = [a^-, a^+]$ *and* $b = [b^-, b^+]$ *be two interval numbers, and let* $l_a = a^+ - a^-$ *and* $l_b = b^+ - b^-$, *then the possibility degree of* $a \ge b$ *is defined as*

$$p(a \ge b) = \frac{\min\{l_a + l_b, \max(b^+ - a^-, 0)\}}{l_a + l_b} \quad (2)$$

which has the following properties:

$$0 \le p(a \ge b) \le 1, \quad p(a \ge b) + p(b \ge a) = 1,$$

$$p(a \ge a) = 0.5.$$

Based on the operational laws of interval numbers, we can prove the following conclusion easily, which will be very useful to group decision making based on interval fuzzy preference relations:

Theorem 2 *Let* $B_l = (b_{ij}^{(l)})_{n\times n}$ $(b_{ij}^{(l)} = [b_{lij}^-, b_{lij}^+],\ l = 1,2,\cdots,m)$ *be* m *interval fuzzy preference relations, then the aggregated matrix of* B_l $(l = 1,2,\cdots,m)$:

$$B = \lambda_1 B_1 \oplus \lambda_2 B_2 \oplus \cdots \oplus \lambda_m B_m \quad (3)$$

is also an interval fuzzy preference relation, where $B = (b_{ij})_{n\times n}$ $(b_{ij} = [b_{ij}^-, b_{ij}^+])$, $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ *is the weight vector of* B_l $(l = 1,2,\cdots,m)$, *and* $b_{ij} = \sum_{l=1}^{m} \lambda_l b_{ij}^{(l)}$, $\lambda_l \ge 0$, $l = 1,2,\cdots,m$, $\sum_{l=1}^{m} \lambda_l = 1$.

4. An Approach to Group Decision Making based on Interval Fuzzy Preference Relations

Consider a group decision making problem, let $D = \{d_1, d_2, \cdots, d_m\}$ be a set of decision makers, and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ be the weight vector of decision makers, where $\lambda_l \ge 0$, $l = 1,2,\cdots,m$, $\sum_{l=1}^{m} \lambda_l = 1$. The decision makers d_l $(l=1,2,\cdots,m)$ compare the given n alternatives $x_i (i = 1,2,\cdots,n)$, and construct interval fuzzy preference relations $B_l = (b_{ij}^{(l)})_{n\times n}$ $(b_{ij}^{(l)} = [b_{lij}^-, b_{lij}^+], l = 1,2,\cdots,m)$, respectively. To get the most desirable alternative, in the following, we develop a practical approach to group decision making based on interval fuzzy preference relations:

Step 1 Utilize (2) to aggregate the interval fuzzy preference relations $B_l = (b_{ij}^{(l)})_{n\times n}$ $(l = 1,2,\cdots, m)$ into a collective interval fuzzy preference relation $B = (b_{ij})_{n\times n}$, where $b_{ij} = [b_{ij}^-, b_{ij}^+]$, $i, j = 1,2,\cdots,n$.

Step 2 Provide the lower limit α_0 of the similarity index according to the actual situation (in general, $0.9 \le \alpha_0 \le 1$), and utilize (1) to calculate the similarity index value $si(B_l, B)$ $(l = 1,2,\cdots,m)$ of the interval fuzzy preference relation B_l $(l = 1,2,\cdots,m)$ and the aggregated matrix B. If $si(B_l, B) \ge \alpha_0$, for all $l = 1,2,\cdots,m$, then the group opinion is of acceptable consistency; otherwise, return the interval fuzzy preference relation B_l with $si(B_l, B) < \alpha_0$ to the decision maker d_l for reassessment until the similarity index value $si(B_l, B) \ge \alpha_0$.

Step 3 Utilize the error-propagation principle (Yoon 1989) to derive the priority vector of the aggregated matrix B. To do so, the following procedure is involved:

1) Calculate the mean fuzzy preference relation $\overline{B} = \left(\overline{b}_{ij}\right)_{n\times n}$ and the error matrix $\delta = \left(\delta_{ij}\right)_{n\times n}$ of $B = \left(b_{ij}\right)_{n\times n}$, where

$$\overline{b}_{ij} = \frac{1}{2}\left(b_{ij}^- + b_{ij}^+\right), \quad \delta_{ij} = \frac{1}{2}\left(b_{ij}^+ - b_{ij}^-\right), \quad i, j = 1,2,\cdots,n \tag{4}$$

2) Calculate the priority vector $\overline{w} = (\overline{w}_1, \overline{w}_2, ..., \overline{w}_n)^T$ of the mean fuzzy preference relation $\overline{B}$ by using the priority formula for fuzzy preference relation (Xu 2001b):

$$\overline{w}_i = \left(\sum_{j=1}^{n} \overline{b}_{ij} + \frac{n}{2} - 1\right) / n(n-1), \quad i = 1,2,\cdots,n \tag{5}$$

3) Calculate the error vector $\Delta w = (\Delta w_1, \Delta w_2, \cdots, \Delta w_n)^T$ of $\overline{w} = \left(\overline{w}_1, \overline{w}_2, \cdots, \overline{w}_n\right)^T$ (due to the imprecise assessments of $\overline{b}_{ij}$ $(i, j = 1,2,\cdots,n)$)using the error-propagation formula

$$\Delta w_i = \frac{1}{n(n-1)}\sqrt{\sum_{j=1}^{n} \delta_{ij}^2}, \quad i = 1,2,\cdots,n \tag{6}$$

and thus get the priority vector $w = \left(w_1, w_2, \cdots, w_n\right)^T$ of the aggregated matrix B, where $w_i = [\overline{w}_i - \Delta w_i, \overline{w}_i + \Delta w_i]$, $i = 1,2,\cdots,n$.

Step 4 To rank these interval weights w_i $\left(i = 1,2,\cdots,n\right)$, we first compare each w_i with all w_i $\left(i = 1,2,\cdots,n\right)$ by using the possibility degree formula (2), and get

$$p\left(w_i \ge w_j\right) = \frac{\min\left\{2(\Delta w_i + \Delta w_j), \max(\overline{w}_i + \Delta w_i - (\overline{w}_j - \Delta w_j), 0)\right\}}{2(\Delta w_i + \Delta w_j)} \tag{7}$$

For similarity, we let $p_{ij} = p\left(w_i \ge w_j\right)$, and denote $w_i \underset{p_{ij}}{\ge} w_j$ as the order relation of w_i and w_j, and then construct a possibility degree matrix $P = \left(p_{ij}\right)_{n\times n}$ (obviously, by the properties of the possibility degree formula (2), P is also a fuzzy preference relation, where $p_{ij} \ge 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $i, j = 1,2,\cdots,n$). Summing all elements in each line of matrix P, we have $p_i = \sum_{j=1}^{n} p_{ij}$, $i = 1,2,\cdots,n$, and then rank w_i $\left(i = 1,2,\cdots,n\right)$ in descending order by the values of $p_i\left(i = 1,2,\cdots,n\right)$.

Step 5 Rank all the alternatives $x_i\left(i = 1,2,\cdots,n\right)$ and then select the most desirable one according to w_i $\left(i = 1,2,\cdots,n\right)$.

5. Illustrative Example

Let us suppose an investment company, which wants to invest a sum of money in the best option. There is a panel with five possible alternatives $x_i\ (i=1,2,3,4,5)$ in which to invest the money (Herrera and Herrera-Viedma et al. 2000): 1) x_1 is a car company; 2) x_2 is a food company; 3) x_3 is a computer company; 4) x_4 is an arms company; 5) x_5 is a TV company. There are four decision makers $d_l\ (l=1,2,3,4)$, whose weight vector is $\lambda=(0.4,0.2,0.3,0.1)^T$. These four decision makers provide their preferences over these five decision alternatives, and give four interval fuzzy preference relations $B_l=\left(b_{ij}^{(l)}\right)_{5\times5}$ $(l=1,2,3,4)$ as follows:

$$B_1=\begin{bmatrix}[0.5,0.5] & [0.7,0.8] & [0.3,0.5] & [0.6,0.8] & [0.4,0.6]\\ [0.2,0.3] & [0.5,0.5] & [0.2,0.4] & [0.7,0.9] & [0.4,0.5]\\ [0.5,0.7] & [0.6,0.8] & [0.5,0.5] & [0.8,1] & [0.5,0.7]\\ [0.2,0.4] & [0.1,0.3] & [0,0.2] & [0.5,0.5] & [0.3,0.5]\\ [0.4,0.6] & [0.5,0.6] & [0.3,0.5] & [0.5,0.7] & [0.5,0.5]\end{bmatrix}$$

$$B_2=\begin{bmatrix}[0.5,0.5] & [0.6,0.7] & [0.3,0.4] & [0.6,0.8] & [0.5,0.6]\\ [0.3,0.4] & [0.5,0.5] & [0.3,0.5] & [0.7,0.8] & [0.4,0.6]\\ [0.6,0.7] & [0.5,0.7] & [0.5,0.5] & [0.8,0.9] & [0.6,0.7]\\ [0.2,0.4] & [0.2,0.3] & [0.1,0.2] & [0.5,0.5] & [0.4,0.5]\\ [0.4,0.5] & [0.4,0.6] & [0.3,0.4] & [0.5,0.6] & [0.5,0.5]\end{bmatrix}$$

$$B_3=\begin{bmatrix}[0.5,0.5] & [0.5,0.7] & [0.4,0.5] & [0.7,0.8] & [0.4,0.5]\\ [0.3,0.5] & [0.5,0.5] & [0.3,0.4] & [0.8,0.9] & [0.5,0.7]\\ [0.5,0.6] & [0.6,0.7] & [0.5,0.5] & [0.7,0.9] & [0.6,0.8]\\ [0.2,0.3] & [0.1,0.2] & [0.1,0.3] & [0.5,0.5] & [0.3,0.4]\\ [0.5,0.6] & [0.3,0.5] & [0.2,0.4] & [0.6,0.7] & [0.5,0.5]\end{bmatrix}$$

$$B_4=\begin{bmatrix}[0.5,0.5] & [0.6,0.8] & [0.3,0.5] & [0.7,0.8] & [0.5,0.6]\\ [0.2,0.4] & [0.5,0.5] & [0.3,0.4] & [0.6,0.8] & [0.4,0.7]\\ [0.5,0.7] & [0.6,0.7] & [0.5,0.5] & [0.9,1] & [0.6,0.7]\\ [0.2,0.3] & [0.2,0.4] & [0,0.1] & [0.5,0.5] & [0.3,0.4]\\ [0.4,0.5] & [0.3,0.6] & [0.3,0.4] & [0.6,0.7] & [0.5,0.5]\end{bmatrix}$$

To get the most desirable alternative, the following steps are involved:

Step 1 Utilize (3) to aggregate $B_l=\left(b_{ij}^{(l)}\right)_{5\times5}$ $(l=1,2,3,4)$ into the collective interval fuzzy preference relation B:

$$B=\begin{bmatrix}[0.50,0.50] & [0.61,0.75] & [0.33,0.48] & [0.64,0.80] & [0.43,0.57]\\ [0.25,0.39] & [0.50,0.50] & [0.26,0.42] & [0.72,0.87] & [0.43,0.60]\\ [0.52,0.67] & [0.58,0.74] & [0.50,0.50] & [0.78,0.95] & [0.56,0.73]\\ [0.20,0.36] & [0.13,0.28] & [0.05,0.22] & [0.50,0.50] & [0.32,0.46]\\ [0.43,0.57] & [0.40,0.57] & [0.27,0.44] & [0.54,0.68] & [0.50,0.50]\end{bmatrix}$$

Step 2 Provide the lower limit $\alpha_0=0.9$ of similarity index according to the actual situation, and utilize (1) to calculate the similarity degrees $si(B_l,B)$ $(l=1,2,3,4)$ of the interval fuzzy preference relations B_l $(l=1,2,3,4)$ and the aggregated matrix B:

$$si(B_1,B)=0.9612,\ si(B_2,B)=0.9616,$$

$$si(B_3,B)=0.9476,\ si(B_4,B)=0.9404$$

Since $si(B_l,B)>\alpha_0\ (l=1,2,3,4)$, then the group opinion is of acceptable consistency.

Step 3 Utilize (4) to calculate the mean fuzzy preference relation $\bar{B}$ and the error matrix δ of the aggregated matrix B, and get

$$\bar{B}=\begin{bmatrix}0.500 & 0.680 & 0.405 & 0.720 & 0.500\\ 0.320 & 0.500 & 0.340 & 0.795 & 0.515\\ 0.595 & 0.660 & 0.500 & 0.865 & 0.645\\ 0.280 & 0.205 & 0.135 & 0.500 & 0.390\\ 0.500 & 0.485 & 0.355 & 0.610 & 0.500\end{bmatrix}$$

$$\delta=\begin{bmatrix}0 & 0.070 & 0.075 & 0.080 & 0.070\\ 0.070 & 0 & 0.080 & 0.075 & 0.085\\ 0.075 & 0.080 & 0 & 0.085 & 0.085\\ 0.080 & 0.075 & 0.085 & 0 & 0.070\\ 0.070 & 0.085 & 0.085 & 0.070 & 0\end{bmatrix}$$

Then utilize (5) and (6) to derive the priority vector $\bar{w}$ and the error vector Δw of $\bar{B}$:

$$\overline{w}=(0.2152,0.1985,0.2383,0.1505,0.1975)^T$$

$$\Delta w=(0.0074,0.0078,0.0081,0.0078,0.0078)^T$$

and thus, we get the priority vector of the aggregated matrix B :

$$w=([0.2078,0.2226],[0.1907,0.2063],\\ [0.2302,0.2464],[0.1427,0.1583],\\ [0.1897,0.2053])^T$$

Step 4 To rank these interval weights $w_i\ (i=1,2,3,4,5)$, we first compare each w_i with all $w_i\ (i=1,2,3,4,5)$ by using the possibility degree formula (2), and get the possibility degree matrix:

$$P=\begin{bmatrix} 0.5 & 1 & 0 & 1 & 1 \\ 0 & 0.5 & 0 & 1 & 0.5321 \\ 1 & 1 & 0.5 & 1 & 1 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.4679 & 0 & 1 & 0.5 \end{bmatrix}$$

Summing all elements in each line of matrix P , we have

$$p_1=3.5\ ,\ \ p_2=2.0321\ ,\ \ p_3=4.5$$

$$p_4=0.5\ ,\ \ p_5=1.9679\ .$$

Then we rank $w_i\ (i=1,2,3,4,5)$ in descending order by the values of $p_i\ (i=1,2,3,4,5)$:

$$w_3 \underset{1}{\geq} w_1 \underset{1}{\geq} w_2 \underset{0.5321}{\geq} w_5 \underset{1}{\geq} w_4$$

Step 5 Rank all the alternatives $x_i\ (i=1,2,3,4,5)$ according to w_i (i = 1, 2, 3, 4, 5):

$$x_3 \succ x_1 \succ x_2 \succ x_5 \succ x_4$$

and thus, the most desirable alternative is x_3 .

6. Conclusions

In this paper, we have given an index to measure the similarity degree of two interval fuzzy preference relations, and utilized the similarity index to check the consistency degree of group opinion. By using the lower limit α_0 of similarity index, the decision makers can modify their preference information until a group opinion with acceptable consistency is reached. Based on the error-propagation principle, this paper has developed a practical approach to group decision making with interval fuzzy preference relations. Theoretical analysis and the numerical results have shown that the developed approach is feasible and effective.

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Yunliang Jiang is an Associate Professor of the School of Information Engineering of Huzhou University, and a Ph.D. Candidate of the College of Computer Science of Zhejiang University, China. He obtained the B.S. degree in Mathematics from Zhejiang Normal University in 1989 and the M.E. degree in Computer Science and Technology from Zhejiang University in 1997, respectively. His research interests include information fusion, artificial intelligence, and geographic information system. He has published over 20 papers in journals such as Lecture Notes in Artificial Intelligence, Lecture Notes in Computer Science, Pattern Recognition and Artificial Intelligence (in Chinese), and others.